

- Answer for Section 1 & 2 with workings:

Q1: 42

To find out how many rolls of a fair 6-sided die you would expect to take until you see two 5s in a row, we can break it down into three stages: the initial stage where you haven't rolled a 5 yet, the stage where you've rolled one 5, and the final stage where you've rolled two 5s in a row. Starting from the initial stage, there's a $1/6$ chance you roll a 5 and move to the next stage, and a $5/6$ chance you don't and stay where you are. In the second stage, if you roll another 5 ($1/6$ chance), you're done. But if you roll anything else ($5/6$ chance), you go back to the start. By solving this, we find that on average, it will take 42 rolls to get two 5s in a row.

Q2: 19

Q2 since 1 2 3 4 5 6 7 8 9 10
minimum multiple is $7 \times 8 \times 9 \times 10 = 5040$
 $= 2^3 \times 3^2 \times 5 \times 7$
Remove any factor would fall < 2000
∴ must have 7, 8, 9, or one of 5/10.
∴ suppose E_0 : expected rolls from none of 7, 8, 9, 5/10 appears E_7 : only 5/10
 E_1 : from one of 7, 8, 9 [only]
 E_2 : from two of ———
 E_3 : from three 7, 8, 9, no 5/10
 E_4 : from E_1 coupled with E_5 : E_2 couple with
 E_5 : from 7, 8, 9 and at least one of 5/10
∴ $E_6 = 0$. $E_5 = \frac{1}{10}(E_6+1) + \frac{9}{10}(E_5+1)$
 $E_4 = \frac{2}{10}(E_3+1) + \frac{8}{10}(E_4+1) \Rightarrow 2E_4 = 10 + 2E_5$
 $E_3 = \frac{3}{10}(E_2+1) + \frac{7}{10}(E_3+1) \Rightarrow E_3 = E_6 + 5 = 5$
 $E_2 = \frac{1}{10}(E_1+1) + \frac{2}{10}(E_5+1) + \frac{7}{10}(E_2+1) \Rightarrow 3E_2 = 10 + E_3 + 2E_5$
 $E_1 = \frac{2}{10}(E_0+1) + \frac{2}{10}(E_4+1) + \frac{6}{10}(E_1+1) \Rightarrow 4E_1 = 10 + 2E_0 + 2E_2$
 $E_7 = 1 + \frac{3}{10}E_4 + \frac{7}{10}E_7 \Rightarrow 3E_7 = 10 + 3E_4$
 $E_0 = 1 + \frac{2}{10}E_7 + \frac{3}{10}E_1 + \frac{5}{10}E_0$
∴ substitute $E_6 = 0 \Rightarrow \frac{1}{10}E_5 = 1 \Rightarrow E_5 = 10$
 $\frac{2}{10}E_4 = 1 + 2 \times 3 \Rightarrow E_4 = 15$
 $\Rightarrow E_1 = \frac{25}{6}$ $E_7 = \frac{55}{3}$
 $\therefore E_0 = 1 + \frac{11}{3} + \frac{95}{6} \times \frac{3}{10} + \frac{5}{10}E_0$
 $\Rightarrow E_0 = 9.4166 \times 2 = \frac{113}{6} = 18.83$
∴ at least 19 rolls

Q3: 200

Q3 suppose bid for price m
 1) Calculate the Expected profit P

$$\text{PDF: } f(v) = \frac{1}{1000} \text{ for } v \in [0, 1000]$$

$$P(v, B) = 1.5v - m, \text{ if } m > v.$$

$$\begin{aligned} E(P) &= \int_0^m (1.5v - m) \frac{1}{1000} dv \\ &= \frac{1}{1000} \left(\frac{1.5m^2}{2} - m^2 \right) \\ &= \frac{-0.5m^2}{1000} \end{aligned}$$

to maximize

$$E'(P) = 0 \Rightarrow -0.5m = 0$$

No valid sol.

$$\text{PDF: } f(v) = \frac{1}{900} \text{ for } v \in [100, 1000]$$

$$(2) \text{ now } E(P) = \int_{100}^m (1.5v - m) \frac{1}{900} dv$$

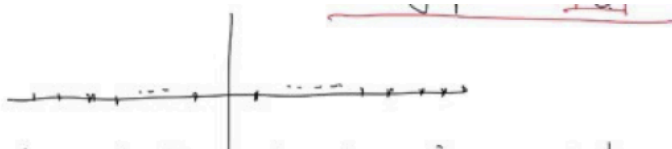
$$= \frac{1}{900} \left(\frac{1.5m^2}{2} - m^2 - \frac{1.5(100)^2}{2} + 100m \right)$$

$$\therefore -0.5B + 100 = 0 \Rightarrow B = 200$$

\therefore Bid 200 to maximize profit.

Q4: 625

Q4.



Since after each collision, the ants remain same speed.
 their new direction are same if we swap the two ants which collide



\therefore simply view them as passing through.

$$\therefore 25 \times 25 = \boxed{625}$$

Q5: 0.5

Q6:

$$\begin{aligned}
 P &= \binom{26}{1} \cdot \frac{1}{2^{26}} \cdot \binom{25}{0} \cdot \frac{1}{2^{25}} + \binom{26}{2} \cdot \frac{1}{2^{26}} \cdot \left(\binom{25}{0} + \binom{25}{1} \right) \cdot \frac{1}{2^{25}} \\
 &+ \dots + \binom{26}{25} \cdot \frac{1}{2^{26}} \cdot \left(\binom{25}{0} + \dots + \binom{25}{24} \right) \cdot \frac{1}{2^{25}} + \binom{26}{26} \cdot \frac{1}{2^{26}} \\
 &= \frac{1}{2^{51}} \cdot \left(\binom{26}{1} + \binom{26}{2} + \dots + \binom{26}{26} \right) \cdot \left(\binom{25}{0} + \binom{25}{1} + \dots + \binom{25}{25} \right) \rightarrow A \\
 &= \frac{1}{2^{51}} \cdot \left(\binom{26}{1} \left(\binom{25}{1} + \dots + \binom{25}{25} \right) + \binom{26}{2} \left(\binom{25}{2} + \dots + \binom{25}{25} \right) \right. \\
 &\quad \left. + \dots + \binom{26}{25} \cdot \binom{25}{25} \right) \quad \downarrow M
 \end{aligned}$$

$$\begin{aligned}
 \therefore \binom{26}{1} &= \binom{26}{25}, \dots \text{etc.} \\
 \therefore 2^{51} \cdot M &\geq \binom{26}{25} \sum_{i=1}^{25} \binom{25}{i} + \binom{26}{24} \sum_{i=2}^{25} \binom{25}{i} + \dots + \binom{26}{1} \sum_{i=25}^{25} \binom{25}{i} \\
 \therefore 2^{51} \cdot P &= 1 \cdot \sum_{i=0}^{25} \binom{25}{i} + \left[\binom{26}{25} \sum_{i=1}^{25} \binom{25}{i} + \dots + \binom{26}{1} \sum_{i=25}^{25} \binom{25}{i} \right] = M \\
 \therefore P &= \frac{1}{2^{51}} \cdot 2^{25} + M \quad \rightarrow \text{replace } \binom{25}{0} \text{ with } \binom{25}{25}, \text{ same onwards.}
 \end{aligned}$$

$$\begin{aligned}
 A &= \frac{1}{2^{51}} \cdot 2^{25} \cdot (2^{26} - 1) & \frac{1}{2^{51}} \cdot 2^{25} + M &= A - M \\
 &= \frac{2^{25}}{2^{51}} & 2M &= A - \frac{2^{25}}{2^{51}} \\
 &M = \frac{2^{25} - 2^{26}}{2^{52}} & M &= \frac{2^{51} - 2^{25}}{2^{52}} - \frac{2^{25}}{2^{52}} \\
 \therefore &= \frac{1}{2} - \frac{1}{2^{26}} \\
 \therefore P &= \frac{1}{2^{26}} + \left(\frac{1}{2} - \frac{1}{2^{26}} \right) = \frac{1}{2} = \boxed{0.5}
 \end{aligned}$$

Q6:6

Q6. since different num.

max num: 12, $\therefore 0 \sim 12 \rightarrow 13 \text{ ppl}$ 12 0

min num: 0

0 1 1 1 1 1 1 1 1

me \downarrow my spouse. other couples

① if my spouse 0: there not possible the other person shakes 12.

② if my spouse 12: there ~ shakes 0

③ \therefore another couple 0, 12 respectively consider 11 & 1 ...

the only possible way: 11 pair with 1. (inference)

\therefore my spouse: 16

- Answer for Section 3:

Jupyter Notebook:

<https://github.com/Blankeeir/NUSInvestSiyi/blob/main/QF/Quant%20Research/qf.ipynb>

Documentation

I Market analysis and Inspiration

- Price trend of cryptocurrencies

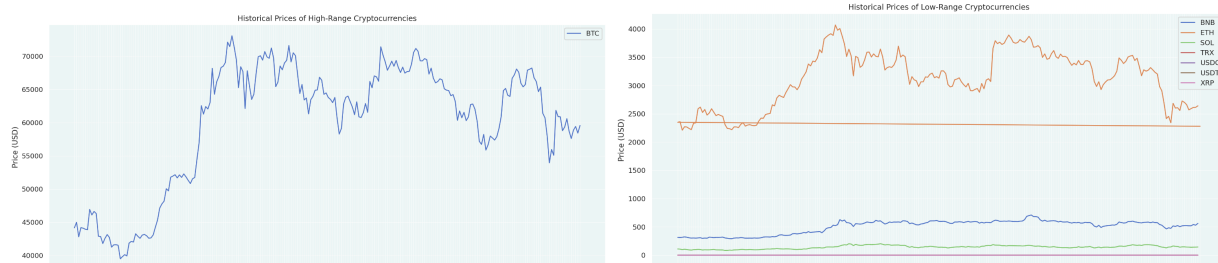
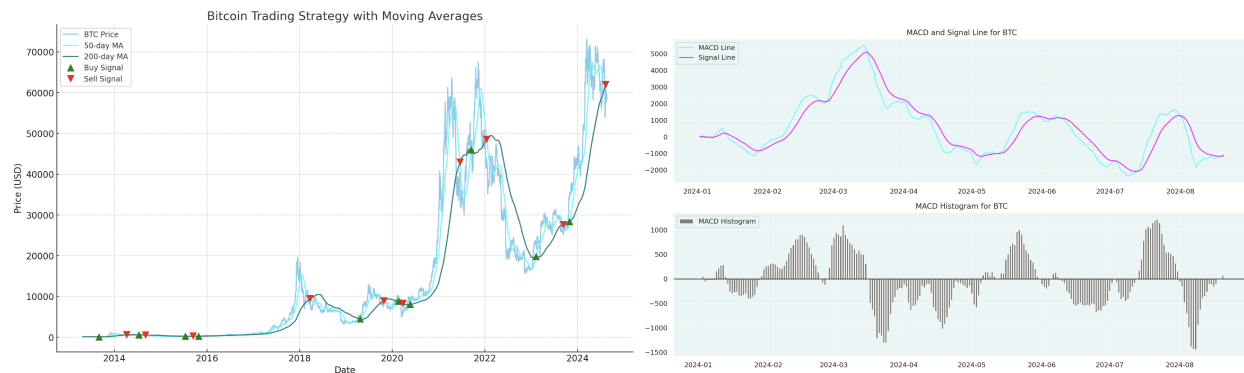


Figure: 2024 crypto currency price trend high-range and low-range

- **Each cryptocurrency** Data includes price, market cap, and volume.
- Inspiration: MACD trading bitcoin



The visualizations above illustrate the MACD (Moving Average Convergence Divergence) and Signal Line for BTC, along with the MACD Histogram:

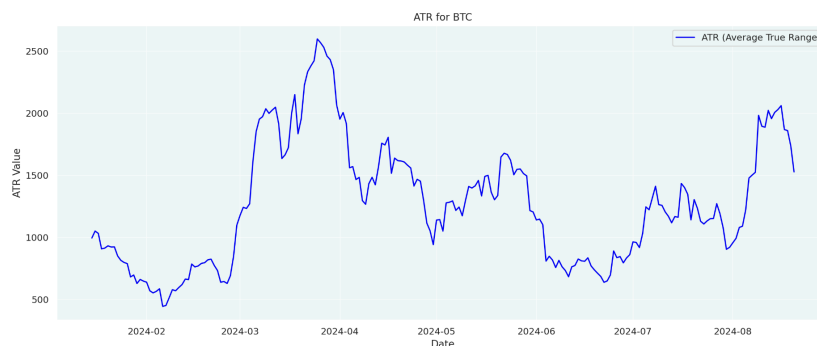
These lines help to identify trends in the price of BTC. When the MACD line crosses above the Signal line, it indicates a bullish signal (potential buy), and when it crosses below, it indicates a bearish signal (potential sell). This histogram shows the difference between the MACD line and the Signal line, providing a clear view of the momentum changes.

However, only MACD cannot take volatility into consideration, which is crucial in cryptocurrency market. Therefore I proposed:
Incorporate ATR Calculation:(Average True Range) for volatility measurement.

II Strategy: (the strategy I used for personal trading, see Appendix for specific detailed parameters)

*Given the price volume and upward trending of BTC, I use it as base trading currency

MACD-ATR strategy



Entry Signal:

- **Buy Signal:** When the MACD line crosses above the Signal line, and the MACD histogram shows a positive value, indicating an upward momentum.
- **Sell Signal:** When the MACD line crosses below the Signal line, and the MACD histogram shows a negative value, indicating downward momentum.
 - **Exit Signal:**
- Exiting the trade when the MACD line crosses back over the Signal line in the opposite direction.
 - **Stop-Loss:**
- Set the stop-loss level based on a percentage of the ATR value, which will account for the volatility of BTC.

- **Hedging:**
- Hedge using USDC when significant volatility is detected (e.g., ATR surpasses a certain threshold), to preserve capital.

Special: Triangular Arbitrage Strategy ** Only when large discrepancy spotted!

Currency Pair Selection: BTC/ETH, ETH/USDT, and BTC/USDT

- Execute trades between three cryptocurrency pairs to profit from discrepancies in exchange rates.
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III Forecasting

	Year	Price
0	2024	\$60,904.68
1	2025	\$63,949.91
2	2026	\$67,147.41
3	2027	\$70,504.78
4	2030	\$81,618.10

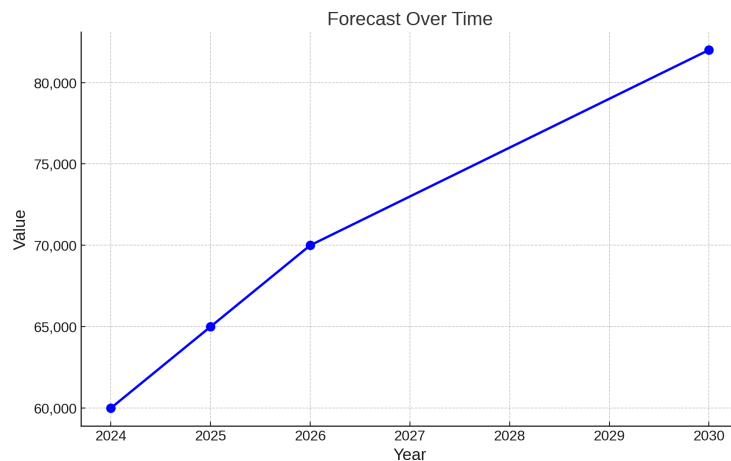
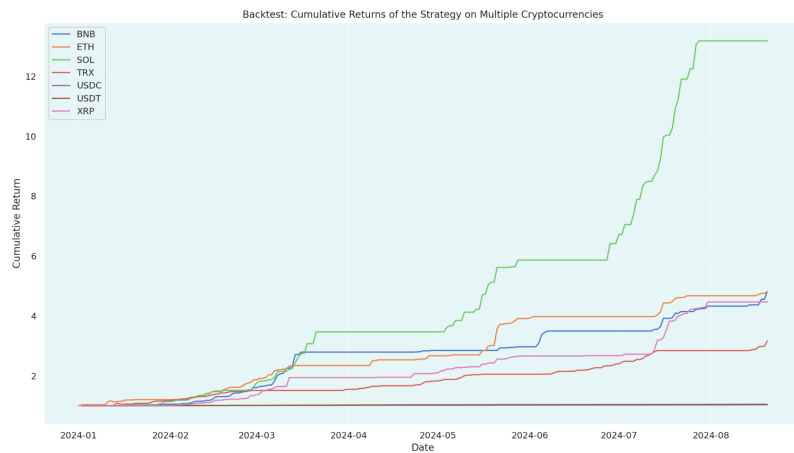
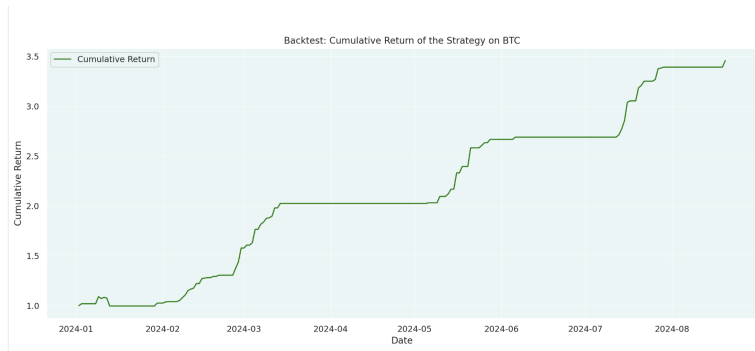


Figure: LSTM prediction for bitcoin price

IV Backtesting

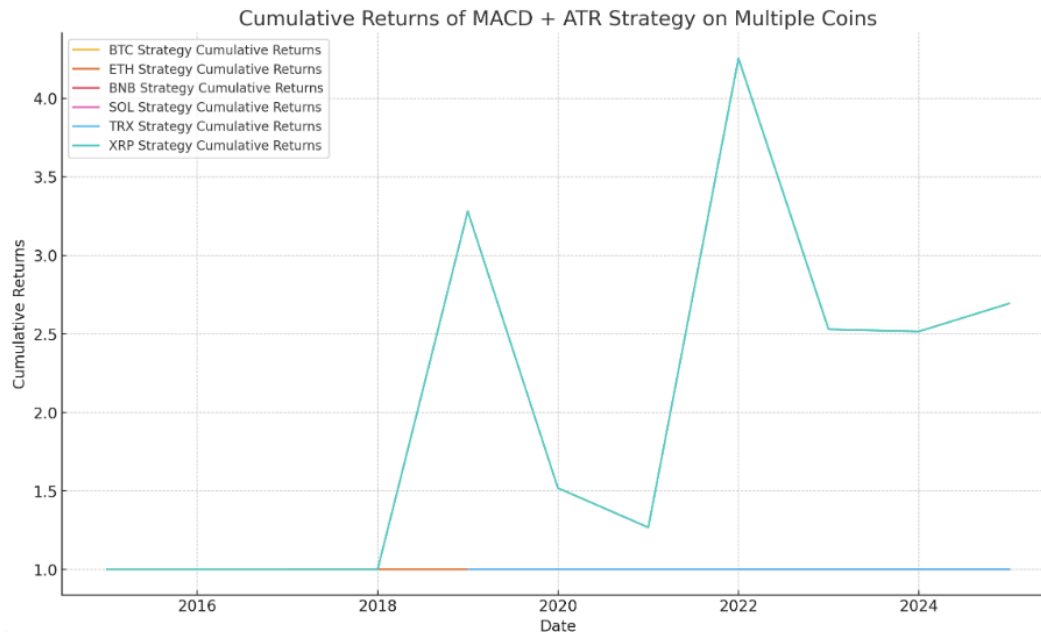
Extend Backtesting to Other Cryptocurrencies: Apply the same strategy to the other cryptocurrencies (ETH, BNB, SOL, etc.).



Different cryptocurrencies exhibit varying levels of success with the strategy, reflecting their unique price movements and volatility characteristics.

The strategy can be adjusted or optimized for specific assets, depending on their historical performance and market conditions.

After implementing algorithm on the historical as well as future data, I graph the cumulative return trend for backtesting:



The plot above shows the cumulative returns of the MACD + ATR strategy applied to multiple cryptocurrencies. It demonstrates how the strategy performed across different coins over time.

XRP (Solana) shows significant returns when the strategy was applied, indicating it might have been particularly responsive to the MACD + ATR approach.

BTC (Bitcoin), ETH (Ethereum), and other coins show either modest or flat returns, suggesting that the strategy may not be as effective for these coins during the given time period.

Appendix

Trading Strategy Parameters:

- FAST EMA 13

Formulas

1. Calculate the 13-period EMA (Fast EMA)

$$EMA_{13}(t) = P(t) \times \frac{2}{13+1} + EMA_{13}(t-1) \times \left(1 - \frac{2}{13+1}\right)$$

Where:

- $P(t)$ is the closing price at time t
- $EMA_{13}(t-1)$ is the EMA at time $t-1$
- Slow EMA 34

2. Calculate the 34-period EMA (Slow EMA)

$$EMA_{34}(t) = P(t) \times \frac{2}{34+1} + EMA_{34}(t-1) \times \left(1 - \frac{2}{34+1}\right)$$

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3. Compute the DIF

$$DIF(t) = EMA_{13}(t) - EMA_{34}(t)$$

4. Calculate the 34-period EMA of the DIF to get the DEA (Signal Line)

$$DEA_{34}(t) = DIF(t) \times \frac{2}{34+1} + DEA_{34}(t-1) \times \left(1 - \frac{2}{34+1}\right)$$

5. Calculate the MACD Histogram

$$MACD\ Histogram(t) = DIF(t) - DEA_{34}(t)$$

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Indicator:

- ATR (Average True Range)
 - Length: 13
 - Smoothing: RMA (Running Moving Average)
 - Purpose: Used to calculate and determine the stop-loss position.

Trading Rule for Long Positions:

1. Identify Trends:
 - Look for two or three consecutive, significant increasing trends in the MACD data. These trends should exhibit a large peak difference (e.g., 6:4 or 7:3).
 - Identify the K-line (candlestick) corresponding to each MACD peak.
2. Confirm Conditions:
 - Check if the lowest point on the K-line shows consecutive decreases. If this condition is met, the K-line following the last peak's next histogram stack in MACD will be your entry point.
3. Set Stop Loss:
 - Subtract the ATR value from the entry point to determine the stop-loss position.
4. Set Profit Target:
 - Apply a profit return rate of 1:2.

Important Notes:

- Ensure that the peak difference in the MACD is significant (e.g., 6:4 or 7:3).
- Be cautious if the graph shows poor or unclear patterns.

Strategy for Sudden Price Movements (Catching the Needle):

Indicators:

- Vegas EMAs: EMA 144 / EMA 169

Steps:

1. Select Target Coin/Stock:
 - Focus on assets with a daily increase of over 10%, as they are more likely to continue increasing.
 - Ensure the asset is in the top 5 of the increase list.
 - Confirm that it has relatively high trading volume.
 - The price should be below \$20.
 - Look for a low turnover rate.
2. Check Position:
 - Use the EMA 200 to determine whether the asset is in a low or high position.

Arbitrage Strategy Flowchart:

