

Exercise 8

Lo I want a maximum so

$$f(x) = -\gamma^T x$$

Lo KKT with no  $h_j$  and  $\mu^*$ . Only one  $g_i$  and  $\lambda^*$

$$g(x) = 1 - x^T Q x$$

$$\mathcal{L}(x^*, \lambda^*) = f(x) - \lambda^* g(x^*); \quad \nabla \mathcal{L}(x^*, \lambda^*) = -\gamma + 2\lambda^* Q x^*$$

$$\text{so } \nabla_x \mathcal{L}(x^*, \lambda^*) = 0 \Leftrightarrow \left[ x^* = \frac{1}{2\lambda^*} Q^{-1} \gamma \right] (1)$$

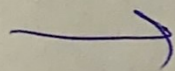
$$\rightarrow \text{impose } \begin{cases} \lambda^* g(x^*) = 0 \\ \lambda^* \geq 0 \end{cases} \Leftrightarrow g(x^*) = 0 \quad (\lambda^* \text{ can't be zero; } \nabla_x \mathcal{L} = \gamma \neq 0)$$

$\rightarrow$  therefore

$$1 = (x^*)^T Q x \Leftrightarrow$$

$$1 = \left( \frac{1}{2\lambda^*} \right)^2 \cdot (\gamma^T Q^{-1}) \cdot Q \cdot (Q^{-1} \gamma) = \left( \frac{1}{2\lambda^*} \right)^2 \gamma^T Q^{-1} \gamma$$

$$\Rightarrow 2\lambda^* = \sqrt{\gamma^T Q^{-1} \gamma}; \text{ using (1)} \Rightarrow \left[ x^* = \frac{1}{\sqrt{\gamma^T Q^{-1} \gamma}} Q^{-1} \gamma \right]$$



continue

$$\text{II) } y^T x^* = \frac{1}{\sqrt{y^T Q^{-1} y}} \quad y^T Q^{-1} y = \sqrt{y^T Q^{-1} y}$$

III) From  $0 = \nabla_x \mathcal{L}(x^*, \lambda^*)$  we obtain

$$y = 2\lambda^* Q x^* \Rightarrow$$

$$(x^T y)^2 = (2\lambda^*)^2 (x^T Q x^*)^2 = (y^T Q^{-1} y) \langle x, x^* \rangle_Q \leq$$

→ With Cauchy-Schwarz

$$\leq (y^T Q^{-1} y) \cdot \langle x, x \rangle_Q \cdot \underbrace{\langle x^*, x^* \rangle_Q}_1 =$$

$$= (y^T Q^{-1} y) (x^T Q x)$$