PDBP

Program Description Based Programming
Scala eXchange

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August 4, 2018

Intro

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  - a `Dotty` *library* `PDBP`

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- this talk is about
  - a `Dotty` *library* `PDBP`
  - inspired by the *function-level programming language* `FP`

John Backus

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- ACM Turing Award Winner 1977

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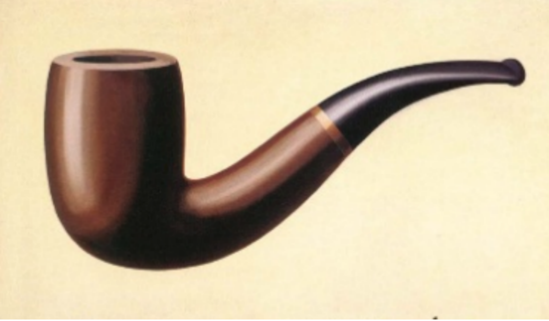
- ACM Turing Award Winner 1977
- *Can programming be liberated from the Von Neumann style?*

What is this?



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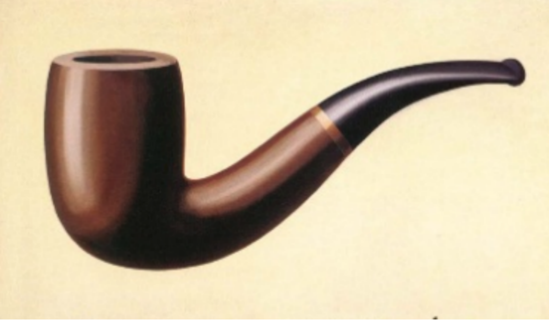

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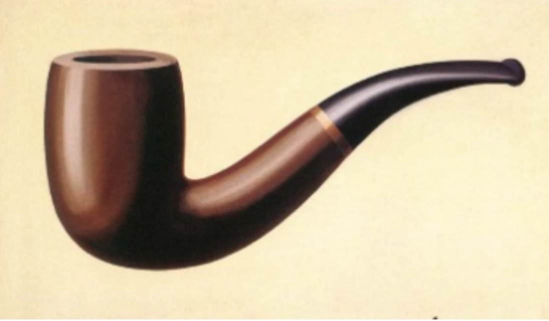

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- A *painting describing* a pipe?

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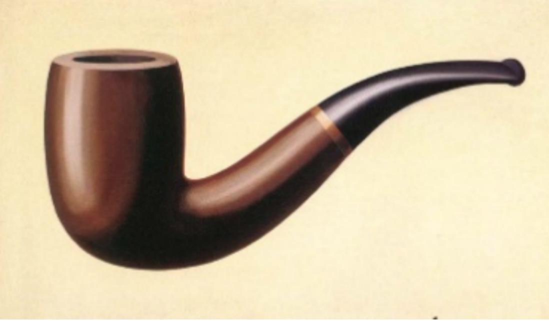

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- A *slide describing* a painting describing a pipe?

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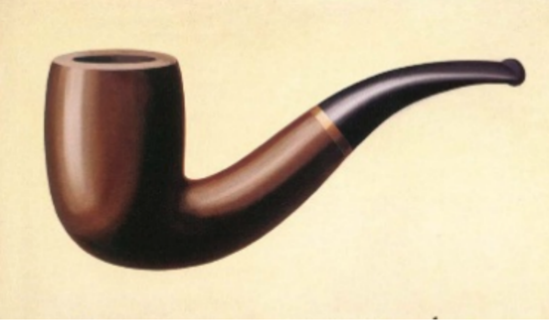

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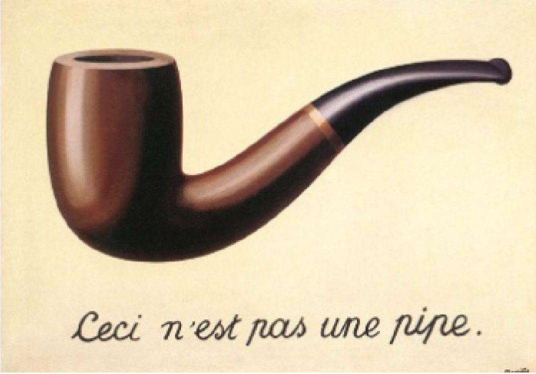
- ...

Ceci n'est pas une pipe



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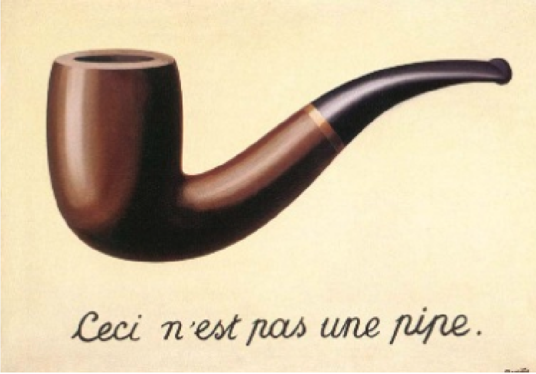

- It is a *painting* by René Magritte *describing* a pipe

This is not a program

```
val ?????????: BigInt >--> BigInt =
  `if`(isZero) {
    one
  } `else` {
    `let` {
      subtractOne >-->
        ?????????
    } `in` {
      multiply
    }
  }
```

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- It is *code describing* a program

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```

- It is *code describing* a program
- Can you think of a more meaningful name?

factorial description

```
val factorial: BigInt >--> BigInt =
  ‘if‘(isZero) {
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`factorial` description

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```

- `factorial` *definition* uses the capabilities *declared* in *type class* `trait Program[>-->[- _, + _]]`

factorial implementations

```
val factorial: BigInt >--> BigInt =
  `if`(isZero) {
    one
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```

- `factorial` *implementations* depend on `implicit object`'s *defining* the capabilities of type class `trait Program[>-->[- _, + _]]`

factorial meanings

```
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    }
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```

- `factorial` *meanings* use *natural transformations* transforming implementations

program

```
val program: Z >--> Y
```

Composition

```
val `z>-->y`: Z >--> Y
val `y>-->x`: Y >--> x

val `z>-->x`: Z >--> X = `z>-->y >--> `y>-->x`
```

`program` (design artifact)

```
 --------------------------------
|   --------- local  ----------  |     ----------
| | Z >--> Y | >--> | Y >--> X | | => | Z >--> X |
|   ---------        ----------  |     ----------
 --------------------------------
```

mainProgram

```
val program: Unit >--> Unit
```

mainProgram

```
val producer: Unit >--> Z
val program: Z >--> Y
val consumer: Y >--> Unit

val mainProgram: Unit >--> Unit =
  producer >--> program >--> consumer
```

`mainProgram` (architectural artifact)

```
         ----------------
        | Unit >--> Unit |
 ------                   ------
| Unit >--> Z >--> Y >--> Unit |
 ---------------------------v---
                            |
                            |       ----------------
     ---- distributed ---    => | Unit >--> Unit |
    |                               ----------------
    |
 ----v--------------------------
| Unit >--> Z >--> Y >--> Unit |
 ------                   ------
        | Unit >--> Unit |
         ----------------
```

FP versus PDBP

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- FP and PDBP

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- FP and PDBP
  - promote *pointfree*, *composition* based, *functional programming*

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  - FP is *heterogeneous* (programs are not objects)
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    `PDBP` *semantics* is *not fixed*
  - `FP` *capabilities* are *fixed*
    `PDBP` *capabilities* are *not fixed*
  - `FP` *effects* are *impure*
    `PDBP` *effects* are *pure*

Semantics

```
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- *production*

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- *production*
    - *recursion* using *stack*

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- *production*
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  - *recursion* using *heap*
- *test*

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- *. . .*

Effects

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- *reading*

Effects

- *reading*
- *writing*

Foundations

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- `trait Computation[C[+ _]]` corresponds to *monads*

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- `mz bind { z => my bind { y => result(ex(z,y)) } }`

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  - expression is used to define function
- `val kleisliArrow:  Z => M[Y] =  z => my(x)`
  - monad is used to define kleisli arrow

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- *monads* can be programmed *pointfree* (*kleisli arrows*)

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- *Constraints Liberate, Liberties Constrain*

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  - `Dotty` *library based* language extensions are *type safe*
- `PDBP` comes with a *program description DSL*

Program Description DSL

```
val factorial: BigInt >--> BigInt =
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    one
  } `else` {
    `let` {
      subtractOne >-->
        factorial
    } `in` {
      multiply
    }
  }
```

Computation Description DSL

```
val factorial: BigInt `=>C` BigInt = { z =>
  isZero(z) bind { b =>
    if (b) {
      one(z)
    } else {
      subtractOne(z) bind { y =>
        factorial(y) bind { x =>
          multiply((y, x))
        }
      }
    }
  }
}
```

Uh! Oh! typo . . . or error?

```
val factorial: BigInt `=>C` BigInt = { z =>
  isZero(z) bind { b =>
    if (b) {
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```

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  - `public` *pointfree arrow API*
    provides *elegance of use* for *application* developers
- the PDBP can live with
  - corresponding implementation constraints

PDBP

```
//       _______         __    __        _______
//      / ___  /\       / /\  / /\      / ___  /\
//     / /__/ / / _____/ / / / /_/__   / /__/ / /
//    / _____/ / / ___  / / / ___  /\ /____  / /
//   / /\____\/ / /__/ / / / /__/ / / \___/ / /
//  /_/ /      /______/ / /______/ /     /_/ /
//  \_\/       \______\/  \______\/      \_\/
//                                           v1.0
//  Program Description Based Programming Library
//  author        Luc Duponcheel        2017-2018
```

PDBP library design decisions

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- *description* separated from *implementations* and *meanings*

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  - `implicit object`'s that `extend trait`'s *define* capabilities

PDBP library design decisions

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  - `trait`'s (*type classes*) that *declare* capabilities
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  - `implicit object`'s that `extend trait`'s *define* capabilities
- *meanings*

`PDBP` library design decisions

- *description* separated from *implementations* and *meanings*

- *description*
  - `trait`'s (*type classes*) that *declare* capabilities
- *implementations*
  - `implicit object`'s that `extend trait`'s *define* capabilities
- *meanings*
  - `implicit object`'s that *define* a *natural transformation*

PDBP library design decisions

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- *dependency injection* by `implicit import`

`PDBP` library design decisions

- *dependency injection* by `implicit import`
- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s

PDBP library design decisions

- *dependency injection* by `implicit import`
- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s
- `object`'s that *extend* those `class`'es `import implicit` `object`'s that *extend* those `trait`'s

`Program` (cfr. *arrow*)

```
trait Program[>-->[- _, + _]]
```

Computation (cfr. *monad*)

```
trait Computation[C[+ _]]
```

Liskov Substitution Principle

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- *impose less*

Liskov Substitution Principle

- *impose less*
- *provide more*

Internet Robustness Principle

Internet Robustness Principle

- *be liberal in what you receive*

Internet Robustness Principle

- *be liberal in what you receive*
- *be generous in what you send*

PDBP library details

Program

```
trait Program[>-->[- _, + _]]
    extends Function[>-->]
    with Composition[>-->]
    with Construction[>-->]
    with Condition[>-->]

    with Aggregation[>-->]
```

`Function`

Function

- `val ‘z>-->y‘ = function(‘z=>y‘)`

Function

- `val `z>-->y` = function(`z=>y`)`
- *pure functions* are *atomic programs*

Function

- `` val `z>-->y` = function(`z=>y`) ``
- *pure functions* are *atomic programs*
  - up to you to define *granularity*

Composition

Composition

- `val ‘z>-->x‘ = ‘z>-->y‘ >--> ‘y>-->x‘`

Construction

Construction

- `val ‘z>-->y&&x‘ = ‘z>-->y‘ & ‘z>-->x‘`

Construction

- val ‘z>-->y&&x‘ = ‘z>-->y‘ & ‘z>-->x‘
- val ‘z&&y>-->x&&w‘ = ‘z>-->x‘ && ‘y>-->w‘

Construction

- ```
  val ‘z>-->y&&x‘ = ‘z>-->y‘ & ‘z>-->x‘
  ```
- ```
  val ‘z&&y>-->x&&w‘ = ‘z>-->x‘ && ‘y>-->w‘
  ```
- ```
  val ‘z>-->x‘ =
     ‘let‘ ‘z>-->y‘ ‘in‘ ‘z&&y>-->x‘
  ```


  ```

Condition

Condition

- `val ‘y||x>-->z‘ = ‘y>-->z‘ | ‘x>-->z‘`

Condition

- `val ‘y||x>-->z‘ = ‘y>-->z‘ | ‘x>-->z‘`
- `val ‘x||w>-->z||y‘ = ‘x>-->z‘ || ‘w>-->y‘`

Condition

- `val ‘y||x>-->z‘ = ‘y>-->z‘ | ‘x>-->z‘`
- `val ‘x||w>-->z||y‘ = ‘x>-->z‘ || ‘w>-->y‘`
- ```
  val ‘y>-->z‘ =
      ‘if‘(‘y>-->b‘) ‘y>-t->z‘ ‘else‘ ‘y>-f->z‘
  ```


  ```

Computation

```
private[pdbp] trait Computation[C[+ _]]
    extends Resulting[C]
    with Binding[C]
    with Program[[-Z, +Y] => Z => C[Y]]

    with Lifting[C]

    with Sequencing[C]
```

Resulting

Resulting

- `val cz = result(z)`

Binding

Binding

- `val cy = cz bind { z => ‘z=>cy‘(y) }`

Binding

- `val cy = cz bind { z => ‘z=>cy‘(y) }`
- `val cy = cz bind { z => result(‘z=>y‘(y)) }`

`Kleisli`

Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`

Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`

- ```
  private[pdbp] trait Computation[C[+ _]]
      extends Resulting[C]
      with Binding[C]
      with Program[Kleisli[C]]

      // ...
  ```


  ```

factorial Helper Functions

```
val isZeroFunction: BigInt => Boolean = { i =>
  i == 0
}

def oneFunction[Z]: Z => BigInt = { z =>
  1
}

val subtractOneFunction: BigInt => BigInt = { i =>
  i - 1
}

val multiplyFunction: (BigInt && BigInt) => BigInt = { (i, j) =>
  i * j
}
```

`factorial` Helper Programs

```
val isZeroHelper: BigInt >--> Boolean =
  function(isZeroFunction)

val subtractOneHelper: BigInt >--> BigInt =
  function(subtractOneFunction)

val multiplyHelper: (BigInt && BigInt) >--> BigInt =
  function(multiplyFunction)

def oneHelper[Z]: Z >--> BigInt =
  function(oneFunction)
```

`factorial` Atomic Programs

```
val isZero: BigInt >--> Boolean =
  isZeroHelper

val subtractOne: BigInt >--> BigInt =
  subtractOneHelper

val multiply: (BigInt && BigInt) >--> BigInt =
  multiplyHelper

def one[Z]: Z >--> BigInt =
  oneHelper
```

`factorial` program

```
val factorial: BigInt >--> BigInt =
  ‘if‘(isZero) {
    one
  } ‘else‘ {
    ‘let‘ {
      subtractOne >-->
        factorial
    } ‘in‘ {
      multiply
    }
  }
```

factorialMain

```
val factorialMain: Unit >--> Unit =
  effectfulReadIntFromConsole >-->
    factorial >-->
    effectfulWriteFactorialOfIntToConsole
```

activeTypes

```
object activeTypes {

  type Active[+Z] = Z

  type ‘=>A‘ = Kleisli[Active]

}
```

activeProgram

```
implicit object activeProgram
    extends Computation[Active]
    with Program[`=>A`] {

  override private[pdbp] def result[Z]: Z => Active[Z] =
    `z=>az`

  override private[pdbp] def bind[Z, Y](
      az: Active[Z],
      `z=>ay`: => (Z => Active[Y])): Active[Y] =
    `z=>ay`(az)

}
```

activeMeaningOfActive

```
implicit object activeMeaningOfActive
    extends MeaningOfActive[Active]()
    with ComputationMeaning[Active, Active]()
    with ProgramMeaning[`=>A`, `=>A`]()
```

FactorialMain

```
import ... mainFactorial
import mainFactorial.factorialMain

import ... activeMeaningOfActive
import activeMeaningOfActive.meaning

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }

}
```

Problems and Solutions

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- Problem: the `factorial` semantics above *is not stack safe*

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Problems and Solutions

- Problem: the `factorial` semantics above *is not stack safe*
- Solution: *transformation* `FreeTransformation` and *meaning* `FreeTransformedMeaning`

- Problem: `effectfulReadIntFromConsole` and `effectfulWriteFactorialOfIntToConsole` *execute effects*
- Solution: `Reading` resp. `Writing` extensions of `Program` with members `read` resp. `write` that *describe effects* and with corresponding *transformation* and *meaning*.

ComputationTransformation

```
private[pdbp] trait ComputationTransformation[
    FC[+ _]: Computation, T[+ _]] {

  private[pdbp] val transform: FC `-U->` T

}
```

ComputationTransformation

```
private[pdbp] trait ComputationTransformation[
    FC[+ _]: Computation, T[+ _]] {

  private[pdbp] val transform: FC `-U->` T

}
```

- *Computation transformations* transform *computations* (and corresponding *kleisli programs*) to *computations* (and corresponding *kleisli programs*)

ProgramMeaning

```
trait ProgramMeaning[
  `>-FP->`[- _, + _]: Program, `>-T->`[- _, + _]] {

  private[pdbp] lazy val binaryTransformation:
      `>-FP->` `-B->` `>-T->`

  lazy val meaning: `>-FP->` `-B->` `>-T->` =
    binaryTransformation

}
```

ComputationMeaning

```
private[pdbp] trait ComputationMeaning[
  FC[+ _]: Computation, T[+ _]]
    extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

  private[pdbp] val unaryTransformation: FC `-U->` T

  // ...

}
```

ComputationMeaning

```
private[pdbp] trait ComputationMeaning[
  FC[+ _]: Computation, T[+ _]]
    extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

  private[pdbp] val unaryTransformation: FC `-U->` T

  // ...

}
```

- *Computation meanings* are *program meanings* for corresponding kleisli programs

FreeTransformed

```
sealed trait Free[C[+ _], +Z]

final case class Transform[C[+ _], +Z]
  (cz: C[Z]) extends Free[C, Z]
final case class Result[C[+ _], +Z]
  (z: Z) extends Free[C, Z]
final case class Bind[C[+ _], -Z, ZZ <: Z, +Y]
  (fczz: Free[C, ZZ], `z=>fcy`: Z => Free[C, Y])
    extends Free[C, Y]

type FreeTransformed[C[+ _]] = [+Z] => Free[C, Z]
```

FreeTransformation

```
private[pdbp]
  trait FreeTransformation[C[+ _]: Computation]
    extends Computation[FreeTransformed[C]]
    with Program[Kleisli[FreeTransformed[C]]]
    with Transformation[C, FreeTransformed[C]] {

  // unfold

  // transform => Transform
  // result => Result
  // bind => Bind

}
```

activeFreeTypes

```
object activeFreeTypes {
  type ActiveFree = FreeTransformed[Active]
  type `=>AF` = Kleisli[ActiveFree]
}
```

activeFreeProgram

```
import ... activeProgram

implicit object activeFreeProgram
    extends Computation[ActiveFree]
    with Program[`=>AF`]

    with FreeTransformation[Active]()
    with ComputationTransformation[Active, ActiveFree]()
```

FreeTransformedMeaning

```
private[pdbp] trait FreeTransformedMeaning[
  FC[+ _]: Computation, T[+ _]](
  implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
  extends ComputationMeaning[FreeTransformed[FC], T] {

  // fold (tail recursive)

  // Transform(fcz) => fcz
  // Result(z) => result(z)
  // Bind(Result(y), y2ftfcz) => fold(y2ftfcz(y))
  // Bind(Bind(fcx, x2ftfcy), y2ftfcz) =>
  //   fold(Bind(fcx, Bind(x2ftfcy, y2ftfcz)))

}
```

activeMeaningOfActiveFree

```
import ... activeMeaningOfActive

implicit object activeMeaningOfActiveFree
    extends FreeTransformedMeaning[Active, Active]()
    with ComputationMeaning[ActiveFree, Active]()

    with ProgramMeaning[‘=>AF‘, ‘=>A‘]()
```

FactorialMain

```
// same factorial and mainFactorial description
// other implementations
import ... mainFactorial
import mainFactorial.factorialMain
// other meaning
import ... activeMeaningOfActiveFree
import activeMeaningOfActiveFree.meaning

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }

}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {

  private[pdbp] def `u>-->r`: Unit >--> R

}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] def `u>-->r`: Unit >--> R =
    `z>-->r`[Unit]

  private[pdbp] def `z>-->r`[Z]: Z >--> R =
    compose(`z>-->u`, `u>-->r`)



}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] def `u>-->r`: Unit >--> R =
    `z>-->r`[Unit]

  private[pdbp] def `z>-->r`[Z]: Z >--> R =
    compose(`z>-->u`, `u>-->r`)

  def read[Z]: Z >--> R = `z>-->r`

}
```

implicit function type

```
type `I=>`[-X, +Y] = implicit X => Y
```

implicit function type

```
type `I=>`[-X, +Y] = implicit X => Y
```

- greatly reduces *reading* boilerplate code

ReadingTransformed

```
type ReadingTransformed[R, C[+ _]] = [+Z] => R ‘I=>‘ C[Z]
```

ReadingTransformation

```
private[pdbp] trait ReadingTransformation[
  R, FC[+ _]: Computation]
    extends ComputationTransformation[
      FC, ReadingTransformed[R, FC]]
    with Computation[ReadingTransformed[R, FC]]

    with Program[Kleisli[ReadingTransformed[R, FC]]]
    with Reading[R, Kleisli[ReadingTransformed[R, FC]]] {

  // ...

}
```

factorialMain

```
val factorialMain: Unit >--> Unit =
  read >-->
    factorial >-->
    effectfulWriteFactorialOfIntToConsole
```

activeReadingTypes

```
object activeReadingTypes {

  type ActiveReading[R] = ReadingTransformed[R, Active]

  type `=>AR`[R] = Kleisli[ActiveReading[R]]

}
```

activeIntReadingProgram

```
import  ... activeProgram

implicit object activeIntReadingProgram
    extends ActiveReadingProgram[BigInt]()
    with Computation[ActiveReading[BigInt]]()
    with Program[‘=>AR‘[BigInt]]()
    with Reading[BigInt, ‘=>AR‘[BigInt]]()

    with ReadingTransformation[BigInt, Active]()
    with ComputationTransformation[Active, ActiveReading[BigInt]]()
```

activeIntReadingMeaningOfActiveIntReading

```
import ... activeIntReadingProgram

implicit object activeIntReadingMeaningOfActiveIntReading
    extends ReadingTransformedMeaning[
      BigInt, Active, Active]()
    with ComputationMeaning[
      ActiveReading[BigInt], ActiveReading[BigInt]]()

    with ProgramMeaning[`=>AR`[BigInt], `=>AR`[BigInt]]()
```

FactorialMain

```
// same factorial description, other implementation
// other factorialMain description
import ... mainFactorialOfIntRead
import mainFactorialOfIntRead.factorialMain
// other meaning
import ... activeIntReadingMeaningOfActiveIntReading
import activeIntReadingMeaningOfActiveIntReading.meaning

object FactorialOfIntReadMain {

  import ... readIntFromConsoleEffect // actual read effect

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }
}
```

factorialMultipliedByIntRead

```
val factorialMultipliedByIntRead: BigInt >--> BigInt =
    (factorial & read) >--> multiply
```

Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {

  private[pdbp] val `w>-->u`: W >--> Unit

  // ...

}
```

Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] val `w>-->u`: W >--> Unit
    = write(identity)

  private[pdbp] def `z>-w->u`[Z]: (Z => W) `I=>` Z >--> Unit =
    compose(function(implicitly), `w>-->u`)

  def write[Z]: (Z => W) `I=>` Z >--> Unit =
     `z>-w->u`

  // ...

}
```

Writing

```
def writeUsing[Z, Y, X](
    `(z&&y)=>x`: ((Z && Y) => X)):
    (Z >--> Y) => ((X => W) `I=>` Z >--> Y) = { `z>-->y` =>
    val `(z&&y)>-->x` = function(`(z&&y)=>x`)
    val `z>-->(x&&y)` =
      `let` {
        `z>-->y`
      } `in` {
        `let` {
          `(z&&y)>-->x`
        } `in` {
          `(z&&y&&x)>-->(x&&y)`
        }
      }
    compose(compose(`z>-->(x&&y)`, left(write),
      `(u&&y)>-->y`)
}
```

Writable

```
private[pdbp] trait Writable[W]
    extends Startable[W]
    with Appendable[W]
```

Startable

```
private[pdbp] trait Startable[W] {

  private[pdbp] val start: W

}
```

Appendable

```
private[pdbp] trait Appendable[W] {

  private[pdbp] val append: W && W => W

}
```

WritingTransformed

```
type WritingTransformed[W, FC[+ _]] = [+Z] => FC[W && Z]
```

WritingTransformation

```
private[pdbp] trait WritingTransformation[
  W: Writable, FC[+ _]: Computation]
    extends ComputationTransformation[
      FC, WritingTransformed[W, FC]]
    with Computation[WritingTransformed[W, FC]]

    with Program[Kleisli[WritingTransformed[W, FC]]]
    with Writing[W, Kleisli[WritingTransformed[W, FC]]] {

  // ...

}
```

ToConsole

```
case class ToConsole(effect: Effect)

type Effect = Unit => Unit
```

WritingToConsoleTransformedMeaning

```
  private[pdbp] trait WritingToConsoleTransformedMeaning[
      FC[+ _]: Computation,
      T[+ _]]
      (implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
      extends ComputationMeaning[WritingTransformed[ToConsole, FC], T]

    // executes console effect

}
```

toConsoleWritable

```
implicit object toConsoleWritable extends Writable[ToConsole] {

  override private[pdbp] val start: ToConsole =
    ToConsole { _ =>
      ()
    }

  override private[pdbp] val append:
      ToConsole && ToConsole => ToConsole = {
    (tc1, tc2) =>
      ToConsole { _ =>
        { tc1.effect(()); tc2.effect(()) }
      }
  }

}
```

infoUtils

```
def infoFunction[Z, Y](string: String): Z && Y => String = {
  case (z, y) =>
    s"INFO -- $currentCalendarInMilliseconds -- $string($z) => $y"
}

def info[
    W: Writable, Z, Y,
    >-->[- _, + _]: [>-->[- _, + _]] => Writing[W, >-->]]
    (string: String):
    (Z >--> Y) => ((String => W) `I=>` Z >--> Y) = {
  val implicitWriting = implicitly[Writing[W, >-->]]
  implicitWriting.writeUsing(infoFunction(string))
}
```

WritingAtomicPrograms

```
val isZero: (String => W) `I=>` BigInt >--> Boolean =
  info("isZero") { isZeroHelper }

val subtractOne: (String => W) `I=>` BigInt >--> BigInt =
  info("subtractOne") { subtractOneHelper }

val multiply: (String => W) `I=>` (BigInt && BigInt) >--> BigInt =
  info("multiply") { multiplyHelper }

def one[Z]: (String => W) `I=>` Z >--> BigInt =
  info("one") { oneHelper }
```

WritingFactorial

```
val factorial: (String => W) ‘I=>‘ BigInt >--> BigInt =
  info("factorial") {
    ‘if‘(isZero) {
      one
    } ‘else‘ {
      ‘let‘ {
        subtractOne >-->
          factorial
      } ‘in‘ {
        multiply
      }
    }
  }
```

factorialMain

```
val factorialMain:
    (String => ToConsole) ‘I=>‘
        ((BigInt => ToConsole) ‘I=>‘ Unit >--> Unit) =
  read >-->
    factorial >-->
    write
```

activeWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeProgram

implicit object activeWritingToConsoleProgram
    extends ActiveWritingProgram[ToConsole]()
    with Computation[ActiveWriting[ToConsole]]()
    with Program[`=>AW`[ToConsole]]()
    with Writing[ToConsole, `=>AW`[ToConsole]]()

    with ComputationTransformation[
      Active, ActiveWriting[ToConsole]]()
    with WritingTransformation[ToConsole, Active]()
```

activeIntReadingWithWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeWritingToConsoleProgram

implicit object activeIntReadingWithWritingToConsoleProgram
    extends ActiveReadingWithWritingProgram[BigInt, ToConsole]()
    with Computation[ActiveReadingWithWriting[BigInt, ToConsole]]()
    with Program[`=>ARW`[BigInt, ToConsole]]()
    with Reading[BigInt, `=>ARW`[BigInt, ToConsole]]()
    with Writing[ToConsole, `=>ARW`[BigInt, ToConsole]]()

    // ...
```

activeIntReadingMeaning OfActiveIntReadingWithWritingToConsole

```
import ... activeMeaningOfActiveWritingToConsole

implicit object
  activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole
    extends ReadingTransformedMeaning[
      BigInt, ActiveWriting[ToConsole], Active]()
    with ComputationMeaning[
      ActiveReadingWithWriting[BigInt, ToConsole],
      ActiveReading[BigInt]]()
    with ProgramMeaning[
      `=>ARW`[BigInt, ToConsole],
      `=>AR`[BigInt]]()
```

FactorialOfIntRead WritingToConsoleWrittenToConsoleMain

```
// other factorial and factorialMain descriptions and implementations
import ... mainFactorialOfIntReadWritingToConsoleWrittenToConsole
import mainFactorialOfIntReadWritingToConsoleWrittenToConsole.factorialMain
// other meaning
import ... activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole
import activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole.meaning

object FactorialOfIntReadWritingToConsoleWrittenToConsoleMain {

  import ... readIntFromConsoleEffect // actual read effect
  import ... writeFactorialOfIntReadFromConsoleToConsoleEffect // actual write
  import ... writeToConsoleEffect // actual write effect via writeUsing

  def main(args: Array[String]): Unit = {
    meaning(factorialMain)(())
  }

}
```

running

```
please type an integer to read
2
INFO -- 2018-08-01 18:00:42.639 -- isZero(2) => false
INFO -- 2018-08-01 18:00:42.645 -- subtractOne(2) => 1
INFO -- 2018-08-01 18:00:42.646 -- isZero(1) => false
INFO -- 2018-08-01 18:00:42.647 -- subtractOne(1) => 0
INFO -- 2018-08-01 18:00:42.647 -- isZero(0) => true
INFO -- 2018-08-01 18:00:42.648 -- one(0) => 1
INFO -- 2018-08-01 18:00:42.648 -- factorial(0) => 1
INFO -- 2018-08-01 18:00:42.649 -- multiply((1,1)) => 1
INFO -- 2018-08-01 18:00:42.649 -- factorial(1) => 1
INFO -- 2018-08-01 18:00:42.650 -- multiply((2,1)) => 2
INFO -- 2018-08-01 18:00:42.650 -- factorial(2) => 2
the factorial value of the integer read is
2
```