E/17/371
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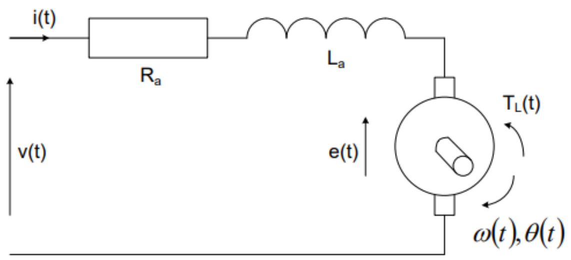
Find the transfer function of the armature-controlled dc motor shown below where the rotor second moment of inertia and the viscose friction coefficient are J, Brespectively.



Apply KVL,

$$v = R_a i + L_a \frac{di}{dt} + e$$

$$e = K_e \omega$$

Assuming coulomb friction is negligible,

$$\tau_i - \tau_L - B\omega = J \frac{d\omega}{dt}$$

$$\tau_i = K_t i$$

Apply Laplace transform assuming initial conditions are zero,

$$V(s) = R_a I(s) + L_a s I(s) + E(s) \quad \text{——} \; (1)$$

$$E(s) = K_e \Omega(s) \quad \text{——} \; (2)$$

$$T_i(s) - T_L(s) - B\Omega(s) = Js\Omega(s) \quad \text{——} \; (3)$$

$$T_i(s) = K_t I(s) \quad \text{——} \; (4)$$

From (3) and (4),

$$I(s) = \frac{(B + Js)\Omega(s) + T_L(s)}{K_t} \quad \text{——} \; (5)$$

From (1) and (2),

$$
\begin{aligned}
V(s) &= (R_a + L_a s) I(s) + K_e \Omega(s) \\
&= (R_a + L_a s)\left[\frac{(B + Js)\,\Omega(s) + T_L(s)}{K_t}\right] + K_a \Omega(s) \\
&= \left[\frac{(R_a + L_a s)(B + Js)}{K_t} + K_a\right]\Omega(s) - \frac{(R_a + L_a s)}{K_t} T_L(s)
\end{aligned}
$$

$$
\left[\frac{(R_a + L_a s)(B + Js) + K_a K_t}{K_t}\right]\Omega(s) = V(s) - \frac{(R_a + L_a s)}{K_t} T_L(s)
$$

$$
\Omega(s) = \left[\frac{K_t}{(R_a + L_a s)(B + Js) + K_a K_t}\right] V(s) - \left[\frac{(R_a + L_a s)}{(R_a + L_a s)(B + Js) + K_a K_t}\right] T_L(s)
$$

The $T_L(s)$ term comes from the noise that is introduced from the outside. Obtain transfer function without considering noise,

$$
H(s) = \frac{\Omega(s)}{V(s)} = \frac{K_t}{(R_a + L_a s)(B + Js) + K_a K_t}
$$