1. **Please use** the following system of linear equations to answer the next 2 questions (1.11 and 1.2.).

$$3x_1 + 5x_2 - 3x_3 = 10$$
$$4x_2 + 4x_3 = -5$$
$$2x_1 - 9x_2 + 6x_3 = 13$$

1.1. Use LU Decomposition Method to get the matrices. L and U. (20 pts)

1.2. Write the Python code to solve the system shown above. You can use **any** one of the methods studied in the class. (20 pts)

2. Please use power method to find the approximated largest eigenvalue of the matrix $\begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$. You can use the initial vector $[1,1]$. (show 3 iterations) (20 pts)

3. Given the following information, write a Python code for computing parameters of the estimation function by using the Least Squares Regression method. (20 pts)

x = np.linspace(0, 1, 101)
y = 1 + x * np.random.random(len(x))
estimation function $\hat{y}$: $\hat{y} = \alpha_1 x + \alpha_2$

4. Find the linear interpolation at x = 3 based on the data x = [2, 5, 6] and y = [2, 3, 1]. (20 pts)

5. Assume that we are using Cubic Spline Interpolation. Given the following data points x = [2, 5, 6] and y = [2, 3, 1], please find 8 cubic functions to set up the system of linear equations. (20 pts)

(hint: if $f(x) = ax^3+bx^2+cx+d$, $f(x)'=3ax^2+2bx+c$ and $f(x)''=6ax+2b$)

6. Given the following data points x = [2, 5, 6] and y = [2, 3, 1], please build 3 polynomials for Lagrange Polynomial Interpolation method. (20 pts)

7. Write a Python code to find an approximated root of the function $f(x) = x^2 + 3x + 1$ using Newton-Raphson method. Please explain each line what it does/means. Use $x_0 = 3$ as a starting point and 1e5 as a tolerance. (hint: $f(x)' = 2x+3$ and newton_raphson can be written as a recursive function.) (40 pts)