

1. Show the series of matrices needed for rotating a 2D object  $45^\circ$  around the point (5, 7)

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix} \text{ Translate x by 5 units and y by 7 units.}$$

$$\begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & 0 \\ \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Rotate } 45^\circ \text{ about the origin}$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{bmatrix} \text{ Translate x back by 5 units and y back by 7 units.}$$

The combined matrix is  $\begin{bmatrix} \cos(\pi/4) & -\sin(\pi/4) & 5(1 - \cos(\pi/4)) + 7\sin(\pi/4) \\ \sin(\pi/4) & \cos(\pi/4) & 7(1 - \cos(\pi/4)) - 5\sin(\pi/4) \\ 0 & 0 & 1 \end{bmatrix}$

2. Show the series of matrices needed for reflecting a 2D object around the line  $y = x + 4$ .

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \text{ Translate y by 4 units so that it passes through the origin.}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ Reflect about the line } y = x$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} \text{ Translate y back 4 units}$$

The combined matrix is  $\begin{bmatrix} 0 & 1 & -4 \\ 1 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

3. Show the series of matrices needed for reflecting a 3D object about the line  $y = 10$ .

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ Translate by 10 units in the y direction}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ Reflect about xz-plane}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ Translate back 10 units in y direction}$$

The combined matrix is  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$