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Exercise 6

↳ We know that the minimum of $f(x)$ in $x_0 + \langle z_1, \dots, z_k \rangle$ is x^k. [1]

↳ We also know that, in general, $\dim \langle z_1, \dots, z_k \rangle = k$ (i.e. if $z^j \neq 0 \ \forall_j$) [2]

↳ then, if $\langle z_k^1, \dots, z_k \rangle = \langle v^0, \dots, v^{k-1} \rangle$, the exercise will be done.[1] Assume that $T_k = \langle v_0, \dots, v^{k-1} \rangle$ and $S_k = \langle z^1, \dots, z^k \rangle$. They tell us that $T_k \leq k$, so to see that $S_k = T_k$ we need to prove, using [2], $S_k \subset T_k$.

↳ We will use hypotesis induction, using $\nabla f = Ax - b$

① First case, $k=1$: $z^1 = -v_0 = -\nabla f(x_0)$

② Second case, $k=2$: $z^2 = -\nabla f(x_1) + \lambda_1 z_1 = b - Ax_1 + \lambda_1 z_1$

if $x_1 = x_0 + \alpha_1 z_1$, then; $= b - Ax_0 + (\lambda_1 - \alpha_1 A) z_1 =$

$(z_1 = -v_0)$ $= -\nabla f(x_0) + \lambda_1 v_0 + \alpha_1 A \cdot v_0 = (\lambda_1 - 1) \cdot v_0 + \alpha_1 v_1 \in T_2$

linear combination

$\longrightarrow$

continues

↳ Let's suppose it's true for $k = 1, 2, \dots k-1$. Then we prove it for k:

↳ We have:

$$z^k = -\nabla f(x^{k-1}) + \underbrace{\lambda_k z^{k-1}}$$

z^{k-1} is conditional of $v_0, \dots, v^{k-2}$ so we know that is already a linear combination.

↳ then

$$\nabla f(x^{k-1}) = Ax^{k-1} - b = A(x^{k-2} + \alpha_{k-1} z^{k-1}) - b =$$

$$= \underbrace{Ax^{k-2} - b}_{(1)} + \underbrace{\alpha_{k-2} \cdot Az^{k-1}}_{(2)}$$

• (2) z^{k-1} is conditional of $v^0, \dots v^{k-2} \Rightarrow$ (2) is conditional of $Av_0, Av^{k-2}, \dots, = v_1, \dots v_{k-1} \Rightarrow (2) \in T_k$

• $(1) = \nabla f(x^{k-2}) = -z^{k-1} + \lambda_{k-2} z^{k-2}$

↳ Hypotesis induction: z^{k-2} is conditional of $v_0, \dots, v^{k-2}$

↳ therefore: $T_{k-1} \in T_k$

$$\Rightarrow z^k \in T_k \underset{(H.I)}{\Rightarrow} S_k \in T_k \Rightarrow S_k = T_k \,]$$

↑ "inside"