Principal Component Analysis

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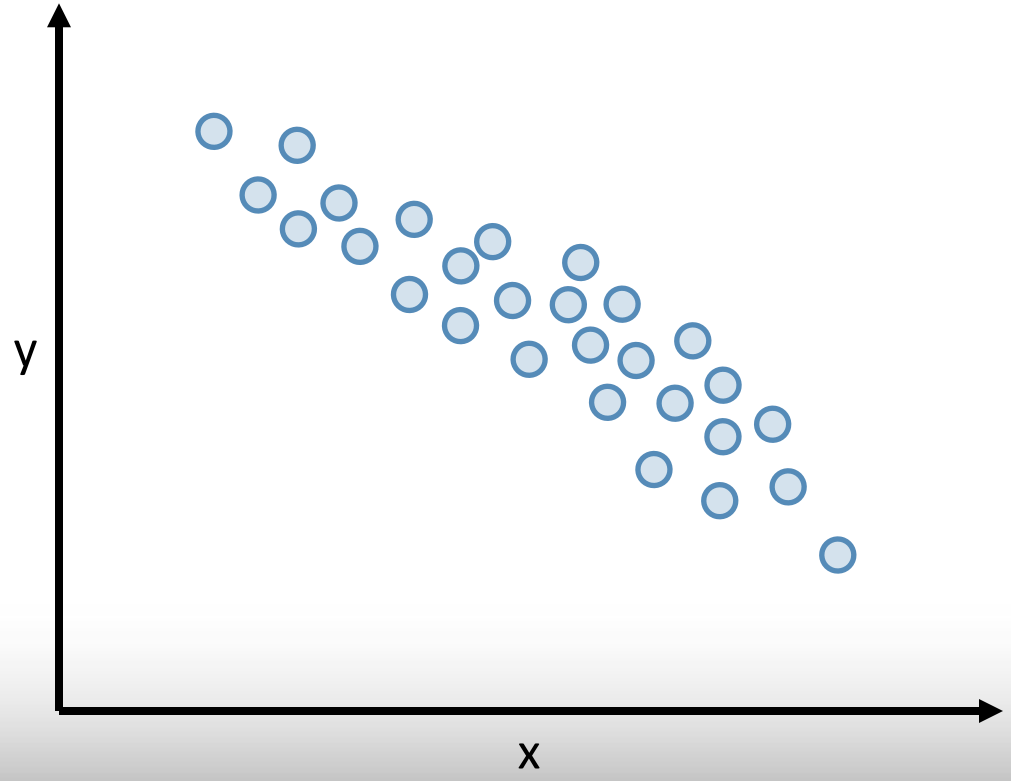
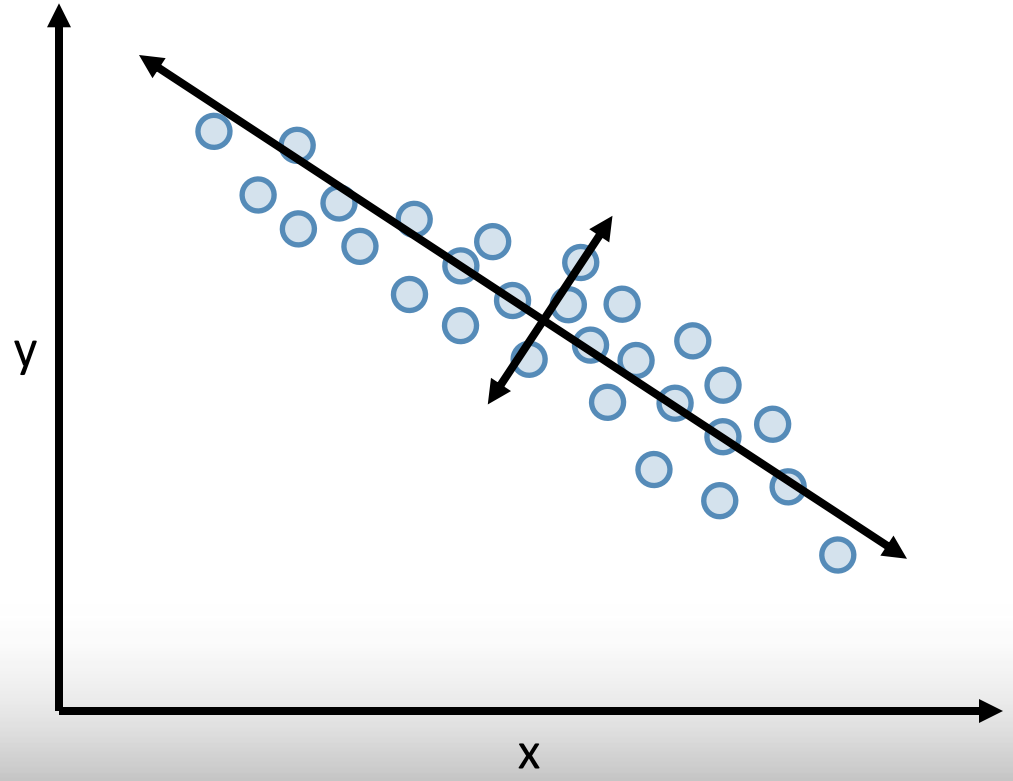
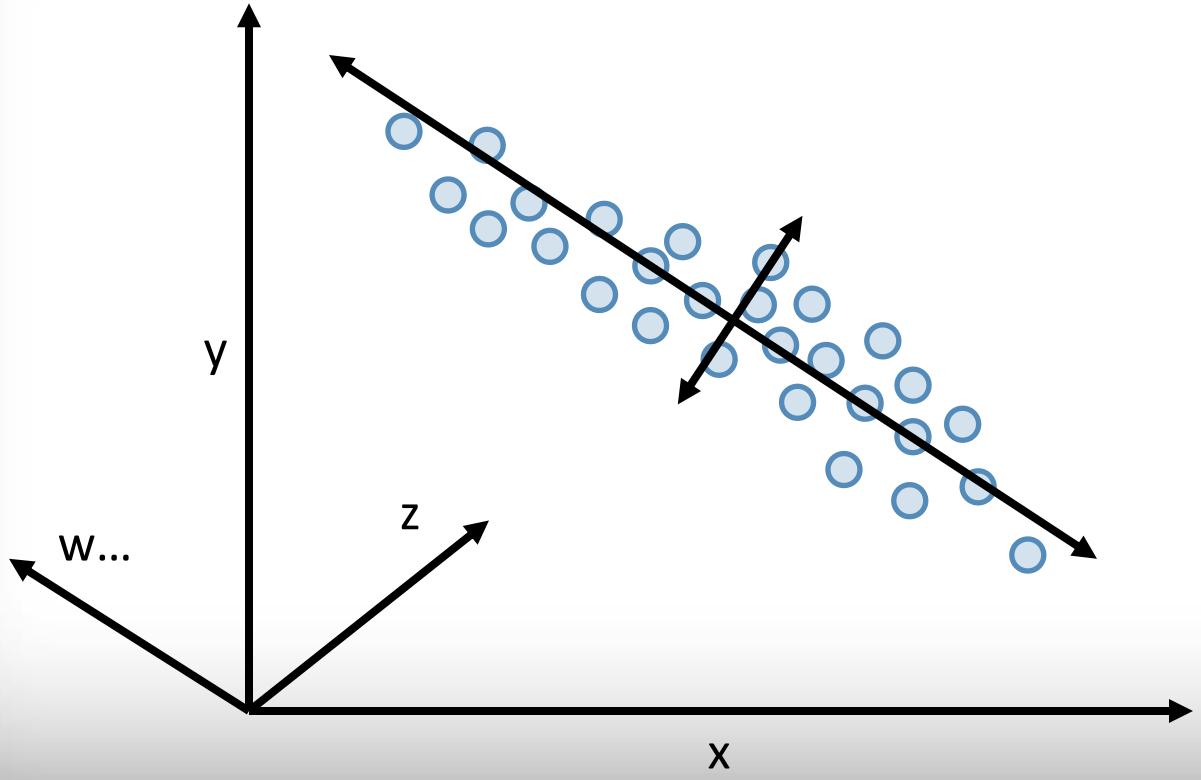
Data Variance



Data Variance



Data Variance



Covariance Matrix

data

$[x_1\ y_1\ z_1\ w_1\ \ldots]$
$[x_2\ y_2\ z_2\ w_2\ \ldots]$
$[x_3\ y_3\ z_3\ w_3\ \ldots]$
$[x_4\ y_4\ z_4\ w_4\ \ldots]$
$[x_5\ y_5\ z_5\ w_5\ \ldots]$
$[x_6\ y_6\ z_6\ w_6\ \ldots]$
$\ldots$
$[x_n\ y_n\ z_n\ w_n\ \ldots]$

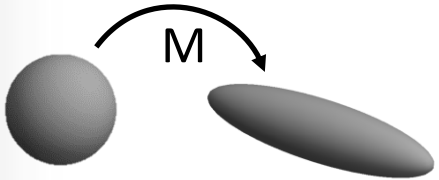
subtract mean

$- [x_m\ y_m\ z_m\ w_m\ \ldots]$

$$\begin{bmatrix} \frac{1}{n}\sum x_i x_i & \frac{1}{n}\sum x_i y_i & \frac{1}{n}\sum x_i z_i & \frac{1}{n}\sum x_i w_i & \cdots \\ \frac{1}{n}\sum y_i x_i & \frac{1}{n}\sum y_i y_i & \frac{1}{n}\sum y_i z_i & \frac{1}{n}\sum y_i w_i & \cdots \\ \frac{1}{n}\sum z_i x_i & \frac{1}{n}\sum z_i y_i & \frac{1}{n}\sum z_i z_i & \frac{1}{n}\sum z_i w_i & \cdots \\ \frac{1}{n}\sum w_i x_i & \frac{1}{n}\sum w_i y_i & \frac{1}{n}\sum w_i z_i & \frac{1}{n}\sum w_i w_i & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

Principal Component Analysis

Large matrix M transforms sphere into an ellipsoid



$A = M^T M$ product creates a square symmetric matrix

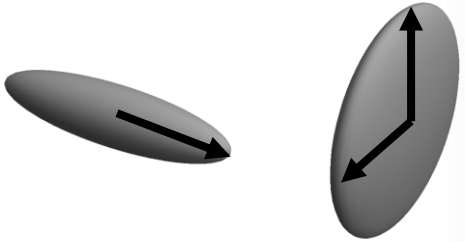
Eigenvector: **x**
Eigenvalue: λ

$$A\mathbf{x} = \lambda\mathbf{x}$$

Matrix A has same effect on vector **x** as multiplying it by scalar value λ

Eigenvalue represents how ellipsoid is stretched and squashed

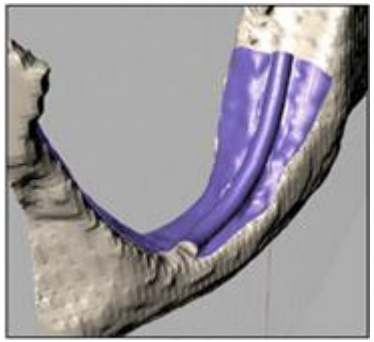
Eigenvector represents what direction the stretching and squashing occur in

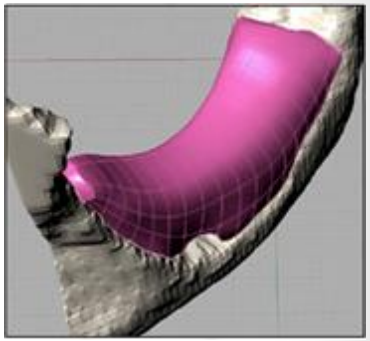


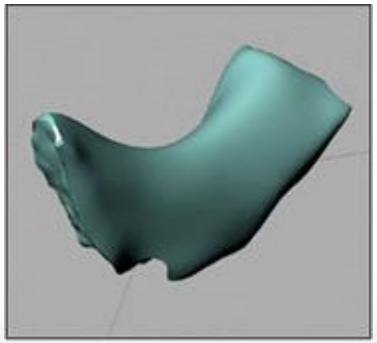
Principal Component Analysis keeps only the eigenvectors corresponding to largest magnitude eigenvalues

Bone Replacement

- Work with Dr. Michael Goldwasser, Russ Jamison, Amy Wagoner Johnson, Matei Stroila, and Ben Grosser
- Can print new bone material to replace damaged bone
- Need specification for new bone material
- Focused on the mandible

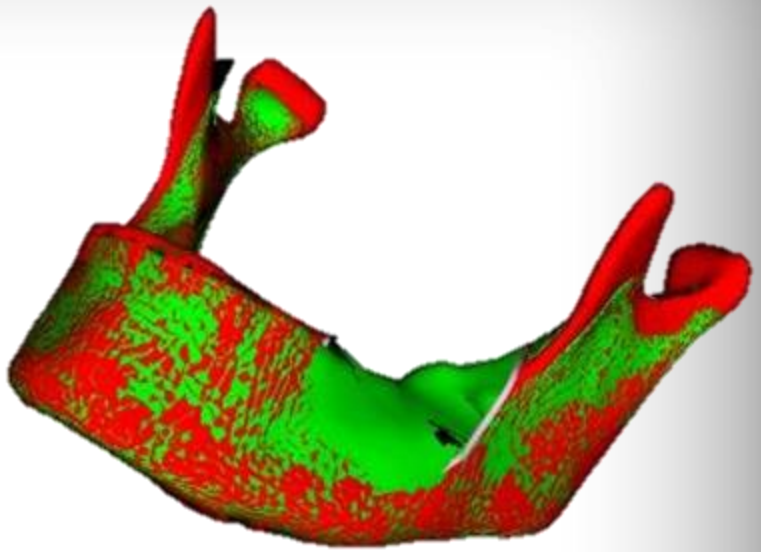






Mandible

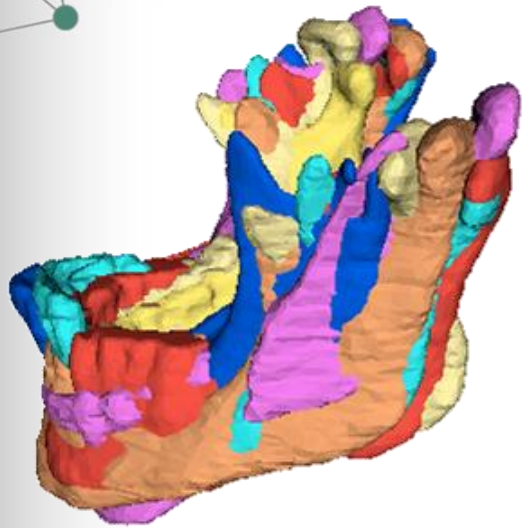
- Can find the shape of the missing bone
  - align the undamaged mandible to the damaged mandible
  - regularized subtraction
- What if we do not have the shape of the undamaged mandible?



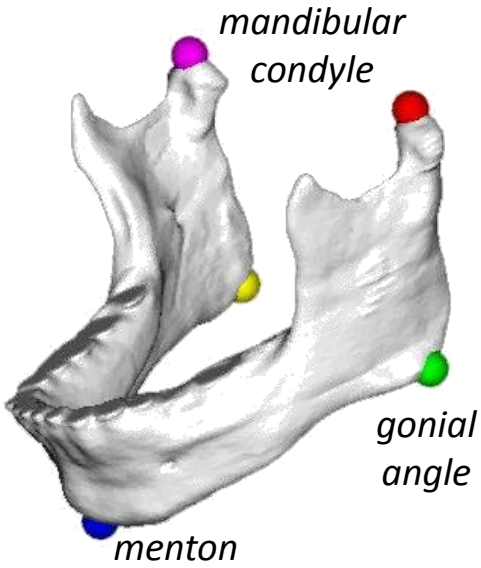
healthy mandible

damaged mandible

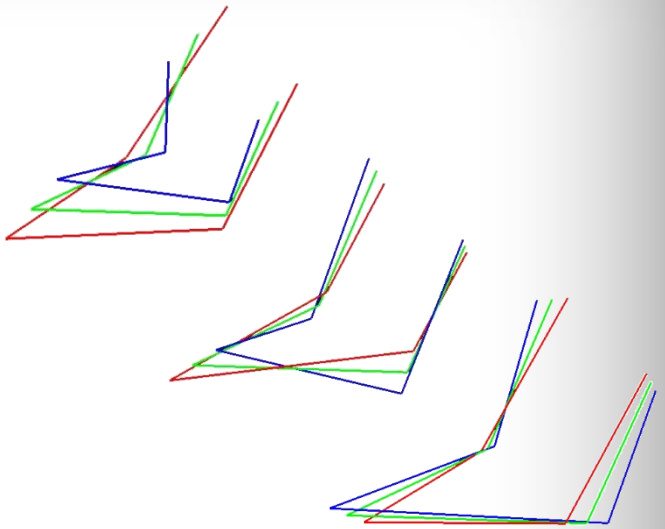
Mandible Variation



Composite from database of mandibles showing variation in shape



Five medical feature points used to measure mandible variation



Principal component analysis reveals three main variations

Space of Mandibles