Clue Deduction: an introduction to satisfiability reasoning

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So Many Topics, So Little Time

- Artificial Intelligence
  - **the really interesting miscellaneous pile of CS**
- Common goals, many diverse techniques
- You can't teach all of Russell & Norvig in a semester.
- Pick and choose, minimalist approach
- *What's a good, minimalist, fun, memorable way to teach concepts of logic?*

Simplicity + Pop Culture Fun

- Keep It Simple:
  - With a limited curricular footprint, minimize to the simplest material with maximal benefit.
  - Propositional logic
- Choose a fun reasoning domain.
  - Clue®, a deductive murder mystery game
  - Top-selling board game for over 60 years
  - Fun, familiar, nostalgic, and *underestimated*

HALL

CLUE

Start
Miss
Scarle

MISS SCARLET

REVOLVER

police evidence
case number 00
CLASSIFIED

HALL

police evidence
case number
CLASSIFIED

The Game of Clue

- 21 cards: 6 suspects, 6 weapons, 9 rooms
- Case file has unknown, random suspect, weapon, and room (SWR)
- Remaining cards dealt to players
- Player *suggests* SWR, first player clockwise that can refute, must show card
- Each player can make 1 SWR *accusation*
- Correct → win; incorrect → lose (& refute)

More Than Child's Play

- Children mark off each dealt/shown card
- Small notepad + instruction = trivial play
- However, consider this:
  - You know player **A** has 3 cards, two of which are **t** and **u**.
  - Player **A** refuted player **B**'s and **C**'s suggestions of (**v**, **w**, **x**) and (**y**, **z**, **x**), respectively, by showing a card.
  - Therefore, … **A**'s 3rd card must be **x**.

Constraint Satisfaction

- Clue reasoning is constraint satisfaction.
- One formulation: Boolean variables c_p denoting "Card c is in place p."
- Given CNF representation of Boolean constraints, reason with SAT solver refutations

Clue Deduction Project

- "Clue Deduction: an introduction to satisfiability reasoning"
  - Website, 28 page project guide, starter code
  - Java, Python[1], and LISP[1]
- Guide outline:
  - Introduction to propositional logic
  - Use of SAT solvers
  - Implementation of expert Clue reasoner

[1]Thanks to Dave Musicant

Basic Logic Concepts

- Concepts: sentences, operators, literals, truth assignments, (un)satisfiability, models, validity, tautologies, entailment, logical equivalence, derivation, soundness, completeness, …

- That's covering a *lot* without FOL.

- Simple, minimalist, high-utility approach
  - Con: No predicates, unification, FOL generalizations.
  - Pro: Time-efficient, experiential learning.

Applying Logic Concepts

- Collection of favorite propositional logic problems (e.g. "Amy says, 'Bob is a liar.' Bob says…")

- Representation

- Conversion to CNF

- Resolution TP

- Automatic TP through use of a …

(1)	$\{\neg A, \neg B\}$		Knowledge base
(2)	$\{B, A\}$		
(3)	$\{\neg B, \neg C\}$		
(4)	$\{C, B\}$		
(5)	$\{\neg C, \neg A\}$		
(6)	$\{\neg C, \neg B\}$		
(7)	$\{A, B, C\}$		
(8)	$\{C\}$		Assumed negation
(9)	$\{\neg A\}$	(5),(8)	Derived clauses
(10)	$\{B\}$	(2),(9)	
(11)	$\{\neg C\}$	(3),(10)	
(12)	$\{\}$	(8),(11)	*Contradiction!*

SAT Solver Black Box

- Simple SATSolver interface to underlying black box (e.g. zchaff, SAT4J[1], etc.)

- Alternatively, students may write their own solver (e.g. simple DPLL, WalkSAT)

- Reductio ad absurdum (proof by contradiction)

[1]Thanks to Daniel Le Berre

ClueReasoner

- Some helper functions provided, e.g.
  - Conversion of (card, place) pair to Gödel number for atomic sentence c_p
- Students convert Clue game facts to CNF and add them to SATSolver KB, e.g.
  - A card cannot be in two places.
  - At least one suspect card is in the case file.
  - Player **A** refuted suggestion (**x**,**y**,**z**).

ClueReasoner

```java
ClueReasoner cr = new ClueReasoner();
String[] myCards = {"wh", "li", "st"};
cr.hand("sc", myCards);
cr.suggest("sc", "sc", "ro", "lo", "mu", "sc");
cr.suggest("mu", "pe", "pi", "di", "pe", null);
cr.suggest("wh", "mu", "re", "ba", "pe", null);
cr.suggest("gr", "wh", "kn", "ba", "pl", null);
cr.suggest("pe", "gr", "ca", "di", "wh", null);
cr.suggest("pl", "wh", "wr", "st", "sc", "wh");
cr.suggest("sc", "pl", "ro", "co", "mu", "pl");
cr.suggest("mu", "pe", "ro", "ba", "wh", null);
cr.suggest("wh", "mu", "ca", "st", "gr", null);
cr.printNotepad();
```

	sc	mu	wh	gr	pe	pl	cf
mu	n	-	-	n	-	-	-
pl	n	Y	n	n	n	n	n
gr	n	n	Y	n	n	n	n
pe	n	-	n	n	-	-	-
sc	n	Y	n	n	n	n	n
wh	Y	n	n	n	n	n	n
kn	n	-	-	-	n	-	-
ca	n	n	n	Y	n	n	n
re	n	-	-	n	-	-	-
ro	n	-	-	-	-	-	-
pi	n	-	n	n	-	-	-
wr	n	-	-	-	-	-	-
ha	n	-	-	-	-	-	-
lo	n	-	-	-	-	-	-
di	n	n	n	n	-	n	-
ki	n	-	-	-	-	-	-
ba	n	-	-	n	n	-	-
co	n	-	-	-	-	-	-
bi	n	-	-	-	-	-	-
li	Y	n	n	n	n	n	n
st	Y	n	n	n	n	n	n

Exploring Further

- One constraint omitted:
  - Player p has exactly n cards.
- Pseudo-Boolean "at least" constraints
- DIY SAT Solver
  - DPLL
  - WalkSAT, Novelty, etc.
- Many starting points for deeper study of Constraint Satisfaction, KR&R topics

Resources

- Clue Deduction:
  http://cs.gettysburg.edu/~tneller/nsf/clue/
- MLExAI:
  http://uhaweb.hartford.edu/compsci/ccli/

The Cal-Clue-ator