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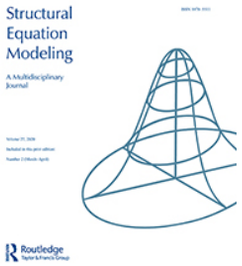

Structural Equation Modeling: A Multidisciplinary Journal

ISSN: 1070-5511 (Print) 1532-8007 (Online) Journal homepage: https://www.tandfonline.com/loi/hsem20

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To cite this article: Daniel McNeish & Tyler H. Matta (2020) Flexible Treatment of Time-Varying Covariates with Time Unstructured Data, Structural Equation Modeling: A Multidisciplinary Journal, 27:2, 298-317, DOI: 10.1080/10705511.2019.1627213

To link to this article: https://doi.org/10.1080/10705511.2019.1627213

Structural Equation Modeling: A Multidisciplinary Journal, 27: 298–317, 2020

ISSN: 1070-5511 print / 1532-8007 online
DOI: https://doi.org/10.1080/10705511.2019.1627213

Flexible Treatment of Time-Varying Covariates with Time Unstructured Data

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Time-varying covariates (TVCs) are a common component of growth models. Though mixed effect models (MEMs) and latent curve models (LCMs) are often seen as interchangeable, LCMs are generally more flexible for accommodating TVCs. Specifically, the standard MEM constrains the effect of TVCs across time-points whereas the typical LCM specification can estimate time-specific TVC effects, can include lagged TVC effects, or constrain some TVC effects based on theoretically appropriate phases. However, when data are time-unstructured, LCMs can have difficulty providing TVC effects whose interpretation aligns with typical research questions. This paper shows how MEMs can be adapted to yield TVC effects that mirror the flexibility of LCMs such that the model likelihoods are identical in ideal circumstances. We then extend this adaptation to the context of time-unstructured data where MEMs tend to be more flexible than LCMs. Examples and software code are provided to facilitate implementation of these methods.

Keywords: Growth model, time-varying covariate, time-unstructured data, unbalanced data, multilevel model, random effects model

Growth modeling has become a standard statistical technique for modeling change in repeated measures over time. For modeling growth, psychologists often resort to models under the umbrella of random effects models such that a mean growth trajectory is estimated for the entire sample but a unique growth curve is estimated for each individual in the data (Bauer & Sterba, 2011; Curran & Bauer, 2011; McNeish & Kelley, 2019). Models in this general family have many aliases but can be broadly grouped into two different classes of methods. The first general method is the latent curve model (LCMs) that treats the repeated measures as multivariate (also known as the "wide" data format) which tend to be fit in general SEM software (Meredith & Tisak, 1990; see Bollen, 2007 for a history of the developments in the SEM framework that lead to the LCM). The second general method is the mixed effect model (MEM; a.k.a. multilevel models) that treats the repeated measures as univariate (also known as the "long" data format) and are generally fit in regression software (Bryk & Raudenbush, 1987; Laird & Ware, 1982; Rao, 1965). Growth modeling with either of these approaches allows researchers to explore both between-individual variability that occurs across people and within-individual variability that occur within people (Curran & Hussong, 2003; Grimm & Stegmann, 2019; Harring & Blozis, 2016).

Covariates of between-individual variability tend to be more commonly included in growth models (though the trend is changing as idiographic intensive longitudinal methods steadily increase in popularity; Baird & Maxwell, 2016; Molenaar & Campbell, 2009). These covariates are variables that are invariant over time (or at least invariant over the observation window of the study) such as biological sex or race and ethnicity. These types of covariates are intuitively referred to as *time-invariant covariates* (TICs) and they explain why intercepts and slopes of individuals' growth curves differ. For instance, if biological sex is a meaningful TIC in a growth model, assuming a dichotomy, this means that males and females may start at different points if the TIC meaningfully predicts the intercept and/or that males

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and females change at different rates if the TIC meaningfully predicts the growth term(s).

Covariates of within-individual variability, on the other hand, are variables whose values are not constant at each time-point such as depression, smoking status or behavior, or employment status.[1] These types of covariates are deemed *time-varying covariates* (TVCs) and explain variability around an individual's growth trajectory. That is, each individual's growth curve is unlikely to perfectly capture their observed data at each time-point. As a result, there is a residual term at each time-point that captures how far each observed data point is from the individual-specific growth curve. This residual may be composed of unexplained aspects such as measurement error or random variability; however, it may also be composed of systematic influences (Bollen, 2002).

For example, if alcohol use was the outcome being modeled, variation around an individual's growth curve may be attributable to their depression at each time-point – when the level of depression is higher, observed alcohol use exceeds what is predicted by the model (i.e., a positive within-person residual) whereas lower levels of depression may lead to observed alcohol use that falls below levels predicted by the model (i.e., a negative within-person residual). Depression (which changes over time) can help to explain the variability around the individual growth trajectory in the outcome variable, alcohol use.

There has been considerable work to understand where the MEM and LCM are equivalent and where they are different (e.g., Bauer, 2003; Curran, 2003; McNeish & Matta, 2018). One area that has been considered to differentiate the two models is the treatment of TVCs. Curran and Bauer (2011) note, "the multilevel model and the latent curve model handle the incorporation of TVCs in a radically different way" (p. 615) and continue on to say, "Future work will do well to consider how the issues … are manifested within both modeling frameworks" (p. 615). By virtue of using the structural equation specification, LCMs have generally been considered more flexible and accommodating of TVCs than MEMs (McCoach & Kaniskan, 2010). However, in the current paper, we show that MEMs can be specified to model TVCs in the same way as LCMs, and under certain conditions related to data collection, the equivalent MEM is actually more flexible than the LCM with respect to estimating TVCs.

To outline the structure of the paper, we first provide didactic detail on the differences between TICs and TVCs in growth models. Then, we review the arguments suggesting why LCMs are considered more flexible in their treatment of TVCs. Next, we discuss the limitations of LCMs for TVCs when studies do not measure individuals at the same time-points. We then show how MEMs can be specified to provide equivalent TVC specifications as LCMs with strongly balanced data (i.e., all individuals have the same sets of points in time and there is no missing data). We then draw on an applied example with a challenging (but common) data structure to show the limitations of LCMs and conventional MEM specifications for TVCs. Finally, we show how MEMs can be specified to handle such data structures while flexibly treating TVCs. We return to the applied example to show how an MEM with such a specification can overcome the limitations of conventional approaches.

[1] Note that a single variable could be differentially treated as a TIC or TVC depending on how and for whom the data are collected. For instance, if Depression was collected only at baseline, it would be treated as a TIC because the value of this variable is constant within individuals. However, if Depression was assessed at each time-point, then it would be treated as a TVC because its values change within each person.

TICS VS. TVCS GRAPHICALLY

To concretize the difference between different types of covariates in growth models, consider a hypothetical plot of an unconditional linear growth model in Figure 1 using alcohol use as an example. Panel A shows the individual-specific growth curves and Panel B shows one individual's predicted growth curve along with their observed data points. In Panel A, there is between-individual variability in that the intercept values are different among the different growth curves (people have different alcohol use values at the first measurement occasion) and the slopes of lines are not parallel (the change in alcohol use differs across people). In Panel B, there is within-individual variability in that each observed data point does not fall directly on the predicted growth curve.

In growth models generally, both of these types of variability are unexplained in nature (similar to error variance in a single-level regression except that growth model variance is partitioned into two separate components). The unconditional model partitions the variability into different components (between-individual and within-individual) but the unconditional model does not give any indication *why* there are differences across individuals or why an individual's data deviate from their individual-specific growth curve. More plainly, in the unconditional model, the growth model can capture that between-individual differences exist but cannot attribute why these differences are present.

TICs can be added to the model to account for reasons why individuals differ. For instance, Panel C shows a conditional growth model where biological sex is included as a TIC (assuming that no one in the data changes sex during the course of the study) where the dashed lines are males and solid lines are females. Now, some between-individual variation that was previously unexplained can be accounted for – males generally start higher on alcohol use and their use grows faster over time compared to females. That is, knowing an individual's biological sex gives a better prediction for what their growth curve looks like.

The same idea applies to within-individual variability except that values of variables fluctuate over time and the variability that is explained is the variability *around* an individual growth curve rather than *between* individual's growth curves. Consider

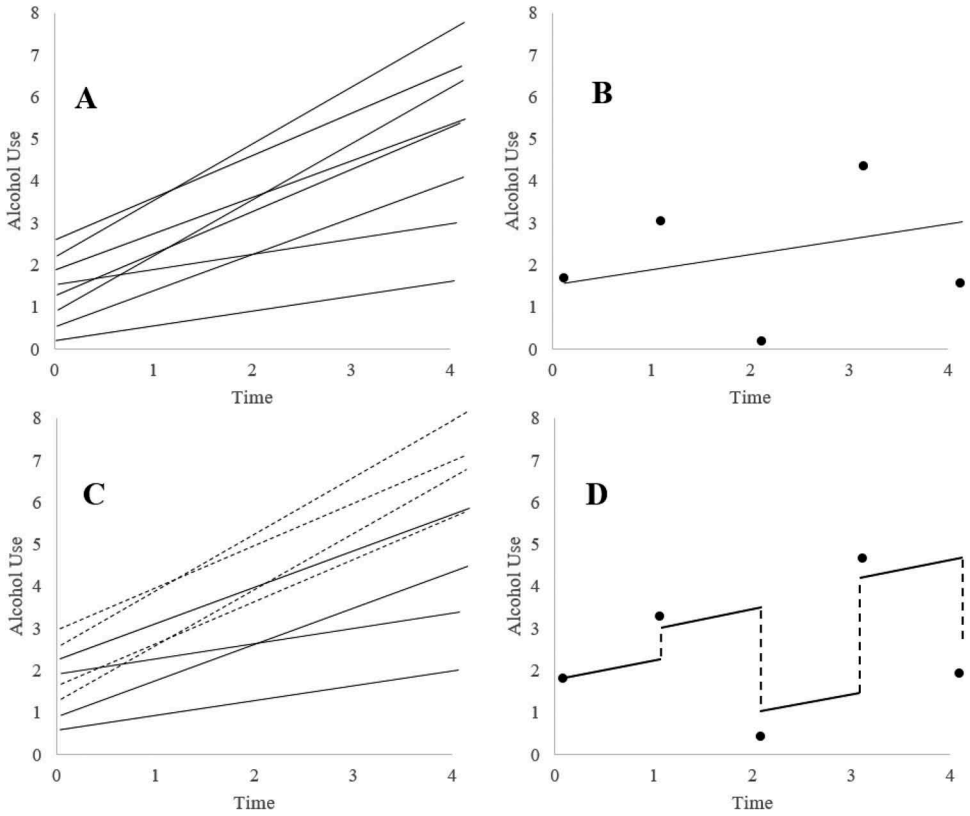

FIGURE 1 Graphical depiction of between-individual variability (panel A) and how time-invariant covariates explain between-individual variability (panel C); and within-individual variability (panel B) and how time-varying covariates explain within-individual variability (panel D).

a situation where depression is the TVC and the effect is positive (higher depression values predict higher alcohol use). Assume that the hypothetical individual in Panel D has relatively high depression values at Time 1 and 3 but their depression value is relatively low at Time 2 and 4. The result is that the prediction line is shifted up at Time 1 and 3 and shifted down at Time 2 and 4. That is, the prediction line is altered based on the time-varying value of depression – the prediction line is broken up in multiple discontinuous segments whose intercepts shift based on the value of the TVC. Note that the observed data points are closer to the prediction line in Panel D than in Panel B because the TVC is explaining the systematic within-individual variability (i.e., error variance in Panel B is explained by time-varying Depression in Panel D). Put into a substantive interpretation, knowing this individual's level of depression results in more accurate predictions about their alcohol use, above and beyond the information given their individual-specific growth curve.

DIFFERENT GROWTH MODEL SPECIFICATIONS

Overview of latent curve models

Latent curve models are confirmatory factor analysis (CFA) models within the SEM framework with an imposed mean structure and constraints to yield estimates of growth (Bollen & Curran, 2006; Meredith & Tisak, 1990). The LCM describes the growth trajectory for the entire sample through a set of factor variables (Chou, Bentler, & Pentz, 1998). In addition to the growth trajectory for the entire sample, the model allows each individual to have a unique growth curve. This subject-specific growth curve is defined by adding an individual's specific factor score to the overall mean trajectory, which is reflected by the factor means.

In SEM matrix notation, the LCM for growth is written as:

$$\begin{aligned}\mathbf{y}_i &= \mathbf{\Lambda}\boldsymbol{\eta}_i + \mathbf{B}\mathbf{z}_i + \boldsymbol{\varepsilon}_i \\ \boldsymbol{\eta}_i &= \boldsymbol{\alpha} + \mathbf{\Gamma}\mathbf{x}_i + \boldsymbol{\zeta}_i\end{aligned} \quad (1)$$

Assuming a balanced and *time-structured* design where all N individuals are measured on the same J occasions and q is the number of growth factors to be estimated, $\mathbf{y}_i$ is a $J \times 1$ vector of the outcome variable for the ith person (where $i = 1, 2, \ldots, N$), $\mathbf{\Lambda}$ is a $J \times q$ matrix of factor loadings that are often pre-specified to fit a specific type of growth trajectory and $\boldsymbol{\eta}_i$ is a $q \times 1$ vector of subject-specific growth factor scores for person i. The $J \times 1$ vector $\mathbf{z}_i$ contains the TVC values, while the $J \times J$ matrix $\mathbf{B}$ includes the associated coefficients for the effect of the TVC on the outcome $\mathbf{y}_i$. Finally, $\boldsymbol{\varepsilon}_i$ is a $J \times 1$ vector of

residuals for person i which are assumed distributed $MVN(\mathbf{0}, \mathbf{\Theta})$. The subject-specific growth factors, $\boldsymbol{\eta}_i$, are defined by a second equation. For each growth factor, there is a $q \times 1$vector of factor means defined as $\boldsymbol{\alpha}$. A $p \times 1$ vector $\mathbf{x}_i$ includes the time-invariant covariate values for person i (for p the number of time-invariant covariates) and $\mathbf{\Gamma}$ is the associated $q \times p$ matrix of coefficients. Finally, $\boldsymbol{\zeta}_i$ is a $q \times 1$ vector of subject-specific deviations from the factor mean for person i and is assumed distributed $MVN(\mathbf{0}, \Psi)$. In the conventional LCM, $\mathbf{\Lambda}$, $\mathbf{\Gamma}$, $\mathbf{B}$, and $\boldsymbol{\alpha}$ have no i subscript because their elements do not vary across individuals. As we note in later sections, methods have been developed in the literature to allow $\mathbf{\Lambda}$ to include an i subscript (e.g., Mehta & Neale, 2005).

TVCs in the latent curve models

Latent curve models treat the outcome as multivariate such that each unique time-point is a separate outcome in the model. Similarly, when a covariate is time varying, each measure is similarly treated as a separate variable (Muthén, 2002). Because each TVC is treated as a separate variable, every element of $\mathbf{B}$ can be estimated such that the TVC at each unique time-point can predict the outcome at any other time-point. This property is what gives credence to the commonly held notion that LCMs are more flexible for modeling TVCs (Muthén & Curran, 1997). To demonstrate, consider $\mathbf{B}$ from Equation 1. For a model of data with five repeated measures, $\mathbf{B}$ can be written as:

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} & \beta_{35} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & \beta_{45} \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} & \beta_{55} \end{bmatrix} \tag{2}$$

where the first subscript is the time point at which the outcome is collected and the second subscript is the time-point at which the TVC is collected (e.g., β_{32} would be the effect of the TVC at Time 2 on the outcome variable at Time 3). Researchers have full reign over which elements of $\mathbf{B}$ should be estimated, set to zero, or constrained to be equal to other parameters in $\mathbf{B}$. For instance, treating $\mathbf{B}$ as unstructured and estimating all 25 unique parameters may not be parsimonious, so it is more common to specify the model such that the effects are only estimated for congruent TVC and outcome variable pairs (i.e., only the diagonal elements of $\mathbf{B}$ are estimated; Grimm, Ram, & Estabrook, 2016, p. 183).

Overview of mixed effect models

The goal of MEMs is similar to that of LCMs – model the mean trajectory of the entire sample and permit subject-specific growth curves to be estimated for each individual (Grimm, Mazza, & Mazzocco, 2016). To accomplish this, MEMs estimate fixed regression coefficients to describe the overall trajectory in addition to random effects to describe subject-specific deviations from the overall trajectory (Laird & Ware, 1982; Stiratelli, Laird, & Ware, 1984). The two frameworks differ with respect to how time is included in the model. In an LCM, time is included by constraining the factor loadings within $\mathbf{\Lambda}$. In MEMs, time is explicitly included as a covariate in the model. This difference stems from how each model is formulated. Whereas the LCM treats repeated measures as multivariate so that each time-point is considered as a separate outcome, the MEM treats the outcome as univariate such that all repeated measures are treated as a single variable.

With continuous outcomes, the linear MEM can be written in matrix notation as:

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta} + \mathbf{Z}_i\mathbf{u}_i + \varepsilon_i, \tag{3}$$

where $\mathbf{y}_i$ is an $J_i \times 1$ vector of responses for individual i, J_i is the number of observations for individual i, $\mathbf{X}_i$ is an $J_i \times p$ design matrix for the covariates belonging to individual i, p is the number of covariates (which includes the intercept), $\boldsymbol{\beta}$ is a $p \times 1$ vector of fixed regression coefficients, $\mathbf{Z}_i$ is an $J_i \times r$ design matrix for the random effects associated with individual i, r is the number of random effects, $\mathbf{u}_i$ is a $r \times 1$ vector of random effects for individual i where $\mathbf{u}_i \sim MVN(\mathbf{0}, \mathbf{G})$, $\varepsilon_i$is a vector of residuals of the observations in individual i where $\varepsilon_i \sim MVN(\mathbf{0}, \mathbf{R}_i)$ and $Cov(\mathbf{u}_i, \varepsilon_i) = \mathbf{0}$.

The correspondence of the parameters in the MEM and the LCM can be directly mapped (Bauer, 2003). The factor means $\boldsymbol{\alpha}$ and TIC paths $\mathbf{\Gamma}$ in LCMs are related to fixed coefficients $\boldsymbol{\beta}$ in MEMs. Additionally, the LCM growth factor scores $\boldsymbol{\eta}_i$ are related to random effects $\mathbf{u}_i$ in MEM. As will be discussed, whereas LCMs possess separate structures for TVCs, the MEM simply appends the TVCs and their effects in the $\mathbf{X}_i$ matrix and $\boldsymbol{\beta}$ vector, respectively.

TVCs in the MEMs

Now consider a model where a TIC and TVC are included. Whereas LCMs are multivariate and reserve separate matrices to delineate TIC effects from TVCs, MEMs are univariate meaning that all effects, whether TIC or TVC, are combined into the same structure. That is, TIC variable values and TVC variable values both go into the fixed effect design matrix $\mathbf{X}_i$ and effects for both TICs and TVCs both go into the fixed effect coefficient vector $\boldsymbol{\beta}$. Equation 4 shows the expanded equations for a MEM that includes one TIC and one TVC for one hypothetical person with five repeated measures:

$$\begin{matrix} \mathbf{y}_i & = & & \mathbf{X}_i & & & \boldsymbol{\beta} & + & \mathbf{Z}_i & & \mathbf{u}_i & + & \varepsilon_i \end{matrix}$$
$$\begin{bmatrix} y_{1i} \\ y_{2i} \\ y_{3i} \\ y_{4i} \\ y_{5i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & TIC_i & TVC_{1i} \\ 1 & 1 & TIC_i & TVC_{2i} \\ 1 & 2 & TIC_i & TVC_{3i} \\ 1 & 3 & TIC_i & TVC_{4i} \\ 1 & 4 & TIC_i & TVC_{5i} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} u_{0i} \\ u_{1i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \\ \varepsilon_{5i} \end{bmatrix} \quad (4)$$

Because Equation 4 shows data for one hypothetical individual, the TIC value has a constant numeric subscript because it is invariant across time (i.e., there is no within-person variation) and only varies between individuals. The numeric subscripts for the TVC vary because the values change over time. The fixed effect coefficient vector has four elements – β_0 through β_3 – corresponding to the effect for the (a) initial status, (b) linear time, (c) the TIC, and (d) the TVC, respectively.

The vital point to note is that MEMs treat the outcome univariately, so the number of columns in $\mathbf{X}_i$ must match the number of rows in $\boldsymbol{\beta}$ for the matrix multiplication to function properly. However, with TVCs, this creates a potentially limiting constraint: namely, the associated matrix manipulations in the estimation process can only produce a single fixed effect coefficient for the TVC (Grimm, 2007). That is, by virtue of the outcome being univariate in MEMs, the effect of the TVC at each time-point will be is constrained to be equal to all other time-points and TVC effects are restricted to apply only to congruent time-points (Stoel, van Den Wittenboer, & Hox, 2003). That is, Equation 4 does not allow the TVC at Time 2 to predict the outcome at Time 3 nor does it allow the TVC effect at Time 2 to differ from the TVC effect at Time 3. This is in opposition to LCMs where the TVC effects occupy their own matrix and can be relaxed or constrained as the researcher sees fit and identification allows.

Though these restrictions may be reasonable in some contexts, they may be overly restrictive in others. Using the running hypothetical example of modeling the growth in alcohol use with depression as a TVC, a one-unit change in depression could conceivably affect alcohol use differently depending on when the data were collected. For example, perhaps higher levels of depression at age 18 leads to higher levels of alcohol use than at age 12 because 18 year-olds have easier access to alcohol or are more likely to have had experience using alcohol. The standard MEM specification defaults to a restricted model that does not allow for this possibility, unlike the LCM where constraints TVC effects are a conscious choice made by the researcher and the viability of constraints across time can be explicitly tested. This problem has not gone unnoticed and some methods have been advanced to more flexibly incorporate TVC effects that need not be constrained across time in MEMs.

Methods for allowing TVC effects to vary across time in MEMs

Interaction terms

An interaction term between time and the TVC can be included in the model to allow the TVC effect to change over time (McCoach & Kaniskan, 2010; Singer & Willett, 2003). Extending the example from Equation 4, the fixed portion of the model would be written as:

$$\begin{matrix} & & & \mathbf{X}_i & & & \boldsymbol{\beta} \end{matrix}$$
$$\begin{bmatrix} 1 & 0 & TIC_i & TVC_{1i} & 0 \times TVC_{1i} \\ 1 & 1 & TIC_i & TVC_{2i} & 1 \times TVC_{2i} \\ 1 & 2 & TIC_i & TVC_{3i} & 2 \times TVC_{3i} \\ 1 & 3 & TIC_i & TVC_{4i} & 3 \times TVC_{4i} \\ 1 & 4 & TIC_i & TVC_{5i} & 4 \times TVC_{5i} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \quad (5)$$

The last column in the design matrix is the multiplicative interaction between Time (the second column) and the TVC (the fourth column). By adding this column to the design matrix, another coefficient, β_4, is estimated to capture this interaction effect, which allows the TVC effect to vary as a linear function of time. Though less restrictive than constraining the effect to equality, such a method would operate under the rather strict assumption that the magnitude of the effect changes as a linear function of time. This assumption may not be an accurate characterization of the data in many circumstances. Singer and Willett (2003) extend this to illustrate how the TVC and its association with time can include random effects so that the effect varies by individual. Random effects for TVCs, however, allow the effect to differ for each *person* but not each *time-point*.

Dummy variables

A dummy variable method has been suggested as a possible method to obtain time-point specific TVC effects (Skrondal & Rabe-Hesketh, 2004, p. 98). Skibbe, Grimm, Bowles, and Morrison (2012) explored this method further; the idea is to implement a series of dummy variables to differentiate the measurement occasions and then include interaction terms for these dummy variables and the TVC. For example, with five measurement occasions, five dummy variables would be

created. The first dummy variable would be equal to 1 for the first occasion and 0 for all others, the second dummy variable would be equal to 1 for the second occasion and 0 for all others, and so forth. Using d to represent these dummy variables and continuing the hypothetical example of five measurement occasions, the fixed portion of the model would be:

The interaction of each dummy variable with the TVC is included but the main effect for the TVC is not (this allows each time-point to be directly estimated rather than using one time-point as a reference category, similar to LCMs). This yields a diagonal matrix for the TVC portion of the design matrix. The resulting fixed effect coefficient vector matches the number of columns of the fixed effect design matrix, which allows for additional fixed coefficients – β_3 through β_7 – to be estimated where β_3 would be the TVC effect at Time = 0, β_4 would be the TVC effect at Time = 1, and so on.

$$\begin{bmatrix} 1 & 0 & TIC_i & d_1 \times TVC_{1i} & 0 & 0 & 0 & 0 \\ 1 & 1 & TIC_i & 0 & d_2 \times TVC_{2i} & 0 & 0 & 0 \\ 1 & 2 & TIC_i & 0 & 0 & d_3 \times TVC_{3i} & 0 & 0 \\ 1 & 3 & TIC_i & 0 & 0 & 0 & d_4 \times TVC_{4i} & 0 \\ 1 & 4 & TIC_i & 0 & 0 & 0 & 0 & d_5 \times TVC_{5i} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \\ \beta_7 \end{bmatrix} \quad (6)$$

Grimm et al. (2016, p. 168) suggest that this approach is only effective when the data are balanced and structured (i.e., when all individuals have the same number of measurement occasions and when these occasions align at the same time-points for all individuals), contexts in which LCMs tend to be more flexible and favorable. These constraints on the treatment of TVCs in MEMs have generally led to the conclusion that flexible treatment of TVCs requires adopting LCMs to growth modeling (Grimm, 2007).

In the next section, we show how the dummy variable method with MEMs can be extended to more challenging models and more challenging data structures than previously thought, ultimately showing that the suggestions provided in Grimm, Ram et al. (2016) may not have incorporated the full potential of the dummy coding method. In the next section, we show how the dummy variable approach with MEMs can be used to yield identical results to LCMs when the study design is balanced and time-structured. Subsequently, we show how the dummy variable approach with MEMs can be extended to unbalanced and time-unstructured data, a common design where LCMs break down, which makes their flexibility for modeling TVCs inconsequential.

DESIGN MATRIX MANIPULATION VIA DUMMY VARIABLES

Demonstration of equivalence with balanced and time-structured data

We will use data from Curran (1997) which feature Peabody Individual Assessment test (PIAT; Dunn & Markwardt, 1970) reading scores for 221 children from a subset of the 1979 National Longitudinal Survey on Youth (NLSY). Time is coded both by wave number (which is balanced and time-structured) and as chronological time to the nearest year (which is semi-continuous and time-unstructured). To illustrate the equivalence of LCMs and the dummy variable TVC approach with MEMs when data are time-structured, we will first model the data using the wave number as the time metric. Each child has four waves and there is no missing data in this subset. The intercept and linear slope will each vary randomly and covary with each other. The residual variance is constrained to be equal across all waves. A measure of the child's Antisocial Behavior at each time-point is used as a TVC in the model. Full model equations are shown in Equation 9 below.

TVC centering

As noted in several recent studies, TVCs coefficients associated with the raw variable contain a mix of between-individual and within-individual information even though these two types of effects are conceptually and empirically different in nature (Curran & Bauer, 2011; Curran, Howard, Bainter, Lane, & McGinley, 2014; Hoffman & Stawski, 2009; Zhang & Wang, 2014). By including a single effect for the TVC, the coefficient necessarily contains a combination of these different components (Wang & Maxwell, 2015), which can lead to misleading or confounded results (Curran, Lee, Howard, Lane, & MacCallum, 2012).

Curran and Bauer (2011) recommend disaggregating the effect of TVCs into between-individual and within-individual components to avoid these potential pitfalls. The standard ways to disaggregate effects in growth models requires three steps: (a) compute the mean of Antisocial Behavior $\left(\overline{Anti}_i\right)$ for each individual across all time-points, (b) subtract the individual

Antisocial Behavior mean from the Antisocial Behavior value at each time-point, $\left(Anti_{ti} - \overline{Anti}_i\right)$, and (c) include the individual mean $\overline{Anti}_i$ as a TIC for the intercept in the model and the person-mean centered value $\left(Anti_{ti} - \overline{Anti}_i\right)$ as the TVC. However, because TVCs are potentially different at each time-point, the TVC itself could be subject to growth over time as well, which renders the standard approach problematic. If there is meaningful growth in the TVC over time, then properly centering the TVC becomes more complex, especially with time-unstructured data that are the focus of this paper and that are discussed shortly (Curran & Bauer, 2011).

To make a connection with disaggregation when there is no growth in the TVC over time, Curran and Bauer (2011) note that the three-step process for centering a TVC with no growth has an alternate specification. If the TVC was modeled as the outcome with a random intercepts model, the individual-specific intercept would be analogous to the individual mean (β_{0i}) and the within-individual error at each time-point (e_{ti}) would be the rescaled TVC value with the individual mean subtracted. To make this clearer using the NLSY data, such a random intercepts model for the TVC disaggregation would be:

$$\begin{aligned} Anti_{ti} &= \beta_{0i} + e_{ti} \\ \beta_{0i} &= \gamma_{00} + u_{0i} \end{aligned} \tag{7}$$

The individual mean for the TVC ($\overline{Anti}_i$) is captured by β_{0i} – the grand mean plus the individual-specific random intercept and the rescaled within-individual TVC ($Anti_{ti} - \overline{Anti}_i$) is captured by e_{ti}.

When there is growth in the TVC, this idea from Equation 7 can be extended by adding Time and a random slope. Again using the NLSY data as an example, the model for TVC disaggregation would be:

$$\begin{aligned} Anti_{ti} &= \beta_{0i} + \beta_{1i} Time_{ti} + e_{ti} \\ \beta_{0i} &= \gamma_{00} + u_{0i} \\ \beta_{1i} &= \gamma_{10} + u_{1i} \end{aligned} \tag{8}$$

The TVC mean for each person is similarly captured by β_{0i} and the rescaled within-individual TVC is also e_{ti}. The β_{0i} and e_{ti} values would similarly be used in place of $\overline{Anti}_i$ and $Anti_{ti} - \overline{Anti}_i$, respectively. However, the within-individual residuals e_{ti} now account for the fact that the TVC is growing over time (by conditioning on β_{1i}) .

Curran and Bauer (2011) note that this method is highly effective with time-structured data (p. 603) and, while there is modest bias with time-unstructured data, estimates of the focal model are vastly improved compared to the standard approach of centering around raw person means (p. 613). Note that effect partitioning in growth models is still an active area of research (e.g., Curran et al., 2014; Howard, 2015; Wang & Maxwell, 2015) and that latent centering (e.g., Lüdtke et al., 2008; Zitzmann, Lüdtke, Robitzsch, & Marsh, 2016) or corrective procedures (Gottfredson, 2019) to account for unreliability of person-means with few time-points may also be beneficial.

Analysis details and estimates

The MEM was fit with full maximum likelihood rather than restricted maximum likelihood in SAS Proc Mixed in order to match the estimation method used in M*plus*, which was used for LCMs. SAS and M*plus* data management and analysis code for fitting this model is provided in the Appendix (our data and code can also be found on the first author's Open Science Framework page, https://osf.io/2x3hm/).[2] In statistical notation, as an LCM, the model can be written as:

$$\begin{bmatrix} R_{1i} \\ R_{2i} \\ R_{3i} \\ R_{4i} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} \eta_{0i} \\ \eta_{1i} \end{bmatrix} + \begin{bmatrix} \beta_{11} & 0 & 0 & 0 \\ 0 & \beta_{22} & 0 & 0 \\ 0 & 0 & \beta_{33} & 0 \\ 0 & 0 & 0 & \beta_{44} \end{bmatrix} \begin{bmatrix} Anti^c_{1i} \\ Anti^c_{2i} \\ Anti^c_{3i} \\ Anti^c_{4i} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{bmatrix}$$
$$\begin{bmatrix} \eta_{0i} \\ \eta_{1i} \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} + \begin{bmatrix} \gamma_{01} \\ 0 \end{bmatrix} \left[\overline{Anti}_i\right] + \begin{bmatrix} \zeta_{0i} \\ \zeta_{1i} \end{bmatrix} \tag{9}$$

where R denotes reading scores, $Anti^c$ denotes antisocial behavior scores centered based on Equation 8, $\overline{Anti}_i$ is the antisocial behavior person-mean based on Equation 8, $\zeta_i \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \psi_{00} & \\ \psi_{10} & \psi_{11} \end{bmatrix}\right)$, and $\varepsilon_i \sim N(\mathbf{0}_4,$

[2] The MEMs discussed are not limited to SAS, but can be estimated in any MEM software. Mplus is used because, to the authors' knowledge, it is currently the only LCM software that permits the definition variable method discussed in one of our subsequent examples.

TABLE 1
Estimates for the NLSY Reading Data Using Wave-Based Time Metric

	MEM		*LCM*	
Parameters	*Estimate*	p-*value*	*Estimate*	p-*value*
Intercept	2.72	<.01	2.72	<.01
Slope	1.08	<.01	1.08	<.01
Antisocial Wave0	.05	.22	.05	.22
Antisocial, Wave1	.03	.35	.03	.35
Antisocial, Wave2	−.04	.30	−.04	.30
Antisocial, Wave3	.08	.13	.08	.13
Antisocial Mean	−.60	.08	−.60	.08
Covariance Parameters				
Var (Int)	.51		.51	
Var(Slope)	.07		.07	
Cov (Int, Slope)	.03		.03	
Residual Var.	.31		.31	
Model Fit				
Deviance	2112.3		2112.3	
AIC	2134.3		2134.3	
BIC	2171.7		2171.7	

diag[θ θ θ θ]). The model can similarly be specified as a MEM based upon Equation 6. Table 1 shows the estimates and *p*-values from each model type. Notably, the estimates are exactly the same (and remain so for more decimal points than we reported) and the likelihoods and *p*-values are identical. The following subsections show how some common but more advanced TVC effect specifications can be adapted to MEMs using dummy variables.

Lagged TVC effects

In addition to only regressing the TVC on the outcome that was collected at the same occasion (e.g., antisocial behavior at wave 1 predicts reading at wave 1), it may also be of interest to regress the TVC at the current time *and* the lag-1 TVC (e.g., predicting reading at wave 2 from antisocial behavior at wave 2 *and* wave 1). Using the NLSY data again with wave as the time metric, this model with lagged TVC effects can be accommodated easily in an LCM when the data are time-structured by virtue of the model's multivariate specification. Figure 2 shows a path diagram for how the model in Equation 9 would be extended to include Lagged TVC effects. Statistically, this simply involves freeing specific off-diagonal elements of **B** in the LCM equations (as shown in Equation 1). In Equation 9, this is equivalent to estimating the first diagonal below the main diagonal.

Grimm, Ram et al. (2016) mention that this type of model cannot generally be fit as a MEM because of concerns with missing values.[3] That is, because of the univariate data structure

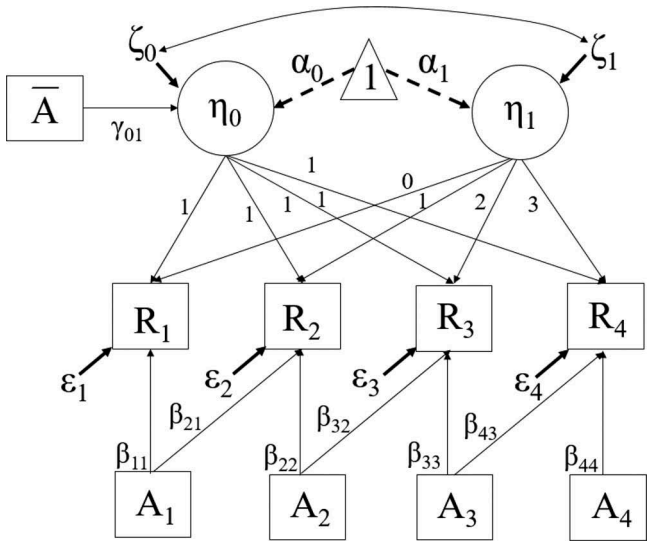

FIGURE 2 Latent curve model path diagram for NLSY data. "*R*" variables are Reading Ability, "*A*" variables are person-centered Antisocial Behavior, and $\bar{A}$ is the person mean of Antisocial Behavior. Numeric values indicate parameters that are constrained. The variance of ε_1 through ε_4 is constrained to be equal. Not pictured are exogenous covariances between all pairs of *A* variables and possible intercepts for A_1 through A_4. β_{21}, β_{32}, and β_{43} are the Lag-1 effects of the TVC.

used by MEMs, the lag-1 column will necessarily be missing for each person for the first measurement occasion. By convention in software used to fit MEMs, this value enters the data as a missing value because it precedes the observation window (i.e., the value of the variable at the previous time-point is not known for the first occasion). Because MEMs use conditional likelihood instead of joint likelihood, missing covariates cannot be accommodated (Allison, 2012). This is in opposition to LCMs and the multivariate structure because there is simply no lag-1 effect specified in the model because no lag-1 TVC variable exists (i.e., in Figure 2, there is simply no lag-1 effect for the first measurement occasion). The differential data structure resolves the issue differently: the lag-1 path for the first measurement occasion is missing from an LCM rather than the data point being missing as in a MEM.

However, an undesirable effect of the conditional likelihood and the univariate structure with MEMs is that all observations for the first measurement occasion would be listwise deleted because the lag-1 TVC value for the first measurement is considered to be missing. However, all lag-1 TVC values must appear in the same column of the design matrix. To demonstrate the point of Grimm, Ram et al. (2016), model with lag-1 TVC effects using software

[3] It is important to note that missing values are a prevalent concern for growth modeling in general, and if a TVC value is missing because the data were not provided rather than because of the structure of the data, then the concerns of Grimm, Ram et al. (2016) are warranted because MEMs will end up listwise deleting in such contexts. Alternatively, a multiple imputation approach could be employed.

Number of observations	
Number of observations read	884
Number of observations used	663
Number of observations not used	221

for MEMs like SAS Proc Mixed using the dummy code approach, the output for the NLSY data reports the following table:

Note that 221 observations are not used and have been listwise deleted – this equates to the first measurement occasion for all 221 individuals in the NLSY data.

Despite this seeming weakness of MEMs and the logical rationale behind why this model tends to fail with MEMs, the design matrix can be "tricked" into keeping all observations in the data even in the presence of missing lag-1 information at the first measurement occasion. Rather than set the lag-1 variable to missing for the first measurement occasion, the value can be set to 0 for each person in the data. This change functions solely to retain these observations without affecting the model estimates. If each person has a 0 value at the first measurement occasion, when dummy codes are added to allow time-varying TVC effects, each person will have all 0 values for the entire column of the design matrix.

For example, consider ID = 34 from the data and their design matrix shown in Equation 10. The underlined zero value would normally be a missing value because it occurred prior to the observations window, but now it has been set to zero. If each person has a 0 value at wave = 0, when dummy codes are added to allow time-varying TVC effects, each person will have all 0 values for the entire $lag-1Anti \times d_1$ column. Therefore, the $lag-1Anti \times d_1$ column will have no variability either within or across individuals, which makes the effect inestimable. However, the matrices are not read as having missing values, so the first measurement occasion is retained, and the lag-1 TVC effect is simply inestimable rather than the first measurement occasions being listwise deleted if the value was treated as missing:
A comparison of the LCM and MEM model with dummy codes for the NLSY data with lag-1 TVC effects at each time-point is shown in Table 2. Note that the MEM retains all observations and that the estimates and likelihoods are identical with what is provided by the LCM, despite cautions in the previous literature that the MEM version of this model would encounter missing data issues.

TABLE 2
Estimates for the NLSY Reading Data Using the Wave-Based Time Metric and Lag-1 Effects

	MEM		*LCM*	
Parameters	*Estimate*	p-*value*	*Estimate*	p-*value*
Intercept	2.72	<.01	2.72	<.01
Slope	1.08	<.01	1.08	<.01
Antisocial Wave0	.08	.10	.08	.10
Antisocial Wave1	.06	.15	.06	.15
Antisocial Wave2	−.06	.27	−.06	.27
Antisocial Wave3	.05	.43	.05	.43
Antisocial Wave1, Lag1	.05	.23	.05	.23
Antisocial Wave2, Lag1	.04	.38	.04	.38
Antisocial Wave3, Lag1	−.05	.42	−.05	.42
Antisocial Mean	−.82	.03	−.82	.03
Covariance Parameters				
Var (Int)	.52		.52	
Var(Slope)	.07		.07	
Cov (Int, Slope)	.03		.03	
Residual Var.	.31		.31	
Model Fit				
Deviance	2109.9		2109.9	
AIC	2137.9		2137.9	
BIC	2185.5		2185.5	

Phase-varying TVC effects

In some cases, it may not be parsimonious to estimate TVC effects for all time-points but instead may make more sense to constrain the TVC effects for some time-points but not others. For example, for a study that follows student's alcohol use from Grade 6 to Grade 12, perhaps the effect of a TVC like depression is different in high school than in middle school. However, the TVC effect may not necessarily be different across grades within the same school type. That is, maybe the TVC effect can be constrained to one value for grades 6 through 8 and constrained to another value for grades 9 through 12 but estimating the TVC effect at each individual time-point is unnecessary. This allows the TVC to vary for different substantively relevant phases of the data but retains some parsimony by applying equality constraints to TVC effects within a phase.

As has been the recurrent theme so far, phase-varying TVC effects are rather simple to estimate as an LCM when the data are time-structured. All that is required is to place a constraint on the TVC effects within each phase. Using the NLSY data again an example, imagine that the first wave belongs to one phase and the last three waves belong to a second phase. The model would be identical to the model that produced the results in Table 1 except that β_{11} would be freely estimated whereas β_{22}, β_{33}, and β_{44} would be constrained to be equal to each other (but not necessarily the same value as β_{11}).

Phase-varying TVCs can also be implemented easily with MEMs using dummy variables. Unlike previous examples where the TVC portion of the fixed effect design matrix took a diagonal structure, phase-varying TVC effects include off-diagonal elements because TVC effects at multiple time-points are constrained to be equal (recall that if all TVC effects are constrained to the same value, then only one column is needed in the design matrix). Using the NLSY data as an example and imagining that

Wave 0 is in Phase 1 and Waves 1 through 3 are in Phase 2, the fixed effect design matrix for ID = 34 used in Equation 10 would be written as:

$$\begin{bmatrix} Int. & Wave & Anti \times Phase\,1 & Anti \times Phase\,2 \\ 1 & 0 & 3 & 0 \\ 1 & 1 & 0 & 6 \\ 1 & 2 & 0 & 4 \\ 1 & 3 & 0 & 5 \end{bmatrix} \quad (11)$$

With this setup, only 2 unique fixed effect coefficients are estimated ($Anti \times Phase\,1$and $Anti \times Phase\,2$) rather than the four that would be needed if TVC effects were estimated at each time-point. Because of the relatively straightforward setup of this model and its similarity to previous models, we will not provide a table of results (though code is included in the Appendix).

DIFFERENCES IN IMPLEMENTATION OF MEMS AND LCMS WHEN TIME IS UNSTRUCTURED

Research designs with repeated measures are often considered as multi-wave, time-structured design with each individual providing data at the same occasions. In empirical data, the design regularly deviates from being time-structured (Blozis & Cho, 2008; Coulombe, Selig, & Delaney, 2016; McCoach, Rambo, & Welsh, 2013; Mehta & Neale, 2005; Mehta & West, 2000; Sterba, 2014). Such *time-unstructured* data can be especially common when time is denoted as chronological age (especially in very young individuals where developmental milestones are spaced closely and age may be recorded in smaller units such as months or weeks). When time is continuous, coarsening it into discretized intervals in order to force measurement occasions to be equivalent for all individuals can have adverse effects on model estimates (Aydin, Leite, & Algina, 2014; Singer & Willett, 2003). The magnitude of the effect is dependent on the degree of the coarsening and the variability in the spacing of the measurement occasions (Coulombe et al., 2016; Singer & Willett, 2003).

Because time is an explicit covariate in MEMs, varying measurement occasions and varying numbers of measurement occasions are not inherently problematic (McCoach et al., 2013; Singer & Willett, 2003; Sterba, 2014). That is, both **X** and **Z** in Equation 4 have an *i* subscript indicating that each individual can possess unique values for time and that these observed time points can vary. For instance, in Equation 5, we used integers from 0 to 4 in the second column which represented time (as a wave). However, time is technically a TVC itself in MEMs and can change for every individual. A different individual's fixed effect design matrix for the model in Equation 4 might be $\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 5 \\ 1 & 8 \end{bmatrix}$ if they were measured at times 0, 2, 3, 5, and 8 instead of at waves 0, 1, 2, 3, and 4 (i.e., the time of the observation can be used instead of the wave of the measurement occasion). The MEM has no issue with estimating the model despite the difference in the timing of data collection. Similarly, another individual may have a fixed effect design matrix of $\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$ if they only provided data at 3 measurement occasions. Again, the MEM does not have any issue estimating parameters from data where individuals have different numbers of occasions. In fact, either design matrix in Equation 4 could be generalized by replacing the scalar values with t_{ji} to represent the *j*th observation for the *i*th individual (with J_i being the total number of observations, which can vary by individuals):

$$\begin{bmatrix} 1 & t_{1i} \\ 1 & t_{2i} \\ \vdots & \vdots \\ 1 & t_{J_i i} \end{bmatrix} \quad (12)$$

On the other hand, the multivariate nature of LCMs means that each time-point is a separate outcome in the model and, practically, requires a unique column in the data. That is, $\mathbf{\Lambda}$ in Equation 1 does not have an *i* subscript, indicating the number and magnitude of the loadings are not permitted to change between people. All individuals are therefore assumed to have an equal number of measurement occasions and each individual provides data at the same time-points. Because time is evoked via constraints on loadings in LCMs rather than as an explicit variable in the model as in MEMs, LCMs tend to be far less flexible in the approaches to accommodate data that deviate from a design that is balanced and time-structured.

Possible options exist for handling time-unstructured data in LCMs. One such method is to include a unique column for each possible time-point and treating the estimation as a missing data problem (Wu, West, & Taylor, 2009). That is, rather than adjusting the number of rows to accommodate time-unstructured data, each possible time point would occupy its own column with many columns being missing values for each person. Consider the earlier example where one person had 5 data points occurring at times 0, 2, 3, 5, and 8, and a second person having 3 data points at 0, 1, and 2. The missing data method would create unique columns for times 0, 1, 2, 3, 5, and 8: the first person would have 5 observed values (0, 2, 3, 5, 8) and 1 missing value (1) whereas the second person would have 3 observed values (0, 1, 2) and 3 missing values (3, 5, 8). As the sample size and the

diversity of measurement occasions increases, the number of unique columns also increases along with the amount of missing data. Ultimately, the method can become intractable if there is too little overlap in the measurement occasions because, unlike MEMs, the missing data method increases the dimensions of the data matrix without increasing the amount of data, leading to potential sparsity issues.

A second method is definition variables, which allow the value of factor loadings to vary by person (a.k.a., the TSCORES option in M*plus*; Mehta & Neale, 2005; Mehta & West, 2000). Though the definition variable method can be effective in accounting for time-unstructured data, it is not identical to the individual-specific design matrices used by MEMs to circumvent the issues presented when data are time-unstructured and may not always provide TVC effects that yield an interpretation consistent with the research question of interest. We elaborate more on the details of this method in the next section.

Instead of using wave-based time with the NLSY data, the subsequent section will use age at data collection as the time metric, which tends to be preferable to account for possible age heterogeneity within waves (e.g., Mehta & West, 2000). This will demonstrate that methods for accommodating time-unstructured data in LCMs are not always effective with TVC effects that are specific to particular time-points or phases. We continue to show how the dummy variable approach in MEMs naturally extends to the context of unbalanced and time-unstructured data.

LCM with definition variable method

As noted previously, LCMs can accommodate highly time-unstructured data with the definition variable method (sometimes referred to by its M*plus* code name, TSCORES). Using the NLSY data with age as the time metric we will show that the results do not quite answer the research question at hand. In the NLSY, children were assessed four times at 9 possible ages (6 through 14, rounded to the nearest year), so while every child was assessed at each of the four waves, they were not assessed at each age.

Figure 3 shows a path diagram of the model with definition variables. The major difference is the values of the loadings from the slope latent variable to the reading scores. Unlike a standard LCM or the path diagram in Figure 2, the loadings are no longer constrained to specific values. Instead, the loadings are now variables (denoted within a diamond, following the precedent from Mehta & West, 2000) with person-specific *i* subscripts to allow the value of the loadings to vary by person and therefore represent differing times at which people were observed. In other words, the values of each of the *T* variables change based on when people were observed, which is coded as a variable in the data.

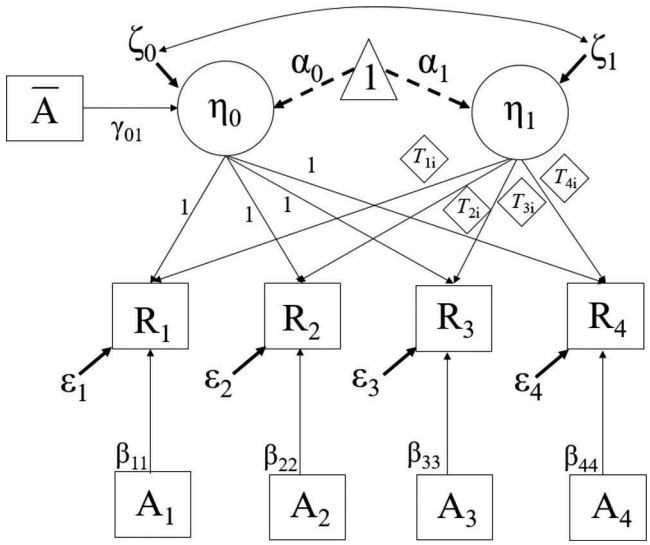

FIGURE 3 Latent curve model path diagram with definition variables for NLSY data. "*R*" variables are Reading Ability, "*A*" variables are person-centered Antisocial Behavior, and $\bar{A}$ is the person mean of Antisocial Behavior. Numeric values indicate parameters that are constrained. The *"T"* variables in diamonds are the definition variables that represent the time at which people were observed and vary for each person given the *i* subscript. The variance of ε_1 through ε_4 is constrained to be equal. Not pictured are exogenous covariances between all pairs of *A* variables and possible intercepts for A_1 through A_4. β_{21}, β_{32}, and β_{43} are the Lag-1 effects of the TVC.

Table 3 shows the results of the LCM using definition variables to accommodate the varying measurement occasions. Notably, the definition variable approach accounts for ages at different measurement occasions, as noted by the different slopes estimate (waves were given about every 2 years, so the slope estimate in Table 3 is half of the estimates in Tables 1 and 2 because the unit is now year instead of two years with wave). However, the model gives four estimates for the effect of antisocial behavior even though there are 9 possible ages. So although the definition variable approach can account for different time points in the growth trajectory, it does *not* give the TVC effects at each time-point.

If researchers are interested in the TVC effect at specific time-points but data are time-unstructured, LCMs – even with the definition variable approach – do not provide this information. Due to LCMs' multivariate specification, the TVC effect – even with the definition variable method – continues to treat the data as if they are wave-based. Definition variables allow the timing of the four waves to be individual-specific, but they do not specify TVC effects for specific times of observation nor do they address scenarios where the number of occasions differs or scenarios where the same time can belong to different waves depending on the individual. For instance, it is not clear what the effect of antisocial behavior on reading is for 7-year-old children in this analysis. A 7-year-old child is likely in the first

TABLE 3
Estimates from LCM Using Definition Variable Method

Parameter	*Estimate*	*p-value*
Mean Structure		
Intercept	2.23	<.01
Slope	.54	<.01
Antisocial, Time 0	.04	.39
Antisocial, Time 1	.04	.22
Antisocial, Time 2	−.05	.26
Antisocial, Time 3	.09	.11
Antisocial Mean	−.71	.03
Covariance Structure		
Var(Int)	.20	
Var(Slope)	.02	
Cov(Int, Slope)	.03	
Var(Residual)	.32	
Model Fit		
Deviance	2042.3	
No. Parameters	11	
BIC	2101.7	

wave, but so are 6-year-old children and some 8-year-old children, so the specific antisocial behavior effect for 7-year-old children cannot be parsed out using this approach in the LCM framework. Put in another way, the first antisocial behavior estimate is the effect at the first time-point, *whenever* that data were collected (6, 7, or 8 years old) but the specific effect at one of those ages cannot be discerned. As we discuss in the next section, the dummy variable approach can be adapted to obtain time-point specific TVC effects with time-unstructured data.

EXTENDING THE DUMMY VARIABLE APPROACH BEYOND BALANCED DATA

We now consider using the dummy variable approach in MEMs to accommodate TVC effects for specific time-points when data are time-unstructured, which has previously been considered infeasible. We compare three models in this section for the time-unstructured NLSY data with age as the time metric: a model with time-specific TVC effects of antisocial behavior for each observed age; a three-phase TVC model for with effects constrained for Age <8 years (Phase 1), Age between 8 and 13 (Phase 2), and Age >13 (Phase 3); and the standard MEM TVC model that constrains TVC effect to be constant across all times.

Table 4 reports the fixed effect and covariance parameter estimates as well as the deviance and BIC for each model of these models. Unlike the definition variable LCM, the time-varying TVC model in the middle of Table 4 yields TVC effects that vary at each of the possible time-points rather than by measurement occasions. Also, note that the deviance of this model is different than the deviance of the definition variable model in Table 3, providing further evidence that the definition variable method is not providing time-point specific information about TVC effect. That is, the MEM approach allows one to specifically inspect the effect of antisocial behavior for 7-year-old children, which is .09 points for each one-unit increase above the person's average antisocial behavior. In the LCM with definition variables, the antisocial behavior effect for a 7-year-old child was not directly estimated. Additionally, the unstructured nature of the data does not cause any issues with respect to missing data – as noted by the SAS output, all observations were included in the analysis.

The ability to estimate each of these effects affords researchers the opportunity to statistically test whether the TVC effects should be constrained, freely estimated, or estimated in phases with time-unstructured data as is standard practice in LCMs when data are time-structured. A likelihood ratio test shows that estimating a TVC effect at each time-point does not fit better than constraining the TVC effect across time[$\chi^2(7) = 7.4, p = .39$] or estimating a three-phase TVC model [$\chi^2(5) = 1.3, p = .93$]. The three-phase TVC model fits significantly better than the constrained TVC model [$\chi^2(2) = 6.1, p = .047$]. If the BIC was used instead, the model that constrains the TVC effects (BIC = 2089.4) would be preferred over the three-phase model (BIC = 2094.1). The disagreement between methods likely arises from close proximity of the likelihood ratio test the .05 cut-off in this example. Nonetheless, the point is that flexibility of the MEM and univariate structure allows these models to be fit and comparisons about the behavior TVCs at different times to be evaluated in ways that LCMs cannot accommodate.

DISCUSSION

Recommendation for phase-varying TVC effects

As the scale of the time metric becomes more precise and increasingly continuous, each person will likely have more idiosyncratic times of data collection. For example, if the time metric was age to the exact day of primary school children, there would be very little overlap in the measurement occasions in the sample. As a consequence, specifying time-point specific TVC effects will divide the data into smaller and smaller pieces as time becomes increasingly continuous. The resulting estimates and standard errors will, therefore, be based on smaller and smaller groups of people, which can limit researchers' ability to find meaningful effects and can lead to anomalous estimates influenced by a single data point. Although time-varying TVC effects can be estimated when data are time-unstructured, the interpretation of these effects becomes less and less meaningful as data approach a continuous-time design where no measurement occasion is shared by any two individuals (i.e., there may be little substantive meaning between the effect of antisocial behavior for a child who is 6 years, 3 months compared to 6 years, 4 months).

TABLE 4
Comparison of Estimates and Fit from ME Model with Time-Specific TVC Effects, Constrained TVC Effects, and a Three-Phase TVC Effect

	Constrained TVC			*Varying TVC*			*Three-Phase TVC*	
Parameter	*Estimate*	p-*value*	*Age*	*Estimate*	p-*value*	*Phase*	*Estimate*	p-*value*
Fixed Effects								
Intercept	2.23	<.01		2.23	<.01		2.23	<.01
Slope	.54	<.01		.54	<.01		.55	<.01
Antisocial Mean	−.61	.03		−.76	.01		−.75	.01
Antisocial TVC	.03	.18		–	–		–	–
			6	.07	.31	1	.09	.07
			7	.09	.10			
			8	.00	.97	2	.00	.93
			9	.02	.67			
			10	−.01	.86			
			11	−.07	.39			
			12	.01	.89			
			13	.06	.47			
			14	.23	.03	3	.23	.01
Covariance Parameters								
Var(Int)	.21			.21			.21	
Var(Slope)	.02			.03			.02	
Cov(Int, Slopes)	.03			.02			.03	
Residual Var.	.32			.31			.31	
Model Fit								
Deviance	2046.2			2038.8			2040.1	
No. Parameters	8			15			10	
BIC	2089.4			2125.1			2094.1	

Using two dummy codes, lag-1 effects with time-unstructured data can also be included in MEMs. With lag-1 effects in time-unstructured data, an added complication occurs because there are potentially multiple lag-1 effects for each time-point. For instance, for those who have data collected at age 8, the lag-1 effect could be from age 6 or age 7, depending on when the child was last tested. A unique lag-1 effect could be estimated depending on whether the lag-1 time is age 6 or age 7 (i.e., there are two lag-1 effects for age 8). As time becomes more continuous, the number of possible lag-1 effects per time-point increases, which makes estimation and interpretation more difficult as the number of possible effects rapidly increases.

Essentially, though MEMs may allow researchers to estimate different TVC effects for as many time-points are present, the underlying theory should guide where the substantively interesting differences in the TVCs may occur. Therefore, with time-unstructured data, we encourage researchers to consider the phase-varying TVC approach to retain TVC effects that optimally balance theory and parsimony. This allows researchers to apply their substantive knowledge to the modeling process to group time-points into meaningful sets rather than allowing the model to estimate a large number of TVC or lagged coefficients, each of which may be based on few individuals.

Though it is possible to obtain unique TVC effects at each time-point with time-unstructured data, doing so entails interpretational challenges whose magnitude increases the further the design deviates from being balanced and time-structured. The phase varying covariate approach also does not necessitate any manipulation of the time metric in order to reduce the number of parameters in the model. For instance, in the NLSY data, the phases of a phase varying TVC model would be the same regardless of whether age was included to the nearest year or nearest month. Adopting a phase varying approach could be beneficial if the growth trajectory required a more granular time metric because an LCM would be unlikely to support such data and an MLM with time-point specific TVC effects would be rather unruly to estimate and interpret.

In LCMs, the issue of idiosyncratic TVC effects is not present – LCM's reliance of wave-based designs imposes de facto phases onto the design and the estimation of LCMs simply breaks down as time becomes increasingly continuous (as seen with the definition variable approach). The

phase varying approach to TVCs is a way to balance the interpretational advantages of the LCM with the flexibility of the MEM by allowing the researcher to actively select which phases are of interest while also allowing growth to be on the most granular metric of interest.

Time-varying effects model

The methods we describe in this paper relate to obtaining time-point- or phrase-specific values of TVCs when there are relatively few time points collected per person, as is common in panel data. When there are many time-points collected as in intensive longitudinal data, an alternative approach with a similar goal is the *time-varying effect model* (Tan, Shiyko, Li, Li, & Dierker, 2012). Rather than estimate the coefficient for a TVC as we have outlined, this model uses splines to fit a smooth curve to the coefficient value of TVCs across time. Graphically, the time-varying covariate effect can be visualized with the coefficient value on the vertical axis, time on the horizontal axis, and a smooth curve indicating what the effect is at a particular time point. When panel data have frequent measurements or when a cohort-sequential design is applied such that there is good coverage along the span of the observation window, the time-varying effect model may be worth considering alongside or instead of the phase-varying TVC model we outline here (Lanza, Vasilenko, & Russell, 2016). However, the time-varying effect model would likely be difficult to fit the data with which we work in the illustrative examples in this paper given the small number of time-points.

Missing data

When we compared the LCM and MEM dummy code approaches for time-structured data, the results were likelihood equivalent. This finding will hold with complete data, but may not replicate with incomplete data, depending on the location of the incomplete data. The incongruence with incomplete data is due to the differential options that each framework can apply to accommodate missingness. In the multivariate LCM, the joint likelihood is used meaning that missing covariates are not inherently problematic and that full-information methods can be applied for missingness anywhere in the model to yield consistent estimates under the missing at random assumption (Bauer, 2003). In MEMs, the conditional likelihood is typically used meaning that full-information methods can be applied on data that are missing on the outcome but not to data that are missing in the covariates (McNeish, 2018). As alluded to previously when discussing coding of lag-1 variables in MEM software, missingness in the covariates results in listwise deletion unless researchers actively apply alternative missing data methods like multiple imputations for the covariates (Allison, 2012; Enders, 2010).

Relatedly, different number of measurement occasions per person are treated as missing by LCM but not by MEMs, which could also lead to differences in the likelihoods. As noted in Equation 12, the univariate structure of MEMs allows the model to adapt to a different number of measurement occasions per person – a participant with fewer measurement occasions is not necessarily seen as having missing data. On the other hand, the multivariate structure of LCMs would treat participants with fewer measurement occasions as having missing data and full-information missing data methods would be applied to accommodate this situation. The differential handling of these commonly encountered data structures would not always yield likelihood-equivalent results that we produced in our examples with more idealistic data structures. We chose such ideal data to demonstrate the equality of the likelihoods, but want to reiterate that this may not always hold in empirical data where missing values can produce different results in situations where we have shown equality of the likelihoods.

CONCLUDING REMARKS

Though TVCs are commonly included in many growth modeling applications, including such effects in a model can be unintuitively difficult when data stray from the ideal of being balanced and time-structured. LCMs are known for their flexibility with TVCs but researchers most benefit from this flexibility in the context of time-structured data. Unfortunately, time-structured data tend to be the exception in longitudinal data in psychology (Finkel, Reynolds, McArdle, Gatz, & Pedersen, 2003; Sterba, 2014). Viable methods to accommodate time-unstructured data such as the definition variable method or standard MEMs have limitations that make estimating TVC effects that vary over time unexpectedly challenging. Methods to work around issues inherent with TVCs in MEMs have often stopped short of their full potential and these methods have been proposed to apply only to optimal data structures.

To date, the context of estimating TVC effects that vary over time has represented a puzzling methodological black hole where multiple methods seemingly fit the task, only to falter with common data structures. However, as we show in this paper, previously proposed methods can be extended to satisfy the time-unstructured context so that TVCs can be treated in a more flexible fashion. Along with this flexibility comes increased difficulty in interpretation, so researchers may wish to group TVC effects into substantively meaningful phases related to time in order to yield more parsimonious and readily interpretable models that do not require manipulation of the time metric.

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APPENDIX
SAS AND M*PLUS* CODE FOR NLSY EXAMPLES

Center TVC, Accounting for Growth

```
*Fit growth model to TVC, save residuals (e) and random effects;
proc mixed data=NLSY;
class id;
model anti = kidage /s outp=e;
random int kidage /sub=id type=un s;
ods output solutionr=beta0;
run;
*This gives lots of output, subsequent code saves only the relevant information;

*only keep the individual-specific intercepts;
data beta0; set beta0;
beta0=estimate;
if effect='Intercept' then output;
keep id beta0;run;

*only keep the time-point specific residuals;
data e ; set e;
drop stderrpred pred df alpha lower upper;run;

*merge the random intercepts, residuals, and original data together;
data NLSY1; merge e beta0;
by id; run;
*in this data, 'Beta0' is the person average for antisocial behavior and 'resid' is the
centered TVC value for antisocial behavior;

DATA NLSY1; SET NLSY1;
AG=kidage-6; *center time so that 0 is lowest possible value;
TIMEA=AG; *dummy duplicate variable for time (as age);
TIMEO=OCCASION; *dummy duplicate variable for time (as wave);
RUN;
```

Flip Data from Long to Wide for Use in M*plus*

```
DATA W; SET NLSY1;
KEEP ID OCCASION READ kidage Beta0 Resid;
RUN;

*Transpose outcome variable, Reading Scores;
PROC TRANSPOSE DATA=W OUT=WIDER PREFIX=READ;
BY ID;
ID OCCASION; VAR READ;RUN;

*Transpose time as chronological age;
PROC TRANSPOSE DATA=W OUT=WIDEA PREFIX=AGE;
BY ID;
ID OCCASION;
VAR kidage;RUN;

*Transpose centered antisocial behavior;
```

```
PROC TRANSPOSE DATA=W OUT=WIDEN PREFIX=ANTI;
BY ID;
ID OCCASION;
VAR resid; RUN;

*Transpose person-mean of antisocial behavior;
PROC TRANSPOSE DATA=W OUT=WIDEM PREFIX=ANTIM;
BY ID;
VAR beta0; RUN;

*save only the relevant variables;
DATA WIDER; SET WIDER; DROP _label_ _name_;RUN;
DATA WIDEN; SET WIDEN; DROP _label_ _name_;RUN;
DATA WIDEA; SET WIDEA; DROP _label_ _name_;RUN;
DATA WIDEM; SET WIDEM; DROP _LABEL_ _nAME_ ANTIm2-ANTIM4;RUN;

*Merge into one wide dataset;
DATA WIDE; MERGE WIDER WIDEN WIDEA WIDEM; BY ID;RUN;

DATA WIDE; SET WIDE;
AG0=AGE0-6;
AG1=AGE1-6;
AG2=AGE2-6;
AG3=AGE3-6;
DROP AGE0-AGE3;RUN;
```

MEM with Time-Point Specific TVC effects, Wave as Time Metric (Table 1)

```
PROC MIXED DATA= NLSY1 METHOD=ML;
CLASS ID TIMEO;
MODEL READ = OCCASION resid*TIMEO beta0 /S;
RANDOM INT OCCASION /SUB=ID TYPE=UN;RUN;
```

LCM with Time-Point Specific TVC effects, Wave as Time Metric (Table 1)

```
DATA: FILE= [Location]

VARIABLE:
NAMES = id READ1-READ4 ANTI1-ANTI4 AM AGE1-AGE4;
USEVAR = READ1-READ4 ANTI1-ANTI4 am;

MODEL:
      I S | READ1@0 READ2@1 READ3@2 READ4@3;
      I; S; I WITH S;
      [I S];
      i on AM;
      READ1 ON ANTI1;
      READ2 ON ANTI2;
      READ3 ON ANTI3;
      READ4 ON ANTI4;
  READ1-READ4 (VAR);
```

MEM with Time-Point Specific and Lag-1 TVC Effects, Wave as Time Metric (Table 2)

```
data NLSY1; set NLSY1;
BY id;
```

```
antil = LAG(resid);
IF FIRST.id THEN DO;
antil=0;*set the first value to 0 for each new ID;
END;run;

PROC MIXED DATA= NLSY1 METHOD=ML;
CLASS ID TIMEO;
MODEL READ = OCCASION resid*TIMEO TIMEO*ANTIL beta0/S;
RANDOM INT OCCASION /SUB=ID TYPE=UN;RUN;
```

LCM with Time-Point Specific and Lag-1 TVC Effects, Wave as Time Metric (Table 2)

```
DATA: FILE= [Location]

VARIABLE:
NAMES = id READ1-READ4 ANTI1-ANTI4 AM AGE1-AGE4;
USEVAR = READ1-READ4 ANTI1-ANTI4 am;

MODEL:
      I S | READ1@0 READ2@1 READ3@2 READ4@3;
      I; S; I WITH S; I on AM;
      [I S];
      READ1 ON ANTI1;
      READ2 ON ANTI1 ANTI2;
      READ3 ON ANTI2 ANTI3;
      READ4 ON ANTI3 ANTI4;
READ1-READ4 (VAR);
```

LCM with Definition Variables, Age as Time Metric (Table 3)

```
DATA: FILE= [Location]

VARIABLE:
NAMES = id READ1-READ4 ANTI1-ANTI4 AM AGE1-AGE4;
USEVAR = READ1-READ4 ANTI1-ANTI4 AGE1-AGE4 am;
TSCORES=AGE1-AGE4;

ANALYSIS: TYPE=RANDOM;

MODEL:
      I S | READ1 READ2 READ3 READ4 AT AGE1-AGE4;
      I; S; I WITH S; I on AM;
      [I S];
      READ1 ON ANTI1;
      READ2 ON ANTI2;
      READ3 ON ANTI3;
      READ4 ON ANTI4;
READ1-READ4 (VAR);
```

MEM with Constrained TVC Effects, Age as Time Metric (Table 4, first column)

```
PROC MIXED DATA= NLSY1 METHOD=ML;
CLASS ID TIMEA;
MODEL READ = AG resid beta0/S;
```

```
RANDOM INT AG /SUB=ID TYPE=UN;RUN;
```

MEM with Time-Point Specific TVC effects, Age as Time Metric (Table 4, middle column)

```
PROC MIXED DATA= NLSY1 METHOD=ML;
CLASS ID TIMEA;
MODEL READ = AG resid*TIMEA beta0/S;
RANDOM INT AG /SUB=ID TYPE=UN;RUN;
```

MEM with Phase-Varying TVC effects, Age as Time Metric (Table 4, right column)

```
data NLSY1; set NLSY1;
if kidage < 8 then p1=1; else p1=0;
if 7 < kidage < 14 then p2=1; else p2=0;
if kidage > 13 then p3=1; else p3=0;;run;

PROC MIXED DATA= NLSY1 METHOD=ML;
CLASS ID TIMEA;
MODEL READ = AG resid*P1 resid*P2 resid*P3 beta0 /S;
RANDOM INT AG /SUB=ID TYPE=UN;RUN;
```