Optimization

Master on Fundamentals of Data science
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Exercise 10

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Prove that the number of faces of dimension p of a n-dimensional simplex is equal to

$$\binom{n+1}{p+1} = \frac{(n+1)!}{(p+1)!(n-p)!}$$

By definition a simplex S of dimension n is a convex polyhedron of $n+1$ vertices.

Also, by definition, a face of dimension p of the simplex $(0 \leq p \leq n)$ is any p-simplex defined by $p+1$ vertices of S. Also, by convexity, is a subset of S's.

Consequently, there's as p-faces as subsets of $p+1$ elements that we can build with the $n+1$ vertices of S. In combinatorial syntaxis, this means "the number of combinations of $n+1$ elements choosing by $p+1$ in $p+1$", also expressed as:

$$(C_{p+1}^{n+1}) := \binom{n+1}{p+1} := \frac{(n+1)!}{(p+1)!((n+1)-(p+1))!} := \frac{(n+1)!}{(p+1)!(n-p)!}$$