$histogram :: Map Int (Bag word) \rightarrow Map word Int$ histogram = manReduce groupOnBags additive GroupOnIntegers histogramMan histogramReducewhere additiveGroupOnIntegers = Group (+) $(\lambda n \rightarrow -n)$ 0 histogramMap _ = foldBag groupOnBags ($\lambda n \rightarrow \text{singletonBag}(n, 1)$) histogramReduce _ = foldBag additiveGroupOnIntegers id -- Precondition: -- For every $key_1 :: k_1$ and $key_2 :: k_2$, the terms mapper key_1 and reducer key_2 are homomorphisms. $mapReduce :: Group \ v_1 \to Group \ v_3 \to (k_1 \to v_1 \to Bag \ (k_2, v_2)) \to (k_2 \to Bag \ v_2 \to v_3) \to Map \ k_1 \ v_1 \to Map \ k_2 \ v_3$ $mapReduce\ group_1\ group_3\ mapper\ reducer = reducePerKey \circ groupByKey \circ mapPerKey$ where $mapPerKey = foldMap \ group_1 \ groupOnBags$ mapper $(\lambda(key, val) \rightarrow \text{singletonMap } key \text{ (singletonBag } val))$ groupByKey = foldBag (groupOnMaps groupOnBags) $reducePerKey = foldMap groupOnBags (groupOnMaps group_3) (\lambda key bag \rightarrow singletonMap key (reducer key bag))$