Extremal Graph Theory - Financial Risk Assessment

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Abstract

The goal of this paper is to provide: Visualizations of trades over time that enforce the concepts about diversification and risk mitigation explored in the paper Give a better understanding on where individuals can begin to get into stock trading without the worry (maybe start with the SP 500 and choose stocks from there?)

Contents

1 Introduction

1.1 Interest

Explain why this interection of fiance and combinatorics is crucial for risk management and investment strategies.

1.2 Brief History

Introdfuce the evolution of graph theory applications in financial markets.

1.3 Motivation

Discuss why modern porfolio theory benefits from advanced mathematical tools such as combinatorics and graph theory.

2 Background

This is the first part of the main body of the paper. Here you will define the key concepts and terms that will be used throughout the paper.

2.1 Definitions

Theorem 2.1 (Extremal Graph Theorem) *Let G be a graph with n vertices and m edges. Then, if G does not contain a subgraph isomorphic to K_{r+1}, the complete graph on $r+1$ vertices, then $m \leq \frac{r}{2}(n-1)$.*

Definition 2.2 (Spearman Rank Coefficent)

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2-1)}$$

Go into detail about the spearman rank coefficent

Definition 2.3 (Welsch-Powwell Algorithm)

Go into detail about the coloring alogirthm
Briefly define graph theory terms that will be used (vertices, edges, etc.).
Convert the concept of a portfolio from a spreadsheet to a graph with vertices and edges. This concept is the central point of the paper.

2.2 Data Collection and Processing

For the purpose of this project, I took the time to develop a custom API that allows me to quickly export historical data for a given stock[1]. The API contains an endpoint that allows users to generate a CSV file for any given stock ticker and date range, providing 20 years of historical data. The benefit of developing a custom API is that it allows the developer full control over data-cleaning and preparation on the server side to enforce uniformity and ease-of-use. The fields we are going to use are as follows:

- **Date - The date of the stock price.**
- **Open - The opening price of the stock on that date.**
- **High - The highest price the stock reached on that date.**
- **Low - The lowest price the stock reached on that date.**
- **Close - The closing price of the stock on that date.**
- **Volume - The number of shares traded on that date.**

3 Portfolio Optimization

Optimization and Diversification Extremal Graph Theory

- Theoretical Framework: Explain the extremal graph theorem.
- Application: Demonstrate how this theorem can predict the maximum or minimum number of edges under certain conditions, which translates to understanding the limits of diversification in a portfolio.
- Examples: Provide hypothetical examples of portfolios and how the theorem applies.

4 Risk Assessment

Coloring algorithms for risk assesment and management

- Concept Introduction: Explain what graph coloring is and the significance of using different colors.
- Implementation: How coloring can be used to represent different levels of risk or different asset classes.
- Practical Example: A case study where coloring helps in decision-making about asset allocation or identifying over-concentrated sectors

[1]Full documentation and a link to the source code of the custom API can be found on my website, linked in the references section.

5 Holding Vizualization

Correlation Graphs for Portfolio Holdings

- Graph Construction: Discuss how to build a graph where vertices represent assets and edges represent correlations between returns.
- Analysis Techniques: Use threshold levels to add/remove edges or use weights to show the strength of correlations.
- Visualization: Include a section on how these graphs can visually represent portfolio diversification and the interconnections between assets.

6 Conclusion

- Summary: Recap how graph theory enhances portfolio management.
- Future Directions: Suggest how further research could integrate other combinatorial techniques or advanced graph theory concepts.
- Open Problems: Pose any unresolved questions or potential for new research that your paper hints at.

References

[ANHF11] M. J. Ablowitz, S. D. Nixon, T. P. Horikis, and D. J. Frantzeskakis, *Perturbations of dark solitons*, Proc. R. Soc. A Vol **467** (2011), 2597-2621.

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