

## SAD2 exam report

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## Introduction

Ranking has become an increasingly important problem, with ever-growing datasets both in the industry and in academia. In the 21. century gathering large data sets is no longer considered novel. More and more people are getting access to the internet and more content like films, music is created and evaluated by them. Ranking provides what might be the simplest type of recommendation system: recommend the items that score best on a global total ranking, independently of the recipient of the recommendation.

In this project, we analyze different approaches to keeping a ranked data set, while receiving live updates to the data. We choose algorithms to analyze based on factors like: time and space requirements, and the quality of the solutions in case of approximation algorithms.

## Ordering movie ratings in a dynamic setting

To motivate our approach, we present our project through the following concrete problem: An IMDB like service maintains a list of movies, and lets users rank movies on some scale (say  $[1, 10] \cap \mathbb{N}$ ). We want to provide the set of movies, sorted by user-supplied ratings. It is likely that ratings of individual movies will span a large portion of the valid range, with some movies getting a few maximum ratings, but a low overall score. It is therefore necessary to use an aggregate for the ranking, such as the average user-rating for each movie. For simplicity, we assume that ratings cannot be changed or deleted, only added. This makes it possible to maintain running averages, as opposed to the full set of ratings, without integrity loss

If ratings are added frequently, we can consider the input — the set of  $(user, rating, movie)$  triplets — to be *dynamic*. The input is dynamic in the sense that the *true* global ranking may change over time, with every added user-supplied rating. In a static setting, where the global rank does not evolve over time, there are simple algorithms that solve our problem. E.g. sorting the full data set would do. In a dynamic setting, where the data evolves over time, those algorithms would need to be re-run to maintain current solutions (as the underlying input changes, so does the global ranking, and hence the correct solution). Hence, there may be better algorithms that maintain and evolve the solution according to the input.

We will start by analyzing the most direct approach of maintaining a binary tree of the movies, updating it with each rating in the input. We will then explore different algorithms in light of the trade-offs between memory consumption, correctness, and currency. In particular it should be possible to achieve a memory bound lower than  $m$  by sacrificing correctness, achieve both by sacrificing currency, i.e. allowing the output to become somewhat outdated as the underlying data set evolves. We note that in our IMDB like scenario, the running time of getting a total ordering is irrelevant. The running time per rating is all that matters, though they will often be closely related.

Following is a list of techniques that we deemed promising at the project outset, and hence wished explore the effectiveness of with respect to our problem:

## Online sorting

From Algorithms and Theory of Computation Handbook:

An algorithm that must process each input in turn, without detailed knowledge of future inputs.

Not every online algorithm has an offline counterpart. And often also online algorithms cannot match the performance of offline algorithms. But in our scenario we want to create competitive, suitable solution - the online algorithm seems to be great fit.

- Storing in right order each input batch on heap.
- Taking advantage of algorithm like Insertion Sort
- Partial sorting, Odds algorithm - which can be valuable for defining optimal stopping
- By combining techniques listed above.

With  $n$  inputs we are creating a heap - which takes  $n \log n$  operations. This gives us a required data. Then if the next input arrives the algorithm inserts each of new input with  $\log n$ . If the item is already on the heap then we are changing its average. So each input is processed in  $\log$  input size. Which is very good score but we have to store each unique movie.

Insertion sorts allows us to easily distribute results into multiple locations which could be advantage for very large sets.

Odds algorithm, Partial sorting - approach is tempting but it is compromising the quality of output. Also if we managed to utilize an algorithm based on partitioning we might be able to parallelize it. Which can be more important factor for efficiency than algorithm cost.

## A Stream Based Approach

Streaming algorithms provide excellent solutions to many problems where data sets are large enough that we wish (or need) to sacrifice correctness for low memory usage and time consumption. From Ikononovska-Zelke:

"Streaming algorithms drop the demand of random access to the input. Rather, the input is assumed to arrive in arbitrary order as an input stream. Moreover, streaming algorithms are designed to settle for a working memory that is much smaller than the size of the input."

To be precise, they require that the size of the working memory is sublinear in both the cardinality of the stream and the universe. Due to this nature of streaming algorithms they are not commonly used for problems that require analysing parts of non-constant, non-parameterized size of the data set, for each given input. Hence an approach to solve our problem — sorting — based on streaming algorithms will provide some interesting trade-offs.

We begin with a definition of our stream, and a simple algorithm for our problem. Our input is a turnstile stream. The universe  $U$  is the set of movies

in our database, and each stream item is a pair  $(j, r) \in U \times ([1, 10] \cap \mathbb{N})$ . With our definition of movie ranks we get a strict turnstile stream — in fact the delta  $r$  for every stream element is positive.

The simplest algorithm to solve our problem is then simply calculating the frequency vector for the stream, and sort it when our algorithm is queried. However, this algorithm is not very satisfactory. The working memory is sub-linear in the cardinality of the stream, but not in the universe size. It does not provide a current solution either, as we have to sort the frequency vector when queried. On the positive side, the solution provided by the algorithm is correct. To be precise, this algorithm would require  $O(m)$  working memory, and time for each stream item,  $m$  being the number of distinct movies in the stream, or the universe size  $|U|$ . A query would then require  $O(n \log(m))$  time, which is unacceptable.

1. Maintaining ordering with each item
2. Approximation based on sampling
3. sketching approach to approximation

For 1 — maintaining ordering — we diverge from the classical definition of streaming algorithms. Both by using linear memory, and by using non-constant time for each stream item. Although the latter is common, it is unusual to use time non-constant in the stream size. We note that this solution will be equivalent to maintaining an ordered set of running averages, and is thus the same as the online-sorting approach. Knowing that for most stream items  $(j, r)$ , the movie represented by  $j$  will already be in the ordered set, it might be possible to achieve insertion time linear in the number of inversions needed to reorder the set, though it is not clear that this improves on the  $\log(n)$  insertion time in binary search trees..

For 2 — sampling the stream — there seem to be two obvious approaches; Sampling as normal (reservoir sampling) over the stream, or sampling over the movies, deliberately making sure that all movies are represented in the sample, or that the number of samples... Neither seem to be much good

Finally 3 — sketching — looks good. We can use `Count-Min sketch` to get approximate point-queries (although we would query all points, or at least max-k points). It may even be possible to combine this solution with 1, to avoid sorting the samples when queried, though this requires further analysis.