PDBP

Program Description Based Programming
Scala eXchange

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August 2, 2018

Intro

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  - a `Dotty` *library* PDBP

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- this talk is about
  - a `Dotty` *library* PDBP
  - inspired by the *function-level programming language* FP
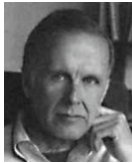
John Backus

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- ACM Turing Award Winner 1977

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- *Can programming be liberated from the Von Neumann style?*

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What is this?

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- A *pipe*?

4

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- A *painting describing* a pipe?

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- . . .

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Ceci n'est pas une pipe

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Ceci n'est pas une pipe

- It is a *painting* by René Magritte *describing* a pipe

This is not a program

```
val xxxxxxxxx: BigInt >--> BigInt =
  'if'(isZero) {
    one
  } 'else' {
    'let' {
      subtractOne >-->
        xxxxxxxxx
    } 'in' {
      multiply
    }
  }
```

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```

- It is *code describing* a program

factorial description

```
val factorial: BigInt >--> BigInt =
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```

- factorial *uses* capabilities *declared* in
  type class `trait Program[>-->[- _, + _]]`

factorial meanings

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```

- different `implicit` `object`'s *define* capabilities of
  type class `trait Program[>-->[- _, + _]]`

program

```
val program: Z >--> Y
```

mainProgram

```
val program: Unit >--> Unit
```

Composition

```
val 'z>-->y': Z >--> Y
val 'y>-->x': Y >--> x

val 'z>-->x': Z >--> X = 'z>-->y >--> 'y>-->x'
```

program

```
 ------------------------------
|    ---------         ----------  |       ----------
| | Z >--> Y | >--> | Y >--> X | | => | Z >--> X |
|    ---------         ----------  |       ----------
 ------------------------------
```


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mainProgram

```
val producer: Unit >--> Z
val program: Z >--> Y
val consumer: Y >--> Unit

val mainProgram: Unit >--> Unit =
  producer >--> program >--> consumer
```


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mainProgram

```
            ----------------
          | Unit >--> Unit |
  ------                      ------
| Unit >--> Z >--> Y >--> Unit |
  ----------------------------
                              v
                              |     ----------------
      --------????--------  => | Unit >--> Unit |
      |                         ----------------
      v
  ----------------------------
| Unit >--> Z >--> Y >--> Unit |
  ------                      ------
          | Unit >--> Unit |
            ----------------
```

FP versus PDBP

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- FP and PDBP promote *pointfree functional programming*

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  PDBP is a *library*

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    - FP *semantics* is *fixed*
      PDBP *semantics* is *not fixed*
    - FP *capabilities* are *fixed*
      PDBP *capabilities* are *not fixed*
    - FP *effects* are *impure*
      PDBP *effects* are *pure*

Semantics

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val factorial: BigInt >--> BigInt =
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- *production*

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  - *recursion* using *stack*

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    - *recursion* using *stack*
    - *recursion* using *heap*

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- *test*

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  - ...

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Effects

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- *reading*

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- *writing*

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- `trait Computation[C[+ _]]` corresponds to *monads*

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  generalize *functions*

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    - `mz bind { z => my bind { y => result(ex(z, y)) } }`

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- *arrows* can also be programmed *pointful* (*arrow calculus*)

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- *Constraints Liberate, Liberties Constrain*

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  - Dotty is a Scalable language
  - Dotty *library based* language extensions are *type safe*
- PDBP comes with a *program description DSL*

Program description DSL

```
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  - `public` *pointfree arrow API*
    provides *elegance of use* for *application* developers
- the PDBP can live with
  - corresponding implementation constraints

PDBP

```
//          _____           __    __       _____
//         / ___  \         / /\  / /\     / ___  \
//        / /__/ / / _____/ / / / / /_/__   / /__/ / /
//       / _____/ / / ___  / / / ___  /\ /____  / /
//      / /\____\/ / /__/ / / / /__/ / / \___/ / /
//     /_/ /      /_____/ / /_____/ /     /_/ /
//     \_\/       \_____\/  \_____\/      \_\/
//                                              v1.0
//  Program Description Based Programming Library
//  author        Luc Duponcheel         2017-2018
```

PDBP library design decisions (cfr. `Haskell`)

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  - `implicit object`'s (`extend trait`'s) *define* capabilities
- *library level* meaning
  - *natural transformations*

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PDBP library design decisions (cfr. `Haskell`)

- *dependency injection* by `implicit import`

- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s

- `object`'s that *extend* those `class`'es `import implicit` `object`'s that *extend* those `trait`'s

Program (cfr. *arrow*)

```
trait Program[>-->[- _, + _]]
```

Computation (cfr. *monad*)

```
trait Computation[C[+ _]]
```

Liskov Substitution Principle

Liskov Substitution Principle

- *impose less*

Liskov Substitution Principle

- *impose less*
- *provide more*

Internet Robustness Principle

Internet Robustness Principle

- *be liberal in what you receive*

Internet Robustness Principle

- *be liberal in what you receive*
- *be generous in what you send*

PDBP library details

Program

```
trait Program[>-->[- _, + _]]
    extends Function[>-->]
    with Composition[>-->]
    with Construction[>-->]
    with Condition[>-->]

    with Aggregation[>-->]
```

Function

Function

-        `val 'z>-->y' = function('z=>y')`

Function

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- pure functions are *atomic programs*

Function

- `val 'z>-->y' = function('z=>y')`
- pure functions are *atomic programs*
  - up to you to define granularity

Composition

Composition

- `val 'z>-->x' = 'z>-->y' >--> 'y>-->x'`

Construction

Construction

- `val ‘z>-->y&&x‘ = ‘z>-->y‘ & ‘z>-->x‘`

Construction

- val `z>-->y&&x` = `z>-->y` & `z>-->x`
- val `z&&y>-->x&&w` = `z>-->x` && `y>-->w`

Construction

-      `val ‘z>-->y&&x‘ = ‘z>-->y‘ & ‘z>-->x‘`
-      `val ‘z&&y>-->x&&w‘ = ‘z>-->x‘ && ‘y>-->w‘`
-      `val ‘z>-->x‘ =`
         `‘let‘ ‘z>-->y‘ ‘in‘ ‘z&&y>-->x‘`

Condition

Condition

-     `val 'y||x>-->z' = 'y>-->z' | 'x>-->z'`

Condition

- val ‘y||x>-->z‘ = ‘y>-->z‘ | ‘x>-->z‘
- val ‘x||w>-->z||y‘ = ‘x>-->z‘ || ‘w>-->y‘

Condition

- `val 'y||x>-->z' = 'y>-->z' | 'x>-->z'`
- `val 'x||w>-->z||y' = 'x>-->z' || 'w>-->y'`
- `val 'y>-->z' =`
  `'if'('y>-->b') 'y>-t->z' 'else' 'y>-f->z'`

Computation

```
private[pdbp] trait Computation[C[+ _]]
    extends Resulting[C]
    with Binding[C]
    with Program[[-Z, +Y] => Z => C[Y]]

    with Lifting[C]

    with Sequencing[C]
```

Resulting

```
val cz = result(z)
```

Binding

Binding

- `val cy = cz bind { z => 'z=>cy'(y) }`

Binding

- `val cy = cz bind { z => ‘z=>cy‘(y) }`

- `val cy = cz bind { z => result(‘z=>y‘(y)) }`

Kleisli

Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`

Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`

- ```
  private[pdbp] trait Computation[C[+ _]]
        extends Resulting[C]
        with Binding[C]
        with Program[Kleisli[C]]

        // ...
  ```

HelperFunctions

```
val isZeroFunction: BigInt => Boolean =
{ i =>
    i == 0
}
def oneFunction[Z]: Z => BigInt =
{ z =>
    1
}
val subtractOneFunction: BigInt => BigInt =
{ i =>
    i - 1
}
val multiplyFunction: (BigInt && BigInt) => BigInt =
{ (i, j) =>
    i * j
}
```


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HelperPrograms

```
val isZeroHelper: BigInt >--> Boolean =
  function(isZeroFunction)

val subtractOneHelper: BigInt >--> BigInt =
  function(subtractOneFunction)

val multiplyHelper: (BigInt && BigInt) >--> BigInt =
  function(multiplyFunction)

def oneHelper[Z]: Z >--> BigInt =
  function(oneFunction)
```

AtomicPrograms

```
val isZero: BigInt >--> Boolean =
  isZeroHelper

val subtractOne: BigInt >--> BigInt =
  subtractOneHelper

val multiply: (BigInt && BigInt) >--> BigInt =
  multiplyHelper

def one[Z]: Z >--> BigInt =
  oneHelper
```

factorial

```
val factorial: BigInt >--> BigInt =
  `if`(isZero) {
    one
  } `else` {
    `let` {
      subtractOne >-->
        factorial
    } `in` {
      multiply
    }
  }
```

factorialMain

```
val factorialMain: Unit >--> Unit =
  producer >-->
    factorial >-->
    consumer
```

activeTypes

```
object activeTypes {

  type Active[+Z] = Z

  type '=>A' = Kleisli[Active]

}
```

activeProgram

```
implicit object activeProgram
    extends Computation[Active]
    with Program['=>A'] {

  override private[pdbp] def result[Z]: Z => Active[Z] =
    'z=>az'

  override private[pdbp] def bind[Z, Y](
      az: Active[Z],
      'z=>ay': => (Z => Active[Y])): Active[Y] =
    'z=>ay'(az)

}
```

activeMeaningOfActive

```
implicit object activeMeaningOfActive
    extends MeaningOfActive[Active]()
    with ComputationMeaning[Active, Active]()
    with ProgramMeaning['=>A', '=>A']()
```

FactorialMain

```scala
import ... activeMeaningOfActive
import activeMeaningOfActive.meaning

import ... mainFactorial
import mainFactorial.factorialMain

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }
}
```

Problems and Solutions

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- Problem: obvious `factorial` meaning
  implementing `>-->` as '`=>A`' *is not stack safe*
  - Solution: `FreeTransformation` and
    `FreeTransformedMeaning`
- Problem: obvious `producer` and `consumer` *execute effects*
  - Solution: `Reading` resp. `Writing` extensions of `Program` with
    members `read` resp. `write` that *describe effects*

NaturalBinaryTypeConstructorTransformation

```
trait '-B->'['>-F->'[- _, + _], '>-T->'[- _, + _]] {

  def apply[Z, Y]: Z '>-F->' Y => Z '>-T->' Y

}
```

NaturalUnaryTypeConstructorTransformation

```scala
private[pdbp] trait `-U->`[F[+ _], T[+ _]]
    extends `-B->`[Kleisli[F], Kleisli[T]] {

  private[pdbp] def apply[Z](fz: F[Z]): T[Z]

  // ...
  }

}
```

ComputationTransformation

```
private[pdbp] trait ComputationTransformation[
    FC[+ _]: Computation, T[+ _]] {

  private[pdbp] val transform: FC '-U->' T

}
```

ProgramMeaning

```
trait ProgramMeaning[
  '>-FP->'[- _, + _]: Program, '>-T->'[- _, + _]] {

  private[pdbp] lazy val binaryTransformation:
      '>-FP->' '-B->' '>-T->'

  lazy val meaning: '>-FP->' '-B->' '>-T->' =
    binaryTransformation

}
```

ComputationMeaning

```scala
private[pdbp] trait ComputationMeaning[
  FC[+ _]: Computation, T[+ _]]
    extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

  private[pdbp] val unaryTransformation: FC '-U->' T

  // ...

}
```

FreeTransformed

```
sealed trait Free[C[+ _], +Z]

final case class Transform[C[+ _], +Z]
  (cz: C[Z]) extends Free[C, Z]
final case class Result[C[+ _], +Z]
  (z: Z) extends Free[C, Z]
final case class Bind[C[+ _], -Z, ZZ <: Z, +Y]
  (fczz: Free[C, ZZ], `z=>fcy`: Z => Free[C, Y])
    extends Free[C, Y]

type FreeTransformed[C[+ _]] = [+Z] => Free[C, Z]
```

FreeTransformation

```
private[pdbp]
  trait FreeTransformation[C[+ _]: Computation]
    extends Computation[FreeTransformed[C]]
    with Program[Kleisli[FreeTransformed[C]]]
    with Transformation[C, FreeTransformed[C]] {

  // unfold
  // transform => Transform
  // result => Result
  // bind => Bind

}
```

activeFreeTypes

```
object activeFreeTypes {

  type ActiveFree = FreeTransformed[Active]

  type '=>AF' = Kleisli[ActiveFree]

}
```

activeFreeProgram

```
import ... activeProgram

implicit object activeFreeProgram
    extends Computation[ActiveFree]
    with Program['=>AF']
    with FreeTransformation[Active]()
    with ComputationTransformation[Active, ActiveFree]()
```

FreeTransformedMeaning

```scala
private[pdbp] trait FreeTransformedMeaning[
  FC[+ _]: Computation, T[+ _]](
  implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
  extends ComputationMeaning[FreeTransformed[FC], T] {

  // tailrecFold
  // Transform(fcz) => fcz
  // Result(z) => result(z)
  // Bind(Result(y), y2ftfcz) => tailrecFold(y2ftfcz(y))
  // Bind(Bind(fcx, x2ftfcy), y2ftfcz) =>
  //   tailrecFold(Bind(fcx, Bind(x2ftfcy, y2ftfcz)))

}
```

activeMeaningOfActiveFree

```
import ... activeMeaningOfActive

implicit object activeMeaningOfActiveFree
    extends FreeTransformedMeaning[Active, Active]()
    with ComputationMeaning[ActiveFree, Active]()
    with ProgramMeaning['=>AF', '=>A']()
```

FactorialMain

```
import ... activeMeaningOfActiveFree
import activeMeaningOfActiveFree.meaning

import ... mainFactorial
import mainFactorial.factorialMain

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }
}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {


  private[pdbp] def 'u>-->r': Unit >--> R



}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] def `u>-->r`: Unit >--> R =
    `z>-->r`[Unit]

  private[pdbp] def `z>-->r`[Z]: Z >--> R =
    compose(`z>-->u`, `u>-->r`)



}
```

Reading

```
trait Reading[R, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] def `u>-->r`: Unit >--> R =
    `z>-->r`[Unit]

  private[pdbp] def `z>-->r`[Z]: Z >--> R =
    compose(`z>-->u`, `u>-->r`)

  def read[Z]: Z >--> R = `z>-->r`

}
```


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ReadingTransformed

```
type 'I=>'[-X, +Y] = implicit X => Y

type ReadingTransformed[R, C[+ _]] = [+Z] => R 'I=>' C[Z]
```

ReadingTransformation

```
private[pdbp] trait ReadingTransformation[
  R, FC[+ _]: Computation]
    extends ComputationTransformation[
      FC, ReadingTransformed[R, FC]]
    with Computation[ReadingTransformed[R, FC]]
    with Program[Kleisli[ReadingTransformed[R, FC]]]
    with Reading[R, Kleisli[ReadingTransformed[R, FC]]] {

  // ...

}
```

factorialMain

```
val factorialMain: Unit >--> Unit =
  read >-->
    factorial >-->
    consumer
```

activeReadingTypes

```
object activeReadingTypes {

  type ActiveReading[R] = ReadingTransformed[R, Active]

  type '=>AR'[R] = Kleisli[ActiveReading[R]]

}
```

activeIntReadingProgram

```
import  ... activeProgram

implicit object activeIntReadingProgram
    extends ActiveReadingProgram[BigInt]()
    with Computation[ActiveReading[BigInt]]()
    with Program['=>AR'[BigInt]]()
    with Reading[BigInt, '=>AR'[BigInt]]()
    with ReadingTransformation[BigInt, Active]()
    with ComputationTransformation[Active, ActiveReading[BigInt]]()
```

activeIntReadingProgram

```
import ... activeIntReadingProgram

implicit object activeIntReadingMeaningOfActiveIntReading
    extends ReadingTransformedMeaning[BigInt, Active, Active]()
    with ComputationMeaning[ActiveReading[BigInt], ActiveReading[BigI
    with ProgramMeaning['=>AR'[BigInt], '=>AR'[BigInt]]()
```

FactorialMain

```
import ... activeIntReadingMeaningOfActiveIntReading
import activeIntReadingMeaningOfActiveIntReading.meaning

import ... mainFactorialOfIntRead
import mainFactorialOfIntRead.factorialMain

object FactorialOfIntReadMain {

  import ... readIntFromConsoleEffect

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())

  }
}
```


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factorialMultipliedByIntRead

```
val factorialMultipliedByIntRead: BigInt >--> BigInt =
    (factorial & read) >--> multiply
```

Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {


  private[pdbp] val 'w>-->u': W >--> Unit




  // ...

}
```

Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {
  this: Function[>-->] & Composition[>-->] =>

  private[pdbp] val 'w>-->u': W >--> Unit
    = write(identity)

  def write[Z]: (Z => W) 'I=>' Z >--> Unit =
    compose(function(implicitly), 'w>-->u')

  // ...

}
```

Writing

```
def writeUsing[Z, Y, X](
    '(z&&y)=>x': ((Z && Y) => X)):
    (Z >--> Y) => ((X => W) 'I=>' Z >--> Y) = { 'z>-->y' =>
    val '(z&&y)>-->x' = function('(z&&y)=>x')
    val 'z>-->(x&&y)' =
      'let' {
        'z>-->y'
      } 'in' {
        'let' {
          '(z&&y)>-->x'
        } 'in' {
          '(z&&y&&x)>-->(x&&y)'
        }
      }
    compose(compose('z>-->(x&&y)', left(write)),
      '(u&&y)>-->y')
}
```

Writable

```
private[pdbp] trait Writable[W]
    extends Startable[W]
    with Appendable[W]
```

Startable

```
private[pdbp] trait Startable[W] {

  private[pdbp] val start: W

}
```

Appendable

```
private[pdbp] trait Appendable[W] {

  private[pdbp] val append: W && W => W

}
```

WritingTransformed

```
type WritingTransformed[W, FC[+ _]] = [+Z] => FC[W && Z]
```

WritingTransformation

```
private[pdbp] trait WritingTransformation[
  W: Writable, FC[+ _]: Computation]
    extends ComputationTransformation[
      FC, WritingTransformed[W, FC]]
    with Computation[WritingTransformed[W, FC]]
    with Program[Kleisli[WritingTransformed[W, FC]]]
    with Writing[W, Kleisli[WritingTransformed[W, FC]]] {

  // ...

}
```

ToConsole

```scala
case class ToConsole(effect: Effect)

type Effect = Unit => Unit
```

ToConsole

```
implicit object toConsoleWritable extends Writable[ToConsole] {

  override private[pdbp] val start: ToConsole =
    ToConsole { _ =>
      ()
    }

  override private[pdbp] val append:
      ToConsole && ToConsole => ToConsole = {
    (tc1, tc2) =>
      ToConsole { _ =>
        { tc1.effect(()); tc2.effect(()) }
      }
  }

}
```

infoUtils

```scala
def infoFunction[Z, Y](string: String): Z && Y => String = {
  case (z, y) =>
    s"INFO -- $currentCalendarInMilliseconds -- $string($z) => $y"
}

def info[
    W: Writable, Z, Y,
    >-->[- _, + _]: [>-->[- _, + _]] => Writing[W, >-->]]
    (string: String):
    (Z >--> Y) => ((String => W) `I=>` Z >--> Y) = {
  val implicitWriting = implicitly[Writing[W, >-->]]
  implicitWriting.writeUsing(infoFunction(string))
}
```

WritingAtomicPrograms

```
val isZero: (String => W) 'I=>' BigInt >--> Boolean =
  info("isZero") { isZeroHelper }

val subtractOne: (String => W) 'I=>' BigInt >--> BigInt =
  info("subtractOne") { subtractOneHelper }

val multiply: (String => W) 'I=>' (BigInt && BigInt) >--> BigInt =
  info("multiply") { multiplyHelper }

def one[Z]: (String => W) 'I=>' Z >--> BigInt =
  info("one") { oneHelper }
```

WritingFactorial

```
val factorial: (String => W) `I=>` BigInt >--> BigInt =
  info("factorial") {
    `if`(isZero) {
      one
    } `else` {
      `let` {
        subtractOne >-->
          factorial
      } `in` {
        multiply
      }
    }
  }
```

factorialMain

```
val factorialMain:
    (String => ToConsole) 'I=>'
        ((BigInt => ToConsole) 'I=>' Unit >--> Unit) =
  read >-->
    factorial >-->
    write
```

activeWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeProgram

implicit object activeWritingToConsoleProgram
    extends ActiveWritingProgram[ToConsole]()
    with ComputationTransformation[Active, ActiveWriting[ToConsole]](
    with WritingTransformation[ToConsole, Active]()
    with Writing[ToConsole, '=>AW'[ToConsole]]()
    // ...
```

activeIntReadingWithWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeWritingToConsoleProgram

implicit object activeIntReadingWithWritingToConsoleProgram
    extends ActiveReadingWithWritingProgram[BigInt, ToConsole]()
    with Computation[ActiveReadingWithWriting[BigInt, ToConsole]]()
    with Program['=>ARW'[BigInt, ToConsole]]()
    with Reading[BigInt, '=>ARW'[BigInt, ToConsole]]()
    with Writing[ToConsole, '=>ARW'[BigInt, ToConsole]]()
    // ...
```

activeIntReadingWithWritingToConsoleProgram

```
// ...
with ReadingWithWritingTransformation[BigInt,
                                       ToConsole,
                                       ActiveWriting[ToConsole]]()
with ReadingTransformation[BigInt, ActiveWriting[ToConsole]]()
with ComputationTransformation[
  ActiveWriting[ToConsole],
  ActiveReadingWithWriting[BigInt, ToConsole]]()
```

FactorialOfIntRead WritingToConsoleWrittenToConsoleMain

```scala
import ... activeIntReadingMeaningOfActiveIntReadingWithWritingToConsol
import activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole.me

import ... mainFactorialOfIntReadWritingToConsoleWrittenToConsole
import mainFactorialOfIntReadWritingToConsoleWrittenToConsole.factorial

object FactorialOfIntReadWritingToConsoleWrittenToConsoleMain {

  import ... readIntFromConsoleEffect
  import ... writeFactorialOfIntReadFromConsoleToConsoleEffect
  import ... writeToConsoleEffect

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)(())
```

```scala
  }
```

running

```
please type an integer to read
2
INFO -- 2018-08-01 18:00:42.639 -- isZero(2) => false
INFO -- 2018-08-01 18:00:42.645 -- subtractOne(2) => 1
INFO -- 2018-08-01 18:00:42.646 -- isZero(1) => false
INFO -- 2018-08-01 18:00:42.647 -- subtractOne(1) => 0
INFO -- 2018-08-01 18:00:42.647 -- isZero(0) => true
INFO -- 2018-08-01 18:00:42.648 -- one(0) => 1
INFO -- 2018-08-01 18:00:42.648 -- factorial(0) => 1
INFO -- 2018-08-01 18:00:42.649 -- multiply((1,1)) => 1
INFO -- 2018-08-01 18:00:42.649 -- factorial(1) => 1
INFO -- 2018-08-01 18:00:42.650 -- multiply((2,1)) => 2
INFO -- 2018-08-01 18:00:42.650 -- factorial(2) => 2
the factorial value of the integer read is
2
```