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A MULTI-AGENT ROUGH SET MODEL TOWARD GROUP DECISION ANALYSIS

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Abstract: In this paper, we propose a variable precision rough set analysis of multiple decision tables. Each decision table is regarded as a collected information about an individual opinion. By a rough set analysis of each decision table, we may capture the core opinion of each individual by the lower approximation and marginal opinion by the boundary region. Based on this idea, we propose a rough set approach to capturing the group consensus or agreeable opinions. To this end, we analyze multiple decision tables. In order to treat the error caused by human evaluation as well as to accommodate disagreements among individuals, we apply a variable precision rough set model. To treat the possible difference of evaluated objects among decision tables, we propose two approaches. In both approaches, we define lower and upper approximations under multiple decision tables and multi-agent rough sets are defined by a pair of lower and upper approximations. Moreover we describe a rule induction method under a multi-agent rough set. The approach is applied to real data about preferences of several students among Japanese companies to be employed.

Keywords: *Rough set, Decision table, Group opinion, Variable precision rough set*

1. INTRODUCTION

Recently, the applicability and advantages of rough sets [1] have been demonstrated in the literatures. The rough set analysis has been developed under a single decision table. However, we may have multiple decision tables when the information comes from multiple information sources or when objects are evaluated by multiple decision makers. In such cases, it would be better for obtaining more robust and accurate results to analyze all information provided from multiple decision tables in a lump. When each decision table is obtained from a decision maker, it can be regarded as information about the opinion of the decision maker. Therefore, the analysis of multiple decision tables is important for the investigation of group opinion, agreement in the group and group preference. We focus on the case when multiple decision tables are obtained from many decision makers.

In order to know product designs which are preferred by many customers as decision rules, Enomoto et al. [2] discussed rule induction from multiple decision tables based on rough set analysis. To analyze multiple decision tables, they proposed merging decision rules which is originally proposed by Mori [3] in analysis of single decision tables. However, in their method, we should first enumerate all decision rules from each decision table and then merge decision rules obtained from different decision tables. This requires a formidable computational effort and will be inapplicable when each decision table becomes large. Moreover, it is reported that the results can be different by the order of decision rules to be merged [4]. They proposed a few heuristic methods to order decision rules to be merged [4]. Then this approach includes a brute force method in the enumeration of decision rules from each decision table and also some ad hoc and heuristic methods to order decision rules to be merged.

In order to treat the problem more theoretically, Inuiguchi et al. [5] have discussed rule induction from two decision tables. They extended the discernibility matrix method [6] to the case of two decision tables. They showed that there are a lot of approaches to treat the problem even in two decision tables. However, some of their various approaches cannot be applicable in the real world because they require a lot of computational effort.

In this paper, we propose a new rough set approach to analysis of multiple decision tables. While the previous approaches have focused on induction of decision rules, the proposed approach focuses on the definition of a rough set, i.e., definitions of lower and upper approximations. Since the rough set is defined under multiple decision tables each of which shows an individual decision maker's opinion, the rough set is called a multi-agent rough set in this paper. Given a rough set, we may define a reduct, induce decision rules, and so on. Then a definition of rough set can play a key role in analysis of multiple decision tables.

In order to treat the error caused by human evaluation as well as to accommodate disagreements among decision tables, we introduce the variable precision rough set model. Sets of objects are not assumed to be common among decision tables but sets of attributes and their domains are. Under this assumption, an object can be absent in some decision tables. Depending on the treatment of the absent objects, two kinds of multi-agent rough sets are proposed.

In the next section, we briefly introduce decision tables and variable precision rough sets. We reformulate the

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decision tables using condition attribute patterns. In Section 3, we define multi-agent rough sets under multiple decision tables and a couple of fundamental properties are described. In Section 4, we investigate a rule induction method for decision rules common in multiple decision tables based on the multi-agent rough sets. In Section 5, we apply the proposed approach to real data about preferences of several students among Japanese companies. Finally, conclusions and future research topics are described in Section 6.

2. VARIABLE PRECISION ROUGH SETS AND DECISION TABLE

2.1 Decision Tables

Rough sets proposed by Pawlak [1] has been applied to analysis of decision tables. The rough set analysis utilizes indiscernibility relations tactfully. By the rough set analysis, we can obtain minimal set of condition attributes to classify objects correctly and induce decision rules from a given decision table.

A decision table is composed of a set of objects U, a set of condition attributes C and a decision attribute d. A decision table is denoted by $(U, C\cup\{d\})$. We regard each attribute $a\in C\cup\{d\}$ as a function from U to V_a, where V_a is the set of attribute values a takes. An example of a decision table is given in Table 1. In Table 1, we have $U = \{u_i, i = 1,2,...,10\}$, C = {Design, Function, Size} and d = Dec. (Decision).

Given a decision table $(U, C\cup\{d\})$, we define the condition attribute pattern, or simply, pattern $Inf_c(u)$ of an object $u\in U$ by

$$Inf_C(u) = \bigcup_{a\in C}\{\langle a, a(u)\rangle\} \tag{1}$$

Table 1: An example of decision table

Object	Design	Function	Size	Dec.
u_1	classic	simple	compact	accept
u_2	classic	multiple	compact	accept
u_3	classic	multiple	normal	reject
u_4	modern	simple	compact	reject
u_5	modern	simple	normal	reject
u_6	classic	multiple	compact	accept
u_7	modern	multiple	normal	reject
u_8	classic	simple	compact	accept
u_9	classic	multiple	normal	accept
u_{10}	modern	multiple	normal	reject

Table 2: A decision table described by condition attribute patterns

Pattern	Design	Function	Size	σ
w_1	classic	simple	compact	(2,0)
w_2	classic	multiple	compact	(2,0)
w_3	classic	multiple	normal	(1,1)
w_4	modern	simple	compact	(0,1)
w_5	modern	simple	normal	(0,1)
w_6	modern	multiple	normal	(0,2)

where $a(u)$ shows the attribute value of u with respect to attribute $a\in C\cup\{d\}$. The set V_c^U of all patterns in the given decision table is defined by

$$V_c^U = \{Inf_c(u) \mid u\in U\}. \tag{2}$$

Let V_d be the set of decision attribute values. Then frequency function σ_c and rough membership function μ_c are defined as follows for $w\in V_c^U$ and $v_d\in V_d$,

$$\sigma_C(w, v_d) = |Inf_C^{-1}(w) \cap d^{-1}(v_d)|, \tag{3}$$

$$\mu_C(w, v_d) = \frac{|Inf_C^{-1}(w) \cap d^{-1}(v_d)|}{|Inf_C^{-1}(w)|}, \tag{4}$$

where Inf_c^{-1} and d^{-1} are inverse images of Inf_c and d, respectively, i.e., $Inf_c^{-1}(w) = \{u\in U \mid Inf_c(u) = w\}$ and $d^{-1}(v_d) = \{u\in U \mid d(u) = v_d\}$. $\sigma_c(w,v_d)$ shows the number of objects whose patterns are w and whose decision attribute values are v_d. $\mu_c(w,v_d)$ shows the ratio of objects which take decision attribute value v_d to all objects whose patterns are w. Given $\sigma_c(w,v_d)$ for every $v_d\in V_d$, we obtain $\mu_c(w,v_d)$ as

$$\mu_C(w, v_d) = \frac{\sigma_C(w, v_d)}{\sum_{v_d\in V_d}\sigma_C(w, v_d)}. \tag{5}$$

However $\sigma_c(w,v_d)$ cannot be obtained from $\mu_c(w,v_d)$ for every $v_d\in V_d$. We can rewrite a decision table described by patterns $w_d\in V_c^U$ and frequencies $\{\sigma_c(w,v_d) \mid v_d\in V_d\}$. For example, the decision table shown in Table 1 can be rewritten as a table shown in Table 2. In Table 2, each entry in column 'σ' shows a vector ($\sigma_c(w_j$, accept), $\sigma_c(w_j$, reject)). In rough set analysis, the order of objects appearing in a decision table does not affect the results of the analysis. Then having a decision table described by patterns $w\in V_c^U$ as in Table 2 is equivalent to having a usual decision table as in Table 1. From this fact, we assume that decision tables are given by using patterns in what follows.

2.2 Rough Sets and Variable Precision Rough Sets

In rough set analysis of decision tables, a decision class or a union of decision classes $\hat{X}$ is analyzed. Namely, associated with $\hat{X}$, there is a unique set $X\subseteq V_d$ of decision attribute values such that

$$\hat{X} = \{u\in U \mid d(u)\in X\}. \tag{6}$$

To a set $X \subseteq V_d$ of decision attribute values, a rough membership function μ_c of patterns $w \in V_c^U$ is defined as

$$\mu_c(w,X) = \frac{\sum_{v_d \in X} \sigma_c(w, v_d)}{\sum_{v_d \in V_d} \sigma_c(w, v_d)}. \tag{7}$$

Given a set of decision attribute values $X \subseteq V_d$, lower and upper approximations composing a rough set are defined as sets of patterns instead of objects by;

$$\underline{C}(X) = \{w_i \in V_c^U \mid \mu_c(w_i, X) = 1\}, \tag{8}$$

$$\overline{C}(X) = \{w_i \in V_c^U \mid \mu_c(w_i, X) > 0\}. \tag{9}$$

The relations of $\underline{C}(X)$ and $\overline{C}(X)$ with usual lower and upper approximations $C_*(\hat{X})$ and $C^*(\hat{X})$ are given as

$$C_*(\hat{X}) = Inf_c^{-1}(\underline{C}(d(\hat{X}))) = Inf_c^{-1}(\underline{C}(X)), \tag{10}$$

$$C^*(\hat{X}) = Inf_c^{-1}(\overline{C}(d(\hat{X}))) = Inf_c^{-1}(\overline{C}(X)). \tag{11}$$

A rough set of $\hat{X}$ is often defined by a pair $(C_*(\hat{X}), C^*(\hat{X}))$. In this paper, a pair $(\underline{C}(X), \overline{C}(X))$)is called a rough set of X.

In the rough set defined by a pair $(\underline{C}(X)), \overline{C}(X))$, patterns w_i included in lower approximation $\underline{C}(X)$ satisfy $\mu_c(w_i, X) = 1$. This implies that all objects having a pattern w_i take a common decision attribute value in X. However, from the consideration of possible errors in observation, evaluation, and so on, this requirement to be a member of lower approximation $\underline{C}(X)$ is too rigorous especially when the size of the given decision table is large. With such errors, we may obtain an empty lower approximation which deteriorates the effectiveness of the rough set analysis. In order to overcome this inconvenience, variable precision rough sets [7] have been proposed by relaxing the requirement to be a member of the lower approximation.

Let $\varepsilon_1 \in [0, 0.5)$ be an admissible level of classification error, lower and upper approximations in a variable precision rough set (VPRS) of X is defined as a set of patterns by

$$\underline{C}_{\varepsilon_1}(X) = \{w_i \in V_c^U \mid \mu_c(w_i, X) \geq 1 - \varepsilon_1\}, \tag{12}$$

$$\overline{C}_{\varepsilon_1}(X) = \{w_i \in V_c^U \mid \mu_c(w_i, X) > \varepsilon_1\}. \tag{13}$$

A VPRS of X is defined by a pair $(\underline{C}_{\varepsilon_1}(X), \overline{C}_{\varepsilon_1}(X))$. As can be seen easily, we have $\underline{C}_{\varepsilon_1}(X) = \underline{C}(X)$ and $\overline{C}_{\varepsilon_1}(X) = \overline{C}(X)$ when $\varepsilon_1 = 0$. As ε_1 increases, $\underline{C}_{\varepsilon_1}(X)$ becomes larger and $\overline{C}_{\varepsilon_1}(X)$ becomes smaller.

The following properties hold:

$$\underline{C}_{\varepsilon_1}(X) \subseteq \overline{C}_{\varepsilon_1}(X), \tag{14}$$

$$\overline{C}_{\varepsilon_1}(X) = V_c^U - \underline{C}_{\varepsilon_1}(V_d - X). \tag{15}$$

In this paper, we propose rough sets under multiple decision tables each of which is obtained by human evaluations. In consideration of the error caused by human evaluation as well as the disagreements among decision tables, the VPRS model is applied to the definitions.

3. ROUGH SETS UNDER MULTIPLE DECISION TABLES

3.1 Agreement Ratio

We assume n decision tables evaluated by n decision makers are given. Let $\mathbf{T}$ be a set of decision tables $T_i = (U_i, C \cup \{d\})$, $i = 1,2,...,n$, i.e., $\mathbf{T} = \{T_1, T_2, ..., T_{1n}\}$. We assume that all decision tables T_i, $i = 1,2,...,n$ share a set C of condition attributes and the unique decision attribute d. On the other hand, sets of objects U_i, $i = 1,2,...,n$ can be different among decision tables. Therefore, sets of patterns, $V_c^{U_i} = \{Inf_c(u) \mid u \in U_i\}$, $i=1,2,..., n$ can be also different among decision tables. We define $V_C = \bigcup_{i=1,2,...,n} V_c^{U_i}$ for convenience.

The evaluation can be different among decision makers. Therefore, decision rules behind each decision table may conflict with those behind another decision table. To treat the disagreement among decision tables, we introduce an agreement ratio. The difficulty to define an agreement ratio is in the treatment of patterns absent in a decision table but appears in the other decision tables because of the difference among sets of objects U_i, $i = 1,2,...,n$.

One of conceivable approaches is to define an agreement ratio to each pattern by using decision tables including the pattern. Following this approach, we can define lower and upper agreement ratios to each pattern with respect to a set X of decision attribute values as

$$\underline{\tau}_{\varepsilon_1}(w_i, X) = \frac{\left|\{T_j \in \mathbf{T} \mid w_i \in \underline{C}_{\varepsilon_1}^{T_j}(X)\}\right|}{\left|\{T_j \in \mathbf{T} \mid w_i \in V_C^{U_j}\}\right|}, \tag{16}$$

$$\overline{\tau}_{\varepsilon_1}(w_i, X) = \frac{\left|\{T_j \in \mathbf{T} \mid w_i \in \overline{C}_{\varepsilon_1}^{T_j}(X)\}\right|}{\left|\{T_j \in \mathbf{T} \mid w_i \in V_C^{U_j}\}\right|}, \tag{17}$$

where $\underline{C}_{\varepsilon_1}^{T_j}(X)$ and $\overline{C}_{\varepsilon_1}^{T_j}(X)$ are ε_1 -lower approximation and ε_1 -upper approximation corresponding to decision table T_j. By (15), we have

$$\underline{\tau}_{\varepsilon_1}(w_i, X) \leq \overline{\tau}_{\varepsilon_1}(w_i, X), \tag{18}$$

$$\overline{\tau}_{\varepsilon_1}(w_i, X) \leq 1 - \underline{\tau}_{\varepsilon_1}(w_i, V_d - X). \tag{19}$$

Then we call $\underline{\tau}_{\varepsilon_1}(w_i, X)$ and $\overline{\tau}_{\varepsilon_1}(w_i, X)$ lower agreement ratio and upper agreement ratio, respectively.

3.2 Upper Estimation of a Rough Membership Value

When the number of decision tables including the pattern is small, the lower and upper agreement ratios defined by (16) and (17) possess lower reliability. To overcome this drawback, it is conceivable to use estimated decision attribute values to absent patterns so that all decision tables are used in calculation of agreement ratios to any pattern.

To make such estimation, we propose the upper estimations of rough membership values. Consider a decision table T_i and a pattern $w \notin V_c^{U_i}$. To a pattern $w_k \in V_c^{U_i}$, we define a set of $B_k(w)$ condition attributes by

$$B_k(w) = \{\alpha \mid \exists v_a \in V_a, \langle \alpha, v_a \rangle \in w \cap w_k\}. \tag{20}$$

Using $B_k(w)$, the restriction of w on B_k is defined by

$$w^{\downarrow B_k(w)} = \{\langle \alpha, v_a \rangle \mid \langle \alpha, v_a \rangle \in w \cap w_k\}. \tag{21}$$

We can define a rough membership value $\mu^{T_i}_{B_k(w)}(w^{\downarrow B_k(w)}, X)$ in decision table T_i in the same way as (7) with the exception of a case when $B_k(w) = \emptyset$. Namely,

$$\mu^{T_i}_{B_k(w)}(w^{\downarrow B_k(w)}, X) = \begin{cases} \dfrac{\sum_{v_d \in X} \sigma^{T_i}_{B_k(w)}(w^{\downarrow B_k(w)}, v_d)}{\sum_{v_d \in V_d} \sigma^{T_i}_{B_k(w)}(w^{\downarrow B_k(w)}, v_d)}, & \text{if } B_k(w) \neq \emptyset \\ \dfrac{\left|\{u \in U_i \mid d(u) \in X\}\right|}{|U_i|}, & \text{if } B_k(w) = \emptyset \end{cases} \tag{22}$$

where $\sigma^{T_i}_B$ is a frequency function of decision table T_i with respect to a set $B \subseteq C$ of condition attributes.

Then, the upper estimation of rough membership value of a pattern $w \notin V^{U_i}_C$ can be defined by

$$\hat{\mu}^{T_i}_C(w, X) = \begin{cases} \max_{w_k \in V^{U_i}_C} \mu_{B_k(w)}(w^{\downarrow B_k(w)}, X), & \text{if } w \notin V^{U_i}_C, \\ \mu^{T_i}_C(w, X), & \text{if } w \in V^{U_i}_C, \end{cases} \tag{23}$$

where $\mu^{T_i}_C$ is a rough membership function with respect to decision table T_i. We have $\Sigma_{v_d \in V_d} \hat{\mu}^{T_i}_C(w, \{v_d\}) \geq 1$. In this sense, we call $\hat{\mu}^{T_i}_C(w, X)$ an upper estimation of the rough membership value.

Note that when decision rule 'if an object u satisfies a pattern $w^{\downarrow B_k(w)}$ then u takes a decision attribute value in X' induced from decision table T_i, we have $\mu_{B_k(w)}(w^{\downarrow B_k(w)}, X) = 1$. Moreover, when decision rule 'if an object u satisfies a pattern $w^{\downarrow B_k(w)}$ then u takes a decision attribute value in X' is certain with degree $\alpha \in [0, 1]$ under decision table T_i, we have $\mu_{B_k(w)}(w^{\downarrow B_k(w)}, X) \geq \alpha$. Therefore the upper estimation $\hat{\mu}^{T_i}_C(w, X)$ shows to what extent we certainly infer that an object having a pattern w takes a decision attribute value in X.

3.3 Modified Approximations and Modified Agreement Ratios

Using the upper estimation of rough membership value, we can define $(\varepsilon_1, \varepsilon_2)$ -lower approximation and $(\varepsilon_1, \varepsilon_2)$ -upper approximation as modifications of ε_1 -lower approximation and ε_2 -upper approximation, respectively, by

$$\underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) = \{w_i \in V_C \mid \hat{\mu}^{T_i}_C(w_i, X) \geq 1 - \varepsilon_1, \ \hat{\mu}^{T_i}_C(w_i, U - X) \leq \varepsilon_2\}, \tag{24}$$

$$\overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) = \{w_i \in V_C \mid \hat{\mu}^{T_i}_C(w_i, X) > \varepsilon_2, \ \text{or } \hat{\mu}^{T_i}_C(w_i, U - X) < 1 - \varepsilon_1\}, \tag{25}$$

where we assume $\varepsilon_1 \leq \varepsilon_2 <$ 1- ε_1. From $\hat{\mu}^{T_i}_C(w_i, X) + \hat{\mu}^{T_i}_C(w_i, U - X) \geq 1$, it is possible to have $\hat{\mu}^{T_i}_C(w_i, X) \geq$ 1- ε_1 and $\hat{\mu}^{T_i}_C(w_i, U - X) > \varepsilon_1$ at the same time. If $\hat{\mu}^{T_i}_C(w_i, X) \geq 1 - \varepsilon_1$ and $\hat{\mu}^{T_i}_C(w_i, U - X) > \varepsilon_1$ are satisfied at the same time, objects having pattern w_i may take a decision attribute value in X and simultaneously a decision attribute value in $U - X$ both with high estimated degrees $\hat{\mu}^{T_i}_C(w_i, X)$ and $\hat{\mu}^{T_i}_C(w_i, U - X)$. This is contradictory. In order to avoid such a contradiction, we exclude such a pattern w_i from lower approximation by adding condition $\hat{\mu}^{T_i}_C(U - X \mid w_i) \leq \varepsilon_2$. The definition of $(\varepsilon_1, \varepsilon_2)$ -upper approximation also follows this idea.

$(\varepsilon_1, \varepsilon_2)$ -lower approximation and $(\varepsilon_1, \varepsilon_2)$ -upper approximation satisfy

$$\underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) \supseteq \underline{C}^{T_i}_{\varepsilon_1}(X), \ \overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) \supseteq \overline{C}^{T_i}_{\varepsilon_1}(X), \tag{26}$$

$$\underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) \subseteq \overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X), \tag{27}$$

$$\overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) = V_C - \underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(V_d - X). \tag{28}$$

Note that $\underline{C}^{T_i}_{\varepsilon_1}(X)$ and $\overline{C}^{T_i}_{\varepsilon_1}(X)$ are defined under universe $V^{U_i}_C$ while $\underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X)$ and $\overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X)$ are defined under universe V_c. When $V_c = V^{U_i}_C$, we have $\underline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) = \underline{C}^{T_i}_{\varepsilon_1}(X)$ and $\overline{C}^{T_i}_{\varepsilon_1, \varepsilon_2}(X) = \overline{C}^{T_i}_{\varepsilon_1}(X)$.

Using $(\varepsilon_1, \varepsilon_2)$ -lower approximation and $(\varepsilon_1, \varepsilon_2)$ -upper approximation, for each $w_i \in V_c$ and for a set X of decision attribute values, we can defined the modified lower and upper agreement ratios as

$$\underline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, X) = \frac{\left|\{T_j \in \mathbf{T} \mid w_i \in \underline{C}^{T_j}_{\varepsilon_1, \varepsilon_2}(X)\}\right|}{|\mathbf{T}|}, \tag{29}$$

$$\overline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, X) = \frac{\left|\{T_j \in \mathbf{T} \mid w_i \in \overline{C}^{T_j}_{\varepsilon_1, \varepsilon_2}(X)\}\right|}{|\mathbf{T}|}, \tag{30}$$

From (28), we have

$$\overline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, X) = 1 - \underline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, V_d - X). \tag{31}$$

3.4 Rough Sets under Multiple Decision Tables

Let a pair $(\underline{\tau}_E, \overline{\tau}_E)$ represent a pair of lower and upper agreement ratios, $(\underline{\tau}_{\varepsilon_1}, \overline{\tau}_{\varepsilon_1})$ or a pair of modified lower and upper agreement ratios, $(\underline{\tau}_{\varepsilon_1, \varepsilon_2}, \overline{\tau}_{\varepsilon_1, \varepsilon_2})$. Then we can define (E, δ_1, δ_2) -lower approximation and (E, δ_1, δ_2) -upper approximation as

$$\underline{\mathbf{T}}^{\delta_1, \delta_2}_E(X) = \{w_i \in V_C \mid \underline{\tau}_E(w_i, X) \geq 1 - \delta_1, \ \overline{\tau}_E(w_i, X) \geq 1 - \delta_2\}, \tag{32}$$

$$\overline{\mathbf{T}}^{\delta_1, \delta_2}_E(X) = \{w_i \in V_C \mid \underline{\tau}_E(w_i, X) > \delta_1, \ \text{or } \overline{\tau}_E(w_i, X) > \delta_2\}, \tag{33}$$

where we assume $0 \leq \delta_2 \leq \delta_1 \leq 0.5$. A variable E takes ε_1 or $\{\varepsilon_1, \varepsilon_2\}$. From (19) and (31), we have $\underline{\tau}_{\varepsilon_1}(w_i, X) = 1 - \overline{\tau}_{\varepsilon_1}(w_i, X)$ and $\underline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, X) = 1 - \overline{\tau}_{\varepsilon_1, \varepsilon_2}(w_i, X)$. Therefore, δ_2 in (32) and (33) represents the admissible ratio of decision tables which support the opposite conclusion.

We easily obtain the following properties of (E, δ_1, δ_2) -

lower approximation and (E, δ_1, δ_2) - upper approximation:

$$\underline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X) \subseteq \overline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X), \tag{34}$$

$$\overline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X) = V_C - \underline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(V_d - X). \tag{35}$$

4. COMMON RULE INDUCTION

As we can induce certain and possible rules based on lower and upper approximations of the classical rough sets, we would like to induce rules based on lower and upper approximations of multi-agent rough sets. From lower approximations of multi-agent rough sets, we obtain rules certainly agreed by many decision makers while, from upper approximations we obtain rules possibly agreed by many decision makers . The former could be regarded as common opinions among decision makers while the latter could be seen as negotiable opinions. In this section, we would like to describe a method to induce such rules.

For rule induction, the decision matrix method [6] is proposed and well-known. By the method, we can enumerate all rules. In the decision matrix method, given a lower or upper approximation, objects are partitioned into two classes: one is a set of objects included in the given approximation (positive examples) and the other is the complementary set (negative examples). All sufficient and minimal conditions satisfied with positive examples but unsatisfied with negative examples are obtained as prime implicants of a logical function.

It is conceivable to apply the decision matrix method directly to the proposed lower and upper approximations under multiple decision tables in order to induce certain and possible decision rules common among decision tables. However, due to the application of VPRS model to each decision table, the obtained conditions are not guaranteed the sufficiency to the conclusion of decision rules. To overcome this defect, we modify the definition of decision matrix so that we obtain sufficient conditions. Let P be a set of positive elements, i.e., a lower approximation $\underline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X)$ or an upper approximation $\overline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X)$ under multiple decision tables. To each decision table T_k, we define P_k as a subsets of positive elements defined by

$$P_k = \begin{cases} \underline{C}_{\mathrm{E}}^{T_k}(X), & \text{if } P = \underline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X), \\ \overline{C}_{\mathrm{E}}^{T_k}(X), & \text{if } P = \overline{\mathrm{T}}_{\mathrm{E}}^{\delta_1,\delta_2}(X), \end{cases} \tag{36}$$

For $w_i \in P$ and $w_j \in V_C$, the modified decision matrix $M^{\mathrm{E},\delta_1,\delta_2} = \left(M_{ij}^{\mathrm{E},\delta_1,\delta_2}\right)$ is defined by

$$M_{ij}^{E,\delta_1,\delta_2} = \begin{cases} \left\{(a\ a,(w_i)) \mid a\ (w_i) \neq a\ (w_i)\right\}, \\ \text{if} \exists T_k \in \mathbf{T}, w_i \in P_k, w_j \in V_C^{U_k} - P_k, \\ *, \quad \text{otherwise}, \end{cases} \tag{37}$$

where $a(w_i)$ is a value of attribute a that object w_i takes and * stands for "do not care".

Regarding (a, $a(w_i)$) as condition "$a(x) = a(w_i)$", sufficient conditions of decision rules are obtained as prime implicants of the following logical function f:

$$f = \bigvee_{i:w\in P} \bigwedge_{j:w\notin P} \vee M_{ij}^{\mathrm{E},\delta_1,\delta_2} \tag{38}$$

Note that, unlike the case of classical rough set, the conditions obtained by the proposed method are not minimal. The minimal conditions can be obtained by enumeration of all weaker conditions than the obtained ones if the minimality is required.

5. ANALYSUS OF STUDENT PREFERENCE DATA ON COMPANIES

5.1 Data for the Analysis

As an application of the proposed approach to real observed data and to demonstrate the procedure of the proposed approach, we use data about the preference among Japanese companies for students to be employed. The data was collected by questionnaire shown in Table 3 from 18 university students belonging to laboratories in systems engineering field. Twenty-one companies were selected and each student evaluated 12 companies randomly chosen from the selected 21 companies.

Table 3: Questionnaire

A Questionnaire on Company Images	
Please answer to the questions.	
Company A:	
1. What is your company image ?	
a. spectacular	b. modest
2. How do you feel the business activities ?	
a. innovative	b. conservative
3. What kind of workers do they need ?	
a. professional	b. regular
4. Do you want to work at the company ?	
a. yes	b. no
Company B:	
………	

We regarded questions 1-3 as conditional attributes and question 4 as the decision attribute so that we can analyze the student preference among companies to be employed by company image, business activities and type of job. Then we may have eight condition attribute patterns. An obtained decision table from a student is shown in Table 4 as an example.

Table 4: An Obtained Decision Table

pattern	Image	Activity	Job	(yes, no)
w_1	spectac.	innova.	pro.	(1,0)
w_2	spectac.	innova.	reg.	(1,2)
w_3	spectac.	conserv.	pro.	(0,1)
w_4	spectac.	conserv.	reg.	no data
w_5	mod.	innova.	pro.	(1,0)
w_6	mod.	innova.	reg.	(1,0)
w_7	mod.	conserv.	pro.	(0,5)
w_8	mod.	conserv.	reg.	no data

5.2 Multi-Agent Rough Set Approach without Upper Estimations of Rough Membership Functions

First let us apply the multi-agaent rough set approach without upper estimations of rough membership functions. Changing parameters ε_1, δ_1, δ_2, we calculated lower approximations of accepted companies (companies evaluated "yes") and rejected companies (companies evaluated "no") under 18 decision tables. For the sake of the simplicity, we restrict the variety of δ_2 into $\{0.2, 0.4\}$. The obtained results are shown in Tables 5 and 6. Note that we assumed $\delta_1 \geq \delta_2$ in the definition of our rough set model. The upper part in each cell corresponding to a combination of ε_1 and δ_1 shows a lower approximation of accepted companies while the lower part shows a lower approximation of rejected companies.

Now let us induce several decision rules. We only consider lower approximations in two combinations of parameters ε_1, δ_1 and δ_2.

When ε_1=0.2, δ_1=0.4 and δ_2=0.2, the lower approximations of accepted and rejected companies are $\{w_1\}$ and $\{w_7\}$, respectively. From those lower approximations, through the proposed rule induction method, we obtain the following certain decision rules:

- If *company image* is "spectacular", *business activities* is "innovative" and *type of job* is "professional" then students want to work in the company,
- If company image is "modest", business activities is "conservative" and type of job is "professional" then students do not want to work in the company.

On the other hand, when ε_1=0.4, δ_1=0.45 and δ_2=0.4, the lower approximations of accepted and rejected companies are $\{w_1,w_2,w_6\}$ and $\{w_5,w_7,w_8\}$, respectively. From those lower approximations, we obtain the following certain decision rules:

- If company image is "spectacular" and business activities is "innovative" then students want to work in the company
- If business activities is "innovative" and type of job is "regular" then students want to work in the company,
- If company image is "modest", business activities is "innovative" and type of job is "professional" then students do not want to work in the company,
- If company image is "modest", business activities is "conservative" and type of job is "regular" then students do not want to work in the company,
- If company image is "modest", business activities is "conservative" and type of job is "professional" then students do not want to work in the company.

The last two decision rules can be combined to the following simpler decision rule without change of the lower approximation of rejected companies:

- If company image is "modest" and business activities is "conservative" then students do not want to work in the company.

From those obtained decision rules, we may read the tendency of student preference, i.e., they prefer "spectacu-

Table 5: Lower Approximations (δ_2 = 0.2)

$\delta_1 \backslash \varepsilon_1$	[0, 1/5)	[1/5, 1/4)	[1/4 ,2/5)	[2/5, 1/2)
$[\frac{1}{5}, \frac{5}{17})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\emptyset$	$\{w_8\}$
$[\frac{5}{17}, \frac{1}{3})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{1}{3}, \frac{6}{17})$	$\emptyset$	$\emptyset$	$\emptyset$	$\{w_1\}$
	$\emptyset$	$\emptyset$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{6}{17}, \frac{2}{5})$	$\emptyset$	$\emptyset$	$\emptyset$	$\{w_1\}$
	$\emptyset$	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{2}{5}, \frac{7}{17})$	$\emptyset$	$\{w_1\}$	$\{w_1, w_2\}$	$\{w_1\}$
	$\emptyset$	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{7}{17}, \frac{3}{7})$	$\emptyset$	$\{w_1\}$	$\{w_1, w_2\}$	$\{w_1\}$
	$\{w_7\}$	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{3}{7}, \frac{7}{15})$	$\emptyset$	$\{w_1\}$	$\{w_1, w_2\}$	$\{w_1\}$
	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$
$[\frac{7}{15}, \frac{1}{2})$	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$	$\{w_1\}$
	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$

Table 6: Lower Approximations (δ_2 = 0.4)

$\delta_1 \backslash \varepsilon_1$	[0, 1/5)	[1/5,1/4)	[1/4, 2/5)	[2/5, 1/2)
$[\frac{2}{5}, \frac{7}{17})$	$\{w_6\}$	$\{w_1, w_6\}$	$\{w_1, w_2, w_6\}$	$\{w_1, w_2, w_6\}$
	$\emptyset$	$\{w_7\}$	$\{w_7\}$	$\{w_5, w_7, w_8\}$
$[\frac{7}{17}, \frac{3}{7})$	$\{w_6\}$	$\{w_1, w_6\}$	$\{w_1, w_2, w_6\}$	$\{w_1, w_2, w_6\}$
	$\{w_7\}$	$\{w_7\}$	$\{w_7\}$	$\{w_5, w_7, w_8\}$
$[\frac{3}{7}, \frac{5}{11})$	$\{w_6\}$	$\{w_1, w_6\}$	$\{w_1, w_2, w_6\}$	$\{w_1, w_2, w_6\}$
	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_5, w_7, w_8\}$
$[\frac{5}{11}, \frac{7}{15})$	$\{w_6\}$	$\{w_1, w_6\}$	$\{w_1, w_2, w_6\}$	$\{w_1, w_2, w_3, w_6\}$
	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_5, w_7, w_8\}$
$[\frac{7}{15}, \frac{1}{2})$	$\{w_1, w_6\}$	$\{w_1, w_6\}$	$\{w_1, w_2, w_6\}$	$\{w_1, w_2, w_3, w_6\}$
	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_7, w_8\}$	$\{w_5, w_7, w_8\}$

lar" companies to "modest" companies and "innovative" companies to "conservative" companies. These results are agreeable to our intuition.

5.3 Multi-Agent Rough Set Approach with Upper Estimations of Rough Membership Functions

Let us apply the multi-agent rough set approach with upper estimations of rough membership functions. Applying the upper estimations (23) of rough membership functions for absent condition attribute patterns w_4 and w_8 in Table 4, we obtain (0.5,1) and (1,1), respectively. Similarly, we calculated upper estimations of all absent condition attribute patterns in all other decision tables. We set ε_2=0.5 and $\delta_2 \in \{0.2, 0.4\}$. Changing parameters ε_1, δ_1, we calculated lower approximations of accepted companies (companies evaluated "yes") and rejected companies (companies evaluated "no") under 18 decision tables. The results are shown in Tables 7 and 8.

Comparing with lower approximations if Tables 5 and 6, lower approximations in Tables 7 and 8 are smaller. This is because many of absent condition attribute patterns are evaluated as ambiguous decisions by the proposed estimation. In the current example, the condition attribute patterns given in each table are relatively many with respect to all possible patterns. Therefore, the estimation of rough membership functions does not work very well. However, when condition attributes are many and we cannot ask many questions to each decision maker, such estimation will be important to find candidates of common rules (common opinions).

When ε_1=0.4, ε_2=0.5, δ_1=0.45 and δ_2=0.4, the lower approximations of accepted and rejected companies are

Table 7: Lower Approximations (ε_2 = 0.5 and δ_2 = 0.2)

$\delta_1 \backslash \varepsilon_1$	[0, 1/6)	[1/6, 1/4)	[1/4, 2/5)	[2/5, 1/2)
$[\frac{1}{5}, \frac{5}{18})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$[\frac{5}{18}, \frac{1}{3})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\emptyset$	$\{w_8\}$
$[\frac{1}{3}, \frac{7}{18})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\emptyset$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{7}{18}, \frac{4}{9})$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{4}{9}, \frac{1}{2})$	$\emptyset$	$\emptyset$	$\emptyset$	$\{w_1\}$
	$\{w_7\}$	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$

Table 8: Lower Approximations (ε_2 = 0.5 and δ_2 = 0.4)

$\delta_1 \backslash \varepsilon_1$	[0, 1/6)	[1/6, 2/5)	[2/5, 1/2)
$[\frac{2}{5}, \frac{4}{9})$	$\emptyset$	$\emptyset$	$\emptyset$
	$\emptyset$	$\{w_7\}$	$\{w_7, w_8\}$
$[\frac{4}{9}, \frac{1}{2})$	$\emptyset$	$\emptyset$	$\{w_1\}$
	$\{w_7\}$	$\{w_7\}$	$\{w_7, w_8\}$

$\{w_1\}$ and $\{w_7, w_8\}$, respectively. The following decision rules are obtained from those lower approximations:

- If company image is "spectacular", business activities is "innovative" and type of job is "professional" then students want to work in the company,
- If company image is "modest", business activities is "conservative" and type of job is "regular" then students do not want to work in the company,
- If company image is "modest", business activities is "conservative" and type of job is "professional" then students do not want to work in the company.

The last two decision rules can again be combined to the following simpler decision rule:

- If company image is "modest" and business activities is "conservative" then students do not want to work in the company.

In the case when upper estimations of rough membership functions are applied, we obtain similar decision rules.

Finally, let us see differences from the results by an analysis through merging all decision tables obtained from different students. Merging all decision tables, we obtain a single decision table shown in Table 9.

From Table 9, we recognize the results from the analysis of this table yields similar results as we obtained by the proposed approach. However, the difference can be found in pattern w_8. From Table 9, we may observe that w_8 can be easily a member of lower approximation of rejected companies since the number of "no" answers is more than five times of the number of "yes" answers.

On the contrary, in the proposed method, w_8 does not appear in lower approximation of rejected companies when parameters ε, δ_1 and δ_2 take small values. This implies that some students hate companies of pattern w_8 but others do not hate such companies very much.

Namely, the preference about companies of pattern $w8$ depends on individuals. The individual dependency can be corrupted by merging decision tables while it can be preserved by the proposed approach. In this point, the proposed approach is advantageous.

Table 9: The Merged Decision Table

Pattern	Image	Activity	Job	(yes, no)
w_1	spectac.	innova.	pro.	(32,11)
w_2	spectac.	innova.	reg.	(13,8)
w_3	spectac.	conserv.	pro.	(9,7)
w_4	spectac.	conserv.	reg.	(9,8)
w_5	mod.	innova.	pro.	(7,14)
w_6	mod.	innova.	reg.	(4,2)
w_7	mod.	conserv.	pro.	(9,40)
w_8	mod.	conserv.	reg.	(7,36)

6. CONCLUDING REMARKS

In this paper, we proposed a rough set analysis of multiple decision tables in order to induce common rules. We assume that each decision table shows a decision maker's opinion. The rough set analysis was proposed to obtain common opinions as common rules. In the proposed model include some parameters controlling the inconsistency in each decision table and the conflicts among decision tables. A decision matrix method is extended to extract common decision rules.

Collecting a lot of preference/opinion data from each decision maker is not an easy task. Then the estimation of absent condition attribute patterns will be important to find more candidates of common opinions. In this paper, we apply an estimation method only to condition attribute patterns appear in one of given decision tables.

It would be better to apply the estimation also to absent condition attribute patterns which do not appear in any decision tables. By doing so, we may find more common decision rules since any decision rule cannot be obtained by many rule induction methods based on rough sets such as a decision matrix method and LERS [8] without condition attribute pattern in lower/upper approximations. Moreover, such new decision rules correspond to merged decision rules discussed in [2,3]. It would be a problem to generate absent condition attribute patterns. The Monte Carlo method based on the distribution of existing attribute values would be one of the conceivable way.

For the estimation, we only proposed upper estimations of rough membership functions but we may have other ways. The investigation of other estimation method would be one of future topics.

For the rule induction, we proposed one based on the decision matrix method. However, we may extend LEM2 [8] to induce common rules among many decision tables. This is also a future topic.

Moreover, considering the variety of individual opinions, we would need to classify decision makers by their opinions before the analysis of common decision rules. In [9], we have just proposed a clustering method of decision tables based on rough sets. There are so many ways to define similarity or dissimilarity between decision tables. Such a clustering method would be another future topic.

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