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Let $\mathbf{n} : \mathbb{R}^m \to \mathbb{R}^m$ be the normalization function so that for a given vector $\mathbf{x} \in \mathbb{R}^m$ it outputs $\mathbf{y} = \mathbf{n}(\mathbf{x})$ with $||\mathbf{y}|| = 1$. The computation of each y_i can be done separately with:

$$\begin{array}{rcl} \mathbb{R}^m & \to & \mathbb{R}, \\ n_i(\mathbf{x}) : \mathbf{x} & \mapsto & \frac{x_i}{||\mathbf{x}||}. \end{array} \tag{1}$$

However, as you pointed out, a singularity may appear and a reformulation of n_i is suitable.
In a set-membership context, if $0 \notin [x_i]$, then n_i is equivalent to:

$$n_i^*([\mathbf{x}]) = \frac{\mathrm{sign}([x_i])}{\sqrt{1 + \frac{\sum_{j \neq i} [x_j]^2}{[x_i]^2}}}. \tag{2}$$

If $0 \in [x_i]$, then we can use the following generic formula:

$$n_i([\mathbf{x}]) = \begin{cases} [-1, 1] & \text{if } [x_i] = 0, \\ \dfrac{1}{\sqrt{1 + \dfrac{\sum_{j \neq i} [x_j]^2}{([x_i] \cap [0, \infty])^2}}} \cup \dfrac{-1}{\sqrt{1 + \dfrac{\sum_{j \neq i} [x_j]^2}{([x_i] \cap [-\infty, 0])^2}}} & \text{otherwise.} \end{cases} \tag{3}$$