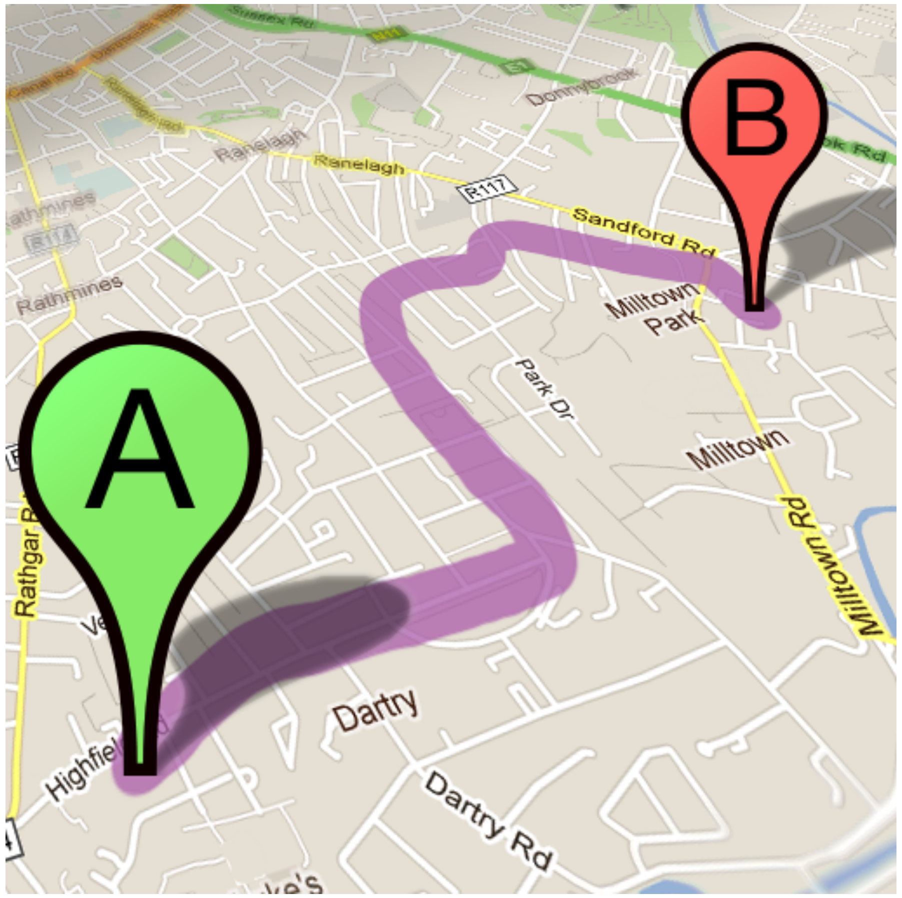
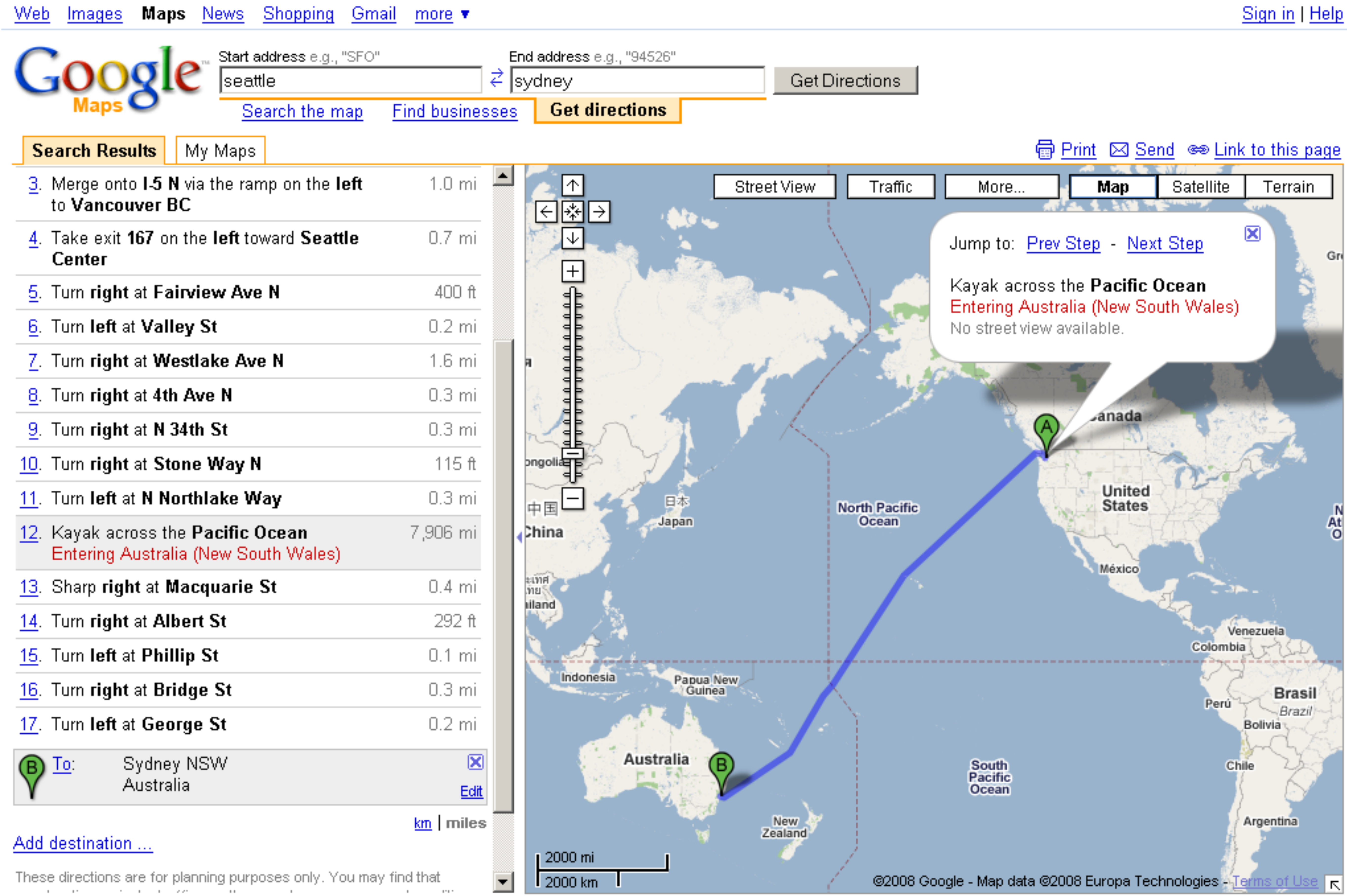
Data Structures in Java

Lecture 18: Unweighted Shortest Paths

11/18/2019

Daniel Bauer




Web Images Maps News Shopping Gmail more ▾
Sign in | Help
Google Maps
Start address e.g., "SFO"
seattle
End address e.g., "94526"
sydney
Get Directions
Search the map
Find businesses
Get directions
Search Results
My Maps
Print
Send
Link to this page
3. Merge onto I-5 N via the ramp on the left to Vancouver BC 1.0 mi
4. Take exit 167 on the left toward Seattle Center 0.7 mi
5. Turn right at Fairview Ave N 400 ft
6. Turn left at Valley St 0.2 mi
7. Turn right at Westlake Ave N 1.6 mi
8. Turn right at 4th Ave N 0.3 mi
9. Turn right at N 34th St 0.3 mi
10. Turn right at Stone Way N 115 ft
11. Turn left at N Northlake Way 0.3 mi
12. Kayak across the Pacific Ocean 7,906 mi
Entering Australia (New South Wales)
13. Sharp right at Macquarie St 0.4 mi
14. Turn right at Albert St 292 ft
15. Turn left at Phillip St 0.1 mi
16. Turn right at Bridge St 0.3 mi
17. Turn left at George St 0.2 mi
B To: Sydney NSW Australia
Edit
km | miles
Add destination ...
These directions are for planning purposes only. You may find that
Street View
Traffic
More...
Map
Satellite
Terrain
Jump to: Prev Step - Next Step
Kayak across the Pacific Ocean
Entering Australia (New South Wales)
No street view available.
A
B
Canada
United States
México
North Pacific Ocean
日本
Japan
中国
China
Indonesia
Papua New Guinea
Australia
New Zealand
South Pacific Ocean
Venezuela
Colombia
Perú
Brasil
Brazil
Bolivia
Chile
Argentina
2000 mi
2000 km
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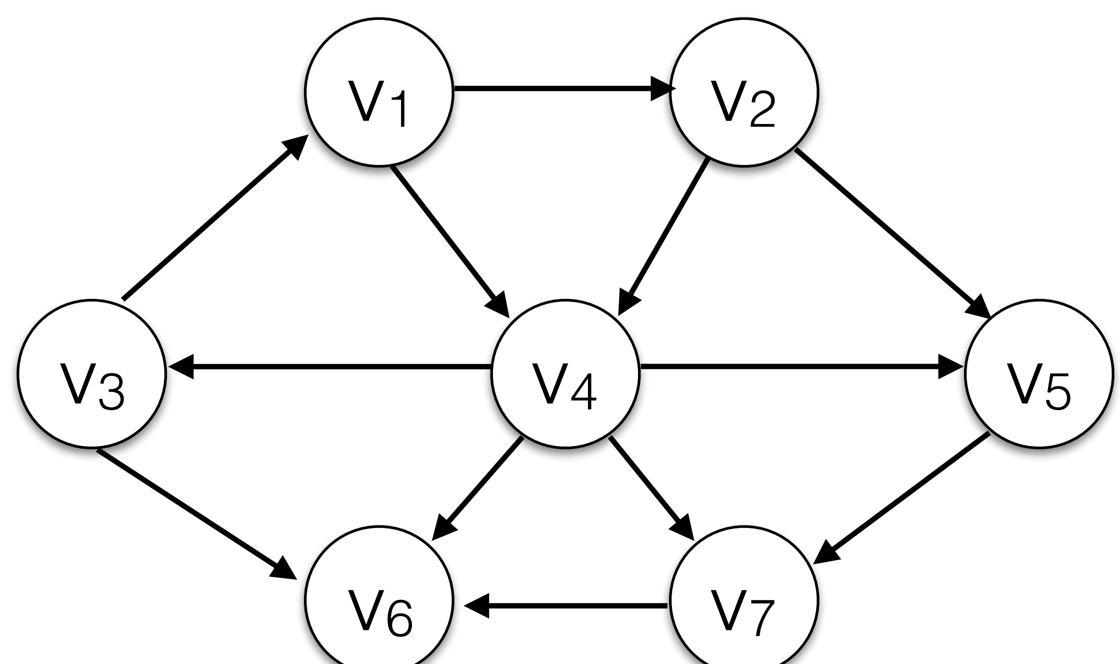
Graph Traversals

- Different ways of exploring graphs:
  - Topological sort for Directed Acyclic Graphs.
  - Depth First Search (a generalization of pre-order traversal on trees to graphs, uses a Stack)
  - Breadth First Search (uses a Queue)
  - Dijkstra's algorithm to find weighted shortest paths (uses a Priority Queue)

Finding Shortest Paths

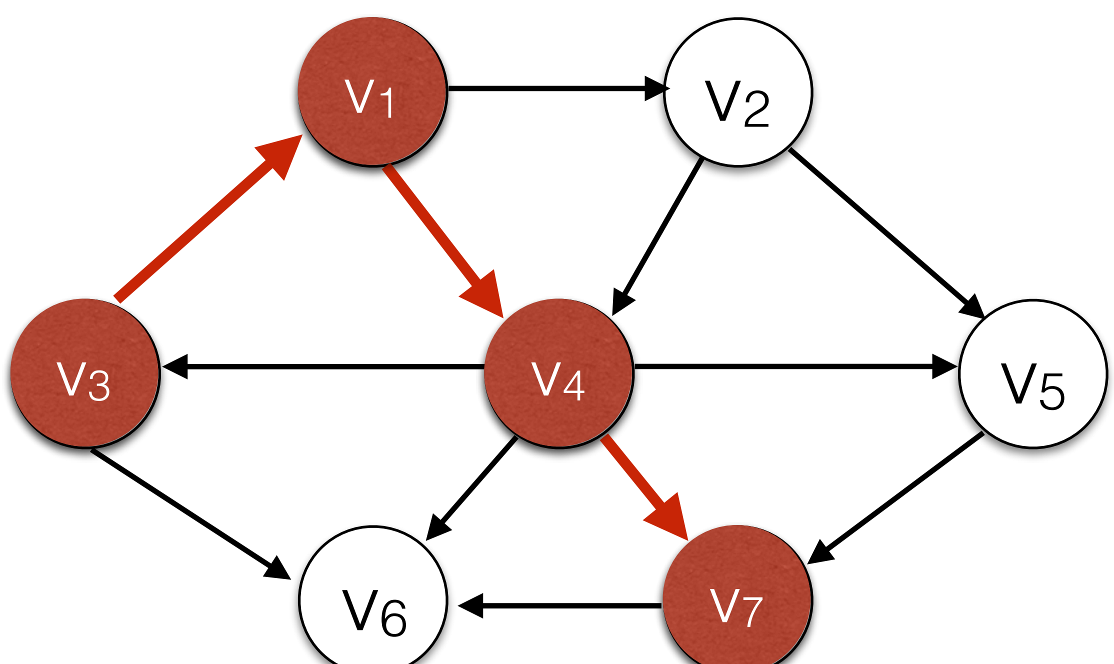
- Goal: Find the shortest path between two vertices s and t.



What is the shortest path between v_3 and v_7?

Finding Shortest Paths

- Goal: Find the shortest path between two vertices s and t.

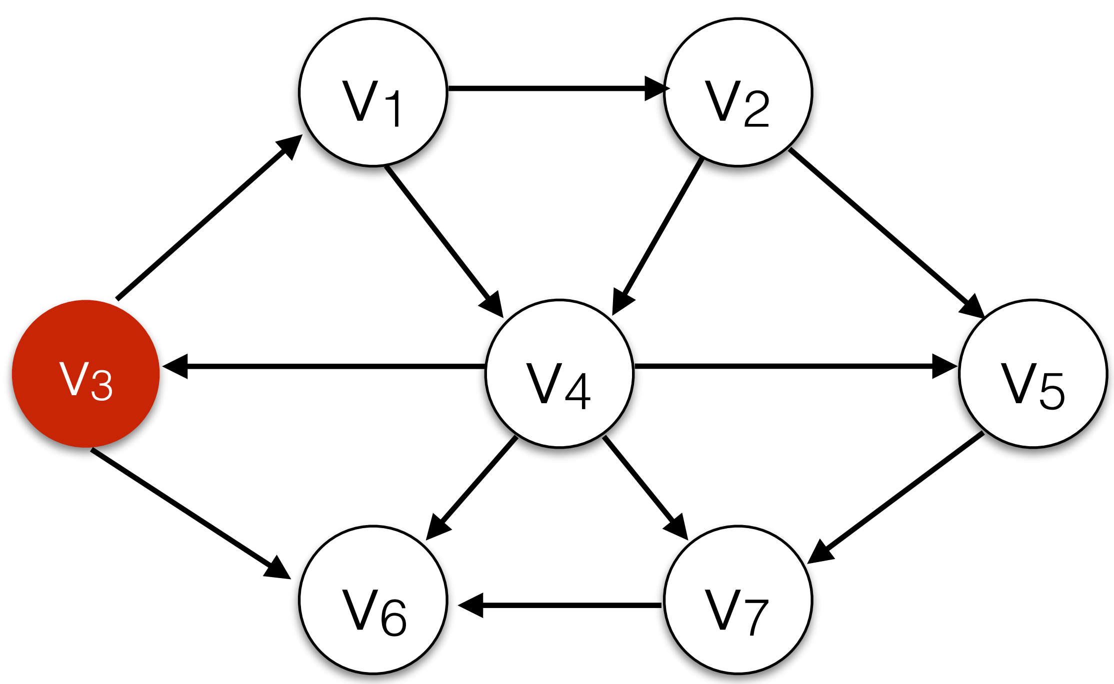


length 3

What is the shortest path between v_3 and v_7?

Finding Shortest Paths

- Goal: Find the shortest path between two vertices s and t.
- It turns out that finding the shortest path between s and ALL other vertices is just as easy. This problem is called **single-source shortest paths**.



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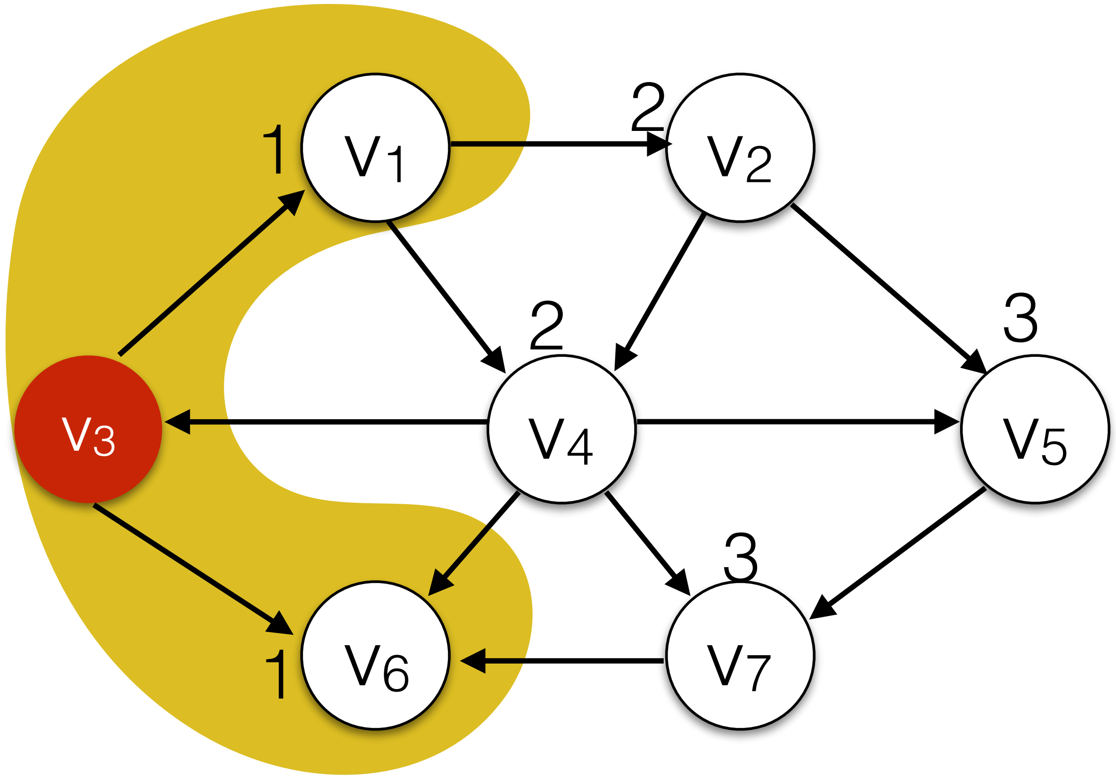

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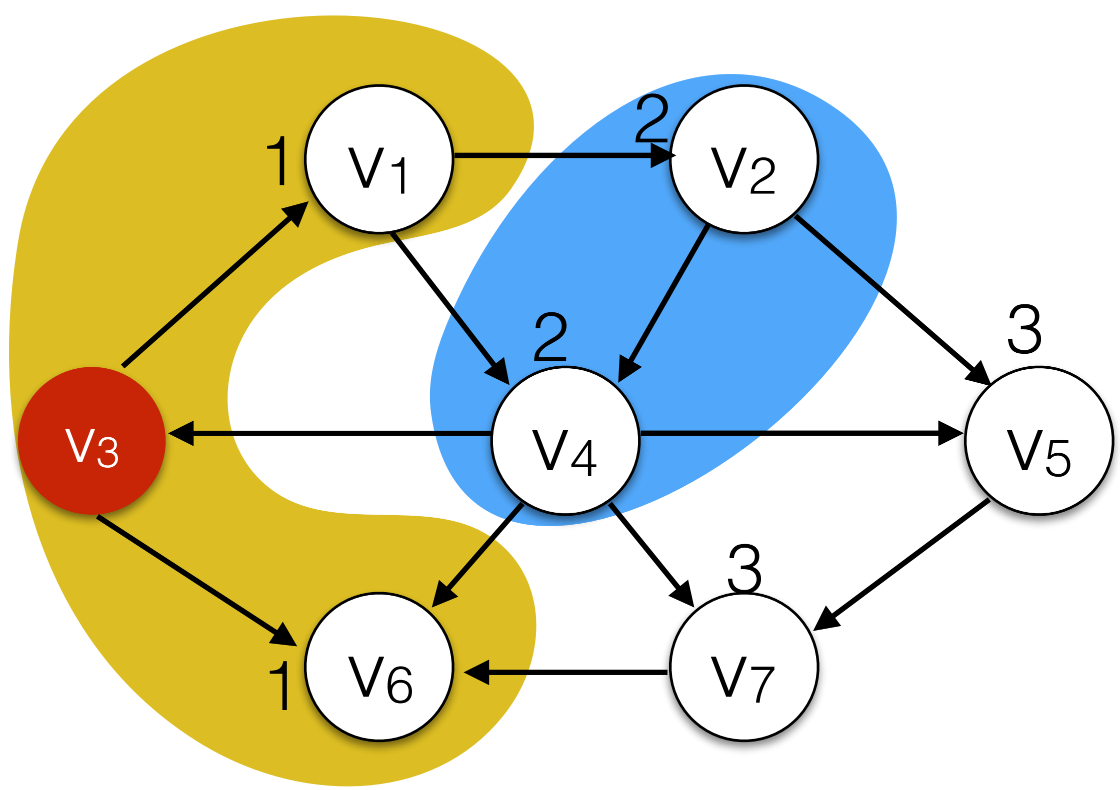

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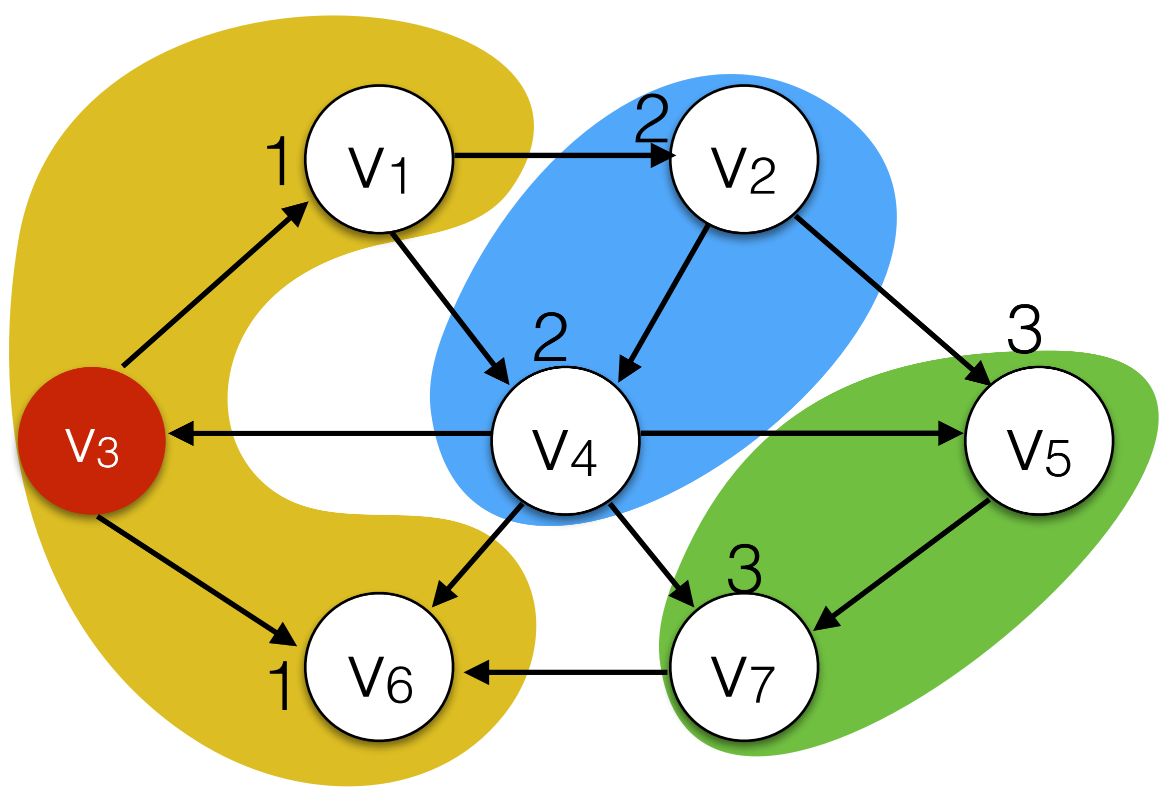
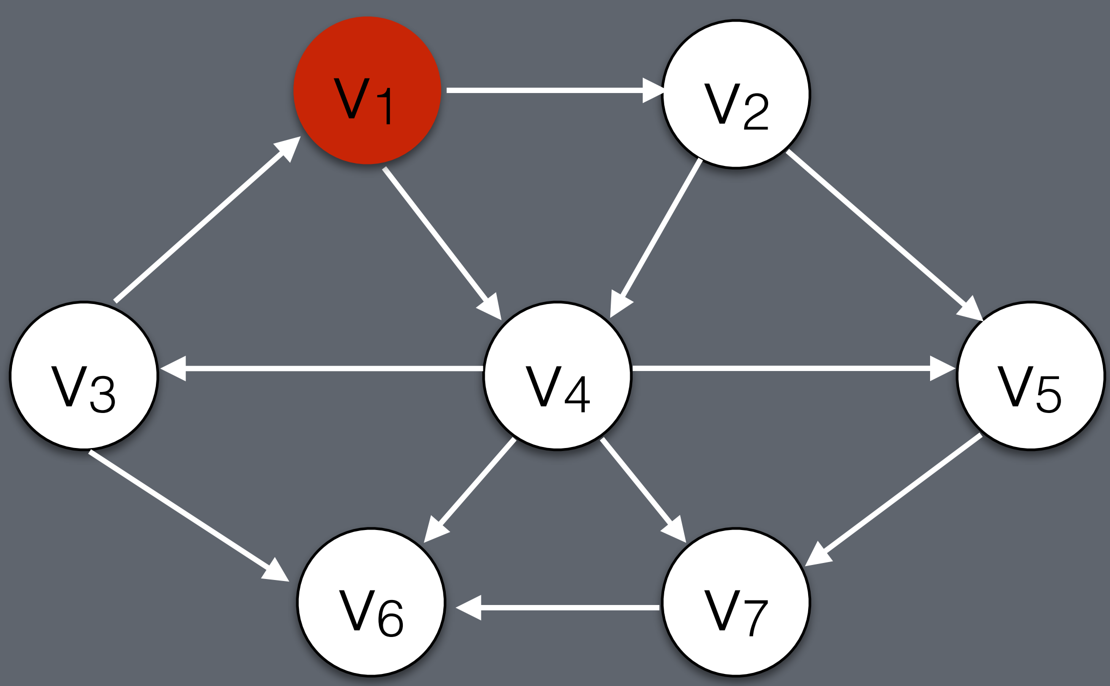


Breadth First Search




```
Queue q
q.enqueue(start)

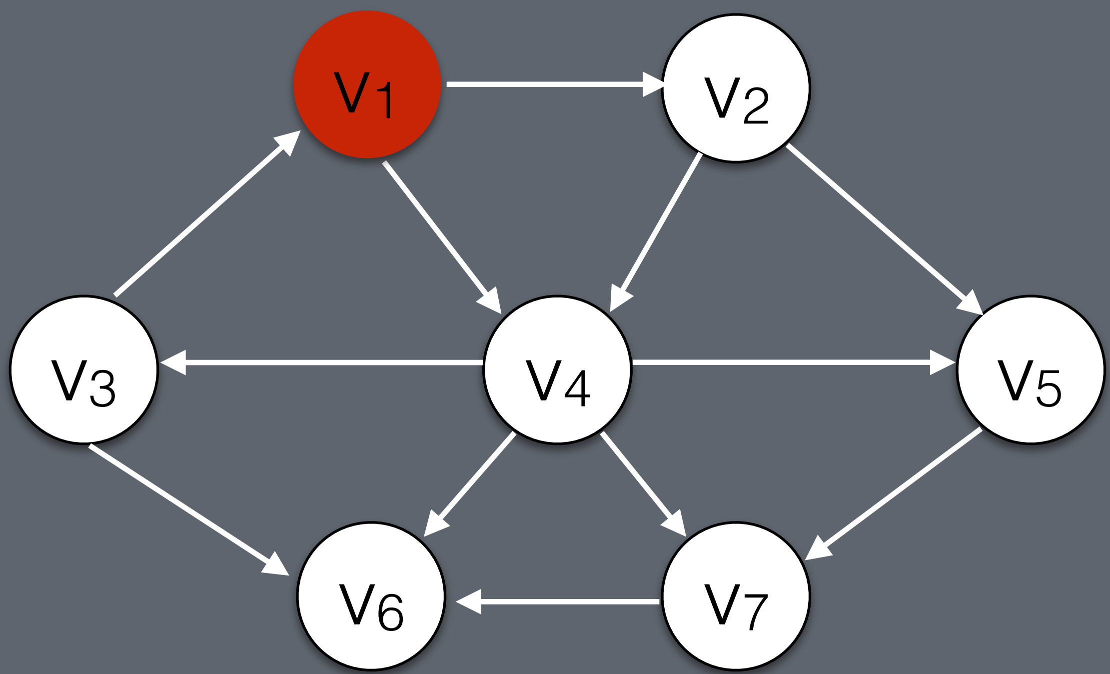

while (q is not empty):
  u = q.dequeue()

  for each v adjacent to u:
    q.enqueue(v)
```

visited.add(v)

Queue: { V_1 }

Breadth First Search




```
Queue q
q.enqueue(start)
Set visited

while (q is not empty):
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visited.add(v)

Queue: { V_1 }

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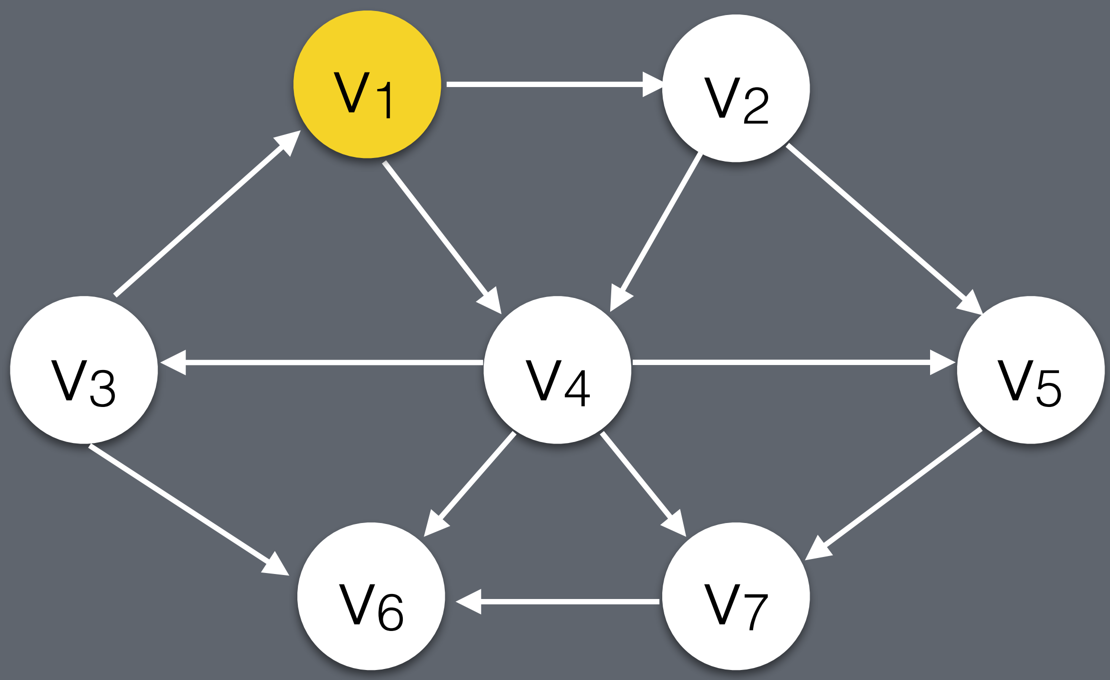


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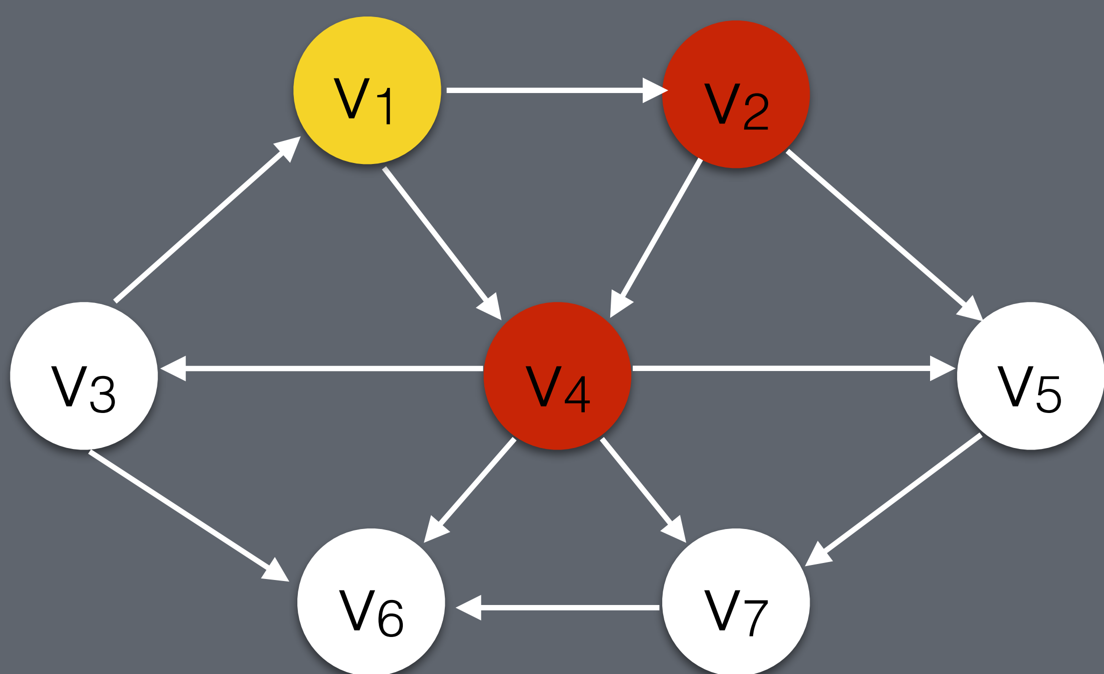
```

Queue: {}

Breadth First Search

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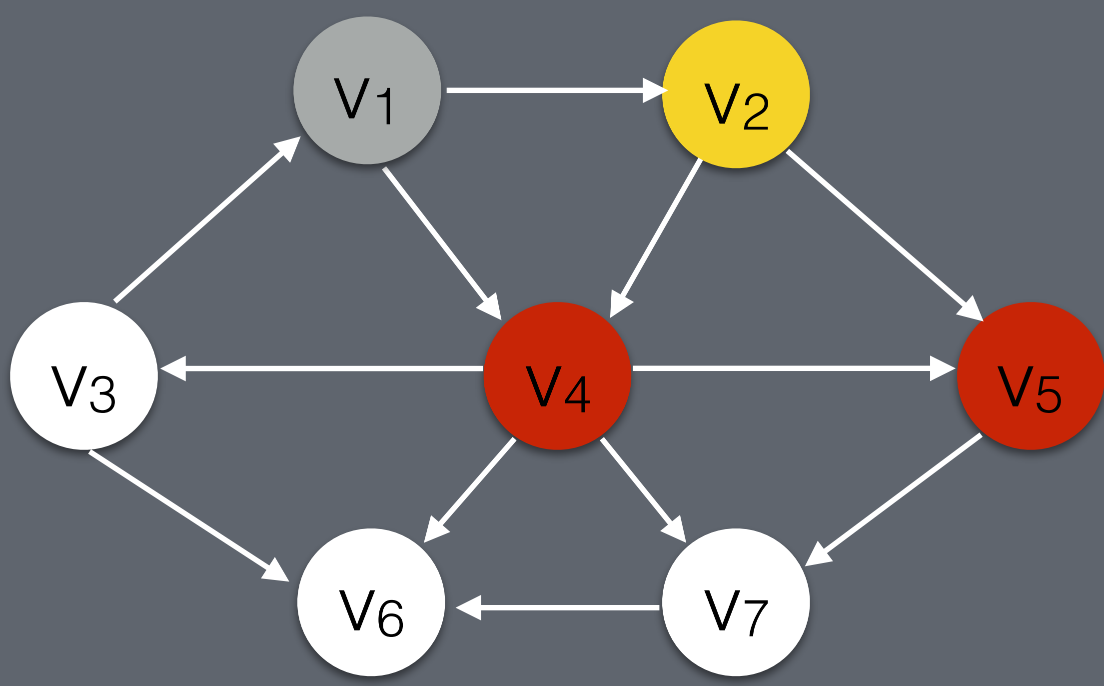
```

Queue: $\{ V_2, V_4 \}$

Breadth First Search

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Set visited

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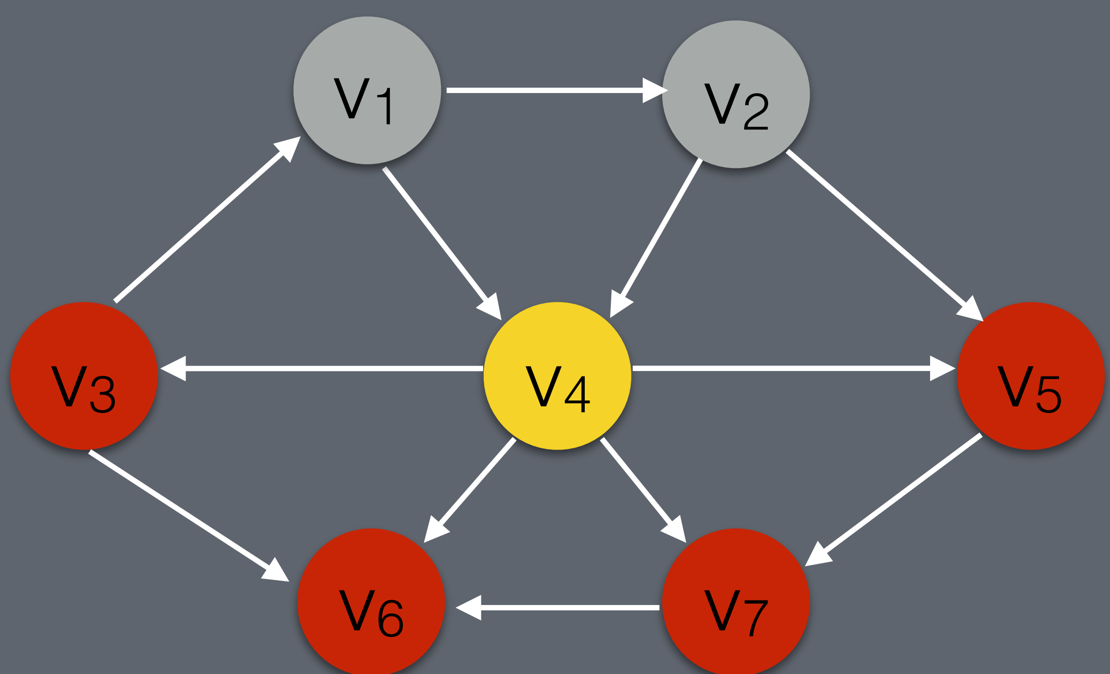
      q.enqueue(v)
```

Queue: $\{ V_4, V_5 \}$

Breadth First Search

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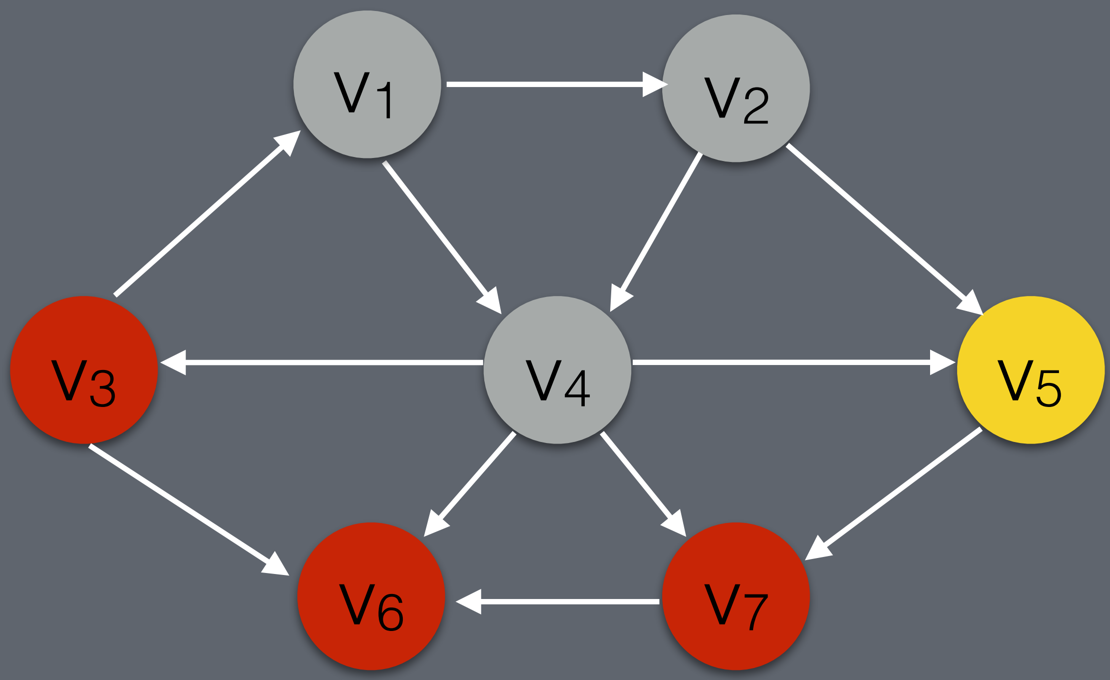
      q.enqueue(v)
```

Queue: $\{V_5, V_3, V_6, V_7\}$

Breadth First Search

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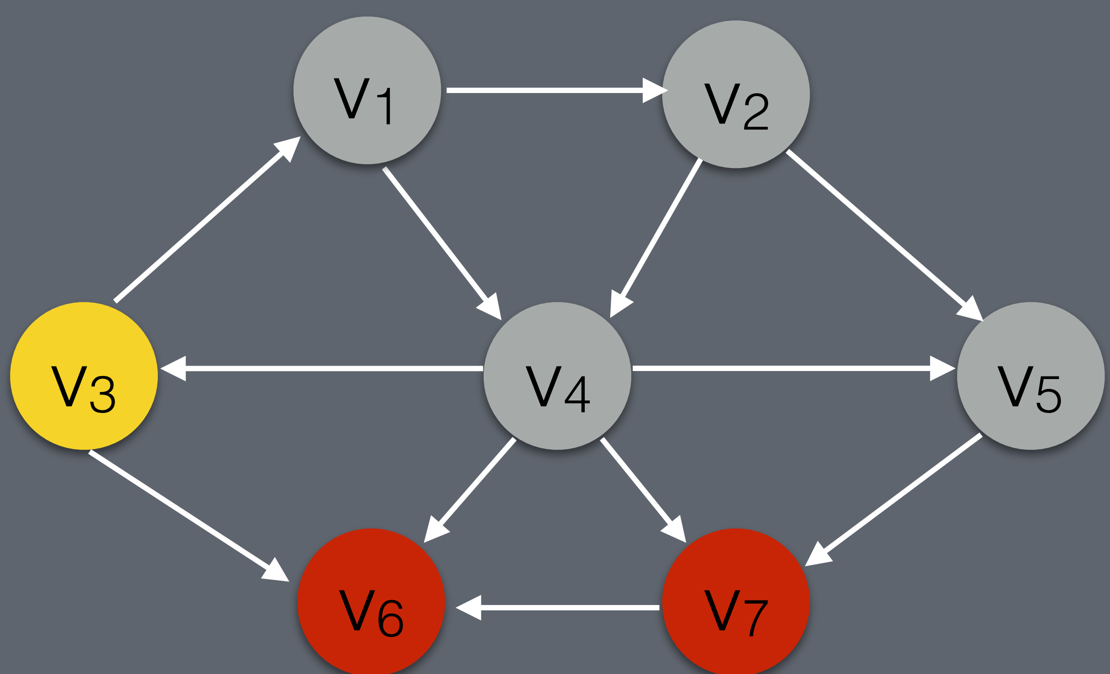
Queue: $\{V_3, V_6, V_7\}$

Breadth First Search

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    if (v is not in visited):
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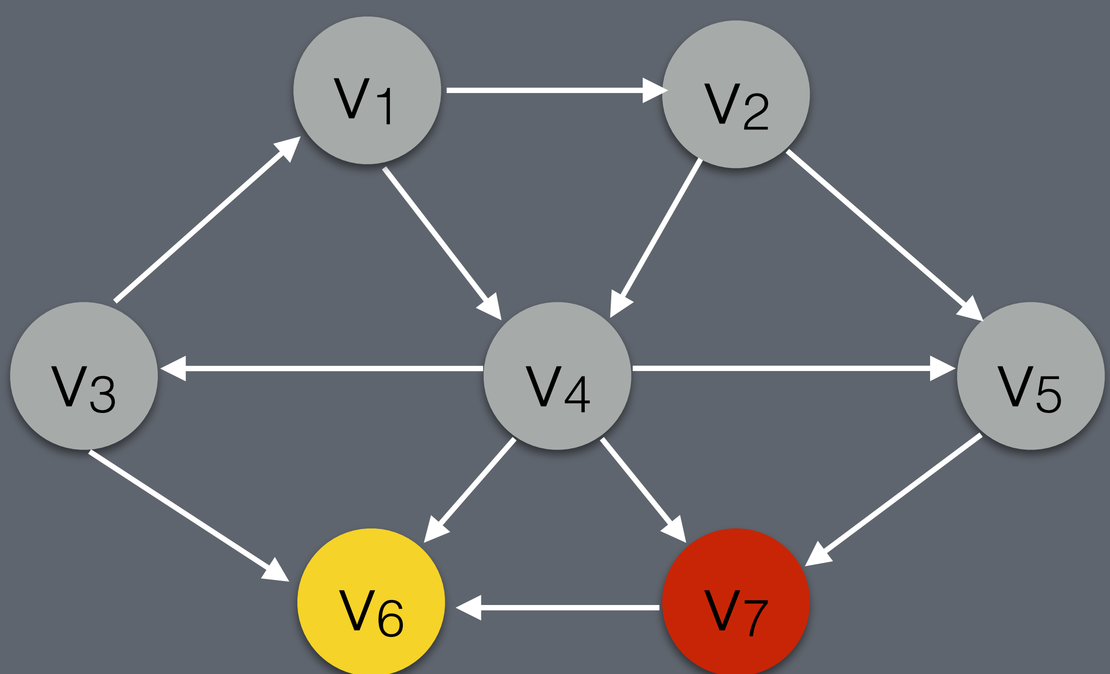
```

Queue: $\{V_6, V_7\}$

Breadth First Search

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Set visited

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  for each v adjacent to u:
    if (v is not in visited):
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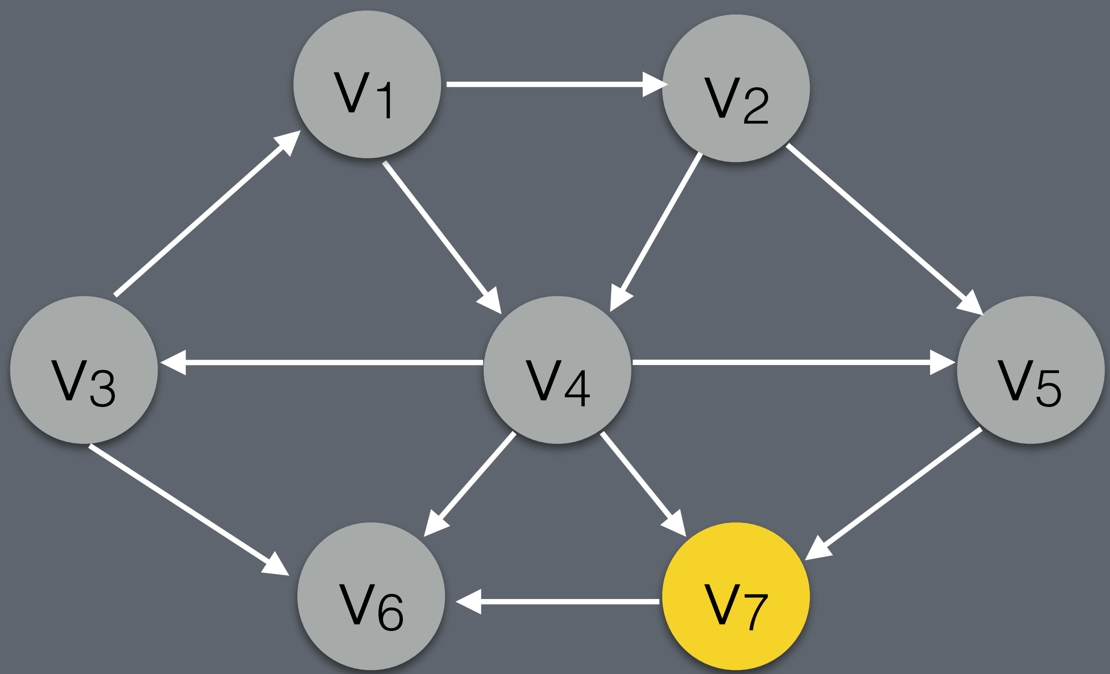
```

Queue: $\{V_7\}$

Breadth First Search

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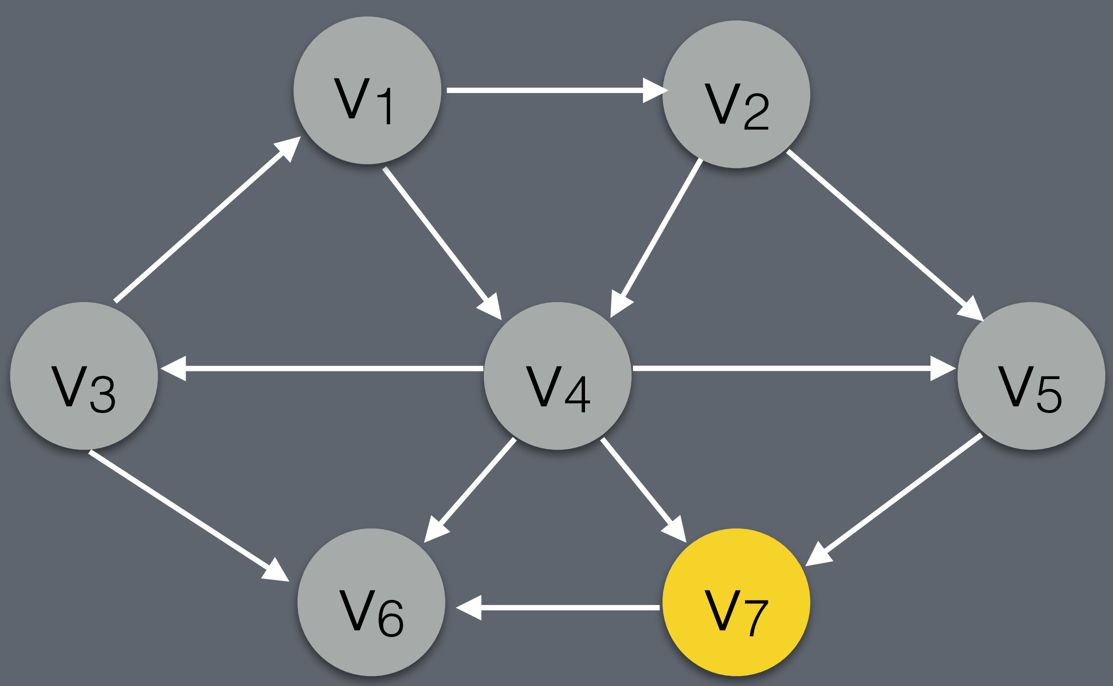
Queue: {}

Breadth First Search

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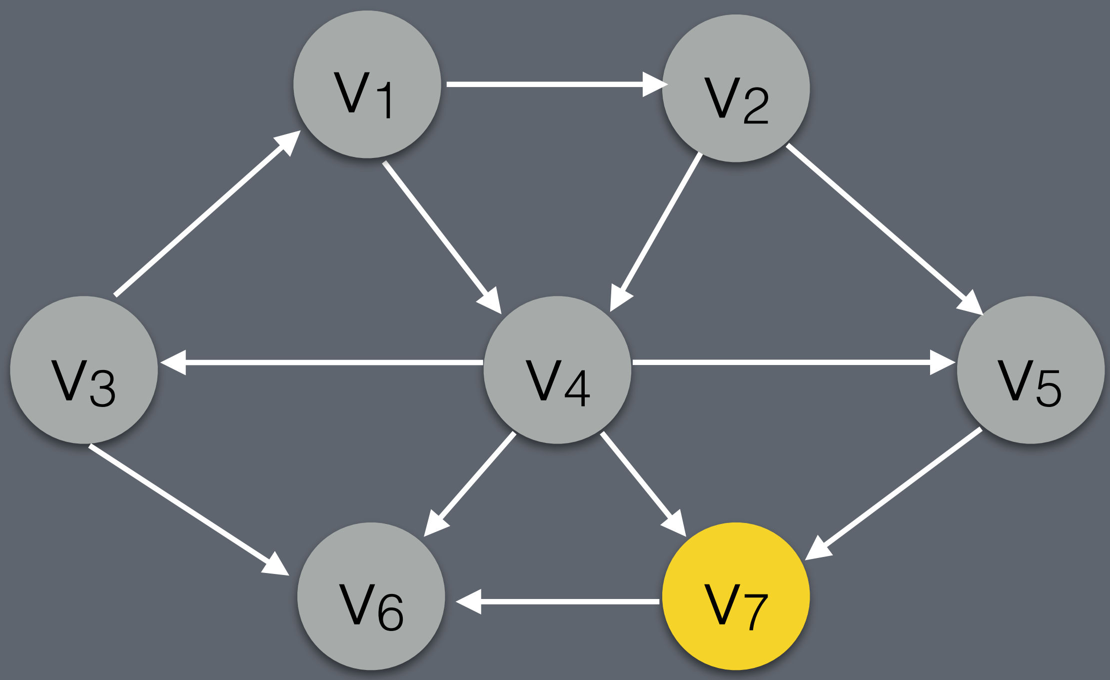
```

Running time: $O(|V|+|E|)$

Breadth First Search




```
Queue q
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Set visited

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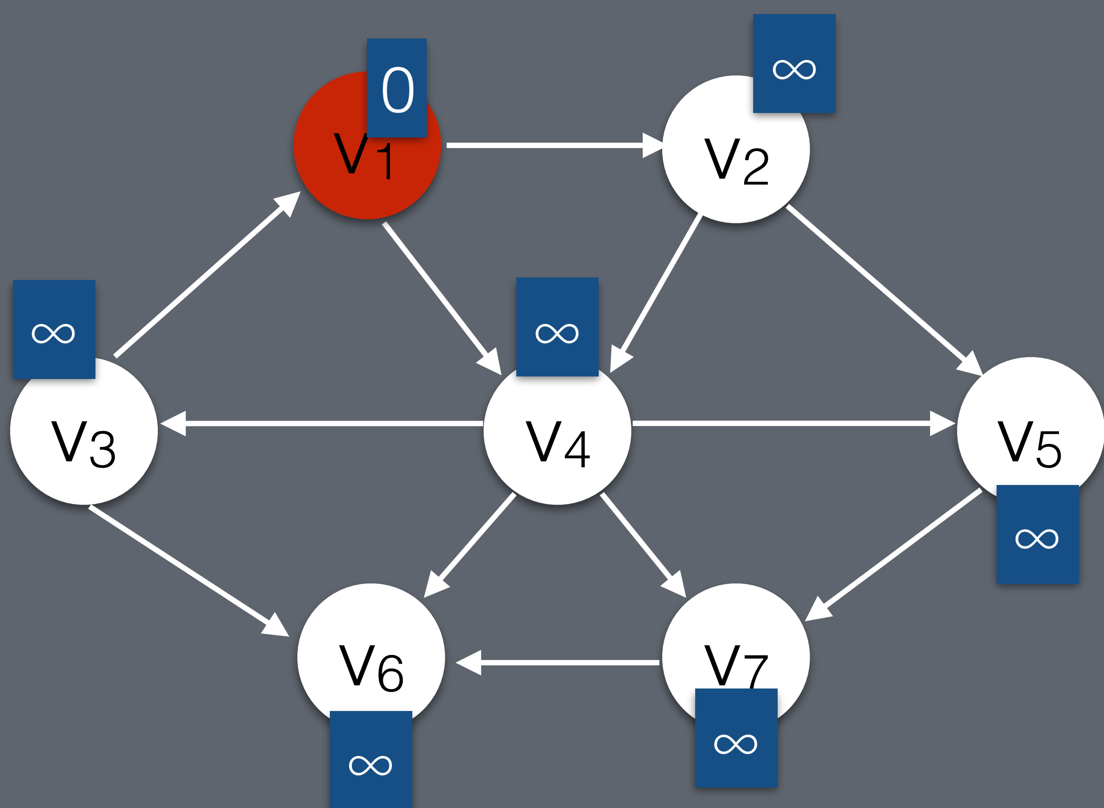
  u = q.dequeue()
```

BFS will traverse the entire graph even without a visited set.

What happens if we use a stack (DFS)?

Running time: O(|V|+|E|)

Unweighted Shortest Paths

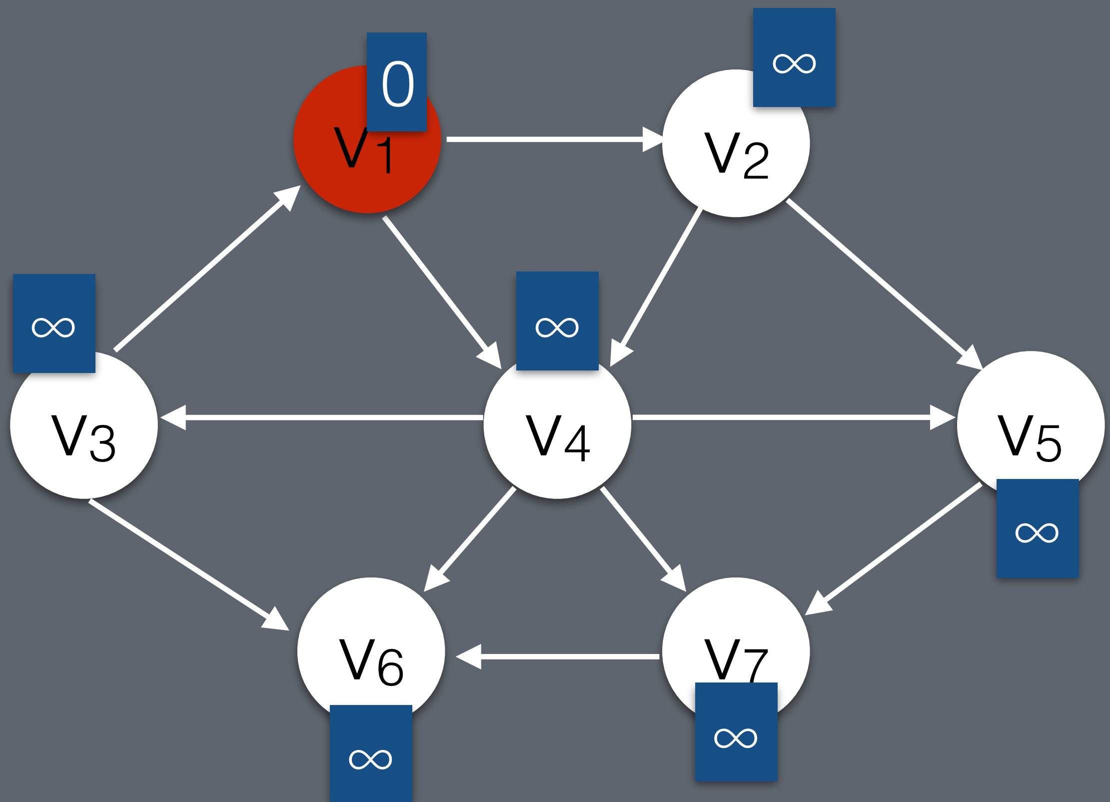


Queue: { V_1 }

```
for all v:
  v.cost = ∞
  v.prev = null

start.cost = 0
```

Unweighted Shortest Paths



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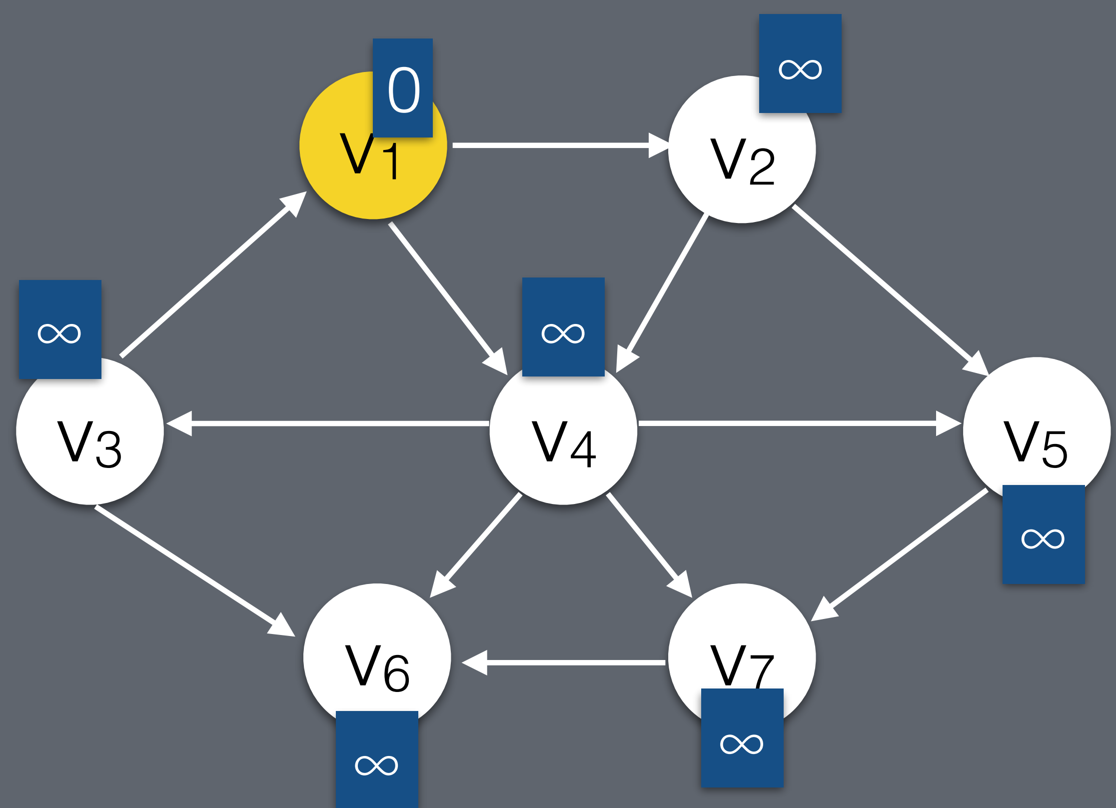
Unweighted Shortest Paths



Queue: {}

```
for all v:
  v.cost = ∞
  v.prev = null

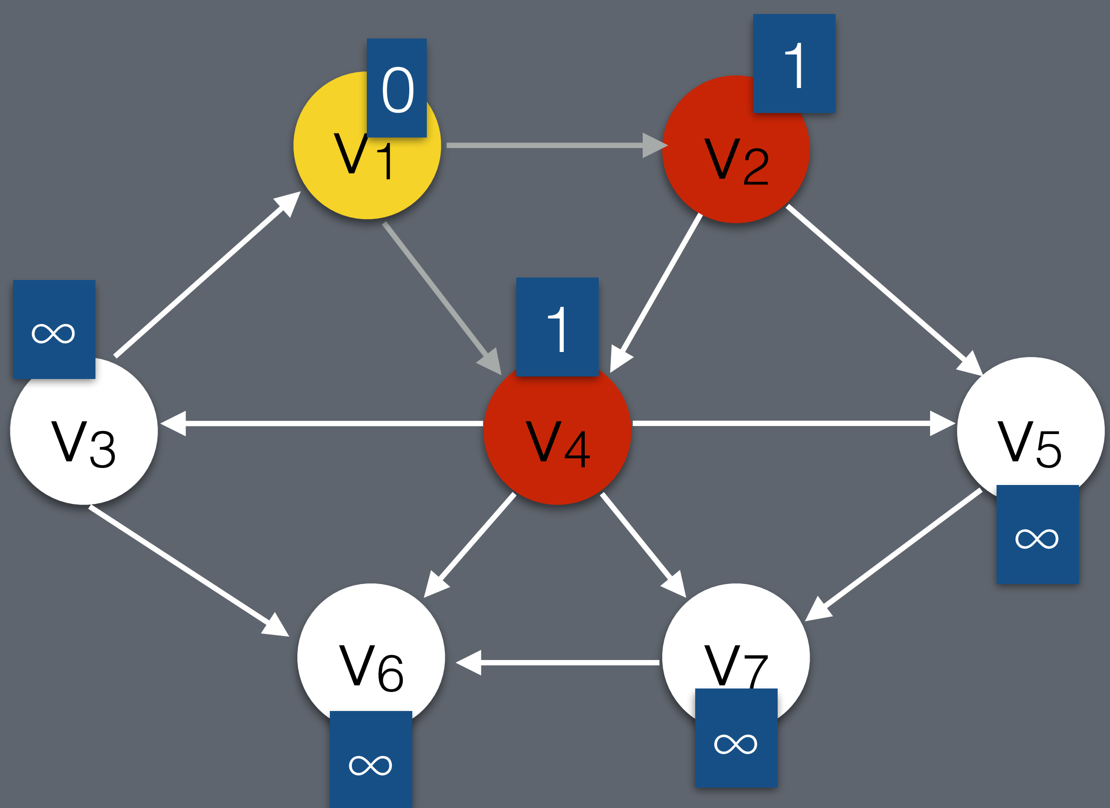
start.cost = 0

Queue q
q.enqueue(start)

while (q is not empty):
  u = q.dequeue()

  for each v adjacent to u:
    if v.cost == ∞:
      v.cost = u.cost + 1
      v.prev = u
      q.enqueue(v)
```

Unweighted Shortest Paths



Queue: { V_2, V_4}

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      v.cost = u.cost + 1
      v.prev = u
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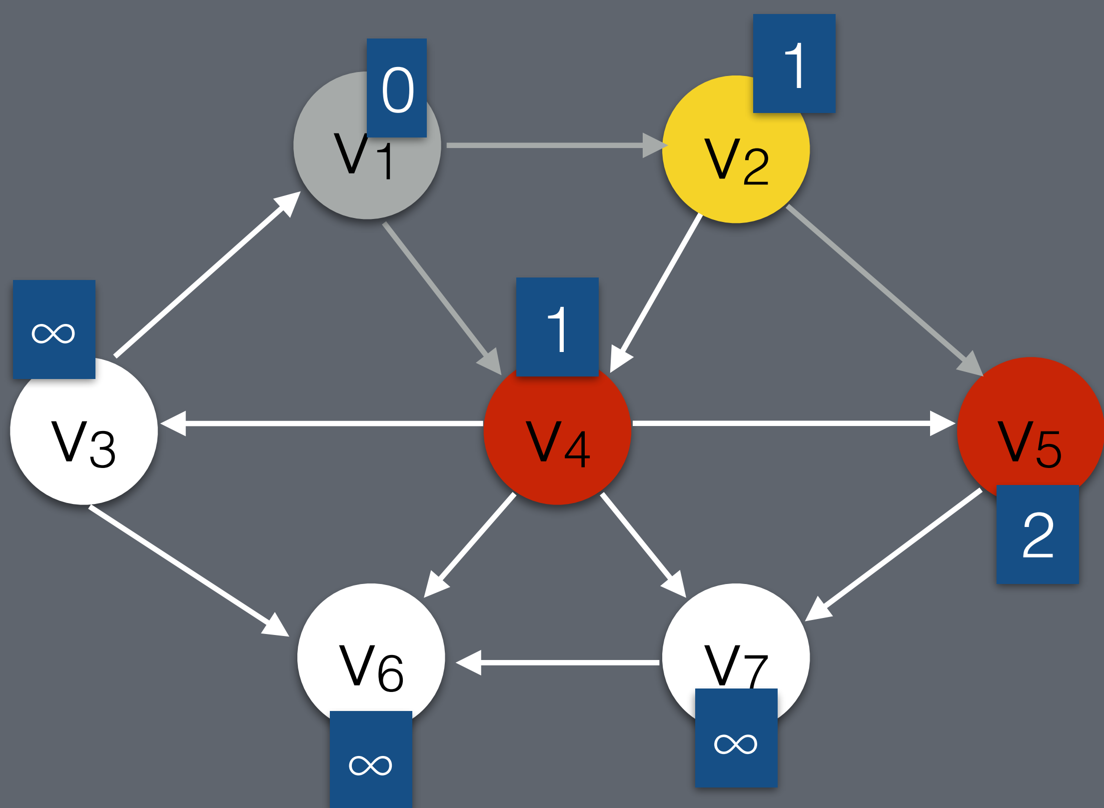
```

Unweighted Shortest Paths



Queue: { V_4, V_5 }

```
for all v:
  v.cost = ∞
  v.prev = null

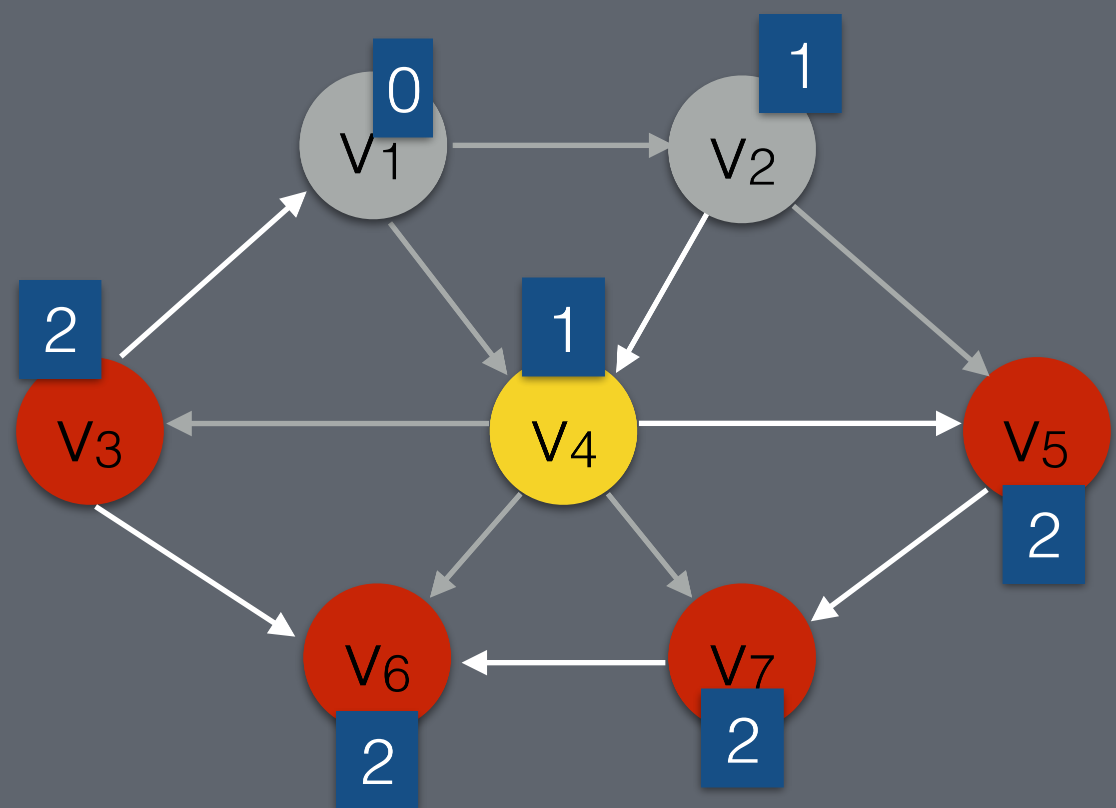
start.cost = 0

Queue q
q.enqueue(start)

while (q is not empty):
  u = q.dequeue()

  for each v adjacent to u:
    if v.cost == ∞:
      v.cost = u.cost + 1
      v.prev = u
      q.enqueue(v)
```

Unweighted Shortest Paths



Queue: $\{V_5, V_3, V_6, V_7\}$

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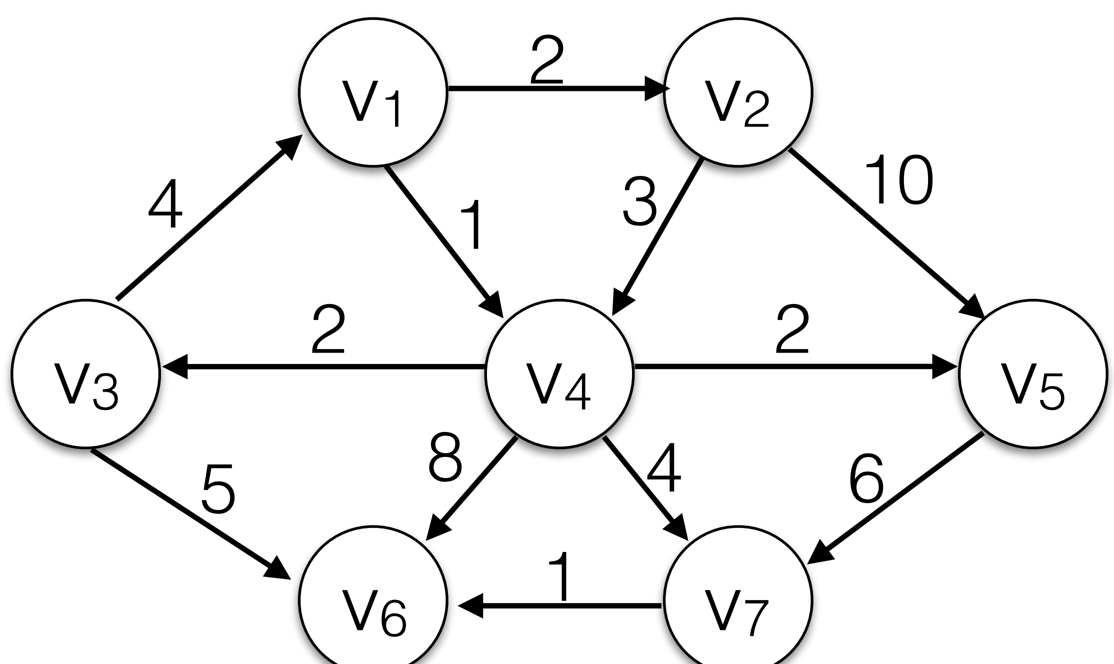
Weighted Shortest Paths

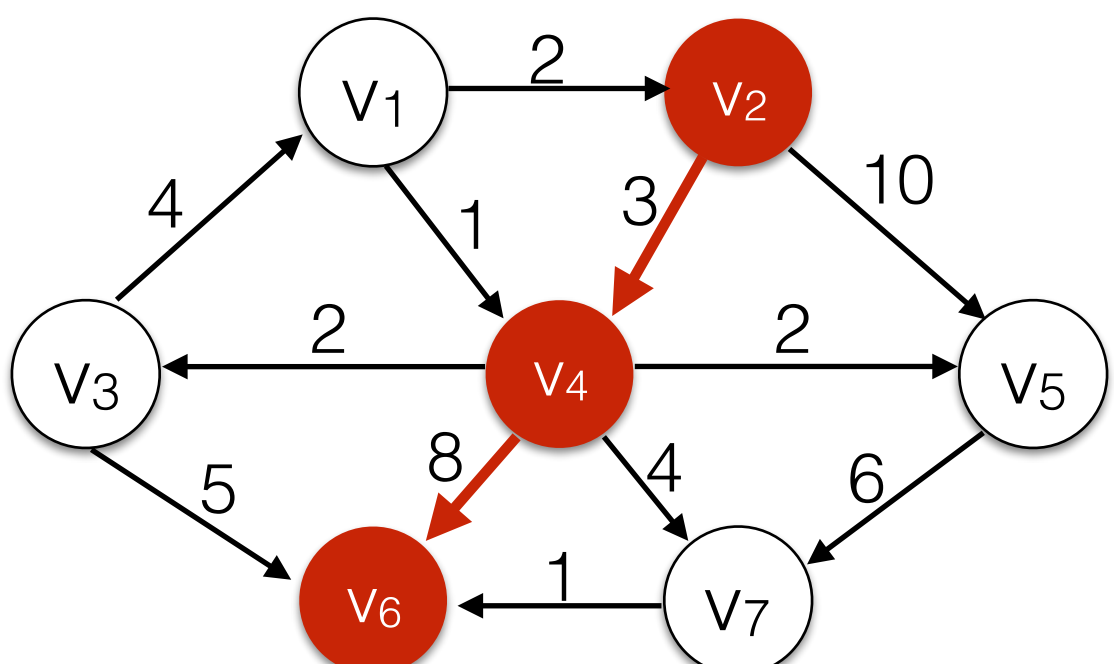
- Goal: Find the shortest path between two vertices s and t.



What is the shortest path between v_2 and v_6?

Weighted Shortest Paths

- Goal: Find the shortest path between two vertices s and t.
- Normal BFS will find this path.

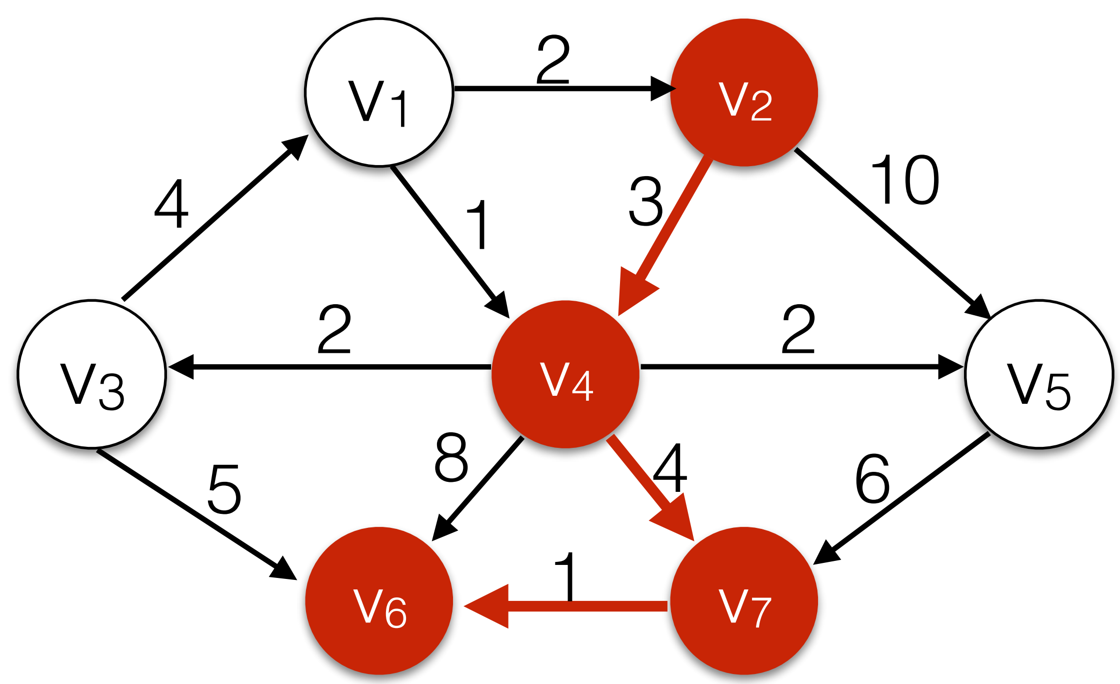


length 2
cost 11

What is the shortest path between v_2 and v_6?

Weighted Shortest Paths

- Goal: Find the shortest path between two vertices s and t.
- This path has a lower cost.



length 3
cost 8

What is the shortest path between v_2 and v_6?