Tema 5

$$T(n) = aT\left(\frac{n}{b}\right) + f(n) \quad \begin{cases} a \geq 1 \\ b > 1 \\ f(n) = n^d \end{cases}, \quad T(n) = \begin{cases} \theta(n^d \log n), & a = b^d \\ \theta(n^d), & a < b^d \\ \theta(n^{\log_b a}), & a > b^d \end{cases}$$

1) $T(n) = 3T\left(\frac{n}{2}\right) + n^2 \rightarrow \begin{bmatrix} a = 3 \\ b = 2 \\ d = 2 \end{bmatrix} \rightarrow 3 < 2^2 = 4$ (caso 2)

$\rightarrow \theta(n^2)$

2) $T(n) = 4T\left(\frac{n}{2}\right) + n^2 \rightarrow \begin{bmatrix} a = 4 \\ b = 2 \\ d = 2 \end{bmatrix} \rightarrow 4 = 2^2$ (caso 1)

$\rightarrow \theta(n^2 \log n)$

3) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0'51} \rightarrow \begin{bmatrix} a = 2 \\ b = 4 \\ d = 0'51 \end{bmatrix} \rightarrow 2 < 4^{0'51}$ (caso 2)

$\rightarrow \theta(n^{0'51})$

4) $T(n) = 2T(n) + n^2 \rightarrow \begin{bmatrix} a = 2 \\ b = 1 \\ d = 2 \end{bmatrix}$ No podemos aplicar el método maestro ya que $b = 1$ y es necesario que $b > 1$.

5) $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Para hallar d, queremos: $\frac{n}{2} = n^d \Leftrightarrow \log\left(\frac{n}{2}\right) = d \log n$

$\Leftrightarrow \frac{\log\left(\frac{n}{2}\right)}{\log n} = d \Leftrightarrow \frac{\log n - \log 2}{\log n} = d \Leftrightarrow 1 - \frac{\log 2}{\log n} = d$

Dado que queremos $d \geq 0 \rightarrow 1 - \frac{\log 2}{\log n} \geq 0$

$\rightarrow \frac{\log 2}{\log n} \leq 1 \rightarrow \log 2 \leq \log n \rightarrow 2 \leq n$

$\Rightarrow \forall n \geq 2, \quad T(n) = 3T\left(\frac{n}{3}\right) + n^{1 - \frac{\log 2}{\log n}} \leq$

$\leq 3T\left(\frac{n}{3}\right) + \underbrace{\frac{1}{2} \cdot n}_{\theta(n^d)} \Rightarrow \begin{bmatrix} a = 3 \\ b = 3 \\ d = 1 - \frac{\log 2}{\log n} \end{bmatrix} \quad 3 > 3^{1 - \overbrace{\frac{\log 2}{\log n}}^{\in (0,1]}} \Big\} \in [1,3)$, $1 - \frac{\log 2}{\log n} \in [0,1)$ $\forall n \geq 2$.

$\Rightarrow$ (caso 3) $3 > 3^{1 - \frac{\log 2}{\log n}} \rightarrow \theta(n^{\log_3 3}) = \theta(n)$

6) $T(n) = 7T\left(\frac{n}{3}\right) + n^2 \rightarrow \begin{bmatrix} a = 7 \\ b = 3 \\ d = 2 \end{bmatrix} \rightarrow 7 < 3^2$ (caso 2)

$\rightarrow \theta(n^2)$

$\Longrightarrow$

7) $T(n) = 2^n \, T\left(\frac{n}{4}\right) + n \rightarrow \begin{bmatrix} a = 2^n \\ b = 4 \\ d = 1 \end{bmatrix} \rightarrow 2^n = a, \; 4 = b^d$

dado que $n \geq 0 \rightarrow a \geq 1$

$2^n = 4 \iff n = 2$ (caso 1) $\rightarrow \Theta(n \log n) = \Theta(2 \log 2)$

$2^n > 4 \iff n > 2$ (caso 3) $\rightarrow \Theta(n^{\log_4 2^n}) = \Theta(n^{n \log_4 2}) = \Theta(n^{2n})$

$2^n < 4 \iff n < 2$ (caso 2) $\rightarrow \Theta(n)$.

Tema 5 (opcionales). Blanca Pieras.

1) $T(n) = 16T(\frac{n}{4}) + n! \leq 16T(\frac{n}{4}) + n^n \rightarrow \begin{bmatrix} a = 16 \\ b = 4 \\ d = n \end{bmatrix}$

$16 = 4^n \Leftrightarrow 4^2 = 4^n \rightarrow n = 2$

CASO 1 $(n = 2) \rightarrow \Theta(2^2 \cdot \log 2) = \Theta(4 \log 2)$

CASO 2 $(n < 2) \rightarrow \Theta(n^n)$

CASO 3 $(n > 2) \rightarrow \Theta(n^{\log_4 16}) = \Theta(n^2)$

2) $T(n) = 4T(\frac{n}{2}) + \log n \leq 4T(\frac{n}{2}) + n \rightarrow$ $a = 4$, $b = 2$, $d = 1$

$(\log n < n \quad \forall n \geq 0)$

$\rightarrow 4 > 2^1$ (caso 3) $\rightarrow \Theta(n^{\log_2 4}) = \Theta(n^2)$

3) $T(n) = T(\frac{n}{2}) + 2^n \leq T(\frac{n}{2}) + n^3 \rightarrow \begin{bmatrix} a = 1 \\ b = 2 \\ d = 3 \end{bmatrix}$ $1 < 2^3$

$2^n < n^3 \quad \forall n \geq 2$

(caso 2)

$\Theta(n^3)$

4) $T(n) = T(|\sqrt{n}|) + 1 = T(\frac{n}{\sqrt{n}}) + n^0$

$\sqrt{n} > 0$
$\forall n$

$\sqrt{n} = \frac{n}{x} \Leftrightarrow x = \frac{n}{\sqrt{n}} \Leftrightarrow x = \sqrt{n} \Leftrightarrow T(|\sqrt{n}|) = T(\frac{n}{\sqrt{n}})$

$\begin{bmatrix} a = 1 \\ b = \sqrt{n} > 1 \quad \forall n > 1 \\ d = 0 \end{bmatrix} \rightarrow 1 = (\sqrt{n})^0$ (caso 1) $\Theta(n^0 \cdot \log n)$

$= \Theta(\log n)$