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Exercise 7

↳ Function $f(x,y) = (x-a)^2 + (y-b)^2 + xy$ is C^∞

↳ The restriction in this case is a compact square with vertices $(0,0)$, $(0,1)$, $(1,1)$, $(1,0)$.

↳ Given that this is a compact square, a minimum (and maximum) must be found. They will be global and inside this square. That's what the theorem of Weierstrass tells us.

↳ those extrems could be:

1. Inside the square. then ∇f would be 0 on that local(s) extreme(s).

2. On the edges of the square. then we can use Lagrange or design an auxiliar function with one variable and compute its derivative and $=0$.

3. On the vertices of the square.

⟶

continues

↳ Case 1:

$\nabla f(p_0) = (2(x-a)+y,\ 2(y-b)+x)^T$

$\nabla f(p_0) = 0 \Rightarrow P_1 = \left(\frac{2}{3}(2a-b),\ \frac{2}{3}(2b-a)\right)$]

Depending on a and b, it might be inside or outside our square.

↳ Case 2:

i) On the edge $x=0$, we take $g_1(z) = f(0,z) = a^2 + (z-b)^2$; $g_1'(z) = 2(z-b)$ which is zero when $z=b$. then $P_2 = (0,b)$]

ii) On the edge $x=1$, we take $g_2(z) = f(1,z) = (1-a)^2 + (y-b)^2 + y$; $g_2'(z) = 2(z-b)+1$

$g_2'(z) = 0 \Leftrightarrow z = \frac{2b-1}{2} \Rightarrow P_4 = \left(1, \frac{2b-1}{2}\right)$]

iii) On the edge $y=0$, for symmetry, $P_3 = (a,0)$]

iv) " " " $y=1$, " ", [$P_5 = \left(\frac{2a-1}{2}, 1\right)$

continues

↳ Case 3:

$P_6 = (0,0)$, $P_7 = (1,0)$, $P_8 = (0,1)$, $P_9 = (1,1)$

→ Let's evaluate this points on f. The one will be the minimum.

$f(p_1) = \frac{1}{3}(4ab - a^2 - b^2)$

$f(p_2) = a^2$ $\quad f(p_3) = b^2$

$f(p_4) = (1-a)^2 + 1/4 + \frac{2b-1}{2}$

$f(p_5) = (1-b)^2 + 1/4 + \frac{2a-1}{2}$

$f(p_6) = a^2 + b^2$

↳ Will never be minimum because $f(p_6) > f(p_2)$ and $> f(p_3)$. If $a = b = 0$ then $p_6 = p_2 = p_3$; in this case this point will be the minimum beause $f(x,y) \geq 0$ always.

$f(p_7) = (1-a)^2 + b^2$

↳ Won't be minimum, $f(p_7) > f(p_3)$. If $a = 1$; when $p_7 = p_3$

→ continue

$f(p_8) = a^2 + (1-b)^2$

↳ Same, $f(p_8) > f(p_2)$. When $b = 1$ then

$p_8 = p_2$

$f(p_9) = (1-a)^2 + (1-b)^2 + 1$

<u>Examples</u>

↳ So we take points 1, 2, 3, 4, 5 and 9.

↳ $a = b = 1$

$p_1 = (2/3, 2/3)$ which is inside our square

$f(p_1) = 2/3$; $f(p_2) = f(p_3) = f(p_9) = 1 > 2/3$;

$f(p_4) = f(p_5) = 3/4 > 2/3$.

↳ $a = 0$

$p_2 = (0, 1)$; $f(p_2) = 0$ and $f(x, y) \geq 0 \ \forall (x, y)$

↳ Symmetry, $b = 0$ then p_3 is the minimum.

↳ $a = b = 2$; p_4 and p_5 are the minimum:

$f(p_4) = f(p_5) = 11/4$; $f(p_2) = f(p_3) = 4 > 11/4$ $f(p_9) = 3 > 11/4$

$f(p_1) \notin$ square

↳ $a = b = 3$; p_9 is the minimum.

$f(p_9) = 9$; p_1, p_2, p_3, p_4, p_5 not in the square.