We had:

$$A^{\lambda} \cdot B^{1-\lambda} \overset{?}{\leq} A\lambda + (1-\lambda)B$$

↳ Let's divide by B (aka multiply by B^{-1}) because $B > 0$

$$A^{\lambda} \cdot B^{1-\lambda} \cdot B^{-1} \overset{?}{\leq} \lambda \frac{A}{B} + (1-\lambda)\frac{B}{B} \Rightarrow$$

$$\Leftrightarrow \quad A^{\lambda} \cdot B^{-1} \overset{?}{\leq} \lambda \cdot R + 1 - \lambda$$

$$\Leftrightarrow \quad \left(\frac{A}{B}\right)^{\lambda} \overset{?}{\leq} \lambda \cdot (R-1) + 1$$

$$\Leftrightarrow \quad R^{\lambda} \overset{?}{\leq} \lambda(R-1) + 1$$

$$\Leftrightarrow \quad \lambda \cdot (1-R) \overset{?}{\leq} 1 - R^{\lambda}$$

$$\Leftrightarrow \quad \lambda \overset{?}{\leq} \frac{1-R^{\lambda}}{1-R}$$

↳ This inequality is true because:

$\lambda \leq 1$; then $R^{\lambda} \leq R$

$$\Rightarrow -R^{\lambda} \geq -R \Rightarrow 1 - R^{\lambda} \geq 1 - R$$

$$\Rightarrow \frac{1-R^{\lambda}}{1-R} \geq 1$$

→ Remark that $0 < R < 1$ then $1 - R > 0$

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Exercise 3

↳ Using the convex condition seen in class:

$$f(\lambda x + (1-\lambda) y) < \lambda f(x) + (1-\lambda) f(y)$$

where $x \neq y$ and $0 < \lambda < 1$

↳ Then:

$$e^{a(\lambda x + (1-\lambda) y)} \overset{?}{\leq} \lambda e^{ax} + (1-\lambda) e^{ay}$$

↳ I take:

$$A = e^{ax} \; ; \; B = e^{ay} \qquad R = \frac{A}{B}$$

where $0 < A < B$; $0 < R < 1$

↳ > 0 because exponential law

and < 1 because if $R = 1 \Rightarrow A = B$

and it will be a trivial case

↳ Then:

$$e^{a(\lambda x + (1-\lambda) y)} \overset{?}{\leq} A \cdot \lambda + (1-\lambda) \cdot B$$

$$\Longleftrightarrow \quad e^{a\lambda x} \cdot e^{a(1-\lambda) y} \overset{?}{\leq} A \cdot \lambda + (1-\lambda) B$$

$$\Longleftrightarrow \quad A^{\lambda} \cdot B^{1-\lambda} \overset{?}{\leq} A\lambda + (1-\lambda) B$$

continues