

A Survey of Probability Concepts

Chapter 5

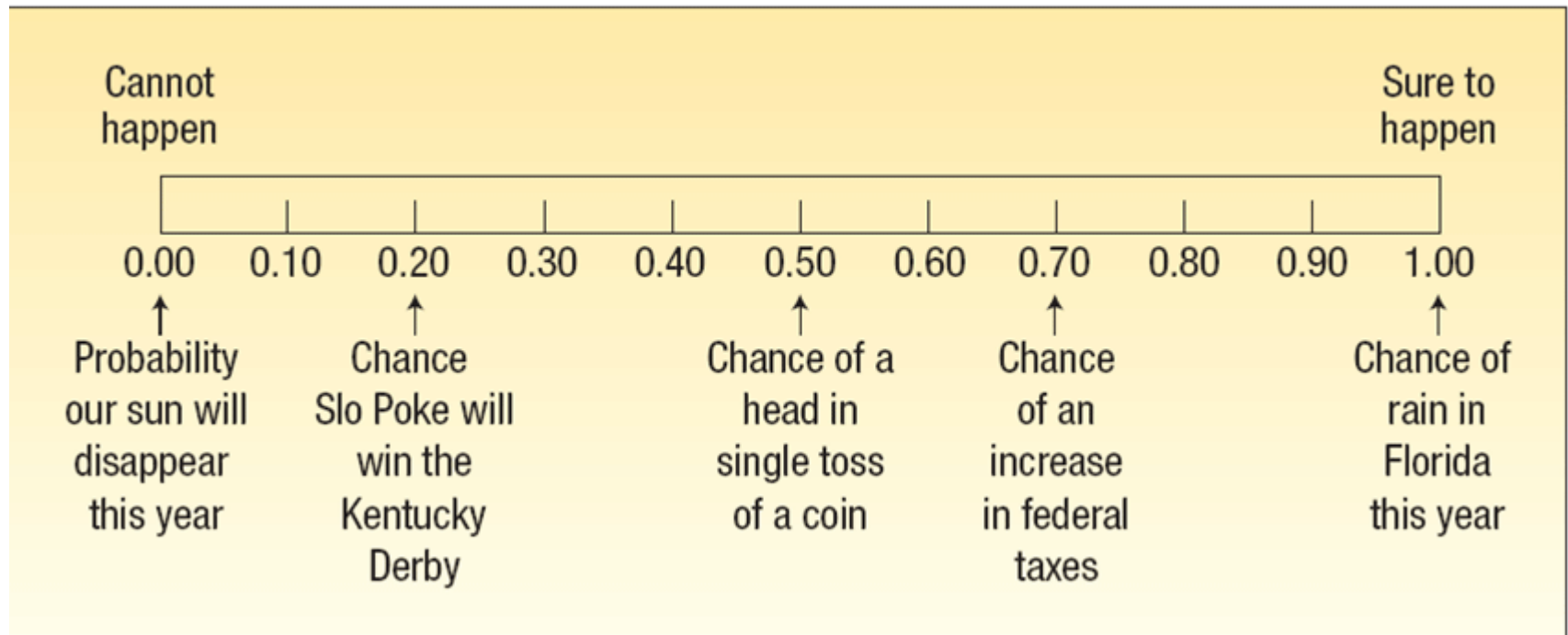


Learning Objectives

- LO1 Define *probability*.
- LO2 Explain the terms *experiment*, *event*, *outcome*,
- LO3 Identify and apply the appropriate approach to assigning probabilities,
- LO4 Calculate probabilities using the *rules of addition*,
- LO5 Define the term *joint probability*,
- LO6 Calculate probabilities using the *rules of multiplication*,
- LO7 Define the term *conditional probability*,
- LO8 Calculate probabilities using a *contingency table*,
- LO9 Calculate a probability using *Bayes' theorem*,
- LO10 Determine the number of outcomes using the appropriate principle of counting.



Probability

PROBABILITY A value between zero and one, inclusive, describing the relative possibility (chance or likelihood) an event will occur.



Experiment, Outcome and Event

- An **experiment** is a process that leads to the occurrence of one and only one of several possible observations.
- An **outcome** is the particular result of an experiment.
- An **event** is the collection of one or more outcomes of an experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None are over 60 One is over 60 Two are over 60 ... 29 are over 60 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

Ways of Assigning Probability

There are three ways of assigning probability:

1. **CLASSICAL PROBABILITY**

Based on the assumption that the **outcomes of an experiment are equally likely.**

2. **EMPIRICAL PROBABILITY**

The probability of an event happening is the fraction of the time similar events happened in the past.

3. **SUBJECTIVE CONCEPT OF PROBABILITY**

The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

Classical Probability

CLASSICAL PROBABILITY

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} \quad [5-1]$$

Consider an experiment of rolling a six-sided die. What is the probability of the event “an **even number** of spots appear face up”?

The possible outcomes are:

a one-spot



a two-spot



a three-spot



a four-spot



a five-spot



a six-spot



There are three “favorable” outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes.

Mutually Exclusive Events and Collectively Exhaustive Events

- Events are **mutually exclusive** if the occurrence of any one event means that **none of the others can occur at the same time**.
- Events are **independent** if the occurrence of one event does not affect the occurrence of another.
- Events are **collectively exhaustive** if at least one of the events must occur when an experiment is conducted.

Empirical Probability

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

Empirical approach to probability is based on what is called the Law of Large Numbers.

Key to establishing probabilities empirically – larger number of observations provides a more accurate estimate of the probability.

LAW OF LARGE NUMBERS Over a large number of trials the empirical probability of an event will approach its true probability.

Empirical Probability - Example

- Here are some examples.
 - Last semester, 80 students registered for Business Statistics 101 at Scandia University. Twelve students earned an A. Based on this information and the empirical approach to assigning a probability, we estimate the likelihood a student will earn an A is .15.
 - Kobe Bryant of the Los Angeles Lakers made 403 out of 491 free throw attempts during the 2009–10 NBA season. Based on the empirical rule of probability, the likelihood of him making his next free throw attempt is .821.

Empirical Probability - Example

On February 1, 2003, the Space Shuttle Columbia exploded. **This was the second disaster** in 123 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

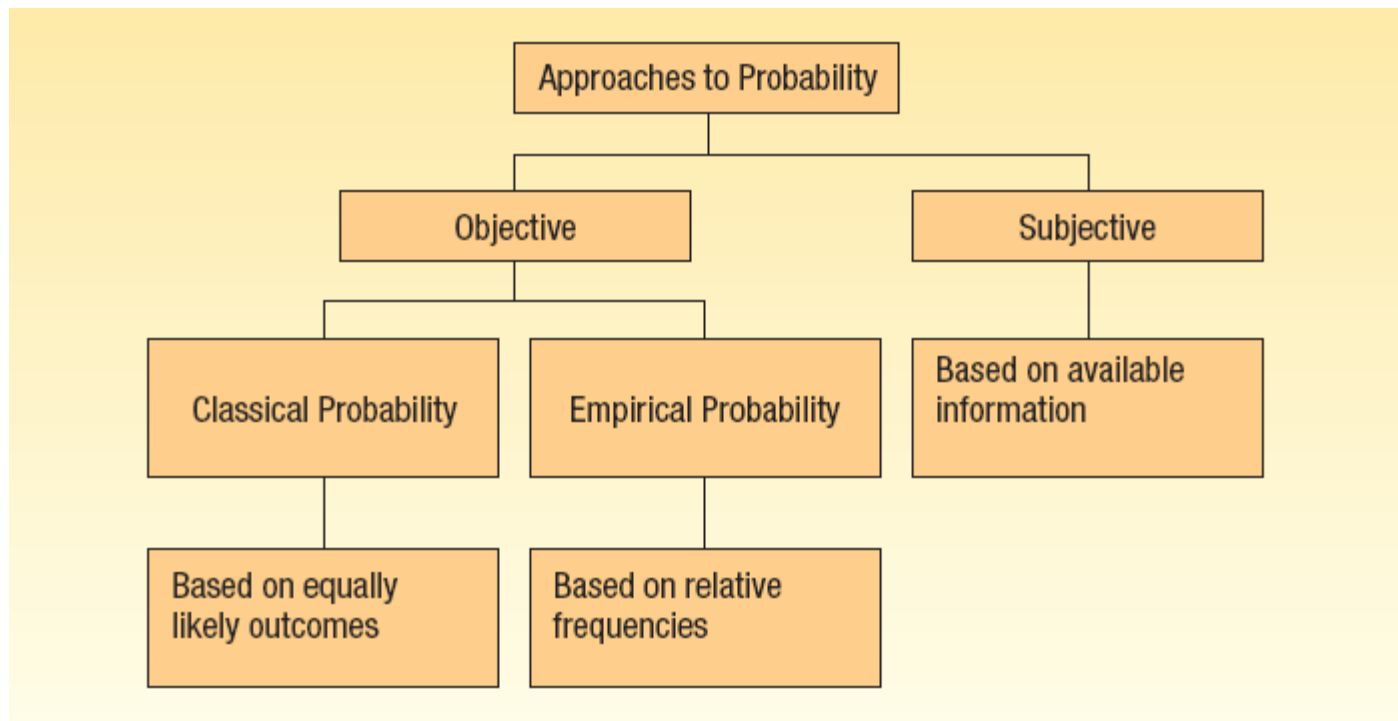
$$\begin{aligned}\text{Probability of a successful flight} &= \frac{\text{Number of successful flights}}{\text{Total number of flights}} \\ &= \frac{121}{123} = 0.98\end{aligned}$$

Subjective Probability - Example

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

- If there is little or no past experience or information on which to base a probability, it may be arrived at subjectively.
- Illustrations of subjective probability are:
 1. Estimating the likelihood the New England Patriots will play in the Super Bowl **next year**.
 2. Estimating the likelihood you will be married before **the age of 30**.
 3. Estimating the likelihood the U.S. budget deficit will be reduced by half in **the next 10 years**.

Summary of Types of Probability



Self-Review

Self-Review 5-2



1. One card will be randomly selected from a standard 52-card deck. What is the probability the card will be a queen? Which approach to probability did you use to answer this question?
2. The Center for Child Care reports on 539 children and the marital status of their parents. There are 333 married, 182 divorced, and 24 widowed parents. What is the probability a particular child chosen at random will have a parent who is divorced? Which approach did you use?
3. What is the probability that the Dow Jones Industrial Average will exceed 12,000 during the next 12 months? Which approach to probability did you use to answer this question?

Rules of Addition for Computing Probabilities

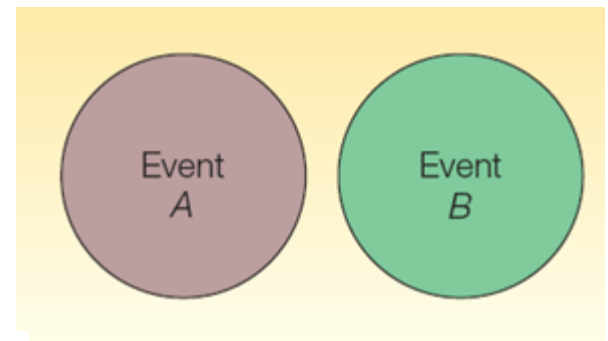
Rules of Addition

- Special Rule of Addition - If two events A and B are **mutually exclusive**, the probability of one or the other event's occurring equals the sum of their probabilities.

SPECIAL RULE OF ADDITION

$$P(A \text{ or } B) = P(A) + P(B)$$

[5-2]



- The General Rule of Addition - If A and B are two events that are not mutually exclusive, then $P(A \text{ or } B)$ is given by the following formula:

GENERAL RULE OF ADDITION

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

[5-4]



Addition Rule – Mutually Exclusive Events

Example

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4,000 packages filled in the past month revealed:

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	<i>A</i>	100	.025
Satisfactory	<i>B</i>	3,600	.900
Overweight	<i>C</i>	300	.075
		<u>4,000</u>	<u>1.000</u>

←

100
4,000

What is the probability that a particular package will be either underweight or overweight?

$$P(A \text{ or } C) = P(A) + P(C) = .025 + .075 = .10$$

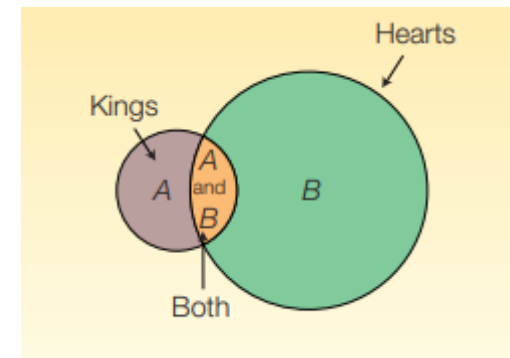
Addition Rule – Not Mutually Exclusive Events

Example

What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?

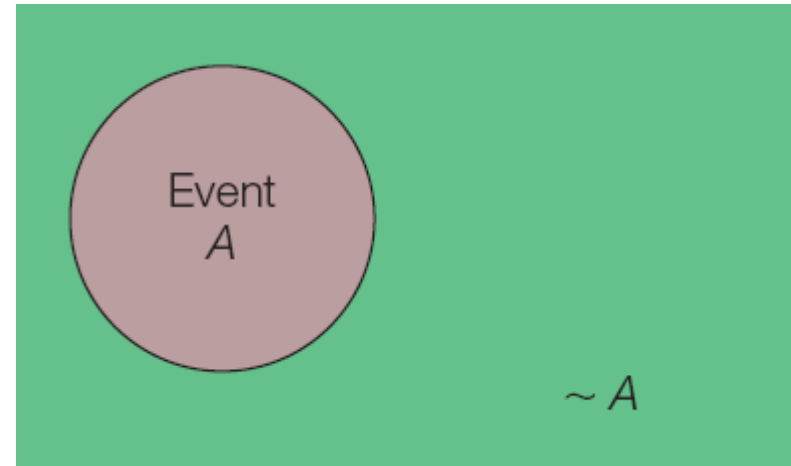
Card	Probability	Explanation
King	$P(A) = 4/52$	4 kings in a deck of 52 cards
Heart	$P(B) = 13/52$	13 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B) = 1/52$	1 king of hearts in a deck of 52 cards

$$\begin{aligned}
 P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\
 &= 4/52 + 13/52 - 1/52 \\
 &= 16/52, \text{ or } .3077
 \end{aligned}$$



The Complement Rule

The **complement rule** is used to determine the probability of an event occurring by subtracting the probability of the event *not* occurring from 1.



COMPLEMENT RULE

$$P(A) = 1 - P(\sim A)$$

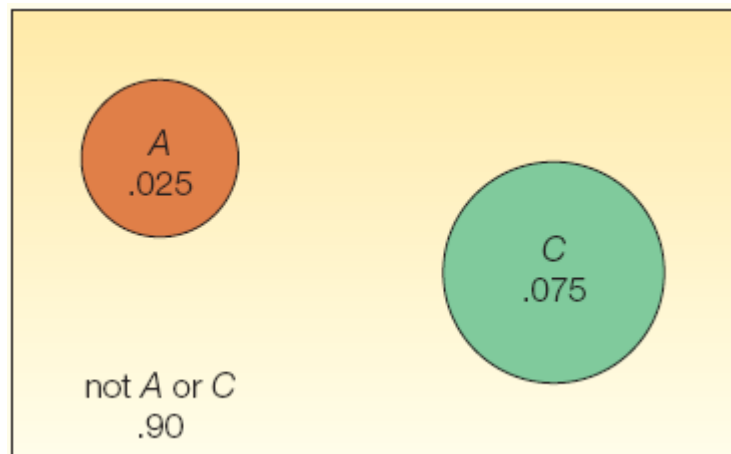
[5-3]

or $P(A) = 1 - P(\sim A)$.

The Complement Rule - Example

An automatic Shaw machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. Use the complement rule to show the probability of a satisfactory bag is .900

Weight	Event	Number of Packages	Probability of Occurrence
Underweight	<i>A</i>	100	.025
Satisfactory	<i>B</i>	3,600	.900
Overweight	<i>C</i>	300	.075
		<u>4,000</u>	<u>1.000</u>



$$\begin{aligned}
 P(B) &= 1 - P(\sim B) \\
 &= 1 - P(A \text{ or } C) \\
 &= 1 - [P(A) + P(C)] \\
 &= 1 - [.025 + .075] \\
 &= 1 - .10 \\
 &= .90
 \end{aligned}$$

The General Rule of Addition

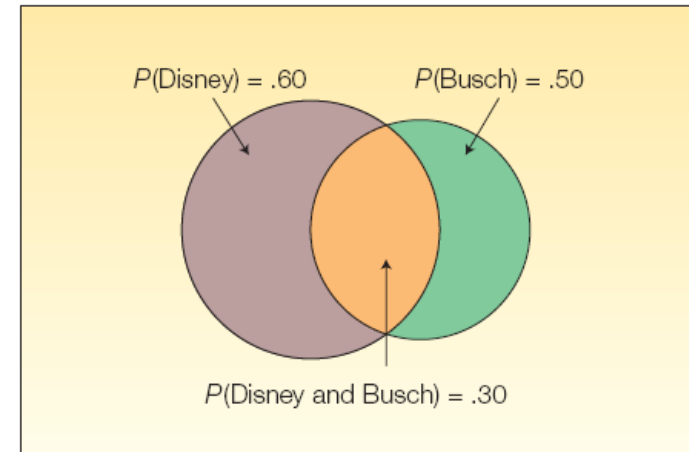
GENERAL RULE OF ADDITION

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

[5-4]

The Venn Diagram shows the result of a survey of 200 tourists who visited Florida during the year. The survey revealed that 120 went to Disney World, 100 went to Busch Gardens and 60 visited both.

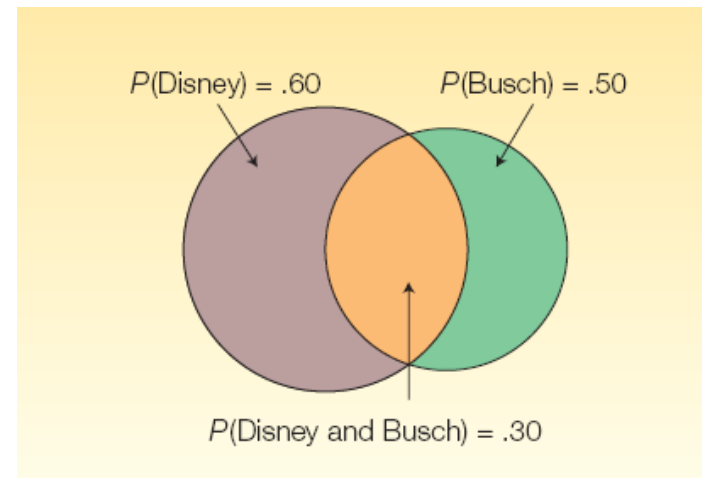
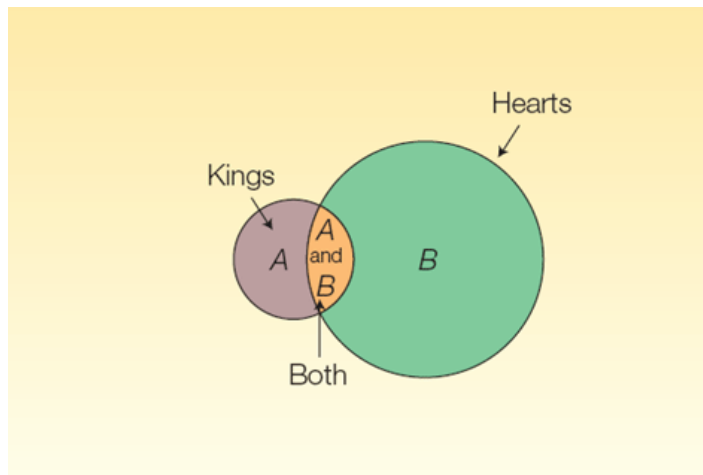
What is the probability a selected person visited either Disney World or Busch Gardens?



$$\begin{aligned} P(\text{Disney or Busch}) &= P(\text{Disney}) + P(\text{Busch}) - P(\text{both Disney and Busch}) \\ &= 120/200 + 100/200 - 60/200 \\ &= .60 + .50 - .30 = .80 \end{aligned}$$

Joint Probability – Venn Diagram

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.



Special Rule of Multiplication

- The **special rule of multiplication** requires that two events A and B are ***independent***.
- Two events A and B are independent if the occurrence of one has no effect on the probability of the occurrence of the other.
- This rule is written:

SPECIAL RULE OF MULTIPLICATION

$$P(A \text{ and } B) = P(A)P(B)$$

[5–5]

Multiplication Rule-Example

A survey by the American Automobile Association (AAA) revealed 60 percent of its members made airline reservations last year. Two members are selected at random. What is the probability both made airline reservations last year?

Solution:

The probability the first member made an airline reservation last year is .60, written as $P(R_1) = .60$

The probability that the second member selected made a reservation is also .60, so $P(R_2) = .60$.

Since the number of AAA members is very large, it can be assumed that R_1 and R_2 are independent.

$$P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$$

Conditional Probability

- A **conditional probability** is the probability of a particular event occurring, given that another event has occurred.
- The probability of the event A given that the event B has occurred is written **$P(A|B)$** .

General Multiplication Rule

The **general rule of multiplication** is used to find the joint probability of two events when the events are *not independent (dependent)*.

It states that for two events, A and B, the joint probability that both events will happen is found by multiplying the probability that event A will happen by the conditional probability of event B occurring given that A has occurred. Symbolically, the joint probability, $P(A \text{ and } B)$, is found by:

GENERAL RULE OF MULTIPLICATION

$$P(A \text{ and } B) = P(A)P(B|A)$$

[5-6]

General Multiplication Rule - Example

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry.

What is the likelihood both shirts selected are white?



- The event that the first shirt selected is white is W_1 . The probability is $P(W_1) = 9/12$
- The event that the second shirt (W_2) selected is also white. The conditional probability that the second shirt selected is white, given that the first shirt selected is also white, is $P(W_2 | W_1) = 8/11$.
- To determine the probability of 2 white shirts being selected we use formula:

$$P(AB) = P(A) P(B|A)$$
- $P(W_1 \text{ and } W_2) = P(W_1)P(W_2 | W_1) = (9/12)(8/11) = 0.55$

Contingency Tables

A CONTINGENCY TABLE is a table used to classify sample observations according to two or more identifiable characteristics

E.g. A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria—the **number of movies attended** and **gender**.

Movies Attended	Gender		Total
	Men	Women	
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

Contingency Tables - Example

A sample of executives were surveyed about their loyalty to their company. One of the questions was, "If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?" The responses of the 200 executives in the survey were cross-classified with their length of service with the company.

Loyalty	Length of Service				Total
	Less than 1 Year, B_1	1–5 Years, B_2	6–10 Years, B_3	More than 10 Years, B_4	
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

Contingency Tables - Example

Event A_1 - if a randomly selected executive will remain with the company despite an equal or slightly better offer from another company. Since there are 120 executives out of the 200 in the survey who would remain with the company

$$P(A_1) = 120/200, \text{ or } .60$$

Event B_4 - if a randomly selected executive has more than 10 years of service with the company. Thus, $P(B_4|A_1)$ is the conditional probability that an executive with more than 10 years of service would remain with the company. Of the 120 executives who would remain 75 have more than 10 years of service, so

$$P(B_4|A_1) = 75/120$$

$$P(A_1 \text{ and } B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{9,000}{24,000} = .375$$

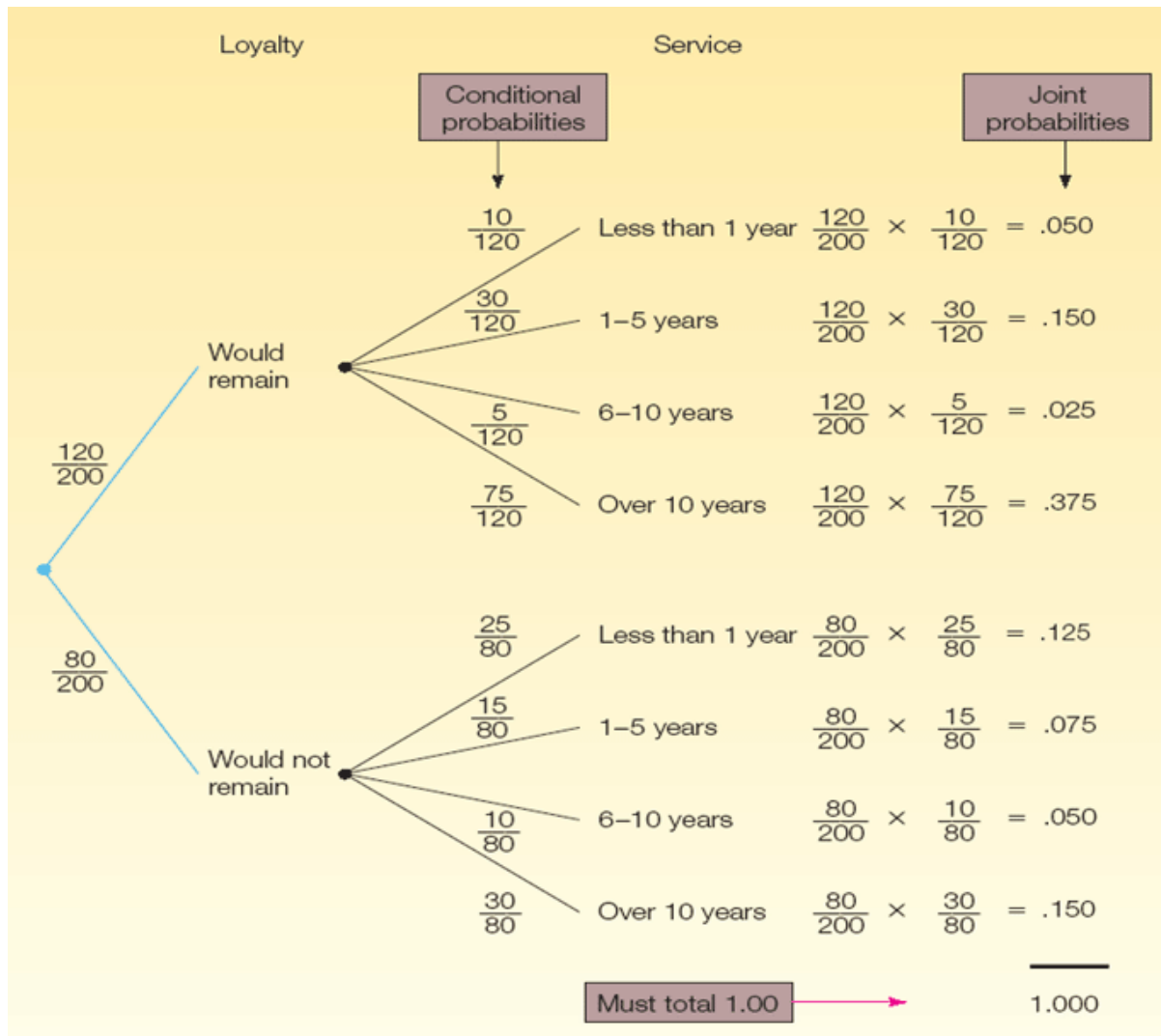
Tree Diagrams

A **tree diagram** is:

1. useful for portraying conditional and joint probabilities.
2. particularly useful for analyzing business decisions involving several stages.
3. a graph that is helpful in organizing calculations that involve several stages.

Each segment in the tree is one stage of the problem. The branches of a tree diagram are weighted by probabilities.

Tree Diagram Example



Bayes' Theorem

- Bayes' Theorem is a method for revising a probability given additional information.
- It is computed using the following formula:

BAYES' THEOREM

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)}$$

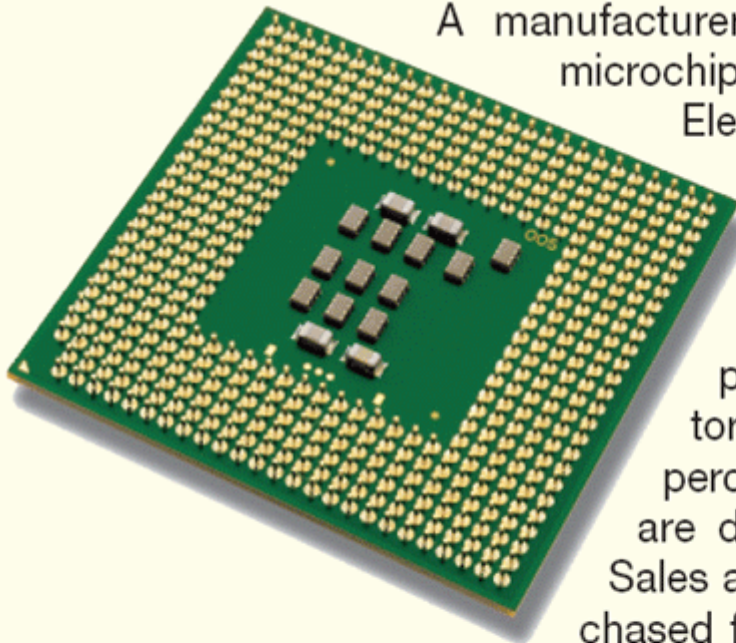
[5-7]

PRIOR PROBABILITY The initial probability based on the present level of information.

POSTERIOR PROBABILITY A revised probability based on additional information.

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

Bayes' Theorem - Example



A manufacturer of DVD players purchases a particular microchip, called the LS-24, from three suppliers: Hall Electronics, Schuller Sales, and Crawford Components. Thirty percent of the LS-24 chips are purchased from Hall Electronics, 20 percent from Schuller Sales, and the remaining 50 percent from Crawford Components. The manufacturer has extensive histories on the three suppliers and knows that 3 percent of the LS-24 chips from Hall Electronics are defective, 5 percent of chips from Schuller Sales are defective, and 4 percent of the chips purchased from Crawford Components are defective.

When the LS-24 chips arrive at the manufacturer, they are placed directly in a bin and not inspected or otherwise identified by supplier. A worker selects a chip for installation in a DVD player and finds it defective. What is the probability that it was manufactured by Schuller Sales?

Bayes' Theorem – Example (*cont.*)

- There are three mutually exclusive and collectively exhaustive events, that is, three suppliers.

A_1 The LS-24 was purchased from Hall Electronics.

A_2 The LS-24 was purchased from Schuller Sales.

A_3 The LS-24 was purchased from Crawford Components.

- The prior probabilities are:

$P(A_1) = .30$ The probability the LS-24 was manufactured by Hall Electronics.

$P(A_2) = .20$ The probability the LS-24 was manufactured by Schuller Sales.

$P(A_3) = .50$ The probability the LS-24 was manufactured by Crawford Components.

- The additional information can be either:

B_1 The LS-24 appears defective, or

B_2 The LS-24 appears not to be defective.

Bayes' Theorem – Example (*cont.*)

- The following conditional probabilities are given.

$P(B_1|A_1) = .03$ The probability that an LS-24 chip produced by Hall Electronics is defective.

$P(B_1|A_2) = .05$ The probability that an LS-24 chip produced by Schuller Sales is defective.

$P(B_1|A_3) = .04$ The probability that an LS-24 chip produced by Crawford Components is defective.

- A chip is selected from the bin. Because the chips are not identified by supplier, we are not certain which supplier manufactured the chip. We want to determine the probability that the defective chip was purchased from Schuller Sales. The probability is written $P(A_2|B_1)$.

Bayes' Theorem – Example (*cont.*)

The probability the defective LS-24 chip came from Schuller Sales can be formally found by using Bayes' theorem. We compute $P(A_2|B_1)$, where A_2 refers to Schuller Sales and B_1 to the fact that the selected LS-24 chip was defective.

$$\begin{aligned} P(A_2|B_1) &= \frac{P(A_2)P(B_1|A_2)}{P(A_1)P(B_1|A_1) + P(A_2)P(B_1|A_2) + P(A_3)P(B_1|A_3)} \\ &= \frac{(.20)(.05)}{(.30)(.03) + (.20)(.05) + (.50)(.04)} = \frac{.010}{.039} = .2564 \end{aligned}$$

Counting Rules – Multiplication

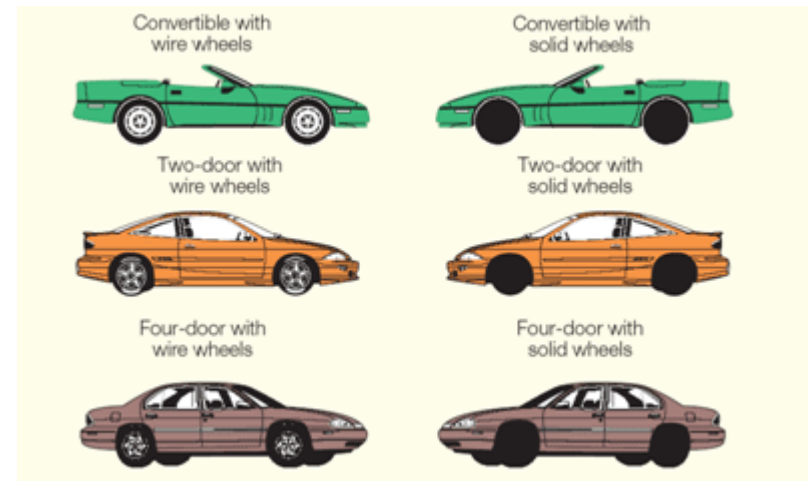
The **multiplication formula** indicates that if there are m ways of doing one thing and n ways of doing another thing, there are $m \times n$ ways of doing both.

Example: Dr. Delong has 10 shirts and 8 ties. How many shirt and tie outfits does he have?

$$(10)(8) = 80$$

Counting Rules – Multiplication: Example

An automobile dealer wants to advertise that for \$29,999 you can buy a convertible, a two-door sedan, or a four-door model with your choice of either wire wheel covers or solid wheel covers. How many different arrangements of models and wheel covers can the dealer offer?



MULTIPLICATION FORMULA

Total number of arrangements = $(m)(n)$

[5–8]

We can employ the multiplication formula as a check (where m is the number of models and n the wheel cover type). From formula (5–8):

$$\text{Total possible arrangements} = (m)(n) = (3)(2) = 6$$

Counting Rules - Permutation

A **permutation** is any arrangement of r objects selected from n possible objects. The order of arrangement is important in permutations.

PERMUTATION FORMULA

$${}_nP_r = \frac{n!}{(n-r)!}$$

[5-9]

where:

n is the total number of objects.

r is the number of objects selected.

Counting - Combination

A **combination** is the number of ways to choose r objects from a group of n objects without regard to order.

COMBINATION FORMULA

$${}_nC_r = \frac{n!}{r!(n - r)!}$$

[5–10]

where:

n is the total number of objects.

r is the number of objects selected.

Combination and Permutation Examples

COMBINATION EXAMPLE

There are 12 players on the Carolina Forest High School basketball team. Coach Thompson must pick five players among the twelve on the team to comprise the starting lineup. How many different groups are possible?

$${}_{12}C_5 = \frac{12!}{5!(12-5)!} = 792$$

PERMUTATION EXAMPLE

Suppose that in addition to selecting the group, he must also rank each of the players in that starting lineup according to their ability.

$${}_{12}P_5 = \frac{12!}{(12-5)!} = 95,040$$