

# [DRAFT] The exDOT Calculus

Samuel Gruetter, Nada Amin, Martin Odersky

EPFL

This document presents gDOT, and the additions made by exDOT are highlighted in gray.

Syntax			
$x, y, z$	Variable	$S, T, U, V, W ::=$	Type
$l ::=$	Label	$\top$	top type
$L$	Type label	$\perp$	bottom type
$m$	Method label	$\{D\}$	one-member record
$t, u ::=$	Term	$x.L$	type selection
$x$	variable	$T \wedge T$	intersection type
$\mathbf{val} \ x = \mathbf{new} \ \{\bar{d}\}; \ t$	new instance	$T \vee T$	union type
$t.m(u)$	method invocation	$x.\mathbf{type}$	singleton type
$d ::=$	Initialization	$\exists(x : T)U$	existential type
$L = T$	field init.	$D ::=$	Declaration
$m(x : T) : U = u$	method init.	$L : S..U$	abstract type decl.
$\Gamma ::= \overline{x : T}$	Environment	$m : S \rightarrow U$	method declaration
$s ::= \overline{x \mapsto \{\bar{d}\}}$	Store		

Reduction	$t \mid s \rightarrow t' \mid s'$
$\frac{x \mapsto \left\{ \overline{L = W} \ \overline{m(z : T) : U = u} \right\} \in s}{x.m_i(y) \mid s \rightarrow [y/z_i]u_i \mid s} \quad (\text{RED-CALL})$	
$\frac{z \notin \text{dom}(s)}{\mathbf{val} \ x = \mathbf{new} \ \{\bar{d}\}; \ t \mid s \rightarrow [z/x]t \mid s, z \mapsto [z/x]\{\bar{d}\}} \quad (\text{RED-NEW})$	
$\frac{t \mid s \rightarrow t' \mid s'}{t.m(u) \mid s \rightarrow t'.m(u) \mid s'} \quad (\text{RED-CALL-1}) \qquad \frac{u \mid s \rightarrow u' \mid s'}{x.m(u) \mid s \rightarrow x.m(u') \mid s'} \quad (\text{RED-CALL-2})$	

<b>Declaration intersection</b>	$\boxed{intersect(D_1, D_2) = D_3}$
$\frac{D_1 = (L : S_1..U_1) \quad D_2 = (L : S_2..U_2)}{intersect(D_1, D_2) = (L : S_1 \vee S_2 .. U_1 \wedge U_2)}$	$\frac{D_1 = (m : S_1 \rightarrow U_1) \quad D_2 = (m : S_2 \rightarrow U_2)}{intersect(D_1, D_2) = (m : S_1 \vee S_2 \rightarrow U_1 \wedge U_2)}$
<b>Declaration union</b>	$\boxed{union(D_1, D_2) = D_3}$
$\frac{D_1 = (L : S_1..U_1) \quad D_2 = (L : S_2..U_2)}{union(D_1, D_2) = (L : S_1 \wedge S_2 .. U_1 \vee U_2)}$	$\frac{D_1 = (m : S_1 \rightarrow U_1) \quad D_2 = (m : S_2 \rightarrow U_2)}{union(D_1, D_2) = (m : S_1 \wedge S_2 \rightarrow U_1 \vee U_2)}$

<b>Membership</b>	$\boxed{\Gamma \vdash T \ni D}$
$\Gamma \vdash \perp \ni (L : \top.. \perp) \quad (\perp\text{-}\ni\text{-TYP})$	
$\Gamma \vdash \perp \ni (m : \top \rightarrow \perp) \quad (\perp\text{-}\ni\text{-MTD})$	
$\Gamma \vdash \{D\} \ni D \quad (\text{RCD-}\ni)$	
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash U \ni D}{\Gamma \vdash x.L \ni D} \quad (\text{SEL-}\ni)$	
$\frac{\Gamma \vdash T_1 \ni D \quad \Gamma \vdash T_2 \not\ni label(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \quad (\wedge\text{-}\ni\text{-1})$	$\frac{\Gamma \vdash T_2 \ni D \quad \Gamma \vdash T_1 \not\ni label(D)}{\Gamma \vdash T_1 \wedge T_2 \ni D} \quad (\wedge\text{-}\ni\text{-2})$
$\frac{\Gamma \vdash T_1 \ni D_1 \quad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \wedge T_2 \ni intersect(D_1, D_2)} \quad (\wedge\text{-}\ni\text{-12})$	
$\frac{\Gamma \vdash T_1 \ni D_1 \quad \Gamma \vdash T_2 \ni D_2}{\Gamma \vdash T_1 \vee T_2 \ni union(D_1, D_2)} \quad (\vee\text{-}\ni)$	
<b>Non-membership</b>	$\boxed{\Gamma \vdash T \not\ni l}$
$\Gamma \vdash \top \not\ni l \quad (\top\text{-}\not\ni)$	
$\frac{l \neq label(D)}{\Gamma \vdash \{D\} \not\ni l} \quad (\text{RCD-}\not\ni)$	
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash U \not\ni l}{\Gamma \vdash x.L \not\ni l} \quad (\text{SEL-}\not\ni)$	
$\frac{\Gamma \vdash T_1 \not\ni l \quad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \wedge T_2 \not\ni l} \quad (\wedge\text{-}\not\ni)$	$\frac{\Gamma \vdash T_1 \ni D \quad \Gamma \vdash T_2 \not\ni label(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni label(D)} \quad (\vee\text{-}\not\ni\text{-1})$
	$\frac{\Gamma \vdash T_2 \ni D \quad \Gamma \vdash T_1 \not\ni label(D)}{\Gamma \vdash T_1 \vee T_2 \not\ni label(D)} \quad (\vee\text{-}\not\ni\text{-2})$
	$\frac{\Gamma \vdash T_1 \not\ni l \quad \Gamma \vdash T_2 \not\ni l}{\Gamma \vdash T_1 \vee T_2 \not\ni l} \quad (\vee\text{-}\not\ni\text{-12})$

<b>Well-formed types</b>		$\boxed{\Gamma; \overline{W} \vdash T \textbf{wf}}$
$\Gamma; \overline{W} \vdash \top \textbf{wf}$	(WF- $\top$ )	
$\Gamma; \overline{W} \vdash \perp \textbf{wf}$	(WF- $\perp$ )	
$\Gamma; \overline{W} \vdash W_i \textbf{wf}$	(WF-HYP)	
$\frac{\Gamma; \overline{W}, \{D\} \vdash D \textbf{wf}}{\Gamma; \overline{W} \vdash \{D\} \textbf{wf}}$	(WF-RCD)	
$\frac{(x : T) \in \Gamma \quad \Gamma; \overline{W} \vdash T \textbf{wf}}{\Gamma; \overline{W} \vdash x.\textbf{type} \textbf{wf}}$	(WF-SELF)	
		$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma; \overline{W} \vdash T \textbf{wf}, S \textbf{wf}, U \textbf{wf}}{\Gamma; \overline{W} \vdash x.L \textbf{wf}}$ (WF-SEL)
		$\frac{\Gamma; \overline{W} \vdash T_1 \textbf{wf}, T_2 \textbf{wf}}{\Gamma; \overline{W} \vdash T_1 \wedge T_2 \textbf{wf}}$ (WF-AND)
		$\frac{\Gamma; \overline{W} \vdash T_1 \textbf{wf}, T_2 \textbf{wf}}{\Gamma; \overline{W} \vdash T_1 \vee T_2 \textbf{wf}}$ (WF-OR)
		$\frac{\Gamma, x : T; \overline{W} \vdash T \textbf{wf}, U \textbf{wf}}{\Gamma; \overline{W} \vdash \exists(x : T)U \textbf{wf}}$ (WF- $\exists$ )
<b>Well-formed declarations</b>		$\boxed{\Gamma; \overline{W} \vdash D \textbf{wf}}$
$\frac{\Gamma; \overline{W} \vdash S \textbf{wf}, U \textbf{wf}}{\Gamma; \overline{W} \vdash L : S..U \textbf{wf}}$	(WF-TMEM)	
		$\frac{\Gamma; \overline{W} \vdash S \textbf{wf}, U \textbf{wf}}{\Gamma; \overline{W} \vdash m : S \rightarrow U \textbf{wf}}$ (WF-MTD)

**Subtyping**

$$\boxed{\Gamma \vdash S <: T}$$

$$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash T <: T} \quad (<:-\text{REFL})$$

$$\frac{\Gamma \vdash T <: U_1 \quad \Gamma \vdash T <: U_2}{\Gamma \vdash T <: U_1 \wedge U_2} \quad (<:-\wedge)$$

$$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash T <: \top} \quad (<:-\top)$$

$$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_1} \quad (<:-\wedge-1)$$

$$\frac{\Gamma \vdash T \mathbf{wf}}{\Gamma \vdash \perp <: T} \quad (\perp-<:)$$

$$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 \wedge T_2 <: T_2} \quad (<:-\wedge-2)$$

$$\frac{\Gamma \vdash D_1 <: D_2}{\Gamma \vdash \{D_1\} <: \{D_2\}} \quad (<:-\text{RCD})$$

$$\frac{\Gamma \vdash T_1 <: U \quad \Gamma \vdash T_2 <: U}{\Gamma \vdash T_1 \vee T_2 <: U} \quad (<:-\vee)$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S <: U}{\Gamma \vdash x.L <: U} \quad (<:-\text{SEL-L})$$

$$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_1 <: T_1 \vee T_2} \quad (<:-\vee-1)$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S <: U}{\Gamma \vdash S <: x.L} \quad (<:-\text{SEL-R})$$

$$\frac{\Gamma \vdash T_1 \mathbf{wf} \quad \Gamma \vdash T_2 \mathbf{wf}}{\Gamma \vdash T_2 <: T_1 \vee T_2} \quad (<:-\vee-2)$$

$$\frac{\Gamma \vdash T_1 <: T_2 \quad \Gamma \vdash T_2 <: T_3}{\Gamma \vdash T_1 <: T_3} \quad (<:-\text{TRANS})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf} \quad \Gamma \vdash T \ni (L : S..U) \quad \Gamma \vdash S \mathbf{wf} \quad \Gamma \vdash U \mathbf{wf}}{\Gamma \vdash S <: U} \quad (<:-\text{HYP})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \mathbf{wf}}{\Gamma \vdash x.\mathbf{type} <: T} \quad (<:-\text{SELF-L})$$

$$\frac{(x : T) \in \Gamma \quad \Gamma \vdash x.\mathbf{type} \mathbf{wf} \quad \Gamma \vdash y.\mathbf{type} \mathbf{wf}}{\Gamma \vdash y.\mathbf{type} <: x.\mathbf{type}} \quad (<:-\text{SELF-R})$$

$$\frac{\Gamma, x : S \vdash T <: U \quad \Gamma \vdash U \mathbf{wf} \quad \Gamma, x : S \vdash S \mathbf{wf}}{\Gamma \vdash \exists(x : S)T <: U} \quad (<:-\exists\text{-L})$$

$$\frac{(x : S') \in \Gamma \quad \Gamma \vdash S' <: S \quad \Gamma \vdash T <: U}{\Gamma \vdash T <: \exists(x : S)U} \quad (<:-\exists\text{-R})$$

**Declaration subtyping**

$$\boxed{\Gamma \vdash D <: D'}$$

$$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (L : S..T) <: (L : S'..T')} \quad (\text{SUBDEC-TYP})$$

$$\frac{\Gamma \vdash S' <: S \quad \Gamma \vdash T <: T'}{\Gamma \vdash (m : S \rightarrow T) <: (m : S' \rightarrow T')} \quad (\text{SUBDEC-MTD})$$

<b>Term Typing</b>		$\boxed{\Gamma \vdash t : T}$
$\frac{(x : T) \in \Gamma \quad \Gamma \vdash T \textbf{wf}}{\Gamma \vdash x : T}$	(TY-VAR)	
$\frac{\Gamma, x : T \vdash \{\bar{d}\} : T \quad \Gamma, x : T \vdash u : U \quad \Gamma \vdash U \textbf{wf}}{\Gamma \vdash \textbf{val } x = \textbf{new } \{\bar{d}\}; u : U}$	(TY-NEW)	
$\frac{\Gamma \vdash t : T \quad \Gamma \vdash u : U \quad \Gamma \vdash V \textbf{wf} \quad \Gamma \vdash T \ni (m : U \rightarrow V)}{\Gamma \vdash t.m(u) : V}$	(TY-CALL)	
$\frac{\Gamma \vdash t : T_1 \quad \Gamma \vdash T_1 <: T_2}{\Gamma \vdash t : T_2}$	(TY-SBSM)	
<b>Initialization Typing</b>		$\boxed{\Gamma \vdash d : D}$
$\frac{\Gamma \vdash T \textbf{wf}}{\Gamma \vdash (L = T) : (L : T..T)}$	(TY-TDEF)	
$\frac{\Gamma \vdash T \textbf{wf} \quad \Gamma \vdash U \textbf{wf} \quad \Gamma, x : T \vdash u : U}{\Gamma \vdash (m(x : T) : U = u) : (m : T \rightarrow U)}$	(TY-MDEF)	
<b>Initialization List Typing</b>		$\boxed{\Gamma \vdash \{\bar{d}\} : T}$
$\Gamma \vdash \{\} : \top$	(TY-NIL)	
$\frac{\text{label}(d) \notin \text{labels}(\bar{d}) \quad \Gamma \vdash \{\bar{d}\} : T \quad \Gamma \vdash d : D}{\Gamma \vdash \{\bar{d}, d\} : T \wedge \{D\}}$	(TY-CONS)	