Bear is a robot. All he wants to do is draw a heart to display his love. Your task is to help Bear express his love and draw his heart. It'll be like a rom-com, but with more math.

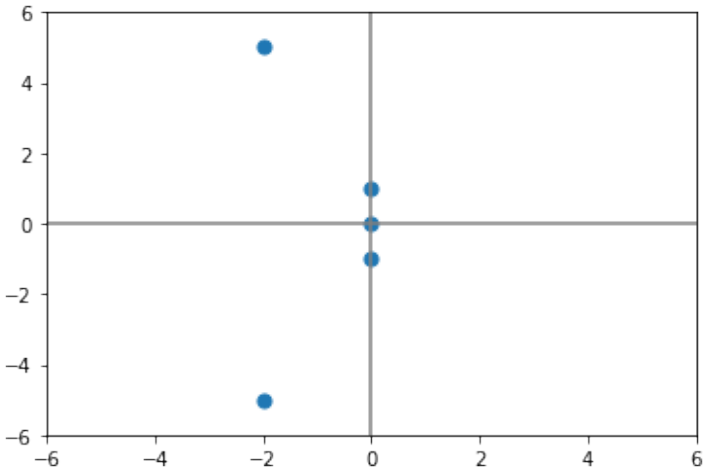
Question 1: A first attempt

Bear decides to use the roots of the function $f(x) = x^5+4x^4+30x^3+4x^2+29x$ to construct his heart.

He's simplified it for you to $f(x) = x(x^2 + 4x + 29)(x^2 + 1)$

a) Find the complex roots of the function $f(x)$

b) Plot the roots on an Argand diagram (y-axis = img, x-axis = real)



hmmmm.... this doesn't look like a heart

Question 2: Polar Bear

Bear reconsidered his choice of points as the last ones didn't look like a heart at all. He decides to use polar co-ords this time to make his heart.

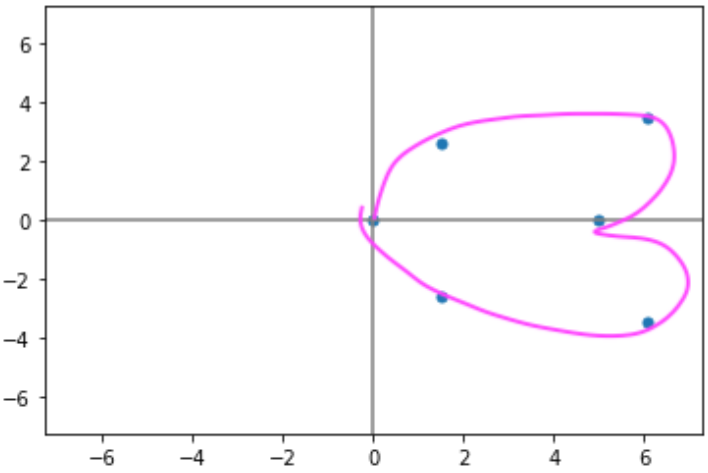
a) The first two points Bear choses are $z = 0, z = 5$. Make a plot using polar co-ords similar to this

b) Add the point $z = 3(sin(60°)i + cos(60°))$ to the plot. This will have polar co-ords (3, 60°), where 3 is the radius and 60° is the angle from the centre point.

c) Add the point $z = 3(sin(-60°)i + cos(-60°))$ to the plot and label the polar co-ords as done above.

d) Add the point $z = 7(sin(30°)i + cos(30°))$ to the plot and label the polar co-ords.

e) Find the complex conjugate of $z = 7(sin(30°)i + cos(30°))$ and add it to the plot and label the polar co-ords.



hmmmm.... still looks a little funky, but bear is happy with this

Question 3: Fix the broken heart

In his excitement bear forgot to position the heart upwards.

a) In order to position it upwards we must rotate his heart. We need to rotate it +90°.

Multiply all of the previous polar points by $1(sin(90°)i + cos(90°))$

Remember the two radius multiply, but the angles add together for multiplication of two polar co-ords

b) Bear wants to rotate the heart +56°. Find a polar complex number that would accomplish this.

c) Bear feels that the heart needs to be bigger. Do the same +56° rotation, but this time make the heart 3x larger.

d) Bear now wants to rotate the heart -60° while making the heart 4x smaller. What polar co-ord would we need to divide the points by to achieve this rotation?