NMPC-based control for Quadrotor trajectory tracking subject to input constraints.

EE372 Progress Report

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Outline

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Problem Statement

- Main Objective:
  Develop a model predictive controller to achieve trajectory tracking for a quadrotor with input constraints.
- Keywords:
  Quadrotor trajectory tracking Control, Multiple shooting, Nonlinear model predictive control, Input constraints

Outline

MPC Strategy Structure

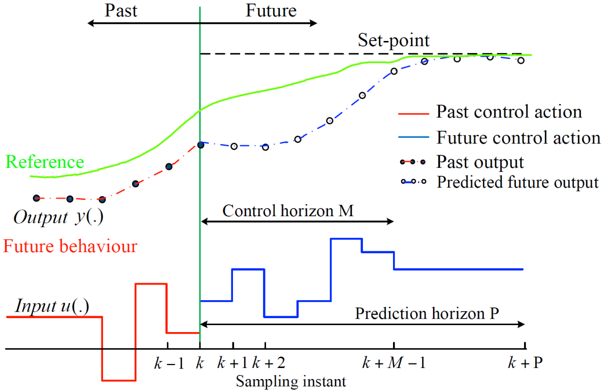
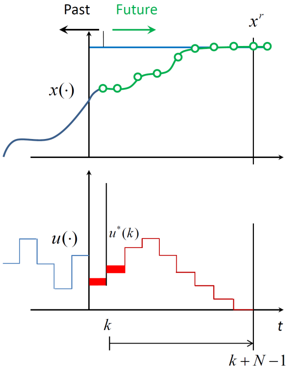


Figure: A general MPC structure [3].

MPC Strategy Structure

MPC Strategy Summary:

1. Prediction based on model
2. Online optimization
3. Receding horizon implementation



MPC Formulation

- **Running costs:** characterizes the control objective

$$\ell(\mathbf{x}, \mathbf{u}) = \|\mathbf{x}_{\mathbf{u}} - \mathbf{x}_r\|_{\mathbf{Q}}^2 + \|\mathbf{u} - \mathbf{u}_r\|_{\mathbf{R}}^2$$

  where **Q** and **R** are the weight matrices specifying the weights on tracking the reference states and penalizing the control input, respectively.
- **Cost function:** Evaluation of the running costs along the whole prediction horizon

$$J_N(\mathbf{x}, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(\mathbf{x}_{\mathbf{u}}(k), \mathbf{u}(k))$$

MPC Formulation

- **Optimal Control Problem (OCP):** to find a minimizing control sequence

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_N(\mathbf{x}_0, \mathbf{u}) = \sum_{k=0}^{N-1} \ell(\mathbf{x}_{\mathbf{u}}(k), \mathbf{u}(k)) \\ \text{s.t.} \quad & \mathbf{x}_{\mathbf{u}}(k+1) = \mathbf{f}(\mathbf{x}_{\mathbf{u}}(k), \mathbf{u}(k)) \\ & \mathbf{x}_{\mathbf{u}}(0) = \mathbf{x}_0 \\ & \mathbf{u}(k) \in \mathcal{U}, \ \forall\, k \in [0, N-1] \\ & \mathbf{x}_{\mathbf{u}}(k) \in \mathcal{X}, \ \forall\, k \in [0, N] \end{aligned} \tag{1}$$

Outline

Quadrotor Dynamic Model

- state variables $\mathbf{x} = (x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})^T$
- control input $\mathbf{u} = (f_1, f_2, f_3, f_4)^T$

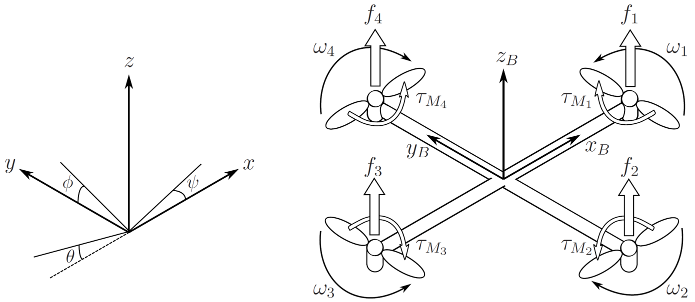


Figure: The inertial and body frames of a quadrotor

Quadrotor Dynamic Model

The equations of motion of a quadrotor are derived using Lagrange's method.

- Translational motion

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \sin\psi\sin\phi + \cos\psi\sin\theta\cos\phi \\ -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ \cos\theta\cos\phi \end{bmatrix} \frac{u_t}{m} + \frac{1}{m}\begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{2}$$

  where u_t is total thrust force.

- Rotational motion $\eta = [\phi, \theta, \psi]^T$

$$\ddot{\boldsymbol{\eta}} = \mathbf{J}^{-1}\left(\boldsymbol{\tau}_\eta - \mathbf{C}(\boldsymbol{\eta}, \dot{\boldsymbol{\eta}})\right) \tag{3}$$

  where $\boldsymbol{\tau}_B = (\tau_\phi, \tau_\theta, \tau_\psi)$.

Quadrotor Dynamic Model

The external force and torques of a quadrotor are $T, \tau_\phi, \tau_\theta$ and τ_ψ representing thrust, roll, pitch, yaw torques, respectively, which can be expressed in terms of the propeller thrusts f_1, f_2, f_3, f_4.

$$\begin{bmatrix} T \\ \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -\ell & 0 & \ell \\ \ell & 0 & -\ell & 0 \\ -\mu & \mu & -\mu & \mu \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \tag{4}$$

where ℓ is the distance between the rotor and the center of mass of the quadrotor and μ is the drag constant.

Quadrotor Dynamic Model

Rewrite the nonlinear equations (2) and (3) into a compact form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{5}$$

A Runga-Kutta method is used to obtain the state at the next time step $\mathbf{x}_{k+1}$ with sampling time h.

$$\begin{aligned}
\hat{\mathbf{x}}_{k+1} &= \mathbf{x}_k + \frac{h}{6}(R_1 + 2R_2 + 2R_3 + R_4) \\
R_1 &= \mathbf{f}(t_k.\mathbf{x}_k) \\
R_2 &= \mathbf{f}(t_k + h/2, \mathbf{x}_k + R_1/2) \\
R_3 &= \mathbf{f}(t_k + h/2, \mathbf{x}_k + R_2/2) \\
R_4 &= \mathbf{f}(t_k + h, \mathbf{x}_k + R_3)
\end{aligned} \tag{6}$$

Overview of numerical methods for optimal control

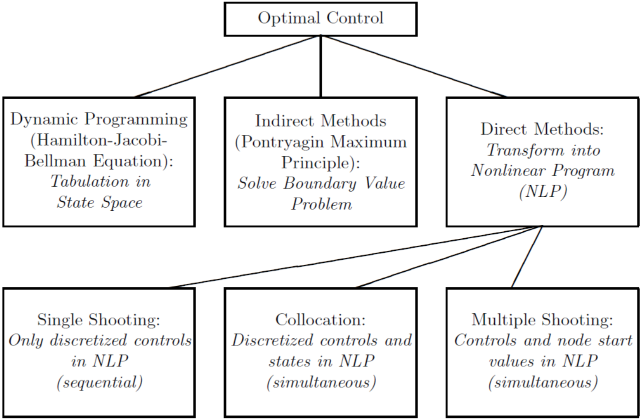


Figure: Numerical methods for optimal control [2].

OCP and NLP

To solve an NMPC problem, the optimization problem (1) needs to be reformulated as a Nonlinear Programming problem (NLP) (7).

$$\begin{aligned} \min_{\mathbf{z}} \quad & f(\mathbf{z}) \\ \text{s.t.} \quad & \mathbf{z} \in \mathcal{Z} \\ & g_i(\mathbf{z}) \leq 0 \ \text{ for each } i \in [1, ..., m] \\ & h_j(\mathbf{z}) = 0 \ \text{ for each } j \in [1, ..., p] \end{aligned} \tag{7}$$

The optimization problem is rewritten as a function of a set of decision variables $\mathbf{z}$. Once NLP formulation is obtained, the NLP can be solved by existing NLP solvers.

OCP and NLP

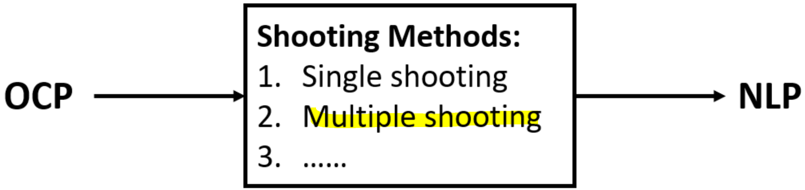
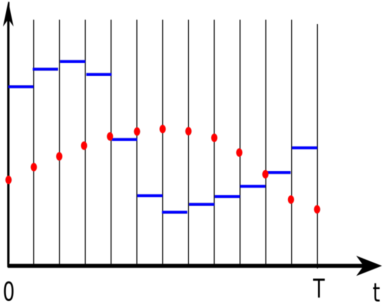
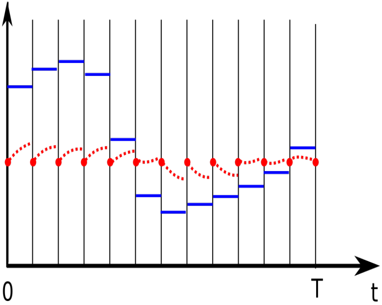


Figure: Relationship between OCP and NLP

Multiple Shooting Method



(a) State trajectory



(b) Control input

Figure: Illustration of multiple shooting [1].

Multiple Shooting Method - cont'd

- Not only control inpus **u** but also state variables **x** are decision variables in the optimization process.
- Additional continuity constraints are added to make sure that there is no mismatch between two states.

$$\mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1} = 0$$

An interior point method is used to solve the NLP obtained from the multiple shooting approach. The open-source package CasADi is used to implement and solve the NLP.

Brief Introduction to CasADi

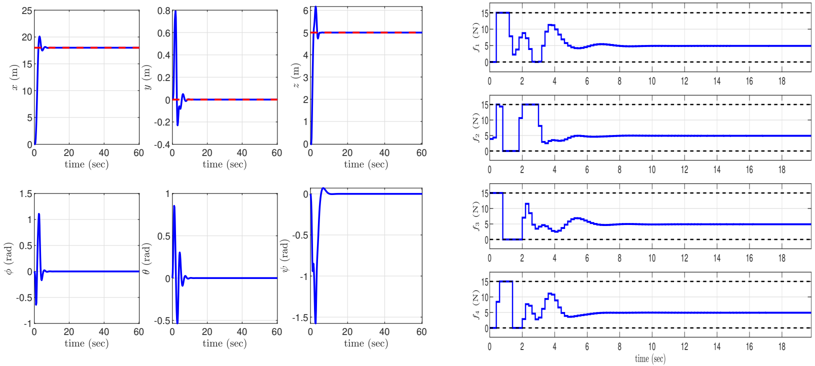
CasADi

What is CasADi?

- An open-source symbolic framework for quick, yet efficient, implementation of derivative based algorithms for dynamic optimization
- Framework for writing OCP solvers - solve NLP efficiently
- 4 standard problems can be handled by CasADi
  i.e., Quadratic Programming (QP), Nonlinear Programming (NLP), Root finding Problem, Initial-value Problem

`https://web.casadi.org/`

Ex: Position Stabilization



(a) State trajectory (b) Control input

Figure: Position stabilization of quadrotor using NMPC

Ex: Position Stabilization - cont'd

DEMO: P2P flight: simulation results

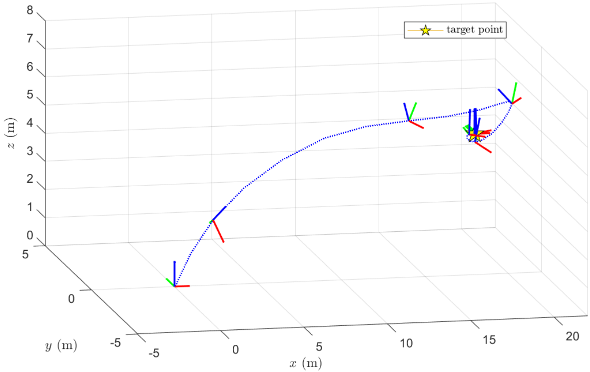
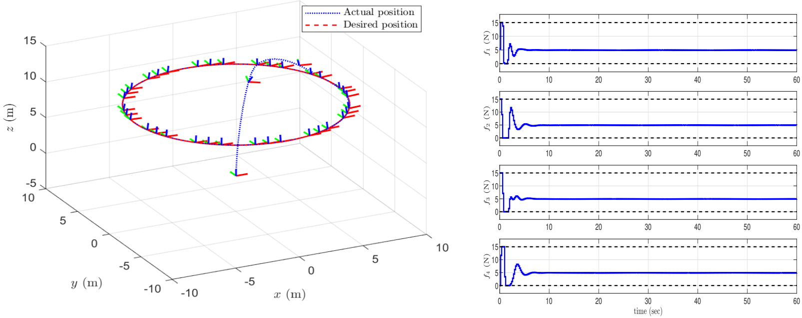


Figure: Position stabilization of quadrotor using NMPC

Ex: Circular Tracking

DEMO: simulation results

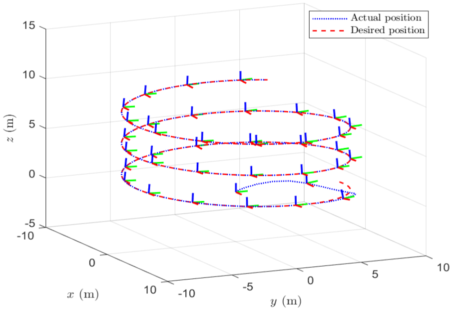


(a) 3D trajectory (b) Control input

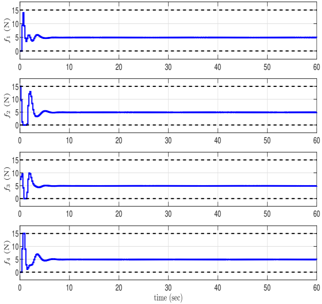
Figure: Circular tracking control of quadrotor using NMPC

Ex: Helix Tracking

DEMO: simulation results



(a) State trajectory

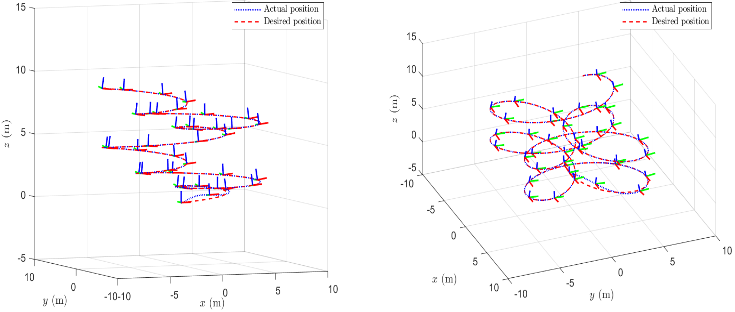


(b) Control input

Figure: Helix tracking control of quadrotor using NMPC

Ex: Complex Helix Tracking

DEMO: simulation results



(a) State trajectory (b) Control input

Figure: Complex Helix tracking control of quadrotor using NMPC

Ex: Complex Helix Tracking - cont'd

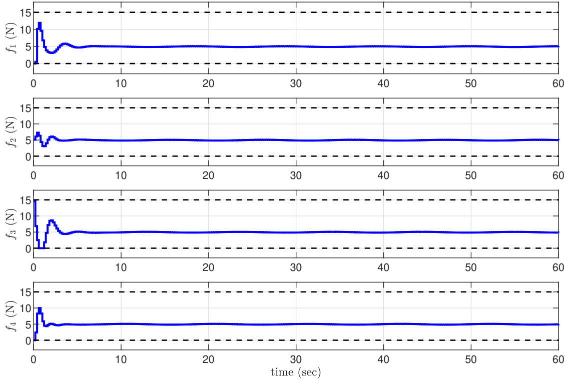


Figure: Complex Helix tracking control of quadrotor using NMPC

Outline

Conclusions

My work in this project

- Formulate quadrotor control as an optimal control problem
- Cast the OCP to NLP as a NLP via Multiple Shooting
- MPC implementation to quadrotor control using CaSAdi
  i.e., position stabilization and trajectory tracking

Future Work

- Stability Analysis, e.g. effect of length of prediction horizon, etc.

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