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Exercise 8

↳ I want a maximum so

$$f(x) = -y^T x$$

↳ KKT with no h_j and μ^*. Only one g_i and λ^*

$$g(x) = 1 - x^T Q x$$

$\mathcal{L}(x^*, \lambda^*) = f(x) - \lambda^* g(x^*)$; $\nabla \mathcal{L}(x^*, \lambda^*) = -y + 2\lambda^* Q^* x^*$

So $\nabla_x \mathcal{L}(x^*, \lambda^*) = 0 \iff \left[x^* = \frac{1}{2\lambda^*} Q^{-1} y \right]$ (1)

→ Impose $\begin{cases} \lambda^* g(x^*) = 0 \\ \lambda^* \geq 0 \end{cases} \iff g(x^*) = 0$ (λ^* can't be zero; $\nabla_x \mathcal{L} = y \neq \vec{0}$

→ therefore

$$1 = (x^*)^T Q x \iff$$

$$1 = \left(\frac{1}{2\lambda^*}\right)^2 \cdot (y^T Q^{-1}) \cdot Q \cdot (Q^{-1} y) = \left(\frac{1}{2\lambda}\right)^2 y^T Q^{-1} y$$

$\Rightarrow 2\lambda^* = \sqrt{y^T Q^{-1} y}$; Using (1) $\Rightarrow \left[x^* = \frac{1}{\sqrt{y^T Q^{-1} y}} Q^{-1} y \right]$

⟶

continues

II) $y^T x^* = \frac{1}{\sqrt{y^T Q^{-1} y}} y^T Q^{-1} y = \sqrt{y^T Q^{-1} y}$

III) From $0 = \nabla_x \mathcal{L}(x^*, \lambda^*)$ we obtain

$y = 2\lambda^* Q x^* \implies$

$$(x^T y)^2 = (2\lambda^*)^2 (x^T Q x^*)^2 = (y^T Q^{-1} y) \langle x, x^* \rangle_Q \leq$$

$\to$ With Cauchy - Schwartz

$$\leq (y^T Q^{-1} y) \cdot \langle x, x \rangle_Q \cdot \underbrace{\langle x^*, x^* \rangle_Q}_{1} =$$

$$= (y^T Q^{-1} y)(x^T Q x)$$