Adaptive Fuzzy Finite-Time Control for Nonstrict-Feedback Nonlinear Systems

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***Abstract*—This article presents an adaptive fuzzy finite-time control (AFFTC) method for nonstrict-feedback nonlinear systems (NFNSs) with unknown dynamics. With the aid of the backstepping technique, by establishing the smooth switch function (SSF), a novel C^1 AFFTC strategy is recursively constructed, which counteracts the effect of nonstrict-feedback structure and unknown dynamics. Different from the reporting finite-time control achievements, the singularity hindrance derived from the differentiating virtual control law is availably surmounted. Moreover, the developed AFFTC strategy can drive the tracking error to converge into a small neighborhood of the origin in a finite time. Simulation results are conducted to substantiate the efficacy of theoretical findings.**

***Index Terms*—Finite-time control (FTC), nonlinear systems, singularity hindrance, smooth switch function (SSF).**

I. Introduction

SINCE THE last century, the adaptive control has become an effective method to solve the stabilization or tracking problem of the complex nonlinear systems [1]. By fusion of the adaptive control with the backstepping technique [2], prosperous meaningful investigations have been uncovered for nonlinear systems [3]–[6]. In [3], the maneuvering control problem was studied for nonlinear systems with the help of the backstepping method. For uncertain systems with hysteresis character, the robust backstepping design strategy was derived [4]. In [5], by fusion of the performance function, the guaranteed performance controller was tactfully presented. In the early works [3]–[6], the nonlinear terms existing systems are known or can be linearly parameterized, and these methods are not applicable to those nonlinear systems containing the absolutely unknown terms. To more effectively dispose of the unknown terms, many intelligent adaptive design strategies [7]–[13] have been presented by virtue of approximation-based neural network (NN) or fuzzy-logic systems (FLSs). In [7], the direct fuzzy design algorithm was presented for strict-feedback systems. By combining the small gain theorem with the backstepping technique, the robust design strategy was proposed for pure feedback nonlinear systems [8]. In [9], the simplified optimized NN design scheme was presented for strict-feedback systems based on the reinforcement learning algorithm. In [10], for nonlinear systems containing unavailable states, the observer-based design approach was described. In [13], the indirect fuzzy tracking control method was presented for canonical nonlinear systems. Moreover, many applications of practical physical systems based on NN or FLS have been exploited, such as robot systems [14]–[17]; chaotic systems [18], [19]; and so on. However, the foregoing design methods only achieve the uniformly ultimately bounded (UUB) results, that is, they merely ensure the system stability when the time goes infinity.

Different from the UUB control effect, the finite-time control (FTC) is recognized as a powerful tool to make the system state come up to the desired value in finite time. Since the finite-time controller contains a fractional power term, it makes the homologous controller have better transient performance and robustness than the UUB approach. Therefore, the research on FTC has acquired compelling interest from plentiful scientific communities. Based on the Lyapunov analysis, the finite-time criteria were described [20]. Motivated by the seminal work [20], tremendous results have been proposed by employing the classical finite-time theory, such as [21]–[24]. In [21], the FTC issue of nonlinear systems was investigated by combining the backstepping methodology. By adding the power technique, the FTC output-feedback approach was detailedly elaborated on [22]. In [23], the fast FTC theory was created by resorting to the sliding-mode technique. Recently, the scholars have made many strides on FTC by combining NN or FLSs with the backstepping technique, with many intelligent FTC approaches being published [25]–[29]. These FTC methods [25]–[29] are merely applicable to the strict feedback or pure feedback nonlinear systems, in which the ith subsystem includes states of itself and the i+1th subsystem. However, for the nonstrict-feedback nonlinear systems (NFNS), every subsystem function includes the entire states. If these FTC schemes [25]–[29] are applied to NFNS, the algebraic loop problem will appear, which shows that the foregoing backstepping schemes are feeble for NFNS. For the sake of addressing this problem, approximation-based fuzzy controllers were constructed for NFNS, which only achieve the UUB objective [30]–[32]. By virtue of the finite-time theory,

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many intelligent FTC strategies of NFNS were proposed by combining the approximation-based schemes [33]–[39].

It is worth noting that the singularity problem will be generated when differentiating the virtual control law in the aforementioned intelligent FTC schemes, that is, these FTC strategies only achieve C^0 continuous. On the other hand, letting $V(x)$ be a Lyapunov function, and the relation of $\dot{V}(x)$ in [33]–[39] satisfying $\dot{V}(x) \leq -\mu V^r(x) + c$ with $\mu > 0, 0 < r < 1$, and bounded constant $c > 0$, which only achieves the slow convergence feature. Moreover, as proved in [23], if the relation of $\dot{V}(x)$ satisfies $\dot{V}(x, t) \leq -\mu_1 V(x, t) - \mu_2 V^r(x, t) + c$, then the fast finite-time character will be obtained. Fortunately, Cui *et al.* [40] first presented the smooth fast FTC method for strict-feedback nonlinear systems. In [40], by delicately switching in virtual control, the singularity hindrance was tactfully overcome. However, the result in [40] is mainly focused on the nonlinear systems satisfying the strict-feedback structure. Therefore, up to now, for NFNS with unknown dynamics, how to design a nonsingular adaptive fuzzy FTC (AFFTC) scheme has not been investigated, which motives us for this work.

Based on the above literature review, this article presents the AFFTC scheme for NFNS with unknown dynamics. The smooth switch function (SSF) is fabricated to avoid the singularity problem. The salient merits of the devised AFFTC are as follows.

1) In contrast to the existing achievements on NFNS [30]–[32], which only guarantee the UUB control performance, a remarkable feature of the presented method is that the finite-time convergence is achieved.
2) Different from the published intelligent FTC strategies [33]–[39] which only achieve slow convergence feature, by fabricating the SSF, the developed AFFTC can make the system output fast converge the desired signal in a finite time.
3) The FLSs are explored to address unknown dynamics of the NFNS. Meanwhile, the nonstrict-feedback structure is effectively addressed by employing the property of the fuzzy basis function.

II. Problem Formulation and Preliminaries

A. System Description

Consider the NFNS as follows:

$$\begin{cases} \dot{x}_i = f_i(x) + g_i(\bar{x}_i)x_{i+1}, & i = 1, \dots, n-1 \\ \dot{x}_n = f_n(x) + g_n(\bar{x}_n)u \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, \dots, x_i]^T \in R^i$, $i = 1, \dots, n$ and $x = [x_1, \dots, x_n]^T \in R^n$ are the system state vectors, $u \in R$ and $y \in R$ are the control input and system output, and $f_i(x)$ and $g_i(\bar{x}_i), i = 1, \dots, n$ are unknown nonlinear functions.

Remark 1: Since the systems (1) satisfy input-affine and $f_i(x)$ are completely unknown dynamics, then the nonadaptive fuzzy control approach cannot be employed to control (1). To address this issue, by means of the FLS, the approximation-based AFFTC strategy is recursively constructed.

To facilitate the next controller design, some necessary assumptions and lemmas will be described as follows.

Assumption 1 [4]: The desired signal y_d and its ith order time derivatives $y_d^{(i)}, i = 1, \dots, n$ are bounded and continuous.

Assumption 2 [7]: There are constants $g_{i1} \geq g_{i0} > 0, i = 1, \dots, n$, so that $g_{i0} \leq g_i(\bar{x}_i) \leq g_{i1}$.

Remark 2: As described in [7], Assumption 2 is sound because $g_i(\cdot)$ deviate from 0 to satisfy the controllable condition. It is worth mentioning that the values of g_{i0} are only required for analysis purpose.

Lemma 1 [23]: For the following nonlinear system:

$$\dot{x}(t) = f(x), \quad x(0) = 0 \tag{2}$$

where $x \in R^n$ is the system state. If the relation of $\dot{V}(x)$ satisfies

$$\dot{V}(x, t) \leq -\mu_1 V(x, t) - \mu_2 V^r(x, t) \tag{3}$$

where $\mu_1 > 0$, $\mu_2 > 0$, and $0 < r < 1$, then the fast finite-time converge peculiarity can be ensured, and the settling time can be denoted as $T \leq (1/[\mu_1(1-r)]) \ln([\mu_1 V^{1-r}(x(0)) + \mu_2]/\mu_2)$.

Lemma 2 [24]: Letting $0 < r = r_1/r_2 < 1$ with r_1 and r_2 being odd integers, then we have the following result:

$$ab^r \leq -\nu a^{1+r} + \tau(a+b)^{1+r} \tag{4}$$

where $\nu = (1/[1+r])(2^{r-1} - 2^{(r-1)(r+1)})$ and $\tau = (1/[1+r])([(1+2r)/(1+r)] + [(2^{-(r-1)^2(r+1)})/(1+r)] - 2^{r-1})$.

Lemma 3 [41]: For nonlinear function $F(Z)$, there is a corresponding FLS $W^T\varphi(Z)$ such that

$$\sup_{Z\in\Omega} \left|F(Z) - W^T\varphi(Z)\right| \leq \bar{\delta}, \quad \bar{\delta} > 0 \tag{5}$$

where $Z \in R^q$ is the input vector, $W = [w_1, w_2, \dots, w_l]^T \in R^l$ denotes weight vector satisfying $\|W\| \leq \bar{W}$, and $\varphi(Z) = [\Phi_1(Z), \dots, and \Phi_l(Z)]^T / \sum_{i=1}^{l} \Phi_i(Z)$ is the fuzzy basis function vector with the property: $0 < \varphi^T(Z)\varphi(Z) \leq 1$.

III. Main Results

A. Controller Design

In this part, we will describe the AFFTC scheme with the aid of the backstepping technique. Define coordinate change as follows:

$$z_1 = x_1 - y_d \tag{6}$$

$$z_i = x_i - \alpha_{i-1}, \quad i = 2, \dots, n \tag{7}$$

where z_i are the virtual errors and α_{i-1} are the virtual control laws.

Step 1: In light of (1) and (6), we have

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{y}_d \\ &= f_1(x) + g_1(x_1)x_2 - \dot{y}_d. \end{aligned} \tag{8}$$

Choose the Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}g_{10}\tilde{\theta}_1^2 \tag{9}$$

where $\tilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ and $\hat{\theta}_1 > 0$ is the estimation of $\theta_1 = g_{10}^{-1}\bar{W}_1^2$.

Differentiating V_1 along (8) yields

$$\begin{aligned}\dot{V}_1 &= z_1(f_1(x) + g_1(x_1)x_2 - \dot{y}_d) - g_{10}\tilde{\theta}_1\dot{\hat{\theta}}_1 \\ &= z_1(F_1 + g_1(x_1)x_2) - g_{10}\tilde{\theta}_1\dot{\hat{\theta}}_1 \end{aligned} \tag{10}$$

where $F_1 = f_1(x) - \dot{y}_d$.

Since F_1 is the unknown function, the FLS $W_1^T\varphi_1(x, y_d)$ can be adopted to approximate it, such that for any constant $\bar{\delta}_1 > 0$

$$F_1 = W_1^T\varphi_1(x, y_d) + \delta_1(x, y_d),\ |\delta_1(x, y_d)| \le \bar{\delta}_1. \tag{11}$$

Substituting (11) into (10) yields

$$\dot{V}_1 = z_1W_1^T\varphi_1(x, y_d) + z_1\delta_1(x, y_d) + z_1g_1(x_1)x_2 - g_{10}\tilde{\theta}_1\dot{\hat{\theta}}_1. \tag{12}$$

Remark 3: Note that the term $W_1^T\varphi_1(x, y_d)$ of (12) includes the entire system state vector $x = [x_1, \ldots, x_n]^T$. If the traditional backstepping technique is deployed, it will generate the algebraic loop problem.

Since the fuzzy basis function satisfies $0 < \varphi_1^T(\cdot)\varphi_1(\cdot) \le 1$, by applying Young's inequality, there are

$$\begin{aligned} z_1W_1^T\varphi_1(x, y_d) &\le \frac{z_1^2\bar{W}_1^2\varphi_1^T(x, y_d)\varphi_1(x, y_d)}{2\eta_1^2} + \frac{\eta_1^2}{2} \\ &\le \frac{z_1^2\bar{W}_1^2\varphi_1^T(x, y_d)\varphi_1(x, y_d)}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} + \frac{\eta_1^2}{2} \\ &\le \frac{z_1^2\bar{W}_1^2}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} + \frac{\eta_1^2}{2} \end{aligned} \tag{13}$$

$$z_1\delta_1(x, y_d) \le \frac{g_{10}z_1^2}{2} + \frac{\bar{\delta}_1^2}{2g_{10}} \tag{14}$$

where $\eta_1 > 0$ and $g_{10} > 0$ are constants.

Putting (13) and (14) into (12) yields

$$\begin{aligned}\dot{V}_1 \le\ & z_1g_1(x_1)x_2 + \frac{z_1^2\bar{W}_1^2}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} + \frac{\eta_1^2}{2} \\ &+ \frac{g_{10}z_1^2}{2} + \frac{\bar{\delta}_1^2}{2g_{10}} - g_{10}\tilde{\theta}_1\dot{\hat{\theta}}_1. \end{aligned} \tag{15}$$

Design the C^1 finite-time virtual control law as follows:

$$\alpha_1 = -\lambda_1z_1 - \frac{z_1}{2} - \frac{z_1\hat{\theta}_1}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} - k_1\rho_1(z_1) \tag{16}$$

where $\lambda_1 > 0$ and $k_1 > 0$ are constants, and $\rho_1(z_1)$ is constructed as

$$\rho_1(z_1) = \begin{cases} z_1^r, & |z_1| \ge \varepsilon_1 \\ \xi_1z_1 + \gamma_1z_1^3, & |z_1| < \varepsilon_1 \end{cases} \tag{17}$$

where $\xi_1 = (1/2)(3 - r)\varepsilon_1^{r-1}$, $\gamma_1 = (1/2)(r - 1)\varepsilon_1^{r-3}$, and $\varepsilon_1 > 0$ is a small constant.

Remark 4: By appropriately selecting ξ_1 and γ_1, the continuity of the virtual control law can be guaranteed. From (17), it is obvious that $\rho_1(\varepsilon_1^+) = \lim_{z_1\to\varepsilon_1^+} z_1^r = \varepsilon_1^r$ and $\rho_1(\varepsilon_1^-) = \lim_{z_1\to\varepsilon_1^-}(\xi_1z_1 + \gamma_1z_1^3) = \varepsilon_1^r$ when $z_1 > 0$. Hence, we have $\rho_1(\varepsilon_1^+) = \rho_1(\varepsilon_1^-)$. Similarly, it can be deduced that $\rho_1(-\varepsilon_1^+) = \rho_1(-\varepsilon_1^-)$. So we can obtain that $\rho_1(z_1)$ is continuous for ε_1 and $-\varepsilon_1$. Likewise, we can obtain $\dot{\rho}_1(\varepsilon_1^+) = \dot{\rho}_1(\varepsilon_1^-)$ and $\dot{\rho}_1(-\varepsilon_1^+) = \dot{\rho}_1(-\varepsilon_1^-)$. Therefore, $\dot{\rho}_1(z_1)$ also is continuous. From the above discussion, we can conclude that (16) satisfies C^1 continuous. Then, the singularity problem is excluded.

Design the following adaptive law:

$$\dot{\hat{\theta}}_1 = -\sigma_{11}\hat{\theta}_1 - \sigma_{12}\hat{\theta}_1^r + \frac{z_1^2}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} \tag{18}$$

where $\sigma_{11} > 0$ and $\sigma_{12} > 0$ are design parameters.

According to $z_2 = x_2 - \alpha_1$ and Assumption 2, we have

$$\begin{aligned} z_1g_1(x_1)x_2 &= z_1g_1(x_1)(\alpha_1 + z_2) \\ &\le -\lambda_1g_{10}z_1^2 - \frac{g_{10}z_1^2}{2} - \frac{g_{10}z_1^2\hat{\theta}_1}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} \\ &\quad - k_1g_1(x_1)z_1\rho_1(z_1) + z_1g_1(x_1)z_2. \end{aligned} \tag{19}$$

Inserting (18) and (19) into (15) yields

$$\begin{aligned}\dot{V}_1 \le\ & -\lambda_1g_{10}z_1^2 - k_1g_1(x_1)z_1\rho_1(z_1) + z_1g_1(x_1)z_2 \\ &+ g_{10}\sigma_{11}\tilde{\theta}_1\hat{\theta}_1 + g_{10}\sigma_{12}\tilde{\theta}_1\hat{\theta}_1^r + \frac{\eta_1^2}{2} + \frac{\bar{\delta}_1^2}{2g_{10}}. \end{aligned} \tag{20}$$

Step i ($2 \le i \le n - 1$): In conjunction with (7), one can obtain

$$\begin{aligned}\dot{z}_1 &= \dot{x}_i - \dot{\alpha}_{i-1} \\ &= f_i(x) + g_i(\bar{x}_i)x_{i+1} - \dot{\alpha}_{i-1}. \end{aligned} \tag{21}$$

Choose the Lyapunov function as

$$V_i = \frac{1}{2}z_i^2 + \frac{1}{2}g_{i0}\tilde{\theta}_i^2 \tag{22}$$

where $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ and $\hat{\theta}_i > 0$ is the estimation of $\theta_i = g_{i0}^{-1}\bar{W}_i^2$.

Calculating $\dot{V}_i$ along (21) yields

$$\begin{aligned}\dot{V}_i &= z_i(f_i(x) + g_i(\bar{x}_i)x_{i+1} - \dot{\alpha}_{i-1}) - g_{i0}\tilde{\theta}_i\dot{\hat{\theta}}_i \\ &= z_i(F_i + g_i(\bar{x}_i)x_{i+1}) - z_{i-1}g_{i-1}(\bar{x}_{i-1})z_i - g_{i0}\tilde{\theta}_i\dot{\hat{\theta}}_i \end{aligned} \tag{23}$$

where $F_i = f_i(x) - \dot{\alpha}_{i-1} + z_{i-1}g_{i-1}(\bar{x}_{i-1})$.

Similar to step 1, F_i can be approximated by the FLS $W_i^T\varphi_i(x, y_d)$ as follows:

$$F_i = W_i^T\varphi_i(x, y_d) + \delta_i(x, y_d),\ |\delta_i(x, y_d)| \le \bar{\delta}_i. \tag{24}$$

Substituting (24) into (23) yields

$$\begin{aligned}\dot{V}_i =\ & z_iW_i^T\varphi_i(x, y_d) + z_i\delta_i(x, y_d) + z_ig_i(\bar{x}_i)x_{i+1} \\ &- z_{i-1}g_{i-1}(\bar{x}_{i-1})z_i - g_{i0}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{25}$$

Due to $0 < \varphi_i^T(\cdot)\varphi_i(\cdot) \le 1$, then we can obtain

$$\begin{aligned} z_iW_i^T\varphi_i(x, y_d) &\le \frac{z_i^2\bar{W}_i^2\varphi_i^T(x, y_d)\varphi_i(x, y_d)}{2\eta_i^2} + \frac{\eta_i^2}{2} \\ &\le \frac{z_i^2\bar{W}_i^2\varphi_i^T(x, y_d)\varphi_i(x, y_d)}{2\eta_i^2\varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} + \frac{\eta_i^2}{2} \\ &\le \frac{z_i^2\bar{W}_i^2}{2\eta_i^2\varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} + \frac{\eta_i^2}{2} \end{aligned} \tag{26}$$

$$z_i\delta_i(x, y_d) \le \frac{g_{i0}z_i^2}{2} + \frac{\bar{\delta}_i^2}{2g_{i0}} \tag{27}$$

where $\eta_i > 0$ and $g_{i0} > 0$ are constants.

Inserting (26) and (27) into (25) yields

$$\dot{V}_i \le z_i g_i(\bar{x}_i)x_{i+1} + \frac{z_i^2 \bar{W}_i^2}{2\eta_i^2 \varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} + \frac{\eta_i^2}{2} + \frac{g_{i0} z_i^2}{2} + \frac{\bar{\delta}_i^2}{2g_{i0}} - z_{i-1}g_{i-1}(\bar{x}_{i-1})z_i - g_{i0}\tilde{\theta}_i\dot{\hat{\theta}}_i. \tag{28}$$

Design the C^1 finite-time virtual control law as follows:

$$\alpha_i = -\lambda_i z_i - \frac{z_i}{2} - \frac{z_i \hat{\theta}_i}{2\eta_i^2 \varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} - k_i \rho_i(z_i) \tag{29}$$

where $\lambda_i > 0$ and $k_i > 0$ are constants and $\rho_i(z_i)$ is constructed as

$$\rho_i(z_i) = \begin{cases} z_i^r, & |z_i| \ge \varepsilon_i \\ \xi_i z_i + \gamma_i z_i^3, & |z_i| < \varepsilon_i \end{cases} \tag{30}$$

where $\xi_i = (1/2)(3-r)\varepsilon_i^{r-1}$, $\gamma_i = (1/2)(r-1)\varepsilon_i^{r-3}$, and $\varepsilon_i > 0$ is a small constant.

Design the following adaptive law:

$$\dot{\hat{\theta}}_i = -\sigma_{i1}\hat{\theta}_i - \sigma_{i2}\hat{\theta}_i^r + \frac{z_i^2}{2\eta_i^2 \varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} \tag{31}$$

where $\sigma_{i1} > 0$ and $\sigma_{i2} > 0$ are design parameters.

According to $z_{i+1} = x_{i+1} - \alpha_i$, one has

$$\begin{aligned} z_i g_i(\bar{x}_i)x_{i+1} &= z_i g_i(\bar{x}_i)(\alpha_i + z_{i+1}) \\ &\le -\lambda_i g_{i0} z_i^2 - \frac{g_{i0} z_i^2}{2} - \frac{g_{i0} z_i^2 \hat{\theta}_i}{2\eta_i^2 \varphi_i^T(\bar{x}_i, y_d)\varphi_i(\bar{x}_i, y_d)} \\ &\quad - k_i g_i(\bar{x}_i) z_i \rho_i(z_i) + z_i g_i(\bar{x}_i) z_{i+1}. \end{aligned} \tag{32}$$

Substituting (31) and (32) into (28) yields

$$\begin{aligned} \dot{V}_i \le& -\lambda_i g_{i0} z_i^2 - k_i g_i(\bar{x}_i) z_i \rho_i(z_i) + z_i g_i(\bar{x}_i) z_{i+1} \\ &- z_{i-1} g_{i-1}(\bar{x}_{i-1}) z_i + g_{i0}\sigma_{i1}\tilde{\theta}_i\hat{\theta}_i + g_{i0}\sigma_{i2}\tilde{\theta}_i\hat{\theta}_i^r \\ &+ \frac{\eta_i^2}{2} + \frac{\bar{\delta}_i^2}{2g_{i0}}. \end{aligned} \tag{33}$$

Step n: From (1) and $z_n = x_n - \alpha_{n-1}$, one has

$$\begin{aligned} \dot{z}_n &= \dot{x}_n - \dot{\alpha}_{n-1} \\ &= f_n(x) + g_n(\bar{x}_n)u - \dot{\alpha}_{n-1}. \end{aligned} \tag{34}$$

Choose the Lyapunov function

$$V_n = \frac{1}{2}z_n^2 + \frac{1}{2}g_{n0}\tilde{\theta}_n^2 \tag{35}$$

where $\tilde{\theta}_n = \theta_n - \hat{\theta}_n$ and $\hat{\theta}_n > 0$ is the estimation of $\theta_n = g_{n0}^{-1}\bar{W}_n^2$.

Computing $\dot{V}_n$, one can obtain

$$\begin{aligned} \dot{V}_n &= z_n(f_n(\bar{x}_n) + g_n(\bar{x}_n)u - \dot{\alpha}_{n-1}) - g_{n0}\tilde{\theta}_n\dot{\hat{\theta}}_n \\ &= z_n(F_n + g_n(\bar{x}_n)u)) - z_{n-1}g_{n-1}(\bar{x}_{n-1})z_n - g_{n0}\tilde{\theta}_n\dot{\hat{\theta}}_n \end{aligned} \tag{36}$$

where $F_n = f_n(\bar{x}_n) - \dot{\alpha}_{n-1} + z_{n-1}g_{n-1}(\bar{x}_{n-1})$.

Similar to step i, there are

$$\begin{aligned} z_n W_n^T \varphi_n(x, y_d) &\le \frac{z_n^2 \bar{W}_n^2 \varphi_n^T(x, y_d)\varphi_n(x, y_d)}{2\eta_n^2} + \frac{\eta_n^2}{2} \\ &\le \frac{z_n^2 \bar{W}_n^2 \varphi_n^T(x, y_d)\varphi_n(x, y_d)}{2\eta_n^2 \varphi_i^T(\bar{x}_n, y_d)\varphi_n^T(\bar{x}_n, y_d)} + \frac{\eta_n^2}{2} \\ &\le \frac{z_n^2 \bar{W}_n^2}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d)\varphi_n(\bar{x}_n, y_d)} + \frac{\eta_n^2}{2} \end{aligned} \tag{37}$$

$$z_n \delta_n(x, y_d) \le \frac{g_{n0} z_n^2}{2} + \frac{\bar{\delta}_n^2}{2g_{n0}} \tag{38}$$

where $\eta_n > 0$ and $g_{n0} > 0$ are constants.

By applying (37) and (38) into (36), there is

$$\dot{V}_n \le z_n g_n(\bar{x}_n)u + \frac{z_n^2 \bar{W}_n^2}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d)\varphi_n(\bar{x}_n, y_d)} + \frac{\eta_n^2}{2} + \frac{g_{n0} z_n^2}{2} + \frac{\bar{\delta}_n^2}{2g_{n0}} - g_{n0}\tilde{\theta}_n\dot{\hat{\theta}}_n. \tag{39}$$

Construct the C^1 finite-time actual control law as

$$u = -\lambda_n z_n - \frac{z_n}{2} - \frac{z_n \hat{\theta}_n}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d)\varphi_n(\bar{x}_n, y_d)} - k_n \rho_n(z_n) \tag{40}$$

where $\lambda_n > 0$ and $k_n > 0$ are constants and $\rho_n(z_n)$ is constructed as

$$\rho_n(z_n) = \begin{cases} z_n^r, & |z_n| \ge \varepsilon_n \\ \xi_n z_n + \gamma_n z_n^3, & |z_n| < \varepsilon_n \end{cases} \tag{41}$$

where $\xi_n = (1/2)(3-r)\varepsilon_n^{r-1}$, $\gamma_n = (1/2)(r-1)\varepsilon_n^{r-3}$, and $\varepsilon_n > 0$ is a small constant.

Select the following adaptive law:

$$\dot{\hat{\theta}}_n = -\sigma_{n1}\hat{\theta}_n - \sigma_{n2}\hat{\theta}_n^r + \frac{z_n^2}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d)\varphi_n(\bar{x}_n, y_d)}. \tag{42}$$

Consequently, we have

$$\begin{aligned} z_n g_n(\bar{x}_n)u \le& -\lambda_n g_{n0} z_n^2 - \frac{g_{n0} z_n^2}{2} - k_n g_n(\bar{x}_n) z_n \rho_n(z_n) \\ &- \frac{g_{n0} z_n^2 \hat{\theta}_n}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d)\varphi_n(\bar{x}_n, y_d)}. \end{aligned} \tag{43}$$

In conjunction with (42) and (43), one can obtain

$$\begin{aligned} \dot{V}_n \le& -\lambda_n g_{n0} z_n^2 - k_n g_n(\bar{x}_n) z_n \rho_n(z_n) + \frac{\eta_n^2}{2} + \frac{\bar{\delta}_n^2}{2g_{n0}} \\ &- z_{n-1}g_{n-1}(\bar{x}_{n-1})z_n + g_{n0}\sigma_{n1}\tilde{\theta}_n\hat{\theta}_n + g_{n0}\sigma_{n2}\tilde{\theta}_n\hat{\theta}_n^r. \end{aligned} \tag{44}$$

B. Stability Analysis

Based on the foregoing discussion, the key results are summarized as follows.

Theorem 1: Let the controlled system (1) satisfy Assumption 1 and the desired signal satisfy Assumption 2. The constructed C^1 FTC law u (40), the virtual control law α_1 (16), α_i (29), the adaptive law $\dot{\hat{\theta}}_1$ (18), $\dot{\hat{\theta}}_i$ (31), and $\dot{\hat{\theta}}_n$ (42) can guarantee the stability of the entirely closed-loop systems, and the tracking errors converge into the compact set in finite time.

Proof: Construct the following Lyapunov function:

$$V = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \sum_{i=1}^{n} \frac{1}{2} g_{i0} \tilde{\theta}_i^2. \tag{45}$$

In conjunction with (20), (33), and (44), there is

$$\dot{V} \le -\sum_{i=1}^{n} \lambda_i g_{i0} z_i^2 - \sum_{i=1}^{n} k_i g_i(\bar{x}_i) z_i \rho_i(z_i) + \sum_{i=1}^{n} g_{i0} \sigma_{i1} \tilde{\theta}_i \hat{\theta}_i + \sum_{i=1}^{n} g_{i0} \sigma_{i2} \tilde{\theta}_i \hat{\theta}_i^r + \sum_{i=1}^{n} \left(\frac{\gamma_i^2}{2} + \frac{\bar{\delta}_i^2}{2 g_{i0}} \right). \tag{46}$$

Motivated by Lemma 2, there are

$$g_{i0} \sigma_{i1} \tilde{\theta}_i \hat{\theta}_i \le -\frac{\sigma_{i1}}{2} g_{i0} \tilde{\theta}_i^2 + \frac{\sigma_{i1}}{2} g_{i0} \theta_i^2 \tag{47}$$

$$g_{i0} \sigma_{i2} \tilde{\theta}_i \hat{\theta}_i^r \le -g_{i0} \sigma_{i2} \nu \tilde{\theta}_i^{1+r} + g_{i0} \sigma_{i2} \tau \theta_i^{1+r}. \tag{48}$$

Combining (47) and (48), one has

$$\dot{V} \le -\sum_{i=1}^{n} \lambda_i g_{i0} z_i^2 - \sum_{i=1}^{n} k_i g_i(\bar{x}_i) z_i \rho_i(z_i) - \sum_{i=1}^{n} \frac{\sigma_{i1}}{2} g_{i0} \tilde{\theta}_i^2 - \sum_{i=1}^{n} g_{i0} \sigma_{i2} \nu \tilde{\theta}_i^{1+r} + c \tag{49}$$

where $c = \sum_{i=1}^{n} (\sigma_{i1}/2) g_{i0} \theta_i^2 + \sum_{i=1}^{n} g_{i0} \sigma_{i2} \tau \theta_i^{1+r} + \sum_{i=1}^{n} ([\eta_i^2/2] + [\bar{\delta}_i^2 / 2 g_{i0}])$.

Based on the definition of $\rho_i(z_i)$ in (30), it requires the following discussion.

Case 1: When $|z_i| < \varepsilon_i, i = 1, \ldots, n$, inserting $\rho_i(z_i) = \xi_i z_i + \gamma_i z_i^3$ into (49) yields

$$\dot{V} \le -\sum_{i=1}^{n} (\lambda_i g_{i0} + k_i g_{i0} \xi_i) z_i^2 - \sum_{i=1}^{n} \frac{\sigma_{i1}}{2} g_{i0} \tilde{\theta}_i^2 + c. \tag{50}$$

Let $\mu = \min\{2(\lambda_i g_{i0} + k_i g_{i0} \xi_i), \sigma_{i1}, i = 1, \ldots, n\}$, then (50) will become

$$\dot{V} \le -\mu V + c. \tag{51}$$

From (51), we can obtain that all internal signals are UUB when $|z_i| < \varepsilon_i$.

Case 2: When $|z_i| \ge \varepsilon_i, i = 1, \ldots, n$, inserting $\rho_i(z_i) = z_i^r$ into (49) yields

$$\dot{V} \le -\sum_{i=1}^{n} \lambda_i g_{i0} z_i^2 - \sum_{i=1}^{n} k_i g_{i0} \left(z_i^2 \right)^{\frac{1+r}{2}} - \sum_{i=1}^{n} \frac{\sigma_{i1}}{2} g_{i0} \tilde{\theta}_i^2 - \sum_{i=1}^{n} g_{i0} \sigma_{i2} \nu \tilde{\theta}_i^{1+r} + c. \tag{52}$$

Let $\mu_1 = \min\{2\lambda_i g_{i0}, \sigma_{i1}\}$ and $\mu_2 = \min\{2^{(1+r/2)} k_i g_{i0}, 2^{(1+r/2)} g_{i0}^{(1-r/2)} \sigma_{i2} \nu\}, i = 1, \ldots, n$, then (52) will become

$$\dot{V} \le -\mu_1 V - \mu_2 V^{\frac{1+r}{2}} + c. \tag{53}$$

For (53), if $V \ge (2c/\mu_2)^{(2/1+r)}$, it can conclude that $\dot{V} \le -\mu_1 V - (\mu_2/2) V^{(1+r/2)}$. Based on Lemma 1, V will fast converge the set $\Omega_1 = \{.V | V \le (2c/\mu_2)^{(2/1+r)}\}$ with setting time

$$T \le \frac{2}{\mu_1 (1-r)} \ln \frac{2\mu_1 V^{\frac{1-r}{2}}(0) + \mu_2}{\mu_2}. \tag{54}$$

Moreover, the following results can be obtained in the finite-time T:

$$|z_i| \le \sqrt{2} \left(\frac{2c}{\mu_2} \right)^{\frac{1}{1+r}}, \quad i = 1, \ldots, n \tag{55}$$

$$\tilde{\theta}_i \le \left(\frac{2}{g_{i0}} \right)^{\frac{1}{2}} \left(\frac{2c}{\mu_2} \right)^{\frac{1}{1+r}}, \quad i = 1, \ldots, n. \tag{56}$$

Remark 5: Many approximation-based finite-time design strategies have also been presented in [25]–[28]. These control methods only were guaranteed to be C^0 continuous. Recently, the smooth nonsingular FTC strategy has been skillfully presented for strict-feedback nonlinear systems [40]. Motivated by the SSF design thought in [40], in this context, the AFFTC is designed for NFNS by combining the property of the fuzzy basis function.

Remark 6: When $r = 1$ and $\varepsilon_i = 0 (i = 1, \ldots, n)$, the above devised AFFTC will become

$$\alpha_1 = -(\lambda_1 + k_1) z_1 - \frac{z_1}{2} - \frac{z_1^2 \hat{\theta}_1}{2\eta_1^2 \varphi_1^T(x_1, y_d) \varphi_1^T(x_1, y_d)} \tag{57}$$

$$\alpha_i = -(\lambda_i + k_i) z_i - \frac{z_i}{2} - \frac{z_i^2 \hat{\theta}_i}{2\eta_i^2 \varphi_i^T(\bar{x}_i, y_d) \varphi_i^T(\bar{x}_i, y_d)} \tag{58}$$

$$u = -(\lambda_n + k_n) z_n - \frac{z_n}{2} - \frac{z_n^2 \hat{\theta}_n}{2\eta_n^2 \varphi_n^T(\bar{x}_n, y_d) \varphi_n^T(\bar{x}_n, y_d)} \tag{59}$$

with the adaptive laws

$$\dot{\hat{\theta}}_i = -(\sigma_{i1} + \sigma_{i2}) \hat{\theta}_i + \frac{z_i^2}{2\gamma_i^2} \| S_i(Z_i) \|^2, \quad i = 1, \ldots, n. \tag{60}$$

Then, it will become the UUB tracking control scheme.

Remark 7: It should be mentioned that by adjusting parameter r in switch function (30), the created C^1 finite-time scheme can make the internal error converge to the small region in a finite time. Different from the UUB control method in Remark 6, the convergence time is unknown.

To better account for the AFFTC thought, the design block program is described in Fig. 1.

IV. Simulation Study

In this section, the validity of the constructed AFFTC will be elaborated via two examples.

Example 1: For the following second-order NFNS:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) + g_1(x_1) x_2 \\ \dot{x}_n = f_2(x_1, x_2) + g_2(x_1, x_2) u \\ y = x_1 \end{cases} \tag{61}$$

where $f_1(x_1, x_2) = 1 - \cos(x_1 x_2)$, $g_1(x_1) = 2.5 + 0.5 \sin(x_1)$, $f_2(x_1, x_2) = x_1^2 e^{x_2}$, and $g_2(x_1, x_2) = 2 + \sin(x_1 x_2)$. The desired signal is governed by $y_d = \sin(t)$.

Choose fuzzy membership functions (see Fig. 2) as

$$\mu_{F_i^l}(x_i) = \exp\left(-0.5 (x_i + 3 - l)^2 \right), \ i = 1, 2, l = 1, \ldots, 5. \tag{62}$$

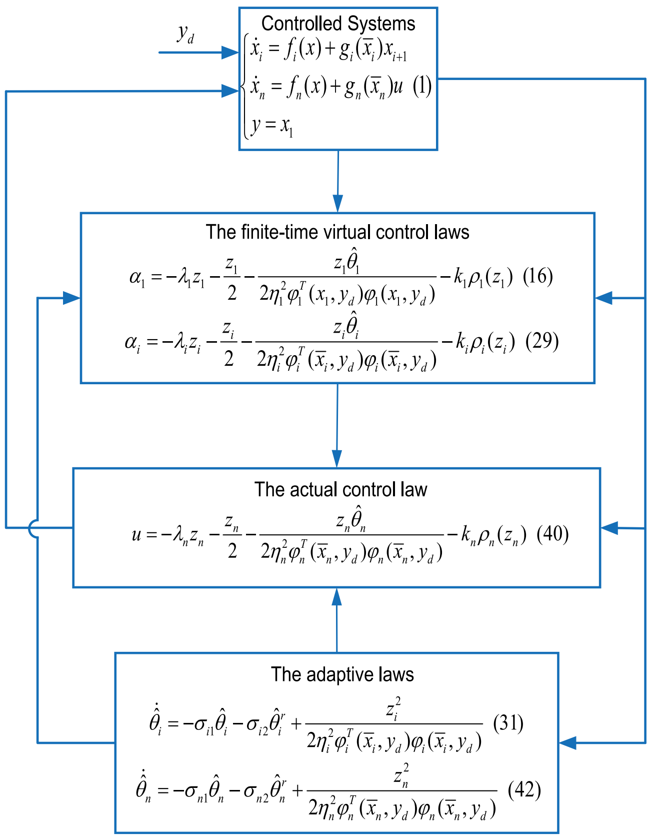

Fig. 1. Block diagram.

Fig. 2. Fuzzy membership functions.

For NFNS (61), the relevant control law and adaptive law are governed by

$$\alpha_1 = -\lambda_1 z_1 - \frac{z_1}{2} - \frac{z_1\hat{\theta}_1}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1^T(x_1, y_d)} - k_1\rho_1(z_1) \tag{63}$$

$$u = -\lambda_2 z_2 - \frac{z_2}{2} - \frac{z_2\hat{\theta}_2}{2\eta_2^2\varphi_2^T(\bar{x}_2, y_d)\varphi_2^T(\bar{x}_2, y_d)} - k_2\rho_2(z_2) \tag{64}$$

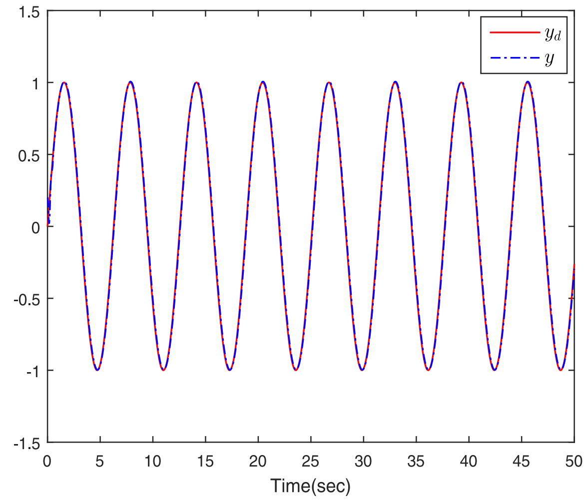

Fig. 3. Output response of y in Example 1.

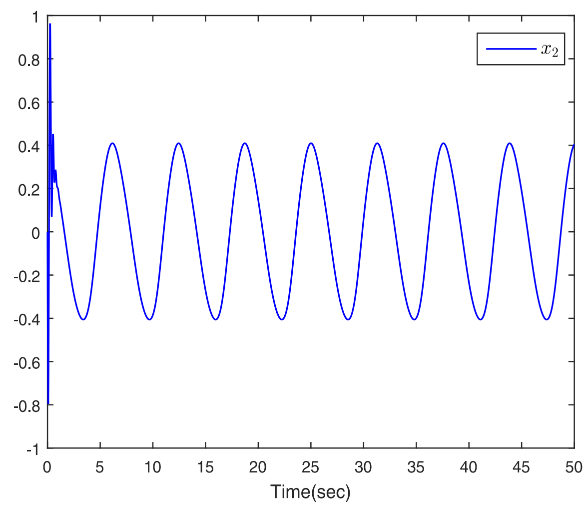

Fig. 4. System state x_2 in Example 1.

$$\dot{\hat{\theta}}_1 = -\sigma_{11}\hat{\theta}_1 - \sigma_{12}\hat{\theta}_1^r + \frac{z_1^2}{2\eta_1^2\varphi_1^T(x_1, y_d)\varphi_1(x_1, y_d)} \tag{65}$$

$$\dot{\hat{\theta}}_2 = -\sigma_{21}\hat{\theta}_2 - \sigma_{22}\hat{\theta}_2^r + \frac{z_2^2}{2\eta_2^2\varphi_2^T(\bar{x}_2, y_d)\varphi_2(\bar{x}_2, y_d)}. \tag{66}$$

The relevant parameters of (63)–(66) are governed by $\lambda_1 = 5$, $\lambda_2 = 4$, $a_1 = 1.8$, $a_2 = 1.8$, $k_1 = 0.6$, $k_2 = 2.7$, $\varepsilon_1 = 0.002$, $\varepsilon_1 = 0.002$, $\sigma_{11} = 0.2$, $\sigma_{12} = 0.2$, $\sigma_{21} = 0.3$, $\sigma_{22} = 0.3$, and $r = 0.7$. The initial values are $x_1(0) = 0.2$, $x_2(0) = 0$, $\hat{\theta}_1(0) = 0.2$, and $\hat{\theta}_2(0) = 0.3$.

Figs. 3–6 display the simulation images. Fig. 3 shows that the devised AFFTC scheme can achieve desirable control performance. Fig. 4 reveals the curve of state x_2. Fig. 5 shows the curves of adaptive law $\hat{\theta}_1$ and $\hat{\theta}_2$. Figs. 6 and 7 denote the curves of virtual control law α_1 and control input u.

Furthermore, we conduct a comparison between the AFFTC method and the UUB control scheme in (57)–(59). Fig. 8 depicts the errors profile using the AFFTC method and UUB control scheme. It can be seen that the devised AFFTC method achieves better tracking performance.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

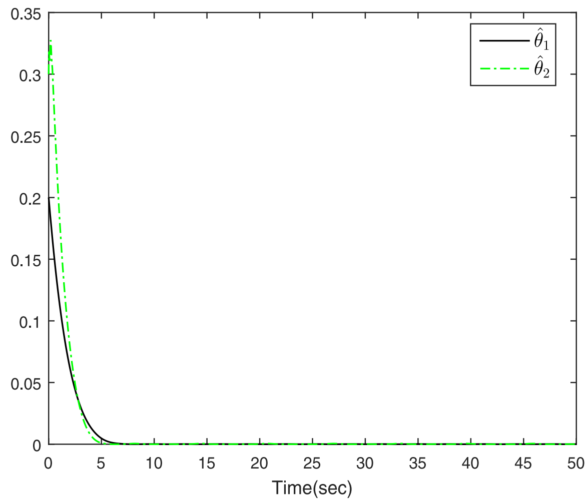

Fig. 5. Adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$ in Example 1.

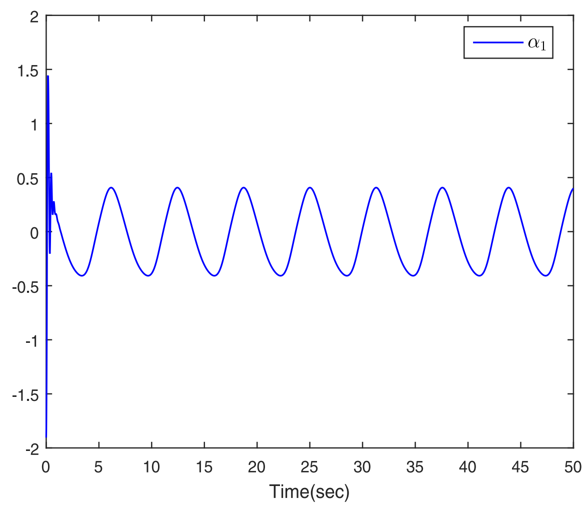

Fig. 6. Virtual control law α_1 in Example 1.

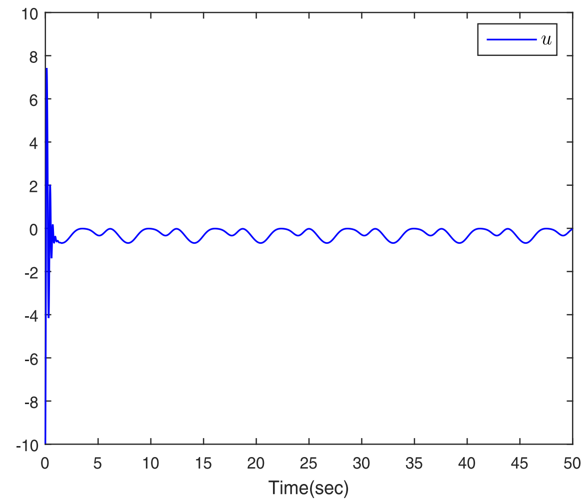

Fig. 7. Control input u in Example 1.

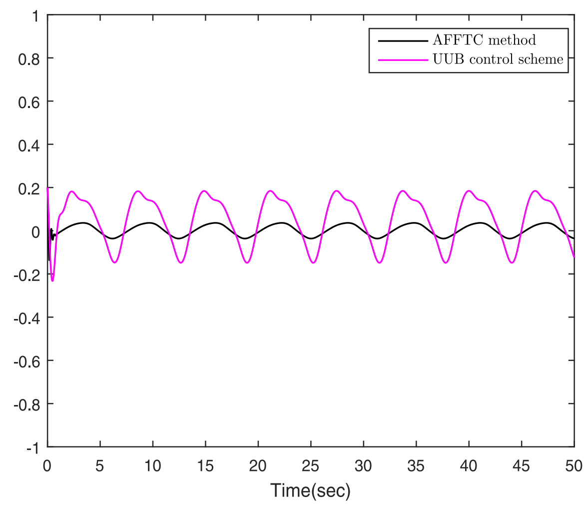

Fig. 8. Curves of tracking errors between the AFFTC method with UUB control scheme of Example 1.

Remark 8: By observation of the simulation images, we can obtain that the devised C^1 AFFTC scheme can drive the system output fast following the desired signal. From Figs. 6 and 7, we can know that the virtual control law α_1 and control input are continuous and bounded. It is worth mentioning that unlike the C^0 AFFTC methods that exist, the singularity issue in the developed method is excluded.

Example 2: Consider an one-link manipulator with the actuator, in Fig. 9, is governed by [39]

$$\begin{cases} M\ddot{q} + C\dot{q} + G\sin(q) = \upsilon \\ B_m\dot{\upsilon} + H_m\upsilon = u - K_m\dot{q} \end{cases} \tag{67}$$

where q, $\dot{q}$, and $\ddot{q}$ are the link position, velocity, and acceleration. u is the control input and υ is the joint dynamic torque. The detailed parameters of the manipulator are listed in Table I.

Letting $x_1 = q$, $x_2 = \dot{q}$, and $x_3 = \upsilon$, then the dynamic equation of (67) can be expressed as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{M}(-Cx_2 - G\sin x_1) + \frac{1}{M}x_3 \\ \dot{x}_3 = \frac{1}{B_m}(-K_m x_2 - H_m x_3) + \frac{1}{B_m}u \end{cases} \tag{68}$$

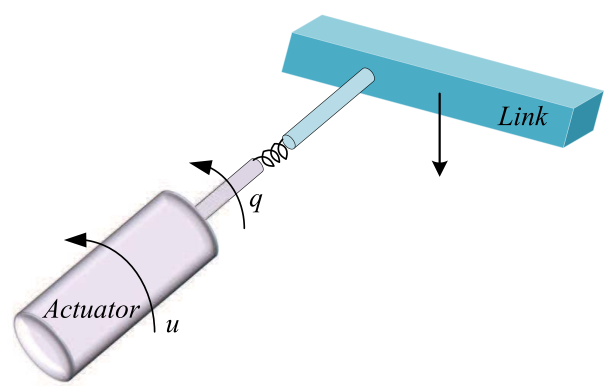

Fig. 9. One-link manipulator.

The desired signal is governed by $y_d = 0.5\sin(0.6t)$ and the relevant parameters are taken as $\lambda_1 = 12$, $\lambda_2 = 8$, $\lambda_3 = 30$, $a_1 = 1.2$, $a_2 = 1.3$, $a_3 = 1.5$, $k_1 = 0.4$, $k_2 = 1.3$, $k_3 = 2.8$, $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.001$, $\varepsilon_3 = 0.001$, $\sigma_{11} = 0.5$, $\sigma_{12} = 0.5$, $\sigma_{21} = 0.8$, $\sigma_{22} = 0.8$, $\sigma_{31} = 0.3$, $\sigma_{32} = 0.3$, and $r = 0.7$.

TABLE I
PARAMETERS OF ONE-LINK MANIPULATOR [39]

Parameter	Description	Value
M	Inertia coefficient	$1\mathrm{kg}\cdot\mathrm{m}^2$
C	Friction coefficient	$1\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s/rad}$
G	Positive gravity constant	2
B_m	Armature inductance	$1\mathrm{H}$
H_m	Armature resistance	1Ω
K_m	Back electromotive force coefficient	$2\mathrm{N}\cdot\mathrm{m/A}$

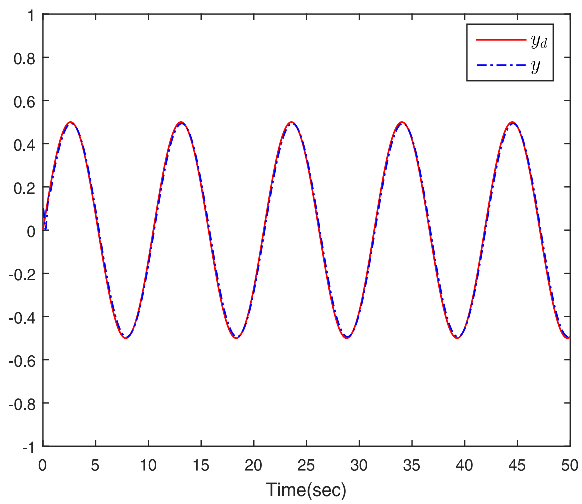

Fig. 10. Output response of y in Example 2.

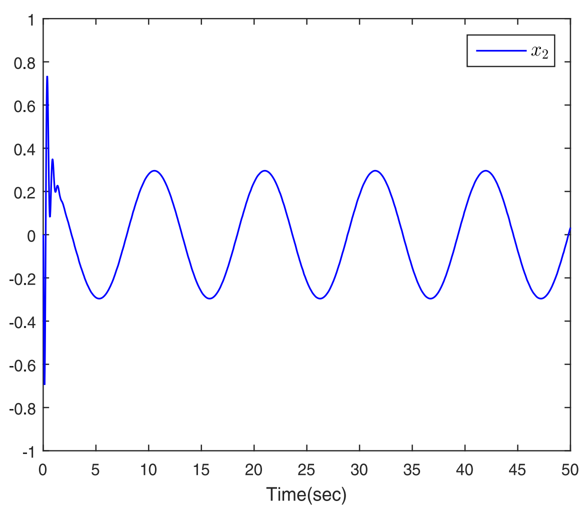

Fig. 11. System state x_2 in Example 2.

The initial values are $x_1(0) = 0.1$, $x_2(0) = 0$, $x_3(0) = 0$, $\hat{\theta}_1(0) = 0.2$, $\hat{\theta}_2(0) = 0.3$, and $\hat{\theta}_2(0) = 0.5$.

Figs. 10–14 display the simulation results. Fig. 10 shows that the devised AFFTC scheme can derive the output y to follow the desired signal y_d. Figs. 11 and 12 depict the curves of state x_2 and x_3. Fig. 13 plots the curve of the adaptive law. Fig. 14 depicts the control input, which is continuous and bounded. The simulation results have been conducted on the practical physical systems, which further illustrates the availability of the presented method.

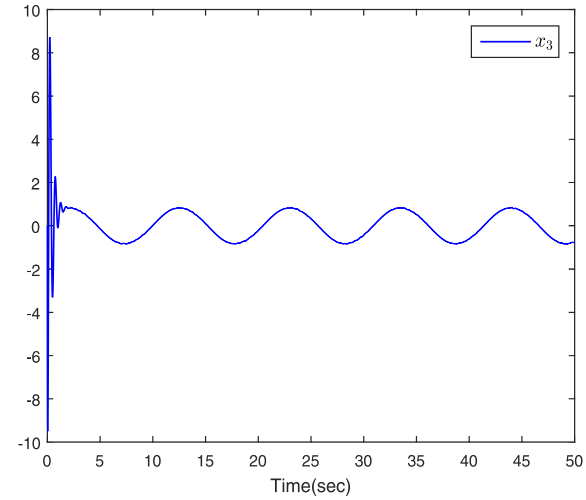

Fig. 12. System state x_3 in Example 2.

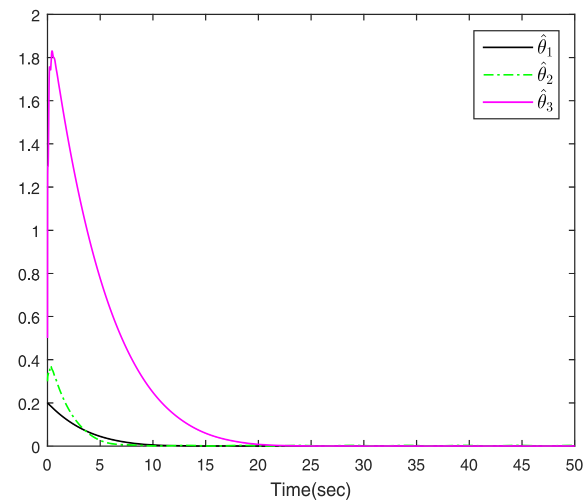

Fig. 13. Adaptive laws $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$ in Example 2.

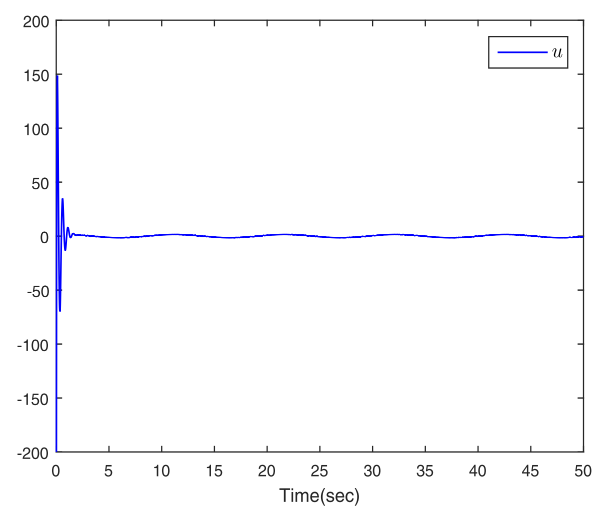

Fig. 14. Control input u in Example 2.

V. CONCLUSION

In this article, the AFFTC method has been presented for NFNS with unknown dynamics. By devising the SSF, the singularity problem is effectively surmounted. Moreover, combing the approximation-based FLS and the backstepping technique, the AFFTC scheme was recursively established. With the aid of Lyapunov analysis, the devised AFFTC scheme can achieve the fast finite-time tracking effect and ensure the boundedness of all internal signals. In this context, all states are immeasurable, and then the output-feedback design scheme will be studied in further development.

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