

[DRAFT] The exDOT Calculus

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This document presents gDOT, and the additions made by exDOT are highlighted in gray.

Syntax			
x, y, z	Variable	$S, T, U, V, W ::=$	Type
$l ::=$	Label	\top	top type
L	Type label	\perp	bottom type
m	Method label	$\{D\}$	one-member record
$t, u ::=$	Term	$x.L$	type selection
x	variable	$T \wedge T$	intersection type
$\mathbf{val} \ x = \mathbf{new} \ \{\bar{d}\}; \ t$	new instance	$T \vee T$	union type
$t.m(u)$	method invocation	$x.\mathbf{type}$	singleton type
$d ::=$	Initialization	$\exists(x : T)U$	existential type
$L = T$	field init.	$D ::=$	Declaration
$m(x : T) : U = u$	method init.	$L : S..U$	abstract type decl.
$\Gamma ::= \overline{x : T}$	Environment	$m : S \rightarrow U$	method declaration
$s ::= \overline{x \mapsto \{\bar{d}\}}$	Store		

Reduction	$t \mid s \rightarrow t' \mid s'$
$\frac{x \mapsto \left\{ \overline{L = W} \ \overline{m(z : T) : U = u} \right\} \in s}{x.m_i(y) \mid s \rightarrow [y/z_i]u_i \mid s} \quad (\text{RED-CALL})$	
$\frac{z \notin \text{dom}(s)}{\mathbf{val} \ x = \mathbf{new} \ \{\bar{d}\}; \ t \mid s \rightarrow [z/x]t \mid s, z \mapsto [z/x]\{\bar{d}\}} \quad (\text{RED-NEW})$	
$\frac{t \mid s \rightarrow t' \mid s'}{t.m(u) \mid s \rightarrow t'.m(u) \mid s'} \quad (\text{RED-CALL-1}) \qquad \frac{u \mid s \rightarrow u' \mid s'}{x.m(u) \mid s \rightarrow x.m(u') \mid s'} \quad (\text{RED-CALL-2})$	

Declaration intersection	$\boxed{\text{intersect}(D_1, D_2) = D_3}$
$\frac{D_1 = (L : S_1..U_1) \quad D_2 = (L : S_2..U_2)}{\text{intersect}(D_1, D_2) = (L : S_1 \vee S_2 .. U_1 \wedge U_2)}$	$\frac{D_1 = (m : S_1 \rightarrow U_1) \quad D_2 = (m : S_2 \rightarrow U_2)}{\text{intersect}(D_1, D_2) = (m : S_1 \vee S_2 \rightarrow U_1 \wedge U_2)}$
Declaration union	$\boxed{\text{union}(D_1, D_2) = D_3}$
$\frac{D_1 = (L : S_1..U_1) \quad D_2 = (L : S_2..U_2)}{\text{union}(D_1, D_2) = (L : S_1 \wedge S_2 .. U_1 \vee U_2)}$	$\frac{D_1 = (m : S_1 \rightarrow U_1) \quad D_2 = (m : S_2 \rightarrow U_2)}{\text{union}(D_1, D_2) = (m : S_1 \wedge S_2 \rightarrow U_1 \vee U_2)}$

Membership	$\boxed{\Gamma \vdash T \ni D}$
$\Gamma \vdash \perp \ni (L : \top.. \perp) \quad (\perp\text{-}\ni\text{-TYP})$	
$\Gamma \vdash \perp \ni (m : \top \rightarrow \perp) \quad (\perp\text{-}\ni\text{-MTD})$	
$\Gamma \vdash \{D\} \ni D \quad (\text{RCD-}\ni)$	
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash U \ni D}{\Gamma \vdash x.L \ni D} \quad (\text{SEL-}\ni)$	
$\frac{\Gamma \vdash T_1 \ni D \quad \Gamma \vdash T_2 \not\ni \text{label}(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \quad (\wedge\text{-}\ni\text{-1})$	$\frac{\Gamma \vdash T_2 \ni D \quad \Gamma \vdash T_1 \not\ni \text{label}(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \quad (\wedge\text{-}\ni\text{-2})$
$\frac{\Gamma \vdash T_1 \ni D_1 \quad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \wedge T_2 \ni \text{intersect}(D_1, D_2)} \quad (\wedge\text{-}\ni\text{-12})$	
$\frac{\Gamma \vdash T_1 \ni D_1 \quad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \vee T_2 \ni \text{union}(D_1, D_2)} \quad (\vee\text{-}\ni)$	
Non-membership	$\boxed{\Gamma \vdash T \not\ni l}$
$\Gamma \vdash \top \not\ni l \quad (\top\text{-}\not\ni)$	
$\frac{l \neq \text{label}(D)}{\Gamma \vdash \{D\} \not\ni l} \quad (\text{RCD-}\not\ni)$	
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash U \not\ni l}{\Gamma \vdash x.L \not\ni l} \quad (\text{SEL-}\not\ni)$	
$\frac{\Gamma \vdash T_1 \not\ni l \quad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \wedge T_2 \not\ni l} \quad (\wedge\text{-}\not\ni)$	$\frac{\Gamma \vdash T_1 \ni D \quad \Gamma \vdash T_2 \not\ni \text{label}(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni \text{label}(D)} \quad (\vee\text{-}\not\ni\text{-1})$
$\frac{\Gamma \vdash T_2 \ni D \quad \Gamma \vdash T_1 \not\ni \text{label}(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni \text{label}(D)} \quad (\vee\text{-}\not\ni\text{-2})$	
$\frac{\Gamma \vdash T_1 \not\ni l \quad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \vee T_2 \not\ni l} \quad (\vee\text{-}\not\ni\text{-12})$	

Well-formed types		$\boxed{\Gamma; \overline{W} \vdash T \mathbf{wf}}$
$\Gamma; \overline{W} \vdash \top \mathbf{wf}$	(WF- \top)	
$\Gamma; \overline{W} \vdash \perp \mathbf{wf}$	(WF- \perp)	
$\Gamma; \overline{W} \vdash W_i \mathbf{wf}$	(WF-HYP)	
$\frac{\Gamma; \overline{W}, \{D\} \vdash D \mathbf{wf}}{\Gamma; \overline{W} \vdash \{D\} \mathbf{wf}}$	(WF-RCD)	
$\frac{(x : T) \in \Gamma \quad \Gamma; \overline{W} \vdash T \mathbf{wf}}{\Gamma; \overline{W} \vdash x.\mathbf{type} \mathbf{wf}}$	(WF-SELF)	
		$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma; \overline{W} \vdash T \mathbf{wf}, S \mathbf{wf}, U \mathbf{wf}}{\Gamma; \overline{W} \vdash x.L \mathbf{wf}}$ (WF-SEL)
		$\frac{\Gamma; \overline{W} \vdash T_1 \mathbf{wf}, T_2 \mathbf{wf}}{\Gamma; \overline{W} \vdash T_1 \wedge T_2 \mathbf{wf}}$ (WF-AND)
		$\frac{\Gamma; \overline{W} \vdash T_1 \mathbf{wf}, T_2 \mathbf{wf}}{\Gamma; \overline{W} \vdash T_1 \vee T_2 \mathbf{wf}}$ (WF-OR)
		$\frac{\Gamma, x : T; \overline{W} \vdash T \mathbf{wf}, U \mathbf{wf}}{\Gamma; \overline{W} \vdash \exists(x : T)U \mathbf{wf}}$ (WF- \exists)
Well-formed declarations		$\boxed{\Gamma; \overline{W} \vdash D \mathbf{wf}}$
$\frac{\Gamma; \overline{W} \vdash S \mathbf{wf}, U \mathbf{wf}}{\Gamma; \overline{W} \vdash L : S..U \mathbf{wf}}$	(WF-TMEM)	
		$\frac{\Gamma; \overline{W} \vdash S \mathbf{wf}, U \mathbf{wf}}{\Gamma; \overline{W} \vdash m : S \rightarrow U \mathbf{wf}}$ (WF-MTD)

Subtyping

$$\boxed{\Gamma \vdash S <: T}$$

$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash T <: T} \quad (<:-\text{REFL})$	$\frac{\Gamma \vdash T <: U_1 \quad \Gamma \vdash T <: U_2}{\Gamma \vdash T <: U_1 \wedge U_2} \quad (<:-\wedge)$
$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash T <: \top} \quad (<:-\top)$	$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_1} \quad (<:-\wedge-1)$
$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash \perp <: T} \quad (\perp-<:)$	$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_2} \quad (<:-\wedge-2)$
$\frac{\Gamma \vdash D_1 <: D_2}{\Gamma \vdash \{D_1\} <: \{D_2\}} \quad (<:-\text{RCD})$	$\frac{\Gamma \vdash T_1 <: U \quad \Gamma \vdash T_2 <: U}{\Gamma \vdash T_1 \vee T_2 <: U} \quad (<:-\vee)$
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S <: U}{\Gamma \vdash x.L <: U} \quad (<:-\text{SEL-L})$	$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 <: T_1 \vee T_2} \quad (<:-\vee-1)$
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S <: U}{\Gamma \vdash S <: x.L} \quad (<:-\text{SEL-R})$	$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_2 <: T_1 \vee T_2} \quad (<:-\vee-2)$
$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_2 <: T_3}{\Gamma \vdash T_1 <: T_3} \quad (<:-\text{TRANS})$	$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S \mathbf{wf} \quad \Gamma \vdash U \mathbf{wf}}{\Gamma \vdash S <: U} \quad (<:-\text{HYP})$
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf}}{\Gamma \vdash x.\mathbf{type} <: T} \quad (<:-\text{SELF-L})$	$\frac{(x : T) \in \Gamma \quad \Gamma \vdash x.\mathbf{type} \mathbf{wf} \quad \Gamma \vdash y.\mathbf{type} \mathbf{wf}}{\Gamma \vdash y.\mathbf{type} <: x.\mathbf{type}} \quad (<:-\text{SELF-R})$
$\frac{\Gamma, x : S \vdash T <: U \quad \Gamma \vdash U \mathbf{wf} \quad \Gamma, x : S \vdash S \mathbf{wf}}{\Gamma \vdash \exists(x : S)T <: U} \quad (<:-\exists\text{-L})$	$\frac{(x : S') \in \Gamma \quad \Gamma \vdash S' <: S \quad \Gamma \vdash T <: U}{\Gamma \vdash T <: \exists(x : S)U} \quad (<:-\exists\text{-R})$

Declaration subtyping

$$\boxed{\Gamma \vdash D <: D'}$$

$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')} \quad (\text{SUBDEC-TYP})$	$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (m : S \rightarrow T) <: (m : S' \rightarrow T')} \quad (\text{SUBDEC-MTD})$
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Term Typing	$\boxed{\Gamma \vdash t : T}$
$\frac{(x : T) \in \Gamma}{\Gamma \vdash T \mathbf{wf}} \quad (\text{TY-VAR})$	$\frac{\Gamma, x : T \vdash \{\bar{d}\} : T \quad \Gamma, x : T \vdash u : U \quad \Gamma \vdash U \mathbf{wf}}{\Gamma \vdash \mathbf{val } x = \mathbf{new } \{\bar{d}\}; u : U} \quad (\text{TY-NEW})$
$\frac{\Gamma \vdash t : T \quad \Gamma \vdash u : U \quad \Gamma \vdash V \mathbf{wf} \quad \Gamma \vdash T \ni (m : U \rightarrow V)}{\Gamma \vdash t.m(u) : V} \quad (\text{TY-CALL})$	$\frac{\Gamma \vdash t : T_1 \quad \Gamma \vdash T_1 <: T_2}{\Gamma \vdash t : T_2} \quad (\text{TY-SBSM})$
$\frac{\Gamma, x : U, y : S \vdash t : T \quad y \notin \text{fv}(t) \quad y \notin \text{fv}(T)}{\Gamma, x : \exists(y : S)U \vdash t : T} \quad (\text{TY-SKOLEM})$	
Initialization Typing	$\boxed{\Gamma \vdash d : D}$
$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash (L = T) : (L : T..T)} \quad (\text{TY-TDEF})$	$\frac{\Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash U \mathbf{wf} \quad \Gamma, x : T \vdash u : U}{\Gamma \vdash (m(x : T) : U = u) : (m : T \rightarrow U)} \quad (\text{TY-MDEF})$
Initialization List Typing	$\boxed{\Gamma \vdash \{\bar{d}\} : T}$
$\Gamma \vdash \{\} : \top \quad (\text{TY-NIL})$	$\frac{\text{label}(d) \notin \text{labels}(\bar{d}) \quad \Gamma \vdash \{\bar{d}\} : T \quad \Gamma \vdash d : D}{\Gamma \vdash \{\bar{d}, d\} : T \wedge \{D\}} \quad (\text{TY-CONS})$