GeeksforGeeks A computer science portal for geeks

Custom Search

Practice GATE CS Placements Videos Contribute

Login/Register

Step by Step Preparation Company Preparation Top Topics Company Specific Practice Software Design Patterns Placements Preparation Course Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New? Leaderboard!! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty evel? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Top Topics Company Specific Practice Software Design Patterns Placements Preparation Course Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Company Specific Practice Software Design Patterns Placements Preparation Course Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New? Leaderboard!! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Practice Software Design Patterns Placements Preparation Course Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Patterns Placements Preparation Course Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Interview Corner Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty evel? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Recent Interview Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Experiences GQ Home Page Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Quiz Corner LMNs Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Hard How to pick a difficulty evel? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Practice Platform What's New? Leaderboard!! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Hard Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Practice Platform What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
What's New ? Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Leaderboard !! Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Topic-wise Practice Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Subjective Problems Difficulty Level - School Difficulty Level - Basic Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Difficulty Level - School Difficulty Level - Basic Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Difficulty Level - Basic Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Difficulty Level - Easy Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty evel? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Difficulty Level - Medium Difficulty Level - Hard How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Difficulty Level - Hard How to pick a difficulty evel? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
How to pick a difficulty level? Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Explore More Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Programming Languages C C++ Java Python SQL Important Quick Links School Programming
Languages C C++ Java Python SQL Important Quick Links School Programming
C++ Java Python SQL Important Quick Links School Programming
Java Python SQL Important Quick Links School Programming
Python SQL Important Quick Links School Programming
SQL Important Quick Links School Programming
Important Quick Links School Programming
School Programming
Operating Systems
DBMS
Computer Networks

1 of 4 2017年08月11日 08:55

Engineering Mathematics
Design Patterns
Common Interview Puzzles
Web Technology
G-Facts
Computer Graphics
Image Processing
Project Ideas

Travelling Salesman Problem | Set 1 (Naive and Dynamic Programming)

Travelling Salesman Problem (TSP): Given a set of cities and distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point.

Note the difference between Hamiltonian Cycle and TSP. The Hamiltonian cycle problem is to find if there exist a tour that visits every city exactly once. Here we know that Hamiltonian Tour exists (because the graph is complete) and in fact many such tours exist, the problem is to find a minimum weight Hamiltonian Cycle.

For example, consider the graph shown in figure on right side. A TSP tour in the graph is 1-2-4-3-1. The cost of the tour is 10+25+30+15 which is 80.

The problem is a famous NP hard problem. There is no polynomial time know solution for this problem.

Following are different solutions for the traveling salesman problem.

Naive Solution:

- 1) Consider city 1 as the starting and ending point.
- 2) Generate all (n-1)! Permutations of cities.
- 3) Calculate cost of every permutation and keep track of minimum cost permutation.
- 4) Return the permutation with minimum cost.

Time Complexity: ?(n!)

Dynamic Programming:

Let the given set of vertices be {1, 2, 3, 4,....n}. Let us consider 1 as starting and ending point of output. For every other vertex i (other than 1), we find the minimum cost path with 1 as the starting point, i as the ending point and all vertices appearing exactly once. Let the cost of this path be cost(i), the cost of corresponding Cycle would be cost(i) + dist(i, 1) where dist(i, 1) is the distance from i to 1. Finally, we return the minimum of all [cost(i) + dist(i, 1)] values. This looks simple so far. Now the question is how to get cost(i)?

To calculate cost(i) using Dynamic Programming, we need to have some recursive relation in terms of sub-problems. Let us define a term C(S, i) be the cost of the minimum cost path visiting each vertex in set S exactly once, starting at 1 and ending at i.

We start with all subsets of size 2 and calculate C(S, i) for all subsets where S is the subset, then we calculate C(S, i) for all subsets S of size 3 and so on. Note that 1 must be present in every subset.

```
If size of S is 2, then S must be {1, i},

C(S, i) = dist(1, i)

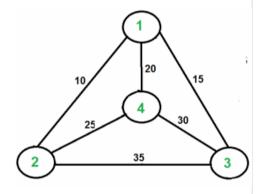
Else if size of S is greater than 2.

C(S, i) = min { C(S-{i}, j) + dis(j, i)} where j belongs to S, j != i and j != 1.
```

For a set of size n, we consider n-2 subsets each of size n-1 such that all subsets don't have nth in them.

Using the above recurrence relation, we can write dynamic programming based solution. There are at most $O(n^*2^n)$ subproblems, and each one takes linear time to solve. The total running time is therefore $O(n^{2*}2^n)$. The time complexity is much less than O(n!), but still exponential. Space required is also exponential. So this approach is also infeasible even for slightly higher number of vertices.

We will soon be discussing approximate algorithms for travelling salesman problem.



Travelling Salesman Problem Set 1 (Naive and Dynamic Programming) Gee
Next Article: Traveling Salesman Problem Set 2
References:
http://www.lsi.upc.edu/~mjserna/docencia/algofib/P07/dynprog.pdf
http://www.cs.berkeley.edu/~vazirani/algorithms/chap6.pdf
Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above
GATE CS Corner Company Wice Coding Practice
GATE CS Corner Company Wise Coding Practice
Dynamic Programming Graph NPHard
Recommended Posts:
Travelling Salesman Problem Set 2 (Approximate using MST)
Backtracking Set 6 (Hamiltonian Cycle)
Greedy Algorithms Set 7 (Dijkstra's shortest path algorithm) Greedy Algorithms Set 5 (Prim's Minimum Spanning Tree (MST))
Dynamic Programming Set 36 (Maximum Product Cutting)
(Login to Rate and Mark)
4.2 Average Difficulty: 4.2/5.0 Based on 37 vote(s) Add to TODO List
Mark as DONE
Writing code in comment? Please use ide.geeksforgeeks.org, generate link and share the link here.
Load Comments Share this post!

3 of 4 2017年08月11日 08:55

@geeksforgeeks, Some rights reserved

Contact Us!

About Us!

Privacy Policy









Careers!

2017年08月11日 08:55 4 of 4