Lab 4: Continuous Random Variables

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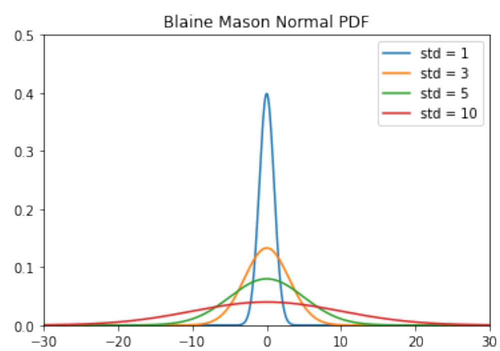
Purpose: In this lab we will use the concept of continuous random variables to softly introduce the concept of The Central Limit Theorem. The intention of this is to relate the concept of density with probability. Throughout this lab I often saw the concept of larger samples from a given distribution being the Integral of a function from a to b.

Introduction

Small confusion exists when looking at the plot of a continuous distribution and random samples from a continuous distribution. This is due to the number of bins and the fact sampling is random observations defined by the distribution itself. There are two distributions that will be reviewed in this lab and at we will observe the convergence of samples to the respective pdf. Those two are distributions are the Normal and Exponential distributions respectively.

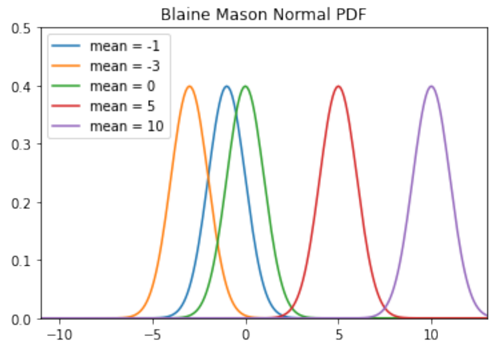
Normal Distribution

1. Graph of $\mathcal{N}[0,1], \mathcal{N}[0,3], \mathcal{N}[0,5], \mathcal{N}[0,10]$



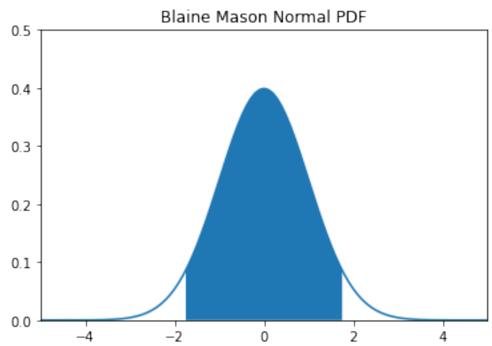
2. Explanation: The data becomes more spread out as the standard deviation increases. The maximum value for Standard normal is approximately 0.4. The maximum value for Normal distribution with mean 0 and standard deviation 5 is approximately .1. The standard deviation being 0 implies all the possible values for the random variable would be a set with a cardinality of 1.

3. Graph of $\mathcal{N}[-1,1], \mathcal{N}[-3,1], \mathcal{N}[0,1], \mathcal{N}[5,1], \mathcal{N}[10,1]$



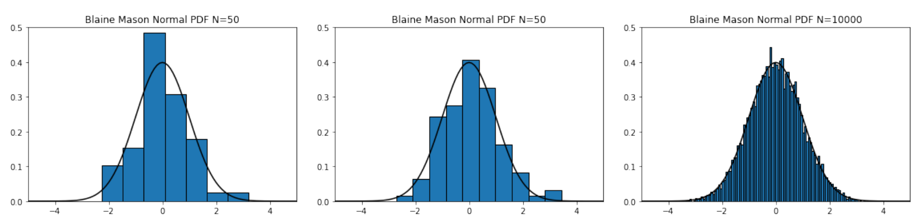
4. Explanation: The center of the distribution moves with respect to the value of the mean.

5. Graph of $Pr(-a < Z < a) = .92$



6. Let $X\ \mathcal{N}(68, 2.5), Pr(64 \leq X \leq 70) = \int_{64}^{70} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2} dx \approx .73$

7. Histograms of n=10, 100, 10000



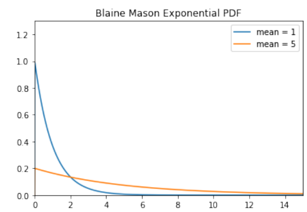
8. Explanation: The histogram converges to fit under the standard normal distribution as the samples increase.

Exponential Distribution

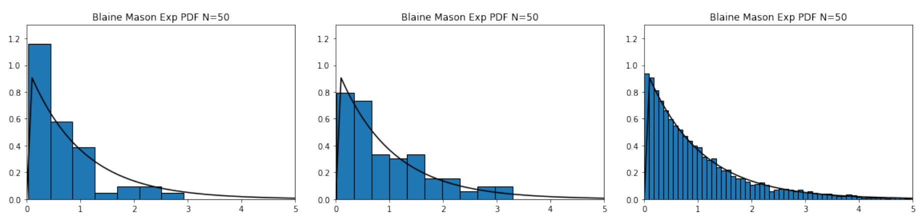
1. Using the PDF of Exponential Distribution $f(x)$

$$\lim_{x->\infty} f(x) \approx 0$$

2. Graphs of $Exp(1), Exp(5)$



3. Let X $Exp(6), Pr(5 \leq X \leq 5) = \int_5^7 \frac{1}{6} e^{\frac{-x}{6}} \approx 0.1232$
4. Threshold: Changes the domain to instead begin at the threshold value instead of always 0.
5. Histograms of n=10, 100, 10000



6. Explanation: The histogram converges to fit under the exponential pdf as the samples increase.

Example of Normal Distribution

Height is Normally distributed since there can be a general height for a given population that most of the data is centered around. This population would vary based on who is sampled and that's where the variance could be determined.

Example of Exponential Distribution

Time between arrival of customers would be Exponentially distributed. If we assume λ customers come in every 3 minutes then using the CDF we can get a probability back that in less than a minute a new customer will arrive.