Bachelor of Software Engineering - Game Programming

GD2P02 – Physics Programming

Angular Motion

Overview

- Angular Motion
  - Circular Motion
  - Centripetal Force
  - Moment of Inertia
  - Angular momentum
  - Inertia Tensor

Reminder for Newton's Laws

1) A body continues in its state of rest or of uniform speed in a straight line unless it is compelled to change that state by forces acting on it (Law of Inertia).

2) The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass. The direction of the acceleration is in the direction of the applied net force ($F = ma$).

3) Whenever one object exerts a force on a second object the second exerts an equal and opposite force on the first.

Circular Motion

- Circular motion explains the motion of objects that move in circles.
- Also known as angular motion.
- Circular movement follows a circular path.

- Uniform Circular motion
  - Constant speed, but not constant velocity!
  - Average Speed = distance / time
    - circumference = 2 * PI * radius

Circular Motion continued…

- Motion around a point
- The larger the radius, the larger the speed.
  - Average speed = 2 * PI * radius / time

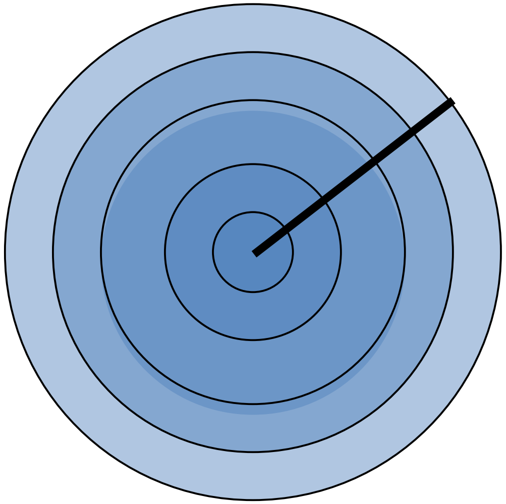

Fig 1: Circles with string rotating from centre.

Velocity Vector

- Speed is constant, but velocity is not!
  - Direction of the velocity vector changes along the motion.
  - Velocity vector is in the same direction the object is moving, tangential to the motion path at any instant time.

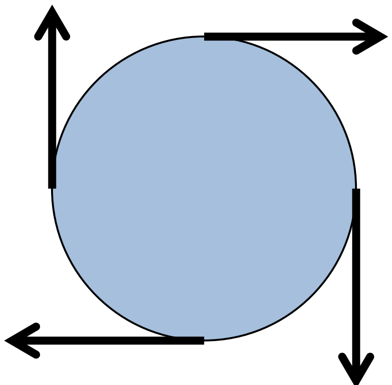

Fig 2: Velocity vectors at various tangents.

Acceleration

- Acceleration is the change in velocity.
  - Therefore, acceleration is present even though speed is constant.
  - Acceleration causes the change in direction!
  - The direction of the acceleration vector is towards the centre of the circle.

Average acceleration = (change in velocity) / time

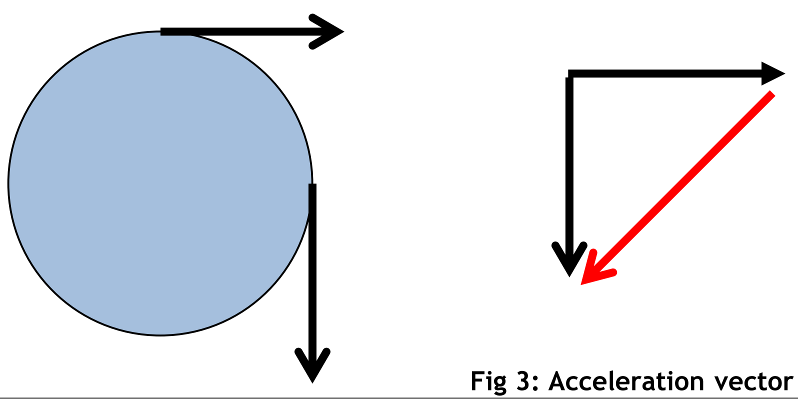

Fig 3: Acceleration vector (red) based upon (vf - vi) vector.

Centripetal Force

- Newton's Second Law: **F = m * a**
- Then, there is a net force in the system that is doing a circular motion.
- Net force is in the same direction with acceleration.
  - Centripetal Force
    - Calls the object back to the centre of the motion path, centre of the circle.
    - Centre seeking
    - Direction of the force is toward the centre
  - Centrifugal Force: not a real force, but it results from inertia. Tendency to fly out… (Newton's Third Law)

Centripetal Force: Angular Distance

- One revolution is 360 degrees.
- Arc length = 2 * PI *radius

  - Theta = arc length / radius
  - $\Theta = s / r$
  - $s = 2 * PI * r$, for one revolution

  Therefore,

  - $\Theta = 2*PI$ radians for a whole revolution

Centripetal Force: Angular Velocity

- How far the object rotates in a given amount of time.
  - The amount of angle the object covers…
  - Angular velocity: ω
  - Average $\omega = \Theta$ / time
    - Θ in here is the angular amount the object has covered within the time frame.
    - Θ is in radians
    - Unit of ω is radians per second.
  - The average angular velocity is:

$$\omega = (\omega_f + \omega_i) / 2$$

Centripetal Force: Angular Acceleration

- Recalling linear acceleration:

  $$a = (v_f - v_i) / t$$

- Average angular acceleration:

  $$\alpha = (\omega_f - \omega_i) / t$$

  - Unit is radians per second per second: radians*sec^{-2}

GD2P02

MEDIA DESIGN SCHOOL GAME DEV

Calculating the Centripetal Force

- Centripetal Force = F_{cent}
- Centripetal Acceleration: $a_{cent} = v^2 / r$

  $a_{cent} = \omega^2 r$

- Using Newton's second law:
  » $F_{cent} = m*a_{cent}$
  » $F_{cent} = m\,(v^2 / r) = m\,(\omega^2 r)$

Moment of Inertia in angular motion

- The resistance to a change in angular velocity about an axis of rotation

$$\tau = I\alpha$$

  - τ: Torque
  - I is the moment of inertia.
  - α is the angular acceleration (in radians per sec^2)

- Moment of inertia of a point mass is:

$$I = mr^2$$

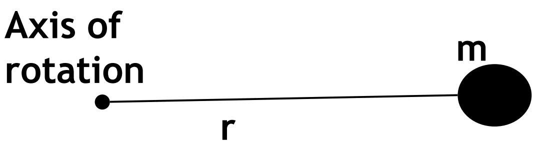

Fig 4: Moment of inertia of a point mass.

MEDIA DESIGN SCHOOL GAME DEV

Angular Momentum

- Linear Momentum: $p = mv$
  - Force is the rate at which linear momentum changes.
  - External force is required to change momentum.
  - No external force, means linear momentum is conserved.
- Angular Momentum: $L = I\omega$
  - The law of conservation of angular momentum
    - Final angular momentum = initial angular momentum

$$I_f\omega_f = I_i\omega_i$$

MEDIA DESIGN SCHOOL GAME DEV

Inertia Tensor

- This is a matrix that holds the moments of inertia information about the three coordinate axes.
- *I*: 3x3 matrix (the inertia tensor)
  - Rate at which the object spins around each axis:

    $< \omega_x, \omega_y, \omega_z >$
  - If each element is zero, the object is not spinning.

- Angular Momentum: $L = I\omega$
  - Vector with three elements, angular momentum around each axis.

Computing Inertia Tensor

- Inertia is different for different shapes.
  - Common shapes: Sphere, pyramid, cube…
    - Use a look up table…
  - Or calculate the inertia tensor matrix from the geometry of the object…
    - http://www.livephysics.com/physical-constants/mechanics-pc/moment-inertia-uniform-objects/

- Total inertia matrix can be calculated from the sum of smaller inertias.
    - Divide the shape into smaller simpler pieces.
- Non moving objects: null matrix!

Kinetic Energy

- Rotational Kinetic Energy:
  - How much work must be done to set the body in motion…
- Linear Kinetic Energy (Translational Kinetic Energy):

  $E_k = (1/2)mv_f^2$
- Rotational Kinetic Energy:

  $E_k = (1/2)I\omega_f^2$

  Where:
  - I = Moment of Inertia
  - ω_f = Angular velocity
- Total E_k: $E_{k(total)} = E_{k(translational)} + E_{k(rotational)}$

Linear and Rotational Motion equations

Linear Kinematics

$$v_f = v_i + at$$

$$d = ½(v_f + v_i)t$$

$$d = v_i t + ½at^2$$

$$v_f^2 = v_i^2 + 2ad$$

Angular Kinematics

$$\omega_f = \omega_i + \alpha t$$

$$\Theta = ½(\omega_f + \omega_i)t$$

$$\Theta = \omega_i t + ½\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Theta$$

- α: angular acceleration
- Θ: angle covered in radians
- ω: angular speed

- **$\alpha = v^2/r$**
- **$\omega * r = V$**

Where, **ω** is angular speed and ***V*** is linear speed.

MEDIA DESIGN SCHOOL GAME DEV

Summary

- Angular Motion
  - Circular Motion
  - Centripetal Force
  - Moment of Inertia
  - Angular momentum
  - Inertia Tensor

GD2P02

MEDIA DESIGN SCHOOL GAME DEV