

PDBP

Program Description Based Programming
Scala eXchange

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Intro



Intro

- this talk is about



Intro

- this talk is about
 - a *Dotty library* PDBP



Intro

- this talk is about
 - a *Dotty library* PDBP
 - inspired by the *function-level programming language* FP



John Backus



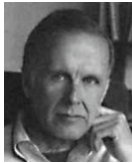
John Backus



- ACM Turing Award Winner 1977



John Backus



- ACM Turing Award Winner 1977
- *Can programming be liberated from the Von Neumann style?*



What is this?



What is this?



- A *pipe*?



What is this?



- A *pipe*?
- A *painting describing* a pipe?



What is this?



- A *pipe*?
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- A *slide describing* a painting describing a pipe?



What is this?



- A *pipe*?
- A *painting describing* a pipe?
- A *slide describing* a painting describing a pipe?
- ...



Ceci n'est pas une pipe



Ceci n'est pas une pipe



- It is a *painting* by René Magritte *describing* a pipe



This is not a program

```
val ??????????: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        ??????????  
    } 'in' {  
      multiply  
    }  
  }  
}
```



This is not a program

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- It is *code describing* a program



This is not a program

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    } 'in' {  
      multiply  
    }  
  }  
}
```

- It is *code describing* a program
- Can you think of a more meaningful name?



factorial description

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        factorial  
    } 'in' {  
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```



factorial description

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val factorial: BigInt >--> BigInt =  
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      multiply  
    }  
  }
```

- `factorial` *definition* uses the capabilities *declared* in *type class* `trait Program[>-->[- _, + _]]`



factorial implementations

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
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factorial implementations

```
val factorial: BigInt >--> BigInt =  
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        factorial  
    } 'in' {  
      multiply  
    }  
  }
```

- factorial *implementations* depend on implicit object's *defining* the capabilities of type class
trait Program[>-->[- _, + _]]



factorial meanings

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
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  } 'else' {  
    'let' {  
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        factorial  
    } 'in' {  
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factorial meanings

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val factorial: BigInt >--> BigInt =  
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  } 'else' {  
    'let' {  
      subtractOne >-->  
        factorial  
    } 'in' {  
      multiply  
    }  
  }
```

- **factorial** *meanings* use *natural transformations* transforming implementations



program

val program: $Z \multimap Y$



Composition

```
val 'z>-->y': Z >--> Y
```

```
val 'y>-->x': Y >--> X
```

```
val 'z>-->x': Z >--> X = 'z>-->y' >--> 'y>-->x'
```



program (design artifact)

```
-----  
| ----- local ----- | -----  
| | Z >--> Y | >--> | Y >--> X | | => | Z >--> X |  
| ----- | ----- | -----  
-----
```



mainProgram

```
val program: Unit >--> Unit
```



mainProgram

```
val producer: Unit >--> Z
val program: Z >--> Y
val consumer: Y >--> Unit

val mainProgram: Unit >--> Unit =
  producer >--> program >--> consumer
```



mainProgram (architectural artifact)

```

      -----
      | Unit >--> Unit |
-----
| Unit >--> Z >--> Y >--> Unit |
-----v-----
                        |
                        |
      ---- distributed ---- => | Unit >--> Unit |
      |                                     -----
      |
      |
-----v-----
| Unit >--> Z >--> Y >--> Unit |
-----
      | Unit >--> Unit |
      -----
```



FP versus PDBP



FP versus PDBP

- FP and PDBP



FP versus PDBP

- FP and PDBP
 - promote *pointfree*, *composition* based, *functional programming*



FP versus PDBP

- FP and PDBP
 - promote *pointfree*, *composition* based, *functional programming*
- FP is a *language*
PDBP is a *library*



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PDBP is a *library*
 - FP *semantics* is *fixed*
PDBP *semantics* is *not fixed*



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 - FP *capabilities* are *fixed*
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 - promote *pointfree*, *composition* based, *functional programming*
- FP is a *language*
PDBP is a *library*
 - FP *semantics* is *fixed*
PDBP *semantics* is *not fixed*
 - FP *capabilities* are *fixed*
PDBP *capabilities* are *not fixed*
 - FP *effects* are *impure*
PDBP *effects* are *pure*



Semantics

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
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Semantics

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```

- *production*



Semantics

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```

- *production*
 - *recursion* using *stack*



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```

- *production*
 - *recursion* using *stack*
 - *recursion* using *heap*



Semantics

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```

- *production*
 - *recursion* using *stack*
 - *recursion* using *heap*
- *test*



Semantics

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```

- *production*
 - *recursion* using *stack*
 - *recursion* using *heap*
- *test*
 - ...



Capabilities



Capabilities

- *manipulating state*



Capabilities

- *manipulating state*
- *handling failure*



Capabilities

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- *handling failure*
- *handling latency*



Capabilities

- *manipulating state*
- *handling failure*
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- *handling control*



Capabilities

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- *handling latency*
- *handling control*
- ...



Effects



Effects

- *reading*



Effects

- *reading*
- *writing*



Foundations



Foundations

- `trait Program[>-->[- _, + _]]` corresponds to *arrows*



Foundations

- `trait Program[>-->[- _, + _]]` corresponds to *arrows*
- `trait Computation[C[+ _]]` corresponds to *monads*



Arrows versus monads



Arrows versus monads

- arrows generalize *functions*



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- *composition* based, *pointfree* functional programming



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- `val 'z=>x' = 'z=>y' andThen 'y=>x'`



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- `val 'z=>x' = 'z=>y' andThen 'y=>x'`
- `val 'z>-->x' = 'z>-->y' >--> 'y>-->z'`



Arrows versus monads



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- monads generalize *expressions*



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- $\{ \text{val } z = ez ; \{ \text{val } y = ey ; ex(z,y) \} \}$



Arrows versus monads

- monads generalize *expressions*
- *binding* based, *pointful* functional programming
- `{ val z = ez ; { val y = ey ; ex(z,y) } }`
- `mz bind { z => my bind { y => mx(z,y) } }`



Arrows versus monads

- monads generalize *expressions*
- *binding* based, *pointful* functional programming
- `{ val z = ez ; { val y = ey ; ex(z,y) } }`
- `mz bind { z => my bind { y => mx(z,y) } }`
- `mz bind { z => my bind { y => result(ex(z,y)) } }`



Arrows versus monads (kleisli arrows)



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- `val function: Z => Y = z => ey(x)`



Arrows versus monads (kleisli arrows)

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 - expression is used to define function



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- `val function: Z => Y = z => ey(x)`
 - expression is used to define function
- `val kleisliArrow: Z => M[Y] = z => my(x)`



Arrows versus monads (kleisli arrows)

- `val function: Z => Y = z => ey(x)`
 - expression is used to define function
- `val kleisliArrow: Z => M[Y] = z => my(x)`
 - monad is used to define kleisli arrow



Arrows versus monads



Arrows versus monads

- *arrows* can be programmed *pointful* (*arrow calculus*)



Arrows versus monads

- *arrows* can be programmed *pointful* (*arrow calculus*)
- *monads* can be programmed *pointfree* (*kleisli arrows*)



Power of expressions



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- *monads* are more *concrete* (less *abstract*) than *arrows*



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 - *monads* allow more *description liberty*



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 - *monads* impose more *implementation constraints*



Power of expressions

- *monads* are more *concrete* (less *abstract*) than *arrows*
 - *monads* allow more *description liberty*
 - *monads* impose more *implementation constraints*
- *Constraints Liberate, Liberties Constrain*



Elegance of use



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- *pointfree* programming is sometimes considered to be more *abstruse* than *pointful* programming



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Elegance of use

- *pointfree* programming is sometimes considered to be more *abstruse* than *pointful* programming
- Dotty comes to the rescue
 - Dotty is a Scalable language
 - Dotty *library based* language extensions are *type safe*
- PDBP comes with a *program description DSL*



Program Description DSL

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        factorial  
    } 'in' {  
      multiply  
    }  
  }
```



Computation Description DSL

```
val factorial: BigInt '=>C' BigInt = { z =>
  isZero(z) bind { b =>
    if (b) {
      one(z)
    } else {
      subtractOne(z) bind { y =>
        factorial(y) bind { x =>
          multiply((y, x))
        }
      }
    }
  }
}
```



Uh! Oh! typo ...or error?

```
val factorial: BigInt =>C' BigInt = { z =>
  isZero(z) bind { b =>
    if (b) {
      one(z)
    } else {
      subtractOne(z) bind { y =>
        factorial(y) bind { x =>
          multiply((z, x))
        }
      }
    }
  }
}
```



PDBP's choice



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- the PDBP library goes for



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 - `private[pdbp]` *pointful monad API*
provides *power of expression* for *library* developers



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 - `public` *pointfree arrow API*
provides *elegance of use* for *application* developers



PDBP's choice

- the PDBP library goes for
 - `private[pdbp]` *pointful monad API*
provides *power of expression* for *library* developers
 - `public` *pointfree arrow API*
provides *elegance of use* for *application* developers
- the PDBP can live with



PDBP's choice

- the PDBP library goes for
 - `private[pdbp]` *pointful monad API*
provides *power of expression* for *library* developers
 - `public` *pointfree arrow API*
provides *elegance of use* for *application* developers
- the PDBP can live with
 - corresponding implementation constraints



PDBP library design decisions



PDBP library design decisions

- *description* separated from *implementations* and *meanings*



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- *description*



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- *description* separated from *implementations* and *meanings*
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 - `trait`'s (*type classes*) that *declare* capabilities



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- *description*
 - `trait`'s (*type classes*) that *declare* capabilities
- *implementations*



PDBP library design decisions

- *description* separated from *implementations* and *meanings*
- *description*
 - `trait`'s (*type classes*) that *declare* capabilities
- *implementations*
 - `implicit object`'s that *extend trait*'s *define* capabilities



PDBP library design decisions

- *description* separated from *implementations* and *meanings*
- *description*
 - `trait`'s (*type classes*) that *declare* capabilities
- *implementations*
 - `implicit object`'s that `extend trait`'s *define* capabilities
- *meanings*



PDBP library design decisions

- *description* separated from *implementations* and *meanings*
- *description*
 - `trait`'s (*type classes*) that *declare* capabilities
- *implementations*
 - `implicit object`'s that *extend trait*'s *define* capabilities
- *meanings*
 - `implicit object`'s that *define* a *natural transformation*



PDBP library design decisions



PDBP library design decisions

- *dependency injection* by `implicit import`



PDBP library design decisions

- *dependency injection* by `implicit import`
- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s



PDBP library design decisions

- *dependency injection* by `implicit import`
- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s
- `object`'s that *extend* those `class`'es `import implicit`
`object`'s that *extend* those `trait`'s



Program (cfr. *arrow*)

```
trait Program[>-->[- _, + _]]
```



Computation (cfr. *monad*)

```
trait Computation[C[+ _]]
```



Liskov Substitution Principle



Liskov Substitution Principle

- *impose less*



Liskov Substitution Principle

- *impose less*
- *provide more*



Internet Robustness Principle



Internet Robustness Principle

- *be liberal in what you receive*



Internet Robustness Principle

- *be liberal in what you receive*
- *be generous in what you send*



PDBP library details



Program

```
trait Program[>-->[- _, + _]]  
  extends Function[>-->]  
  with Composition[>-->]  
  with Construction[>-->]  
  with Condition[>-->]  
  
  with Aggregation[>-->]
```



Function



Function

- `val 'z>-->y' = function('z=>y')`



Function

- `val 'z>-->y' = function('z=>y')`
- *pure functions* are *atomic programs*



Function

- `val 'z>-->y' = function('z=>y')`
- *pure functions* are *atomic programs*
 - up to you to define *granularity*



Composition



Composition

- $\text{val } 'z \multimap x' = 'z \multimap y' \multimap 'y \multimap x'$



Construction



Construction

- `val 'z>-->y&& x' = 'z>-->y' & 'z>-->x'`



Construction

- `val 'z>-->y&&x' = 'z>-->y' & 'z>-->x'`
- `val 'z&&y>-->x&&w' = 'z>-->x' && 'y>-->w'`



Construction

- `val 'z>-->y&&x' = 'z>-->y' & 'z>-->x'`
- `val 'z&&y>-->x&&w' = 'z>-->x' && 'y>-->w'`
- `val 'z>-->x' =
 'let' 'z>-->y' 'in' 'z&&y>-->x'`



Condition



Condition

- $\text{val } 'y||x>-->z' = 'y>-->z' \mid 'x>-->z'$



Condition

- $\text{val } 'y|x>-->z' = 'y>-->z' \mid 'x>-->z'$
- $\text{val } 'x|w>-->z||y' = 'x>-->z' \mid\mid 'w>-->y'$



Condition

- `val 'y||x>-->z' = 'y>-->z' | 'x>-->z'`
- `val 'x||w>-->z||y' = 'x>-->z' || 'w>-->y'`
- `val 'y>-->z' =
 'if'('y>-->b') 'y>-t->z' 'else' 'y>-f->z'`



Computation

```
private[pdbp] trait Computation[C[+ _]]  
  extends Resulting[C]  
  with Binding[C]  
  with Program[[-Z, +Y] => Z => C[Y]]  
  
  with Lifting[C]  
  
  with Sequencing[C]
```



Resulting



Resulting

- `val cz = result(z)`



Binding



Binding

- `val cy = cz bind { z => 'z=>cy'(y) }`



Binding

- `val cy = cz bind { z => 'z=>cy'(y) }`
- `val cy = cz bind { z => result('z=>y'(y)) }`



Kleisli



Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`



Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`
- ```
private[pdbp] trait Computation[C[+ _]]
 extends Resulting[C]
 with Binding[C]
 with Program[Kleisli[C]]

 // ...
```



## factorial Helper Functions

```
val isZeroFunction: BigInt => Boolean = { i =>
 i == 0
}
```

```
def oneFunction[Z]: Z => BigInt = { z =>
 1
}
```

```
val subtractOneFunction: BigInt => BigInt = { i =>
 i - 1
}
```

```
val multiplyFunction: (BigInt && BigInt) => BigInt = { (i, j) =>
 i * j
}
```



## factorial Helper Programs

```
val isZeroHelper: BigInt >--> Boolean =
 function(isZeroFunction)
```

```
val subtractOneHelper: BigInt >--> BigInt =
 function(subtractOneFunction)
```

```
val multiplyHelper: (BigInt && BigInt) >--> BigInt =
 function(multiplyFunction)
```

```
def oneHelper[Z]: Z >--> BigInt =
 function(oneFunction)
```



## factorial Atomic Programs

```
val isZero: BigInt >--> Boolean =
 isZeroHelper
```

```
val subtractOne: BigInt >--> BigInt =
 subtractOneHelper
```

```
val multiply: (BigInt && BigInt) >--> BigInt =
 multiplyHelper
```

```
def one[Z]: Z >--> BigInt =
 oneHelper
```



## factorial program

```
val factorial: BigInt >--> BigInt =
 'if'(isZero) {
 one
 } 'else' {
 'let' {
 subtractOne >-->
 factorial
 } 'in' {
 multiply
 }
 }
```





## factorialMain

```
val factorialMain: Unit >--> Unit =
 effectfulReadIntFromConsole >-->
 factorial >-->
 effectfulWriteFactorialOfIntToConsole
```



## activeTypes

```
object activeTypes {
 type Active[+Z] = Z
 type '=>A' = Kleisli[Active]
}
```



## activeProgram

```
implicit object activeProgram
 extends Computation[Active]
 with Program['=>A'] {

 override private[pdbp] def result[Z]: Z => Active[Z] =
 'z=>az'

 override private[pdbp] def bind[Z, Y](
 az: Active[Z],
 'z=>ay': => (Z => Active[Y])): Active[Y] =
 'z=>ay'(az)

}
```



## activeMeaningOfActive

```
implicit object activeMeaningOfActive
 extends MeaningOfActive[Active]()
 with ComputationMeaning[Active, Active]()
 with ProgramMeaning['=>A', '=>A']()
```



# FactorialMain

```
import ... mainFactorial
import mainFactorial.factorialMain

import ... activeMeaningOfActive
import activeMeaningOfActive.meaning

object FactorialMain {

 def main(args: Array[String]): Unit = {

 meaning(factorialMain)()

 }

}
```



# Problems and Solutions



## Problems and Solutions

- Problem: the **factorial** semantics above *is not stack safe*



## Problems and Solutions

- Problem: the **factorial** semantics above *is not stack safe*
- Solution: *transformation* **FreeTransformation** and *meaning* **FreeTransformedMeaning**





## Problems and Solutions

- Problem: the `factorial` semantics above *is not stack safe*
- Solution: *transformation* `FreeTransformation` and *meaning* `FreeTransformedMeaning`
- Problem: `effectfulReadIntFromConsole` and `effectfulWriteFactorialOfIntToConsole` *execute effects*



## Problems and Solutions

- Problem: the `factorial` semantics above *is not stack safe*
- Solution: *transformation* `FreeTransformation` and *meaning* `FreeTransformedMeaning`
- Problem: `effectfulReadIntFromConsole` and `effectfulWriteFactorialOfIntToConsole` *execute effects*
- Solution: `Reading` resp. `Writing` extensions of `Program` with members `read` resp. `write` that *describe effects* and with corresponding *transformation* and *meaning*.



# ComputationTransformation

```
private[pdbp] trait ComputationTransformation[
 FC[+ _]: Computation, T[+ _]] {

 private[pdbp] val transform: FC '-U->' T

}
```



# ComputationTransformation

```
private[pdbp] trait ComputationTransformation[
 FC[+ _]: Computation, T[+ _]] {

 private[pdbp] val transform: FC '-U->' T

}
```

- *Computation transformations* transform *computations* (and corresponding *kleisli programs*) to *computations* (and corresponding *kleisli programs*)



# ProgramMeaning

```
trait ProgramMeaning[
 '>-FP->'[- _, + _]: Program, '>-T->'[- _, + _]] {

 private[pdbp] lazy val binaryTransformation:
 '>-FP->' '-B->' '>-T->'

 lazy val meaning: '>-FP->' '-B->' '>-T->' =
 binaryTransformation
}
```



# ComputationMeaning

```
private[pdbp] trait ComputationMeaning[
 FC[+ _]: Computation, T[+ _]]
 extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

 private[pdbp] val unaryTransformation: FC '-U->' T

 // ...
}
```



# ComputationMeaning

```
private[pdbp] trait ComputationMeaning[
 FC[+ _]: Computation, T[+ _]]
 extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

 private[pdbp] val unaryTransformation: FC '-U->' T

 // ...
}
```

- *Computation meanings* are *program meanings* for corresponding kleisli programs



# FreeTransformed

```
sealed trait Free[C[+ _], +Z]

final case class Transform[C[+ _], +Z]
 (cz: C[Z]) extends Free[C, Z]
final case class Result[C[+ _], +Z]
 (z: Z) extends Free[C, Z]
final case class Bind[C[+ _], -Z, ZZ <: Z, +Y]
 (fczz: Free[C, ZZ], 'z=>fcy': Z => Free[C, Y])
 extends Free[C, Y]

type FreeTransformed[C[+ _]] = [+Z] => Free[C, Z]
```





# FreeTransformation

```
private[pdbp]
 trait FreeTransformation[C[+ _]: Computation]
 extends Computation[FreeTransformed[C]]
 with Program[Kleisli[FreeTransformed[C]]]
 with Transformation[C, FreeTransformed[C]] {

 // unfold

 // transform => Transform
 // result => Result
 // bind => Bind

 }
```



## activeFreeTypes

```
object activeFreeTypes {
 type ActiveFree = FreeTransformed[Active]
 type '=>AF' = Kleisli[ActiveFree]
}
```



## activeFreeProgram

```
import ... activeProgram

implicit object activeFreeProgram
 extends Computation[ActiveFree]
 with Program['=>AF']

 with FreeTransformation[Active]()
 with ComputationTransformation[Active, ActiveFree]()
```



## FreeTransformedMeaning

```
private[pdbp] trait FreeTransformedMeaning[
 FC[+ _]: Computation, T[+ _]](
 implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
 extends ComputationMeaning[FreeTransformed[FC], T] {

 // fold (tail recursive)

 // Transform(fcz) => fcz
 // Result(z) => result(z)
 // Bind(Result(y), y2ftfcz) => fold(y2ftfcz(y))
 // Bind(Bind(fcx, x2ftfcy), y2ftfcz) =>
 // fold(Bind(fcx, Bind(x2ftfcy, y2ftfcz)))

}
```



## activeMeaningOfActiveFree

```
import ... activeMeaningOfActive

implicit object activeMeaningOfActiveFree
 extends FreeTransformedMeaning[Active, Active]()
 with ComputationMeaning[ActiveFree, Active]()

 with ProgramMeaning['=>AF', '=>A']()
```



# FactorialMain

```
// same factorial and mainFactorial description
// other implementations
import ... mainFactorial
import mainFactorial.factorialMain
// other meaning
import ... activeMeaningOfActiveFree
import activeMeaningOfActiveFree.meaning

object FactorialMain {

 def main(args: Array[String]): Unit = {

 meaning(factorialMain)()

 }

}
```



# Reading

```
trait Reading[R, >-->[- _, + _]] {

 private[pdbp] def 'u>-->r': Unit >--> R

}
```



# Reading

```
trait Reading[R, >-->[- _, + _]] {
 this: Function[>-->] & Composition[>-->] =>

 private[pdbp] def 'u>-->r': Unit >--> R =
 'z>-->r'[Unit]

 private[pdbp] def 'z>-->r'[Z]: Z >--> R =
 compose('z>-->u', 'u>-->r')

}
```





# Reading

```
trait Reading[R, >-->[- _, + _]] {
 this: Function[>-->] & Composition[>-->] =>

 private[pdbp] def 'u>-->r': Unit >--> R =
 'z>-->r'[Unit]

 private[pdbp] def 'z>-->r'[Z]: Z >--> R =
 compose('z>-->u', 'u>-->r')

 def read[Z]: Z >--> R = 'z>-->r'

}
```



## implicit function type

```
type 'I=>'[-X, +Y] = implicit X => Y
```



## implicit function type

```
type 'I=>'[-X, +Y] = implicit X => Y
```

- greatly reduces *reading* boilerplate code



# ReadingTransformed

```
type ReadingTransformed[R, C[+ _]] = [+Z] => R 'I=>' C[Z]
```



# ReadingTransformation

```
private[pdbp] trait ReadingTransformation[
 R, FC[+ _]: Computation]
 extends ComputationTransformation[
 FC, ReadingTransformed[R, FC]]
 with Computation[ReadingTransformed[R, FC]]

 with Program[Kleisli[ReadingTransformed[R, FC]]]
 with Reading[R, Kleisli[ReadingTransformed[R, FC]]] {

 // ...

}
```



## factorialMain

```
val factorialMain: Unit >--> Unit =
 read >-->
 factorial >-->
 effectfulWriteFactorialOfIntToConsole
```



## activeReadingTypes

```
object activeReadingTypes {
 type ActiveReading[R] = ReadingTransformed[R, Active]
 type '=>AR'[R] = Kleisli[ActiveReading[R]]
}
```



## activeIntReadingProgram

```
import ... activeProgram

implicit object activeIntReadingProgram
 extends ActiveReadingProgram[BigInt]()
 with Computation[ActiveReading[BigInt]]()
 with Program['=>AR'[BigInt]]()
 with Reading[BigInt, '=>AR'[BigInt]]()

 with ReadingTransformation[BigInt, Active]()
 with ComputationTransformation[Active, ActiveReading[BigInt]]()
```





# activeIntReadingMeaningOfActiveIntReading

```
import ... activeIntReadingProgram

implicit object activeIntReadingMeaningOfActiveIntReading
 extends ReadingTransformedMeaning[
 BigInt, Active, Active]()
 with ComputationMeaning[
 ActiveReading[BIGInt], ActiveReading[BIGInt]]()

 with ProgramMeaning['=>AR'[BIGInt], '=>AR'[BIGInt]]()
```



# FactorialMain

```
// same factorial description, other implementation
// other factorialMain description
import ... mainFactorialOfIntRead
import mainFactorialOfIntRead.factorialMain
// other meaning
import ... activeIntReadingMeaningOfActiveIntReading
import activeIntReadingMeaningOfActiveIntReading.meaning

object FactorialOfIntReadMain {

 import ... readIntFromConsoleEffect // actual read effect

 def main(args: Array[String]): Unit = {

 meaning(factorialMain)()

 }
}
```



## factorialMultipliedByIntRead

```
val factorialMultipliedByIntRead: BigInt >--> BigInt =
 (factorial & read) >--> multiply
```



# Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {

 private[pdbp] val 'w>-->u': W >--> Unit

 // ...
}
```



# Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {
 this: Function[>-->] & Composition[>-->] =>

 private[pdbp] val 'w>-->u': W >--> Unit
 = write(identity)

 private[pdbp] def 'z>-w->u'[Z]: (Z => W) 'I=>' Z >--> Unit =
 compose(function(implicitly), 'w>-->u')

 def write[Z]: (Z => W) 'I=>' Z >--> Unit =
 'z>-w->u'

 // ...

}
```



## Writing

```
def writeUsing[Z, Y, X](
 'z&&y=>x': ((Z && Y) => X)):
 (Z >--> Y) => ((X => W) 'I=>' Z >--> Y) = { 'z>-->y' =>
 val 'z&&y>-->x' = function('z&&y=>x')
 val 'z>-->(x&&y)' =
 'let' {
 'z>-->y'
 } 'in' {
 'let' {
 'z&&y>-->x'
 } 'in' {
 'z&&y&&x>-->(x&&y)'
 }
 }
 }
 compose(compose('z>-->(x&&y)', left(write),
 'u&&y>-->y')
```



# Writable

```
private[pdbp] trait Writable[W]
 extends Startable[W]
 with Appendable[W]
```



# Startable

```
private[pdbp] trait Startable[W] {
 private[pdbp] val start: W
}
```





# Appendable

```
private[pdbp] trait Appendable[W] {
 private[pdbp] val append: W && W => W
}
```



# WritingTransformed

```
type WritingTransformed[W, FC[+ _]] = [+Z] => FC[W && Z]
```



# WritingTransformation

```
private[pdbp] trait WritingTransformation[
 W: Writable, FC[+ _]: Computation]
 extends ComputationTransformation[
 FC, WritingTransformed[W, FC]]
 with Computation[WritingTransformed[W, FC]]

 with Program[Kleisli[WritingTransformed[W, FC]]]
 with Writing[W, Kleisli[WritingTransformed[W, FC]]] {

 // ...

}
```



# ToConsole

```
case class ToConsole(effect: Effect)
```

```
type Effect = Unit => Unit
```



## WritingToConsoleTransformedMeaning

```
private[pdbp] trait WritingToConsoleTransformedMeaning[
 FC[+ _]: Computation,
 T[+ _]]
 (implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
 extends ComputationMeaning[WritingTransformed[ToConsole, FC], T]

 // executes console effect
}
```



## toConsoleWritable

```
implicit object toConsoleWritable extends Writable[ToConsole] {

 override private[pdbp] val start: ToConsole =
 ToConsole { _ =>
 ()
 }

 override private[pdbp] val append:
 ToConsole && ToConsole => ToConsole = {
 (tc1, tc2) =>
 ToConsole { _ =>
 { tc1.effect(()); tc2.effect(()); }
 }
 }

}
```



## infoUtils

```
def infoFunction[Z, Y](string: String): Z && Y => String = {
 case (z, y) =>
 s"INFO -- $currentCalendarInMilliseconds -- $string($z) => $y"
}

def info[
 W: Writable, Z, Y,
 >-->[- _, + _]: [>-->[- _, + _]] => Writing[W, >-->]]
 (string: String):
 (Z >--> Y) => ((String => W) 'I=>' Z >--> Y) = {
 val implicitWriting = implicitly[Writing[W, >-->]]
 implicitWriting.writeUsing(infoFunction(string))
}
```



## WritingAtomicPrograms

```
val isZero: (String => W) 'I=>' BigInt >--> Boolean =
 info("isZero") { isZeroHelper }
```

```
val subtractOne: (String => W) 'I=>' BigInt >--> BigInt =
 info("subtractOne") { subtractOneHelper }
```

```
val multiply: (String => W) 'I=>' (BigInt && BigInt) >--> BigInt =
 info("multiply") { multiplyHelper }
```

```
def one[Z]: (String => W) 'I=>' Z >--> BigInt =
 info("one") { oneHelper }
```





# WritingFactorial

```
val factorial: (String => W) 'I=>' BigInt >--> BigInt =
 info("factorial") {
 'if'(isZero) {
 one
 } 'else' {
 'let' {
 subtractOne >-->
 factorial
 } 'in' {
 multiply
 }
 }
 }
```



# factorialMain

```
val factorialMain:
 (String => ToConsole) 'I=>'
 ((BigInt => ToConsole) 'I=>' Unit >--> Unit) =
 read >-->
 factorial >-->
 write
```



## activeWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeProgram

implicit object activeWritingToConsoleProgram
 extends ActiveWritingProgram[ToConsole]()
 with Computation[ActiveWriting[ToConsole]]()
 with Program['=>AW'[ToConsole]]()
 with Writing[ToConsole, '=>AW'[ToConsole]]()

 with ComputationTransformation[
 Active, ActiveWriting[ToConsole]]()
 with WritingTransformation[ToConsole, Active]()
```



## activeIntReadingWithWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeWritingToConsoleProgram

implicit object activeIntReadingWithWritingToConsoleProgram
 extends ActiveReadingWithWritingProgram[BigInt, ToConsole]()
 with Computation[ActiveReadingWithWriting[BigInt, ToConsole]]()
 with Program['=>ARW'[BigInt, ToConsole]]()
 with Reading[BigInt, '=>ARW'[BigInt, ToConsole]]()
 with Writing[ToConsole, '=>ARW'[BigInt, ToConsole]]()

// ...
```



## activeIntReadingMeaning OfActiveIntReadingWithWritingToConsole

```
import ... activeMeaningOfActiveWritingToConsole

implicit object
 activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole
 extends ReadingTransformedMeaning[
 BigInt, ActiveWriting[ToConsole], Active]()
 with ComputationMeaning[
 ActiveReadingWithWriting[BiInt, ToConsole],
 ActiveReading[BiInt]]()
 with ProgramMeaning[
 '=>ARW'[BiInt, ToConsole],
 '=>AR'[BiInt]]()
```



# FactorialOfIntRead WritingToConsoleWrittenToConsoleMain

```
// other factorial and factorialMain descriptions and implementations
import ... mainFactorialOfIntReadWritingToConsoleWrittenToConsole
import mainFactorialOfIntReadWritingToConsoleWrittenToConsole.factorialMain
// other meaning
import ... activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole
import activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole.meaning

object FactorialOfIntReadWritingToConsoleWrittenToConsoleMain {

 import ... readIntFromConsoleEffect // actual read effect
 import ... writeFactorialOfIntReadFromConsoleToConsoleEffect // actual write
 import ... writeToConsoleEffect // actual write effect via writeUsing

 def main(args: Array[String]): Unit = {
 meaning(factorialMain)()
 }
}
```



## running

please type an integer to read

2

```
INFO -- 2018-08-01 18:00:42.639 -- isZero(2) => false
INFO -- 2018-08-01 18:00:42.645 -- subtractOne(2) => 1
INFO -- 2018-08-01 18:00:42.646 -- isZero(1) => false
INFO -- 2018-08-01 18:00:42.647 -- subtractOne(1) => 0
INFO -- 2018-08-01 18:00:42.647 -- isZero(0) => true
INFO -- 2018-08-01 18:00:42.648 -- one(0) => 1
INFO -- 2018-08-01 18:00:42.648 -- factorial(0) => 1
INFO -- 2018-08-01 18:00:42.649 -- multiply((1,1)) => 1
INFO -- 2018-08-01 18:00:42.649 -- factorial(1) => 1
INFO -- 2018-08-01 18:00:42.650 -- multiply((2,1)) => 2
INFO -- 2018-08-01 18:00:42.650 -- factorial(2) => 2
the factorial value of the integer read is
```

2

