Philipps Universität Marburg

**Deadline: 17.02.2014**

Submit in box 137 outside the toilets of Kern D.

**Anonymous questionnaire about exercises**
**of the lecture "Programming languages and types"**

- DO NOT write your name or student number (Matrikelnummer) on this page.
- DO NOT staple this page together with other pages of your homework submission.

The questionnaire is for statistics only and is in no way connected to the evaluation of your homework submission. By filling out this questionnaire, you declare to have answered as truthfully as your knowledge permits.

1. *Outside* the exercise session, how many hours did you spend doing the homework?

   _____ hours.

2. In your opinion, how many hours would it take an average student to do the homework?

   _____ hours.

3. How many lecture sessions did you attend this week?

   ☐ None. ☐ One session. ☐ Two sessions.

4. Did you attend the exercise session on 10.02.2014?

   ☐ Yes. ☐ No.

5. Please cross *all* statements that are true about you.

   ☐ I can move the subsumption rule in typing derivations.
   ☐ I understand why it is necessary to use subsumption in a systematic way.
   ☐ I can infer the type annotations of simply typed lambda terms by hand.
   ☐ I implemented systematic type inference for simply typed lambda calculus.

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| | |
|---|---|
| Matrikelnummer: | |
| Name: | |

**Homework 14 of the lecture "Programming languages and types"**

Solutions are available on 17.02.2014 at
`https://www.uni-marburg.de/fb12/ps/teaching/ws13/plt-solutions/`

1. (a) These are some of the typing and subtyping rules of simply-typed lambda calculus with records and subtyping.

$$\frac{x : T \in \Gamma}{\Gamma \vdash x : T}\ \text{(T-Var)} \qquad \frac{\Gamma \vdash t : \{l_i : T_i \quad ^{i \in 1..n}\}}{\Gamma \vdash t.l_j : T_j}\ \text{(T-Proj)}$$

$$\frac{\Gamma, x : T_1 \vdash t_2 : T_2}{\Gamma \vdash (\lambda x : T_1.\ t_2) : T_1 \to T_2}\ \text{(T-Abs)} \qquad \frac{\Gamma \vdash t_1 : T_{11} \to T_{12} \quad \Gamma \vdash t_2 : T_{11}}{\Gamma \vdash t_1\ t_2 : T_{12}}\ \text{(T-App)}$$

$$\frac{\Gamma \vdash t : S \quad S <: T}{\Gamma \vdash t : T}\ \text{(T-Sub)} \qquad \frac{T_1 <: S_1 \quad S_2 <: T_2}{(S_1 \to S_2) <: (T_1 \to T_2)}\ \text{(S-Arrow)}$$

Define the following type synonyms:

$$\begin{aligned}
\text{type Scalar} &= \{\text{magnitude} : \text{Real}\} \\
\text{type Vector} &= \{\text{magnitude} : \text{Real}, \text{direction} : \text{UnitVector}\} \\
\text{type Particle} &= \{\text{velocity} : \text{Vector}\} \\
\text{type Sphere} &= \{\text{velocity} : \text{Vector}, \text{diameter} : \text{Real}\} \\
\text{type Planet} &= \{\text{velocity} : \text{Vector}, \text{diameter} : \text{Real}, \text{mass} : \text{Real}\}
\end{aligned}$$

Starting from $\Gamma_0 = \{\text{earth} : \text{Planet}\}$, fill out the rules used in the typing derivation on page 2.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p : \text{Particle} \in \Gamma_0 \cup \{p : \text{Particle}\}}\ (\text{T-Var})}{\Gamma_0, p : \text{Particle} \vdash p : \text{Particle}}\ (\quad)}{\Gamma_0, p : \text{Particle} \vdash p.\text{velocity} : \text{Vector}}\ (\quad) \quad \dfrac{}{\text{Vector} <: \text{Scalar}}\ (\text{T-RcdWidth})}{\Gamma_0, p : \text{Particle} \vdash p.\text{velocity} : \text{Scalar}}\ (\quad)}{\Gamma_0 \vdash (\lambda p : \text{Particle}.\ p.\text{velocity}) : \text{Particle} \to \text{Scalar}}\ (\quad) \quad \dfrac{\text{Sphere} <: \text{Particle} \qquad \text{Scalar} <: \text{Scalar}}{(\text{Particle} \to \text{Scalar}) <: (\text{Sphere} \to \text{Scalar})}\ (\quad)}{\Gamma_0 \vdash (\lambda p : \text{Particle}.\ p.\text{velocity}) : \text{Sphere} \to \text{Scalar}}\ (\quad)$$

$$\dfrac{\dfrac{}{\Gamma_0 \vdash (\lambda p : \text{Particle}.\ p.\text{velocity}) : \text{Sphere} \to \text{Scalar}}\ (\text{see above}) \quad \dfrac{\dfrac{}{\text{earth} : \text{Planet} \in \Gamma_0}\ (\text{T-Var})}{\Gamma_0 \vdash \text{earth} : \text{Planet}} \quad \dfrac{}{\text{Planet} <: \text{Sphere}}\ (\text{S-RcdWidth})}{\Gamma_0 \vdash \text{earth} : \text{Sphere}}\ (\quad)}{\Gamma_0 \vdash ((\lambda p : \text{Particle}.\ p.\text{velocity})\ \text{earth}) : \text{Scalar}}\ (\quad)$$

(b) How many times is the subsumption rule T-Sub used in the typing derivation on page 2?

(c) Choose *one* from the following tasks and do it:

i. Rewrite the derivation on page 2 so that T-Sub is used nowhere except at the bottom, or right above T-App on the right.

ii. Rewrite the derivation on page 2 so that T-Sub is used only once.

(d) Why is it important that typing derivations can always be rewritten, so that T-Sub is used nowhere except at the bottom or above T-App?

2. Complete the implementation of type inference for simply typed lambda calculus below.
`https://github.com/klauso/PLT2013/blob/master/exercises/ex14/unification.scala`