

CSIT110 / CSIT810

Python

Lecture 12

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Objectives

Understanding of:

- Recursion

Recursion

A recursive function is a function that **calls itself**.

A recursive function usually has two steps:

- **Base step**: deals with **small cases**
- **Recursion step**: how a general case can be **derived from smaller cases**

Factorial function

$$1! = 1$$

$$2! = 2$$

$$3! = 6$$

$$4! = 24$$

$$5! = 120$$

$$6! = 720$$

$$7! = 5040$$

$$8! = 40320$$

$$9! = 362880$$

Factorial function

$$1! = 1 \longrightarrow \text{one factorial}$$

$$2! = 1 \times 2 = 2 \longrightarrow \text{two factorial}$$

$$3! = 1 \times 2 \times 3 = 6$$

$$4! = 1 \times 2 \times 3 \times 4 = 24 \longrightarrow \text{four factorial}$$

If we know $4! = 24$,
how can we calculate $5!$?

$$5! = 4! \times 5 = 24 \times 5 = 120$$

Factorial function

$$1! = 1 \longrightarrow \text{one factorial}$$

$$2! = 1 \times 2 = 2 \longrightarrow \text{two factorial}$$

$$3! = 1 \times 2 \times 3 = 6$$

$$4! = 1 \times 2 \times 3 \times 4 = 24 \longrightarrow \text{four factorial}$$

In general, if we know $\text{factorial}(n-1)$,
we can calculate $\text{factorial}(n)$ as:

$$\text{factorial}(n) = n \times \text{factorial}(n-1)$$

Factorial function

```
# recursive factorial function
def factorial(n):
    if (n==1):
        return 1
    else:
        return n * factorial(n-1)
```

Factorial function

```
# recursive factorial function
```

```
def factorial(n):
```

```
    if (n==1):  
        return 1
```

```
    else:
```

```
        return n * factorial(n-1)
```

base step



Factorial function

```
# recursive factorial function
```

```
def factorial(n):
```

```
    if (n==1):
```

```
        return 1
```

```
    else:
```

```
        return n * factorial(n-1)
```

← recursion
step

Factorial function

```
# recursive factorial function
```

```
def factorial(n):
```

```
    if (n==1):
```

```
        return 1
```

```
    else:
```

```
        return n * factorial(n-1)
```

```
for i in range(1,10):
```

```
    print("{0}! = {1}".format(i, factorial(i)))
```

1! = 1

2! = 2

3! = 6

4! = 24

5! = 120

6! = 720

7! = 5040

8! = 40320

9! = 362880

Fibonacci sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, ...

```
fibonacci(0) = 0
fibonacci(1) = 1
fibonacci(2) = 1
fibonacci(3) = 2
fibonacci(4) = 3
fibonacci(5) = 5
fibonacci(6) = 8
fibonacci(7) = 13
fibonacci(8) = 21
```

Fibonacci sequence

<code>fibonacci(0)</code>	<code>=</code>	0
<code>fibonacci(1)</code>	<code>=</code>	1
<code>fibonacci(2)</code>	<code>=</code>	1
<code>fibonacci(3)</code>	<code>=</code>	2
<code>fibonacci(4)</code>	<code>=</code>	3
<code>fibonacci(5)</code>	<code>=</code>	5
<code>fibonacci(6)</code>	<code>=</code>	8
<code>fibonacci(7)</code>	<code>=</code>	13
<code>fibonacci(8)</code>	<code>=</code>	21

If we know

`fibonacci(6) = 8` and `fibonacci(7) = 13`,

how can we calculate `fibonacci(8)` ?

$$\begin{aligned}\text{fibonacci}(8) &= \text{fibonacci}(6) + \text{fibonacci}(7) \\ &= 8 + 13 = 21\end{aligned}$$

Fibonacci sequence

```
fibonacci(0) = 0
fibonacci(1) = 1
fibonacci(2) = 1
fibonacci(3) = 2
fibonacci(4) = 3
fibonacci(5) = 5
fibonacci(6) = 8
fibonacci(7) = 13
fibonacci(8) = 21
```

In general, we can calculate $\text{fibonacci}(n)$ based on $\text{fibonacci}(n-1)$ and $\text{fibonacci}(n-2)$ as:

$$\text{fibonacci}(n) = \text{fibonacci}(n-1) + \text{fibonacci}(n-2)$$

Fibonacci sequence

```
# recursive function to calculate Fibonacci sequence
def fibo(n):
    if (n==0):
        return 0
    elif (n==1):
        return 1
    else:
        return fibo(n-1) + fibo(n-2)
```

Fibonacci sequence

recursive function to calculate Fibonacci sequence

```
def fibo(n):
```

```
    if (n==0):
```

```
        return 0
```

```
    elif (n==1):
```

```
        return 1
```

```
    else:
```

```
        return fibo(n-1) + fibo(n-2)
```

base step



Fibonacci sequence

```
# recursive function to calculate Fibonacci sequence
```

```
def fibo(n):
```

```
    if (n==0):
```

```
        return 0
```

```
    elif (n==1):
```

```
        return 1
```

```
    else:
```

```
        return fibo(n-1) + fibo(n-2)
```

recursion
step



Fibonacci sequence

recursive function to calculate Fibonacci sequence

```
def fibo(n):  
    if (n==0):  
        return 0  
    elif (n==1):  
        return 1  
    else:  
        return fibo(n-1) + fibo(n-2)  
  
for i in range(0,10):  
    print("fibo({0}) = {1}".format(i, fibo(i)))
```

fibo(0)	=	0
fibo(1)	=	1
fibo(2)	=	1
fibo(3)	=	2
fibo(4)	=	3
fibo(5)	=	5
fibo(6)	=	8
fibo(7)	=	13
fibo(8)	=	21
fibo(9)	=	34

Translate numerical code into words

“0278” → “zero-two-seven-eight”

“5” → “five”

“” → “”

“2000” → “two-zero-zero-zero”

Translate numerical code into words

Break the code into two parts

Part 1: the first character

Part 2: the rest of the string

Translate each part into words

And combine them

“2000”

“2” → **“two”**

“000” → **“zero-zero-zero”**

“two-zero-zero-zero”

Translate numerical code into words

We can only break the code into two parts
if it has at least two digits

base step: if the code is empty or has 1 digit

recursion step: if the code has at least 2 digits

Translate numerical code into words

base step: if the code is empty or has 1 digit

- if the code is empty: easy
- if the code has 1 digit: ???

Translate numerical code into words

- if the code has 1 digit:

```
NUMBER_MAPPING_DICT = {  
    "0": "zero",  
    "1": "one",  
    "2": "two",  
    "3": "three",  
    "4": "four",  
    "5": "five",  
    "6": "six",  
    "7": "seven",  
    "8": "eight",  
    "9": "nine"  
}
```

```
# translate a digit into word using dictionary  
def digit_to_word(digit):  
    word = NUMBER_MAPPING_DICT[digit]  
    return word
```

Translate numerical code into words

```
# translate numerical code into words
def numerical_to_word(numerical_string):
```

```
    if (len(numerical_string) == 0):
```

```
        # empty string
```

base step

```
    elif (len(numerical_string) == 1):
```

```
        # only 1 digit
```

```
    else:
```

```
        # at least 2 digits
```

recursion
step

Translate numerical code into words

```
# translate numerical code into words
def numerical_to_word(numerical_string):
```

```
    if (len(numerical_string) == 0):
```

```
        # empty string
```

```
        return ""
```

```
    elif (len(numerical_string) == 1):
```

```
        # only 1 digit
```

```
    else:
```

```
        # at least 2 digits
```


Translate numerical code into words

```
# translate numerical code into words
def numerical_to_word(numerical_string):

    if (len(numerical_string) == 0):
        # empty string

        return ""

    elif (len(numerical_string) == 1):
        # only 1 digit
        word = digit_to_word(numerical_string)

        return NUMBER_MAPPING_DICT[numerical_string]

    else:
        # at least 2 digits
```

Translate numerical code into words

else:

```
# at least 2 digits

# break the string into two parts:

# the first character
part1 = numerical_string[0]

# substring from the second character
part2 = numerical_string[1:]

# translate the two parts into words
part1_word = numerical_to_word(part1)
part2_word = numerical_to_word(part2)

# combine them
word = part1_word + "-" + part2_word

return word
```

recursion
step



Translate numerical code into words

```
# main program

# ask user for a numerical string
user_input = input("Enter a numerical string: ")

# translate into english words
word = numerical_to_word(user_input)

# display it
print(word)
```