

PDBP

Program Description Based Programming  
Scala eXchange

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# Intro



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- this talk is about



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  - a *Dotty library* PDBP



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- this talk is about
  - a *Dotty library* PDBP
  - inspired by the *function-level programming language* FP



# John Backus



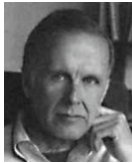
# John Backus



- ACM Turing Award Winner 1977



# John Backus



- ACM Turing Award Winner 1977
- *Can programming be liberated from the Von Neumann style?*





What is this?



What is this?



- A *pipe*?



What is this?



- A *pipe*?
- A *painting describing* a pipe?



What is this?



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- A *slide describing* a painting describing a pipe?



What is this?



- A *pipe*?
- A *painting describing* a pipe?
- A *slide describing* a painting describing a pipe?
- ...



Ceci n'est pas une pipe



Ceci n'est pas une pipe



- It is a *painting* by René Magritte *describing* a pipe



# This is not a program

```
val xxxxxxxxx: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        xxxxxxxxx  
    } 'in' {  
      multiply  
    }  
  }  
}
```





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}
```

- It is *code describing* a program



## factorial description

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        factorial  
    } 'in' {  
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```



## factorial description

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```

- `factorial` *uses* capabilities *declared* in  
type class `trait Program[>-->[- _, + _]]`



## factorial meanings

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
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val factorial: BigInt >--> BigInt =  
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  }  
}
```

- different `implicit object`'s *define* capabilities of type class `trait Program[>-->[- _, + _]]`



program

val program:  $Z \multimap Y$



# mainProgram

```
val program: Unit >--> Unit
```



## Composition

```
val 'z>-->y': Z >--> Y
```

```
val 'y>-->x': Y >--> X
```

```
val 'z>-->x': Z >--> X = 'z>-->y' >--> 'y>-->x'
```





program

-----  
|       -----               -----       |       -----  
| | Z >--> Y | >--> | Y >--> X | | => | Z >--> X |  
|       -----               -----       |       -----  
-----



## mainProgram

```
val producer: Unit >--> Z
val program: Z >--> Y
val consumer: Y >--> Unit

val mainProgram: Unit >--> Unit =
  producer >--> program >--> consumer
```



## mainProgram

```
-----  
| Unit >--> Unit |  
-----  
-----  
| Unit >--> Z >--> Y >--> Unit |  
-----  
                                     v  
                                     |  
-----<????>----- => | Unit >--> Unit |  
|  
v  
-----  
| Unit >--> Z >--> Y >--> Unit |  
-----  
-----  
| Unit >--> Unit |  
-----
```



## FP versus PDBP



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- FP and PDBP promote *pointfree functional programming*



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- FP is a *language*  
PDBP is a *library*



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  - FP *semantics* is *fixed*  
PDBP *semantics* is *not fixed*



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- FP and PDBP promote *pointfree functional programming*
- FP is a *language*  
PDBP is a *library*
  - FP *semantics* is *fixed*  
PDBP *semantics* is *not fixed*
  - FP *capabilities* are *fixed*  
PDBP *capabilities* are *not fixed*
  - FP *effects* are *impure*  
PDBP *effects* are *pure*



# Semantics

```
val factorial: BigInt >--> BigInt =  
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- *production*



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  - *recursion* using *stack*



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  - *recursion* using *heap*
- *test*



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- ...



# Effects



# Effects

- *reading*



# Effects

- *reading*
- *writing*





# Foundations



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- `trait Program[>-->[- _, + _]]` corresponds to *arrows*



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- `trait Computation[C[+ _]]` corresponds to *monads*



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generalize *functions*



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generalize *expressions*



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  - `{ val z = ez ; { val y = ey ; ex(z, y) } }`



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  - `mz bind { z => my bind { y => result(ex(z, y)) } }`



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- *monads* can also be programmed *pointfree* (*kleisli arrows*)



## Monads versus arrows

- *monads* can also be programmed *pointfree* (*kleisli arrows*)
- *arrows* can also be programmed *pointful* (*arrow calculus*)





# Power of expressions



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  - *monads* impose more *implementation constraints*
- *Constraints Liberate, Liberties Constrain*



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## Elegance of use

- *pointfree* programming is sometimes considered to be more *abstruse* than *pointful* programming
- Dotty comes to the rescue
  - Dotty is a Scalable language
  - Dotty *library based* language extensions are *type safe*
- PDBP comes with a *program description DSL*



## Program description DSL

```
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    }  
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```



## PDBP's choice



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- the PDBP library goes for
  - `private[pdbp]` *pointful monad API*  
provides *power of expression* for *library* developers
  - `public` *pointfree arrow API*  
provides *elegance of use* for *application* developers
- the PDBP can live with
  - corresponding implementation constraints



PDBP

[illegible]

## PDBP library design decisions (cfr. Haskell)



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- *description* separated from *meaning*



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  - *implicit object*'s (*extend trait*'s) *define* capabilities



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- *description* separated from *meaning*
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- *library level* meaning



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- *description* separated from *meaning*
- *description*
  - *trait*'s (*type classes*) *declare* capabilities
- *language level* meaning
  - *implicit object*'s (*extend trait*'s) *define* capabilities
- *library level* meaning
  - *natural transformations*



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- *dependency injection* by `implicit import`



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## PDBP library design decisions (cfr. Haskell)

- *dependency injection* by `implicit import`
- *definitions* in `class`'es that *implicitly* depend on `trait`'s (*type classes*) use capabilities *declared* in those `trait`'s
- `object`'s that *extend* those `class`'es `import implicit` `object`'s that *extend* those `trait`'s



## Program (cfr. *arrow*)

```
trait Program[>-->[- _, + _]]
```





## Computation (cfr. *monad*)

```
trait Computation[C[+ _]]
```



# Liskov Substitution Principle



# Liskov Substitution Principle

- *impose less*



# Liskov Substitution Principle

- *impose less*
- *provide more*



# Internet Robustness Principle



# Internet Robustness Principle

- *be liberal in what you receive*



# Internet Robustness Principle

- *be liberal in what you receive*
- *be generous in what you send*



## PDBP library details





# Program

```
trait Program[>-->[- _, + _]]  
  extends Function[>-->]  
  with Composition[>-->]  
  with Construction[>-->]  
  with Condition[>-->]  
  
  with Aggregation[>-->]
```



# Function



# Function

- `val 'z>-->y' = function('z=>y')`



# Function

- `val 'z>-->y' = function('z=>y')`
- pure functions are *atomic programs*



# Function

- `val 'z-->y' = function('z=>y')`
- pure functions are *atomic programs*
  - up to you to define granularity



# Composition



# Composition

- $\text{val } 'z \dashrightarrow x' = 'z \dashrightarrow y' \dashrightarrow 'y \dashrightarrow x'$



# Construction





# Construction

- `val 'z>-->y&& x' = 'z>-->y' & 'z>-->x'`



# Construction

- `val 'z>-->y&&x' = 'z>-->y' & 'z>-->x'`
- `val 'z&&y>-->x&&w' = 'z>-->x' && 'y>-->w'`



# Construction

- `val 'z>-->y&&x' = 'z>-->y' & 'z>-->x'`
- `val 'z&&y>-->x&&w' = 'z>-->x' && 'y>-->w'`
- `val 'z>-->x' =  
    'let' 'z>-->y' 'in' 'z&&y>-->x'`



# Condition



# Condition

- `val 'y||x>-->z' = 'y>-->z' | 'x>-->z'`



# Condition

- $\text{val } 'y|x \multimap z' = 'y \multimap z' \mid 'x \multimap z'$
- $\text{val } 'x|w \multimap z||y' = 'x \multimap z' \mid\mid 'w \multimap y'$



# Condition

- `val 'y||x>-->z' = 'y>-->z' | 'x>-->z'`
- `val 'x||w>-->z||y' = 'x>-->z' || 'w>-->y'`
- `val 'y>-->z' =  
    'if'('y>-->b') 'y>-t->z' 'else' 'y>-f->z'`



# Computation

```
private[pdbp] trait Computation[C[+ _]]  
  extends Resulting[C]  
  with Binding[C]  
  with Program[[-Z, +Y] => Z => C[Y]]  
  
  with Lifting[C]  
  
  with Sequencing[C]
```





# Resulting

```
val cz = result(z)
```



# Binding



# Binding

- `val cy = cz bind { z => 'z=>cy'(y) }`



# Binding

- `val cy = cz bind { z => 'z=>cy'(y) }`
- `val cy = cz bind { z => result('z=>y'(y)) }`



# Kleisli



# Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`



# Kleisli

- `type Kleisli[C[+ _]] = [-Z, + Y] => Z => C[Y]`
- ```
private[pdbp] trait Computation[C[+ _]]  
  extends Resulting[C]  
  with Binding[C]  
  with Program[Kleisli[C]]  
  
  // ...
```



# HelperFunctions

```
val isZeroFunction: BigInt => Boolean =  
{ i =>  
    i == 0  
}  
def oneFunction[Z]: Z => BigInt =  
{ z =>  
    1  
}  
val subtractOneFunction: BigInt => BigInt =  
{ i =>  
    i - 1  
}  
val multiplyFunction: (BigInt && BigInt) => BigInt =  
{ (i, j) =>  
    i * j  
}
```





# HelperPrograms

```
val isZeroHelper: BigInt >--> Boolean =  
    function(isZeroFunction)
```

```
val subtractOneHelper: BigInt >--> BigInt =  
    function(subtractOneFunction)
```

```
val multiplyHelper: (BigInt && BigInt) >--> BigInt =  
    function(multiplyFunction)
```

```
def oneHelper[Z]: Z >--> BigInt =  
    function(oneFunction)
```



# AtomicPrograms

```
val isZero: BigInt >--> Boolean =  
  isZeroHelper
```

```
val subtractOne: BigInt >--> BigInt =  
  subtractOneHelper
```

```
val multiply: (BigInt && BigInt) >--> BigInt =  
  multiplyHelper
```

```
def one[Z]: Z >--> BigInt =  
  oneHelper
```



# factorial

```
val factorial: BigInt >--> BigInt =  
  'if'(isZero) {  
    one  
  } 'else' {  
    'let' {  
      subtractOne >-->  
        factorial  
    } 'in' {  
      multiply  
    }  
  }  
}
```



# factorialMain

```
val factorialMain: Unit >--> Unit =  
  producer >-->  
    factorial >-->  
    consumer
```



## activeTypes

```
object activeTypes {  
  type Active[+Z] = Z  
  type '=>A' = Kleisli[Active]  
}
```



# activeProgram

```
implicit object activeProgram
  extends Computation[Active]
  with Program['=>A'] {

  override private[pdbp] def result[Z]: Z => Active[Z] =
    'z=>az'

  override private[pdbp] def bind[Z, Y](
    az: Active[Z],
    'z=>ay': => (Z => Active[Y])): Active[Y] =
    'z=>ay'(az)

}
```



## activeMeaningOfActive

```
implicit object activeMeaningOfActive
  extends MeaningOfActive[Active]()
  with ComputationMeaning[Active, Active]()
  with ProgramMeaning['=>A', '=>A']()
```



# FactorialMain

```
import ... activeMeaningOfActive
import activeMeaningOfActive.meaning

import ... mainFactorial
import mainFactorial.factorialMain

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)()

  }
}
```





# Problems and Solutions



## Problems and Solutions

- Problem: obvious **factorial** meaning  
implementing  $\rightarrow\rightarrow\rightarrow$  as ' $\Rightarrow A$ ' *is not stack safe*



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implementing  $\rightarrow$  as ' $\Rightarrow A$ ' *is not stack safe*
  - Solution: **FreeTransformation** and  
**FreeTransformedMeaning**



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- Problem: obvious **factorial** meaning implementing  $\rightarrow$  as ' $\Rightarrow A$ ' *is not stack safe*
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- Problem: obvious **producer** and **consumer** *execute effects*



## Problems and Solutions

- Problem: obvious **factorial** meaning implementing  $\>-->$  as ' $\Rightarrow A$ ' *is not stack safe*
  - Solution: **FreeTransformation** and **FreeTransformedMeaning**
- Problem: obvious **producer** and **consumer** *execute effects*
  - Solution: **Reading** resp. **Writing** extensions of **Program** with members **read** resp. **write** that *describe effects*



## NaturalBinaryTypeConstructorTransformation

```
trait '-B->'['>-F->'[- _, + _], '>-T->'[- _, + _]] {  
  def apply[Z, Y]: Z '>-F->' Y => Z '>-T->' Y  
}
```



## NaturalUnaryTypeConstructorTransformation

```
private[pdbp] trait '-U->'[F[+ _], T[+ _]]  
  extends '-B->'[Kleisli[F], Kleisli[T]] {  
  
  private[pdbp] def apply[Z](fz: F[Z]): T[Z]  
  
  // ...  
}  
  
}
```



# ComputationTransformation

```
private[pdbp] trait ComputationTransformation[  
  FC[+ _]: Computation, T[+ _]] {  
  
  private[pdbp] val transform: FC '-U->' T  
  
}
```





# ProgramMeaning

```
trait ProgramMeaning[  
  '>-FP->'[- _, + _]: Program, '>-T->'[- _, + _]] {  
  
  private[pdbp] lazy val binaryTransformation:  
    '>-FP->' '-B->' '>-T->'  
  
  lazy val meaning: '>-FP->' '-B->' '>-T->' =  
    binaryTransformation  
  
}
```



# ComputationMeaning

```
private[pdbp] trait ComputationMeaning[
  FC[+ _]: Computation, T[+ _]]
  extends ProgramMeaning[Kleisli[FC], Kleisli[T]] {

  private[pdbp] val unaryTransformation: FC '-U->' T

  // ...

}
```



# FreeTransformed

```
sealed trait Free[C[+ _], +Z]

final case class Transform[C[+ _], +Z]
  (cz: C[Z]) extends Free[C, Z]
final case class Result[C[+ _], +Z]
  (z: Z) extends Free[C, Z]
final case class Bind[C[+ _], -Z, ZZ <: Z, +Y]
  (fczz: Free[C, ZZ], 'z=>fcy': Z => Free[C, Y])
  extends Free[C, Y]

type FreeTransformed[C[+ _]] = [+Z] => Free[C, Z]
```



# FreeTransformation

```
private[pdbp]
  trait FreeTransformation[C[+ _]: Computation]
    extends Computation[FreeTransformed[C]]
    with Program[Kleisli[FreeTransformed[C]]]
    with Transformation[C, FreeTransformed[C]] {

    // unfold
    // transform => Transform
    // result => Result
    // bind => Bind

  }
```



## activeFreeTypes

```
object activeFreeTypes {  
  type ActiveFree = FreeTransformed[Active]  
  type '=>AF' = Kleisli[ActiveFree]  
}
```



## activeFreeProgram

```
import ... activeProgram

implicit object activeFreeProgram
  extends Computation[ActiveFree]
  with Program['=>AF']
  with FreeTransformation[Active]()
  with ComputationTransformation[Active, ActiveFree]()
```



# FreeTransformedMeaning

```
private[pdbp] trait FreeTransformedMeaning[
  FC[+ _]: Computation, T[+ _]](
  implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
  extends ComputationMeaning[FreeTransformed[FC], T] {

  // tailrecFold
  // Transform(fcz) => fcz
  // Result(z) => result(z)
  // Bind(Result(y), y2ftfcz) => tailrecFold(y2ftfcz(y))
  // Bind(Bind(fcx, x2ftfcy), y2ftfcz) =>
  //   tailrecFold(Bind(fcx, Bind(x2ftfcy, y2ftfcz)))

}
```



## activeMeaningOfActiveFree

```
import ... activeMeaningOfActive

implicit object activeMeaningOfActiveFree
  extends FreeTransformedMeaning[Active, Active]()
  with ComputationMeaning[ActiveFree, Active]()
  with ProgramMeaning['=>AF', '=>A']()
```





# FactorialMain

```
import ... activeMeaningOfActiveFree
import activeMeaningOfActiveFree.meaning

import ... mainFactorial
import mainFactorial.factorialMain

object FactorialMain {

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)()

  }
}
```



# Reading

```
trait Reading[R, >-->[- _, + _]] {  
  
  private[pdbp] def 'u>-->r': Unit >--> R  
  
}
```



# Reading

```
trait Reading[R, >-->[- _, + _]] {  
  this: Function[>-->] & Composition[>-->] =>  
  
  private[pdbp] def 'u>-->r': Unit >--> R =  
    'z>-->r'[Unit]  
  
  private[pdbp] def 'z>-->r'[Z]: Z >--> R =  
    compose('z>-->u', 'u>-->r')  
  
}
```



# Reading

```
trait Reading[R, >-->[- _, + _]] {  
  this: Function[>-->] & Composition[>-->] =>  
  
  private[pdbp] def 'u>-->r': Unit >--> R =  
    'z>-->r'[Unit]  
  
  private[pdbp] def 'z>-->r'[Z]: Z >--> R =  
    compose('z>-->u', 'u>-->r')  
  
  def read[Z]: Z >--> R = 'z>-->r'  
  
}
```



## ReadingTransformed

```
type 'I=>'[-X, +Y] = implicit X => Y
```

```
type ReadingTransformed[R, C[+ _]] = [+Z] => R 'I=>' C[Z]
```



# ReadingTransformation

```
private[pdbp] trait ReadingTransformation[
  R, FC[+ _]: Computation]
  extends ComputationTransformation[
    FC, ReadingTransformed[R, FC]]
  with Computation[ReadingTransformed[R, FC]]
  with Program[Kleisli[ReadingTransformed[R, FC]]]
  with Reading[R, Kleisli[ReadingTransformed[R, FC]]] {

  // ...

}
```



# factorialMain

```
val factorialMain: Unit >--> Unit =  
  read >-->  
    factorial >-->  
    consumer
```



## activeReadingTypes

```
object activeReadingTypes {  
  type ActiveReading[R] = ReadingTransformed[R, Active]  
  type '=>AR'[R] = Kleisli[ActiveReading[R]]  
}
```





## activeIntReadingProgram

```
import ... activeProgram

implicit object activeIntReadingProgram
  extends ActiveReadingProgram[BigInt]()
  with Computation[ActiveReading[BigInt]]()
  with Program['=>AR'[BigInt]]()
  with Reading[BigInt, '=>AR'[BigInt]]()
  with ReadingTransformation[BigInt, Active]()
  with ComputationTransformation[Active, ActiveReading[BigInt]]()
```



## activeIntReadingProgram

```
import ... activeIntReadingProgram

implicit object activeIntReadingMeaningOfActiveIntReading
  extends ReadingTransformedMeaning[BigInt, Active, Active]()
  with ComputationMeaning[ActiveReading[BigInt], ActiveReading[BigInt]]()
  with ProgramMeaning['=>AR'[BigInt], '=>AR'[BigInt]]()
```



# FactorialMain

```
import ... activeIntReadingMeaningOfActiveIntReading
import activeIntReadingMeaningOfActiveIntReading.meaning

import ... mainFactorialOfIntRead
import mainFactorialOfIntRead.factorialMain

object FactorialOfIntReadMain {

  import ... readIntFromConsoleEffect

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)()

  }
}
```



## factorialMultipliedByIntRead

```
val factorialMultipliedByIntRead: BigInt >--> BigInt =  
  (factorial & read) >--> multiply
```



# Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {  
  
  private[pdbp] val 'w>-->u': W >--> Unit  
  
  // ...  
}
```



# Writing

```
trait Writing[W: Writable, >-->[- _, + _]] {  
  this: Function[>-->] & Composition[>-->] =>  
  
  private[pdbp] val 'w>-->u': W >--> Unit  
    = write(identity)  
  
  private[pdbp] def 'z>-w->u'[Z]: (Z => W) 'I=>' Z >--> Unit =  
    compose(function(implicitly), 'w>-->u')  
  
  def write[Z]: (Z => W) 'I=>' Z >--> Unit =  
    'z>-w->u'  
  
  // ...  
  
}
```



## Writing

```
def writeUsing[Z, Y, X](
  'z&&y=>x': ((Z && Y) => X)):
  (Z >--> Y) => ((X => W) 'I=>' Z >--> Y) = { 'z>-->y' =>
  val 'z&&y>-->x' = function('z&&y=>x')
  val 'z>-->(x&&y)' =
    'let' {
      'z>-->y'
    } 'in' {
      'let' {
        'z&&y>-->x'
      } 'in' {
        'z&&y&&x>-->(x&&y)'
      }
    }
  }
  compose(compose('z>-->(x&&y)', left('z>-w->u'),
    'u&&y>-->y')
```



# Writable

```
private[pdbp] trait Writable[W]  
  extends Startable[W]  
  with Appendable[W]
```





# Startable

```
private[pdbp] trait Startable[W] {  
  private[pdbp] val start: W  
}
```



# Appendable

```
private[pdbp] trait Appendable[W] {  
    private[pdbp] val append: W && W => W  
}
```



# WritingTransformed

```
type WritingTransformed[W, FC[+ _]] = [+Z] => FC[W && Z]
```



## WritingTransformation

```
private[pdbp] trait WritingTransformation[
  W: Writable, FC[+ _]: Computation]
  extends ComputationTransformation[
    FC, WritingTransformed[W, FC]]
  with Computation[WritingTransformed[W, FC]]
  with Program[Kleisli[WritingTransformed[W, FC]]]
  with Writing[W, Kleisli[WritingTransformed[W, FC]]] {

  // ...

}
```



# ToConsole

```
case class ToConsole(effect: Effect)
```

```
type Effect = Unit => Unit
```



# ToConsole

```
implicit object toConsoleWritable extends Writable[ToConsole] {  
  
  override private[pdbp] val start: ToConsole =  
    ToConsole { _ =>  
      ()  
    }  
  
  override private[pdbp] val append:  
    ToConsole && ToConsole => ToConsole = {  
    (tc1, tc2) =>  
      ToConsole { _ =>  
        { tc1.effect(()); tc2.effect() }  
      }  
  }  
  
}
```



## infoUtils

```
def infoFunction[Z, Y](string: String): Z && Y => String = {  
  case (z, y) =>  
    s"INFO -- $currentCalendarInMilliseconds -- $string($z) => $y"  
}  
  
def info[  
  W: Writable, Z, Y,  
  >-->[- _, + _]: [>-->[- _, + _]] => Writing[W, >-->]]  
  (string: String):  
    (Z >--> Y) => ((String => W) 'I=>' Z >--> Y) = {  
  val implicitWriting = implicitly[Writing[W, >-->]]  
  implicitWriting.writeUsing(infoFunction(string))  
}
```



## WritingAtomicPrograms

```
val isZero: (String => W) 'I=>' BigInt >--> Boolean =  
  info("isZero") { isZeroHelper }
```

```
val subtractOne: (String => W) 'I=>' BigInt >--> BigInt =  
  info("subtractOne") { subtractOneHelper }
```

```
val multiply: (String => W) 'I=>' (BigInt && BigInt) >--> BigInt =  
  info("multiply") { multiplyHelper }
```

```
def one[Z]: (String => W) 'I=>' Z >--> BigInt =  
  info("one") { oneHelper }
```





# WritingFactorial

```
val factorial: (String => W) 'I=>' BigInt >--> BigInt =  
  info("factorial") {  
    'if'(isZero) {  
      one  
    } 'else' {  
      'let' {  
        subtractOne >-->  
          factorial  
      } 'in' {  
        multiply  
      }  
    }  
  }
```



## factorialMain

```
val factorialMain:  
  (String => ToConsole) 'I=>'  
    ((BigInt => ToConsole) 'I=>' Unit >--> Unit) =  
  read >-->  
    factorial >-->  
    write
```



## WritingTransformation

```
private[pdbp] trait WritingTransformation[W: Writable, FC[+ _]: Computation]
  extends ComputationTransformation[FC, WritingTransformed[W, FC]]
  with Computation[WritingTransformed[W, FC]]
  with Program[Kleisli[WritingTransformed[W, FC]]]
  with Writing[W, Kleisli[WritingTransformed[W, FC]]] {

  // uses Writing capabilities of W

}
```



## FreeTransformedMeaning

```
private[pdbp] trait WritingToConsoleTransformedMeaning[FC[+ _]: ComputationMeaning, T[+ _]](implicit toBeTransformedMeaning: ComputationMeaning[FC, T])
  extends ComputationMeaning[WritingTransformed[ToConsole, FC], T] {

  //...

  override private[pdbp] def apply[Z](wtfcz: WTFC[Z]): T[Z] = {
    def executing(wtfcz: WTFC[Z]): FC[Z] =
      bind(wtfcz, {
        case (ToConsole(effect), z) =>
          effect()
          result(z)
      })
    toBeTransformedMeaning.unaryTransformation(executing(wtfcz))
  }
}
```



## activeWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeProgram

implicit object activeWritingToConsoleProgram
  extends ActiveWritingProgram[ToConsole]()
  with ComputationTransformation[Active, ActiveWriting[ToConsole]]()
  with WritingTransformation[ToConsole, Active]()
  with Writing[ToConsole, '=>AW'[ToConsole]]()
  // ...
```



## activeIntReadingWithWritingToConsoleProgram

```
import ... toConsoleWritable
import ... activeWritingToConsoleProgram

implicit object activeIntReadingWithWritingToConsoleProgram
  extends ActiveReadingWithWritingProgram[BigInt, ToConsole]()
  with Computation[ActiveReadingWithWriting[BigInt, ToConsole]]()
  with Program['=>ARW'[BigInt, ToConsole]]()
  with Reading[BigInt, '=>ARW'[BigInt, ToConsole]]()
  with Writing[ToConsole, '=>ARW'[BigInt, ToConsole]]()
  // ...
```



## activeIntReadingWithWritingToConsoleProgram

```
// ...  
with ReadingWithWritingTransformation[BigInt,  
                                     ToConsole,  
                                     ActiveWriting[ToConsole]]()  
with ReadingTransformation[BigInt, ActiveWriting[ToConsole]]()  
with ComputationTransformation[  
  ActiveWriting[ToConsole],  
  ActiveReadingWithWriting[BigInt, ToConsole]]()
```



## activeIntReadingMeaningOfActiveIntReadingWithV

```
object implicits {  
  
  import ... activeMeaningOfActiveWritingToConsole  
  
  implicit object activeIntReadingMeaningOfActiveIntReadingWithWritingT  
    extends ReadingTransformedMeaning[BigInt,  
                                     ActiveWriting[ToConsole],  
                                     Active]()  
    with ComputationMeaning[ActiveReadingWithWriting[BigInt, ToConsol  
                           ActiveReading[BigInt]]]()  
    with ProgramMeaning['=>ARW'[BigInt, ToConsole], '=>AR'[BigInt]]()  
  
}
```





## activeIntReadingMeaningOfActiveIntReadingWithV

```
object implicits {  
  
  import ... activeMeaningOfActiveWritingToConsole  
  
  implicit object activeIntReadingMeaningOfActiveIntReadingWithWritingT  
    extends ReadingTransformedMeaning[BigInt,  
                                     ActiveWriting[ToConsole],  
                                     Active]()  
    with ComputationMeaning[ActiveReadingWithWriting[BigInt, ToConsol  
                           ActiveReading[BigInt]]]()  
    with ProgramMeaning['=>ARW'[BigInt, ToConsole], '=>AR'[BigInt]]()  
  
}
```



# FactorialOfIntRead

## WritingToConsoleWrittenToConsoleMain

```
import ... activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole
import activeIntReadingMeaningOfActiveIntReadingWithWritingToConsole.me

import ... mainFactorialOfIntReadWritingToConsoleWrittenToConsole
import mainFactorialOfIntReadWritingToConsoleWrittenToConsole.factorial

object FactorialOfIntReadWritingToConsoleWrittenToConsoleMain {

  import ... readIntFromConsoleEffect
  import ... writeFactorialOfIntReadFromConsoleToConsoleEffect
  import ... writeToConsoleEffect

  def main(args: Array[String]): Unit = {

    meaning(factorialMain)()
  }
}
```



## running

please type an integer to read

2

```
INFO -- 2018-08-01 18:00:42.639 -- isZero(2) => false
INFO -- 2018-08-01 18:00:42.645 -- subtractOne(2) => 1
INFO -- 2018-08-01 18:00:42.646 -- isZero(1) => false
INFO -- 2018-08-01 18:00:42.647 -- subtractOne(1) => 0
INFO -- 2018-08-01 18:00:42.647 -- isZero(0) => true
INFO -- 2018-08-01 18:00:42.648 -- one(0) => 1
INFO -- 2018-08-01 18:00:42.648 -- factorial(0) => 1
INFO -- 2018-08-01 18:00:42.649 -- multiply((1,1)) => 1
INFO -- 2018-08-01 18:00:42.649 -- factorial(1) => 1
INFO -- 2018-08-01 18:00:42.650 -- multiply((2,1)) => 2
INFO -- 2018-08-01 18:00:42.650 -- factorial(2) => 2
the factorial value of the integer read is
```

2

