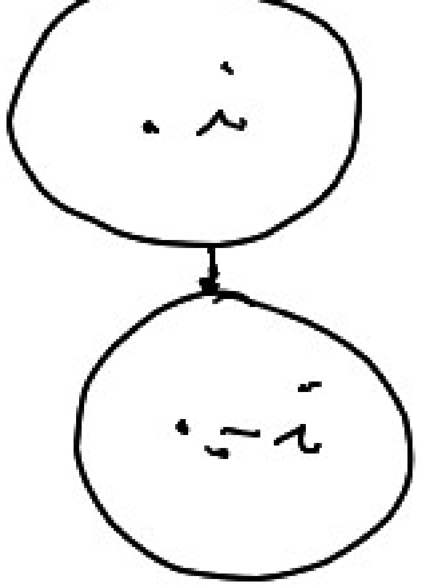
(3) $\dfrac{1}{1+z^2} = \dfrac{1}{(i-z)}\,\dfrac{-1}{(i+z)}$



$\gamma : [-\pi, \pi] \longrightarrow \mathbb{C}$

$\gamma(t) = i + e^{it}$

$\gamma'(t) = ie^{it}$

$(*) = \int_{-\pi}^{\pi} \dfrac{2+\sin t}{\cos^2 t + (2+\sin t)^2}\, dt > 0$

$\Downarrow$

$\int_\gamma \dfrac{1}{1+z^2} \neq 0 \Rightarrow \dfrac{1}{1+z^2}$ no admite primitiva en ese abierto

$$\int_\gamma \frac{dz}{1+z^2} = \int_{-\pi}^{\pi} \frac{ie^{it}\,dt}{1+(i+e^{it})^2} = \int_{-\pi}^{\pi} \frac{i\cancel{e^{it}}\,dt}{2i\cancel{e^{it}} + (e^{it})^2} = i\int_{-\pi}^{\pi} \frac{dt}{2i+e^{it}} =$$

$$1+z^2 = 1+(i+e^{it})^2 = \cancel{1} + \cancel{(-1)} + 2ie^{it} + (e^{it})^2$$

$$z = i+e^{it}$$

$$2i+e^{it} = 2i + \cos t + i\sin t$$

$$= i\int_{-\pi}^{\pi} \frac{dt}{\cos t + i(2+\sin t)} = i\int_{-\pi}^{\pi} \frac{\cos t - i(2+\sin t)}{(\cos^2 t + (2+\sin t)^2)}\, dt$$

$\rightsquigarrow$ parte real es $(*)$