[DRAFT] The exDOT Calculus

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This document presents gDOT, and the additions made by exDOT are highlighted in gray.

Syntax

x, y, z	Variable
$l ::=$	Label
$\quad L$	Type label
$\quad m$	Method label
$t, u ::=$	Term
$\quad x$	variable
$\quad \mathbf{val}\ x = \mathbf{new}\ \{\overline{d}\};\ t$	new instance
$\quad t.m(u)$	method invocation
$d ::=$	Initialization
$\quad L = T$	type definition
$\quad m(x : T) : U = u$	method definition
$\Gamma ::= \overline{x : T}$	Environment
$s ::= \overline{x \mapsto \{\overline{d}\}}$	Store

$S, T, U, V, W ::=$	Type
$\quad \top$	top type
$\quad \bot$	bottom type
$\quad \{D\}$	one-member record
$\quad x.L$	type selection
$\quad T \wedge T$	intersection type
$\quad T \vee T$	union type
$\quad x.\mathbf{type}$	singleton type
$\quad \exists(x : T)U$	existential type
$D ::=$	Declaration
$\quad L : S..U$	abstract type decl.
$\quad m : S \to U$	method declaration

Reduction $\boxed{t \,|\, s \ \to\ t' \,|\, s'}$

$$\frac{x \mapsto \left\{\overline{L = W}\ \overline{m(z : T) : U = u}\right\} \in s}{x.m_i(y) \,|\, s \ \to\ [y/z_i]u_i \,|\, s} \quad (\textsc{Red-Call})$$

$$\frac{z \notin dom(s)}{\mathbf{val}\ x = \mathbf{new}\ \{\overline{d}\};\ t \,|\, s \ \to\ [z/x]t \,|\, s, z \mapsto [z/x]\{\overline{d}\}} \quad (\textsc{Red-New})$$

$$\frac{t \,|\, s \ \to\ t' \,|\, s'}{t.m(u) \,|\, s \ \to\ t'.m(u) \,|\, s'} \quad (\textsc{Red-Call-1}) \qquad \frac{u \,|\, s \ \to\ u' \,|\, s'}{x.m(u) \,|\, s \ \to\ x.m(u') \,|\, s'} \quad (\textsc{Red-Call-2})$$

Declaration intersection $\boxed{\mathit{intersect}(D_1, D_2) = D_3}$

$$\frac{D_1 = (L : S_1..U_1) \qquad D_2 = (L : S_2..U_2)}{\mathit{intersect}(D_1, D_2) = (L : S_1 \vee S_2 \;..\; U_1 \wedge U_2)} \qquad \frac{D_1 = (m : S_1 \to U_1) \qquad D_2 = (m : S_2 \to U_2)}{\mathit{intersect}(D_1, D_2) = (m : S_1 \vee S_2 \;\to\; U_1 \wedge U_2)}$$

Declaration union $\boxed{\mathit{union}(D_1, D_2) = D_3}$

$$\frac{D_1 = (L : S_1..U_1) \qquad D_2 = (L : S_2..U_2)}{\mathit{union}(D_1, D_2) = (L : S_1 \wedge S_2 \;..\; U_1 \vee U_2)} \qquad \frac{D_1 = (m : S_1 \to U_1) \qquad D_2 = (m : S_2 \to U_2)}{\mathit{union}(D_1, D_2) = (m : S_1 \wedge S_2 \;\to\; U_1 \vee U_2)}$$

Membership $\boxed{\Gamma \vdash T \ni D}$

$$\Gamma \vdash \bot \ni (L : \top..\bot) \qquad (\bot\text{-}\ni\text{-}\textsc{typ})$$

$$\Gamma \vdash \bot \ni (m : \top \to \bot) \qquad (\bot\text{-}\ni\text{-}\textsc{mtd})$$

$$\Gamma \vdash \{D\} \ni D \qquad (\textsc{rcd}\text{-}\ni)$$

$$\frac{(x : T) \in \Gamma \qquad \Gamma \vdash T \ni (L : S..U) \qquad \Gamma \vdash U \ni D}{\Gamma \vdash x.L \ni D} \qquad (\textsc{sel}\text{-}\ni)$$

$$\frac{\Gamma \vdash T_1 \ni D \qquad \Gamma \vdash T_2 \not\ni \mathit{label}(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \qquad (\wedge\text{-}\ni\text{-}1)$$

$$\frac{\Gamma \vdash T_2 \ni D \qquad \Gamma \vdash T_1 \not\ni \mathit{label}(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \qquad (\wedge\text{-}\ni\text{-}2)$$

$$\frac{\Gamma \vdash T_1 \ni D_1 \qquad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \wedge T_2 \ni \mathit{intersect}(D_1, D_2)} \qquad (\wedge\text{-}\ni\text{-}12)$$

$$\frac{\Gamma \vdash T_1 \ni D_1 \qquad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \vee T_2 \ni \mathit{union}(D_1, D_2)} \qquad (\vee\text{-}\ni)$$

Non-membership $\boxed{\Gamma \vdash T \not\ni l}$

$$\Gamma \vdash \top \not\ni l \qquad (\top\text{-}\not\ni)$$

$$\frac{l \neq \mathit{label}(D)}{\Gamma \vdash \{D\} \not\ni l} \qquad (\textsc{rcd}\text{-}\not\ni)$$

$$\frac{(x : T) \in \Gamma \qquad \Gamma \vdash T \ni (L : S..U) \qquad \Gamma \vdash U \not\ni l}{\Gamma \vdash x.L \not\ni l} \qquad (\textsc{sel}\text{-}\not\ni)$$

$$\frac{\Gamma \vdash T_1 \not\ni l \qquad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \wedge T_2 \not\ni l} \qquad (\wedge\text{-}\not\ni)$$

$$\frac{\Gamma \vdash T_1 \ni D \qquad \Gamma \vdash T_2 \not\ni \mathit{label}(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni \mathit{label}(D)} \qquad (\vee\text{-}\not\ni\text{-}1)$$

$$\frac{\Gamma \vdash T_2 \ni D \qquad \Gamma \vdash T_1 \not\ni \mathit{label}(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni \mathit{label}(D)} \qquad (\vee\text{-}\not\ni\text{-}2)$$

$$\frac{\Gamma \vdash T_1 \not\ni l \qquad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \vee T_2 \not\ni l} \qquad (\vee\text{-}\not\ni\text{-}12)$$

Well-formed types $\boxed{\Gamma; \overline{W} \vdash T \ \mathbf{wf}}$

$$\Gamma; \overline{W} \vdash \top \ \mathbf{wf} \qquad (\text{WF-}\top)$$

$$\Gamma; \overline{W} \vdash \bot \ \mathbf{wf} \qquad (\text{WF-}\bot)$$

$$\Gamma; \overline{W} \vdash W_i \ \mathbf{wf} \qquad (\text{WF-HYP})$$

$$\frac{\Gamma; \overline{W}, \{D\} \vdash D \ \mathbf{wf}}{\Gamma; \overline{W} \vdash \{D\} \ \mathbf{wf}} \qquad (\text{WF-RCD})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma; \overline{W} \vdash T \ \mathbf{wf}}{\Gamma; \overline{W} \vdash x.\mathbf{type} \ \mathbf{wf}} \qquad (\text{WF-SELF})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma; \overline{W} \vdash T \ \mathbf{wf}\ ,\ S \ \mathbf{wf}\ ,\ U \ \mathbf{wf}}{\Gamma; \overline{W} \vdash x.L \ \mathbf{wf}} \qquad (\text{WF-SEL})$$

$$\frac{\Gamma; \overline{W} \vdash T_1 \ \mathbf{wf}\ ,\ T_2 \ \mathbf{wf}}{\Gamma; \overline{W} \vdash T_1 \wedge T_2 \ \mathbf{wf}} \qquad (\text{WF-AND})$$

$$\frac{\Gamma; \overline{W} \vdash T_1 \ \mathbf{wf}\ ,\ T_2 \ \mathbf{wf}}{\Gamma; \overline{W} \vdash T_1 \vee T_2 \ \mathbf{wf}} \qquad (\text{WF-OR})$$

$$\frac{\Gamma, x : T; \overline{W} \vdash T \ \mathbf{wf}\ ,\ U \ \mathbf{wf}}{\Gamma; \overline{W} \vdash \exists(x : T)U \ \mathbf{wf}} \qquad (\text{WF-}\exists)$$

Well-formed declarations $\boxed{\Gamma; \overline{W} \vdash D \ \mathbf{wf}}$

$$\frac{\Gamma; \overline{W} \vdash S \ \mathbf{wf}\ ,\ U \ \mathbf{wf}}{\Gamma; \overline{W} \vdash L : S..U \ \mathbf{wf}} \qquad (\text{WF-TMEM})$$

$$\frac{\Gamma; \overline{W} \vdash S \ \mathbf{wf}\ ,\ U \ \mathbf{wf}}{\Gamma; \overline{W} \vdash m : S \to U \ \mathbf{wf}} \qquad (\text{WF-MTD})$$

Subtyping $\boxed{\Gamma \vdash S <: T}$

$$\frac{\Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash T <: T}\ (\textsc{<:-refl})$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash T <: \top}\ (\textsc{<:-}\top)$$

$$\frac{\Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash \bot <: T}\ (\bot\textsc{-<:})$$

$$\frac{\Gamma \vdash D_1 <: D_2}{\Gamma \vdash \{D_1\} <: \{D_2\}}\ (\textsc{<:-rcd})$$

$$\frac{\Gamma \vdash x.L\ \mathbf{wf} \quad (x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U)}{\Gamma \vdash x.L <: U}\ (\textsc{<:-sel-l})$$

$$\frac{\Gamma \vdash x.L\ \mathbf{wf} \quad (x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U)}{\Gamma \vdash S <: x.L}\ (\textsc{<:-sel-r})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T\ \mathbf{wf}}{\Gamma \vdash x.\mathbf{type} <: T}\ (\textsc{<:-self-l})$$

$$\frac{\Gamma, x : S \vdash T <: U \quad \Gamma \vdash U\ \mathbf{wf} \quad \Gamma, x : S \vdash S\ \mathbf{wf}}{\Gamma \vdash \exists(x : S)T <: U}\ (\textsc{<:-}\exists\textsc{-l})$$

$$\frac{\Gamma \vdash T <: U_1 \quad \Gamma \vdash T <: U_2}{\Gamma \vdash T <: U_1 \wedge U_2}\ (\textsc{<:-}\wedge)$$

$$\frac{\Gamma \vdash T_1\ \mathbf{wf} \quad \Gamma \vdash T_2\ \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_1}\ (\textsc{<:-}\wedge\textsc{-1})$$

$$\frac{\Gamma \vdash T_1\ \mathbf{wf} \quad \Gamma \vdash T_2\ \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_2}\ (\textsc{<:-}\wedge\textsc{-2})$$

$$\frac{\Gamma \vdash T_1 <: U \quad \Gamma \vdash T_2 <: U}{\Gamma \vdash T_1 \vee T_2 <: U}\ (\textsc{<:-}\vee)$$

$$\frac{\Gamma \vdash T_1\ \mathbf{wf} \quad \Gamma \vdash T_2\ \mathbf{wf}}{\Gamma \vdash T_1 <: T_1 \vee T_2}\ (\textsc{<:-}\vee\textsc{-1})$$

$$\frac{\Gamma \vdash T_1\ \mathbf{wf} \quad \Gamma \vdash T_2\ \mathbf{wf}}{\Gamma \vdash T_2 <: T_1 \vee T_2}\ (\textsc{<:-}\vee\textsc{-2})$$

$$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_2 <: T_3}{\Gamma \vdash T_1 <: T_3}\ (\textsc{<:-trans})$$

$$\frac{(x : y.\mathbf{type}) \in \Gamma \quad \Gamma \vdash x.\mathbf{type}\ \mathbf{wf} \quad \Gamma \vdash y.\mathbf{type}\ \mathbf{wf}}{\Gamma \vdash y.\mathbf{type} <: x.\mathbf{type}}\ (\textsc{<:-self-r})$$

$$\frac{(x : S') \in \Gamma \quad \Gamma \vdash S' <: S \quad \Gamma \vdash T <: U}{\Gamma \vdash T <: \exists(x : S)U}\ (\textsc{<:-}\exists\textsc{-r})$$

Declaration subtyping $\boxed{\Gamma \vdash D <: D'}$

$$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')}\ (\textsc{subdec-typ})$$

$$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (m : S \to T) <: (m : S' \to T')}\ (\textsc{subdec-mtd})$$

Term Typing $\boxed{\Gamma \vdash t : T}$

$$\frac{(x : T) \in \Gamma \qquad \Gamma \vdash T \ \textbf{wf}}{\Gamma \vdash x : T} \quad (\textsc{ty-var})$$

$$\frac{\Gamma, x : T \vdash \{\overline{d}\} : T \qquad \Gamma, x : T \vdash u : U \qquad \Gamma \vdash U \ \textbf{wf}}{\Gamma \vdash \textbf{val}\ x = \textbf{new}\ \{\overline{d}\};\ u : U} \quad (\textsc{ty-new})$$

$$\frac{\Gamma \vdash t : T \qquad \Gamma \vdash u : U \qquad \Gamma \vdash T \ni (m : U \to V) \qquad \Gamma \vdash V \ \textbf{wf}}{\Gamma \vdash t.m(u) : V} \quad (\textsc{ty-call})$$

$$\frac{\Gamma \vdash t : T_1 \qquad \Gamma \vdash T_1 <: T_2}{\Gamma \vdash t : T_2} \quad (\textsc{ty-sbsm})$$

$$\frac{\Gamma, x : U, y : S \vdash t : T \qquad y \notin \mathrm{fv}(t) \qquad y \notin \mathrm{fv}(T)}{\Gamma, x : \exists(y : S)U \vdash t : T} \quad (\textsc{ty-skolem})$$

Initialization Typing $\boxed{\Gamma \vdash d : D}$

$$\frac{\Gamma \vdash T \ \textbf{wf}}{\Gamma \vdash (L = T) : (L : T..T)} \quad (\textsc{ty-tdef})$$

$$\frac{\Gamma \vdash T \ \textbf{wf} \qquad \Gamma \vdash U \ \textbf{wf} \qquad \Gamma, x : T \vdash u : U}{\Gamma \vdash (m(x : T) : U = u) : (m : T \to U)} \quad (\textsc{ty-mdef})$$

Initialization List Typing $\boxed{\Gamma \vdash \{\overline{d}\} : T}$

$$\Gamma \vdash \{\} : \top \quad (\textsc{ty-nil})$$

$$\frac{\mathit{label}(d) \notin \mathit{labels}(\overline{d}) \qquad \Gamma \vdash \{\overline{d}\} : T \qquad \Gamma \vdash d : D}{\Gamma \vdash \{\overline{d}, d\} : T \wedge \{D\}} \quad (\textsc{ty-cons})$$