What is spatial autocorrelation? Reflections on the past 25 years of spatial statistics

Daniel A. Griffith

Abstract

To date a clear, simple, and concise definition of spatial autocorrelation has eluded the literature, although an increasing number of publications concerned with this concept have appeared in recent years. The scope of this problem is explored here, in a search for a meaning of spatial autocorrelation that is understandable to the average numerate geographer. The primary finding is that this concept can take on any one of the following nine different meanings: self-correlation attributable to the geographical ordering of data; a descriptor of the nature and degree of certain types of map pattern; an index of the information content latent in geo-referenced data, especially that information overlooked by classical statistical estimators when applied to spatial data series; a diagnostic tool for spatial model misspecifi- cation; a surrogate for unobserved geographic variables; a nuisance in applying conventional statistical methodology to spatial data series; an indicator of the appropriateness of, and possibly an artifact of, areal unit demarcation; a spatial process mechanism; and a spatial spillover effect. Each definition is spelled out, in turn, and either numerically or conceptually illustrated. One purpose of this exercise is to aid in the dissemination of introductory spatial statistical understanding throughout the geography community. The importance of attaining this goal lies in uses and potential abuses of spatial statistics that may accompany the proliferation of GIS throughout the technical community.

Résumé

RÉSUMÉ.— Qu'est-ce que l'autocorrélation spatiale? Réflexions sur 25 ans de statistiques spatiales. — II n'existe pas à ce jour de définition simple, claire et concise de l' autocorrélation spatiale, bien qu'il y ait eu ces dernières années un nombre croissant de publications en rapport avec ce concept. L'auteur cherche à rendre compréhensible l'autocorrélation spatiale au géographe moyen possédant un minimum de connaissances mathématiques. D apparaît en premier lieu que ce concept peut revêtir neuf significations différentes: la corrélation d'une variable avec elle-même provenant de l'ordonnancement géographique des données; un instrument de description de la nature et de l'intensité d'une structure spatiale; un indicateur de la quantité d'information latente contenue dans les données spatialisées, en particulier l'information qui s'avère toujours négligée dans les estimations statistiques classiques quand elles sont appliquées aux séries de données spatiales; un outil permettant de repérer l'existence de variables significatives, mais non prises en compte dans le modèle; un substitut à des données manquantes; un obstacle pour l'application des méthodologies statistiques conventionnelles à des séries de données spatiales; un indicateur du bien-fondé d'une partition spatiale, voire un artefact introduit par les frontières; un mécanisme d'un processus spatial; enfin, un effet de redistribution sur les lieux avoisinants. L'article détaille tour à tour chacune des définitions avant de les illustrer soit par des exemples numériques soit de manière conceptuelle. La compréhension des statistiques spatiales est essentielle, étant donné l'utilisation croissante et les abus potentiels qui risquent fort de se produire avec la prolifération des SIG parmi la communauté des techniciens.

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Reflections on the past 25 years of spatial statistics

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AREAL UNIT, GEO-REFERENCED DATA, LOCATIONAL INFORMATION, SPATIAL AUTOCORRELATION, SPATIAL MODEL

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AUTOCORRÉLATION SPATIALE, DONNÉES SPATIALISÉES, INFORMATION LOCALISÉE, MODELE SPATIAL, UNITÉS SPATIALES

Introduction

A generic way of describing the concept of spatial autocorrelation, frequently encountered in the literature, refers to either similar or dissimilar values clustering, rather than being randomly located, on a map (see fig. 1), producing pattern that is undetectable outside of a geographic reference. But what does this description actually mean? Many readers of quantitative geography literature are hard pressed to come up with an answer to this question, and find an understandable definition of spatial autocorrelation to be elusive.

Recognition of the problem of spatial autocorrelation dates back as early as 1914 (see Student), with the acknowledgement that for geo-referenced data series observed correlation can be attributed to nothing more than the geographic location of data. For subsequent decades, though, its study was eclipsed by many of the more pressing general needs in modern statistical theory development, including sampling theory. Spatial autocorrelation was noticed in passing here, too, with spatial data being characterized by a modified urn model, where sample selections appear to more closely resemble an experiment in which clusters of grapes, rather than separate balls, are drawn from an urn (after Stephan, 1934). Eventually this conception led to sample designs, especially in agricultural field plot experiments, that sought to neutralize the impact of spatial autocorrelation (see, for instance, Whittle 1956). About mid-century some attention was turned to this problem again, with the formulation of indices to measure spatial autocorrelation (see Geary, 1954; Moran, 1948). In addition, other linear spatial-type statistics were appearing, too, such as the runs statistic (which generalizes to the join count statistics in two dimensions).

These developments helped to illuminate the behavior of spatially autocorrelated data. Their accompanying published results established that it clearly is a geographic phenomenon. Naturally, then, one would expect geographers to have begun devoting considerable research effort to its study. Some did allude to its complicating affects in spatial data analysis (e. g., see Wolpert, 1964, p. 557). But, the first NSF research grant concerning spatial autocorrelation went to an economist, rather than a geographer (see the acknowledgement in Fisher, 1971). In fact, comprehensive discussions of this concept accessible to and aimed at the general geography community did not begin to appear until the late 1980s (see Goodchild, 1986; Griffith, 1987; Odland, 1988). In the interim, geographers continued to comment on the importance of spatial autocorrelation. Tobler (1970), for instance, stressed that all attribute values on a geographic surface are related to each other, but closer values are more strongly related than are more distant ones. Gould (1970) noted that spatial data series are at odds with the classical statistical assumption of independent observations, which posits that the ordering of geo-referenced data does not matter. And, Haining (1980) emphasized that for spatial data there is a better than random chance that one can predict attribute values for a given areal unit from those taken on by its juxtaposed areal units. Each of these descriptions is consistent with the general notion of map pattern.

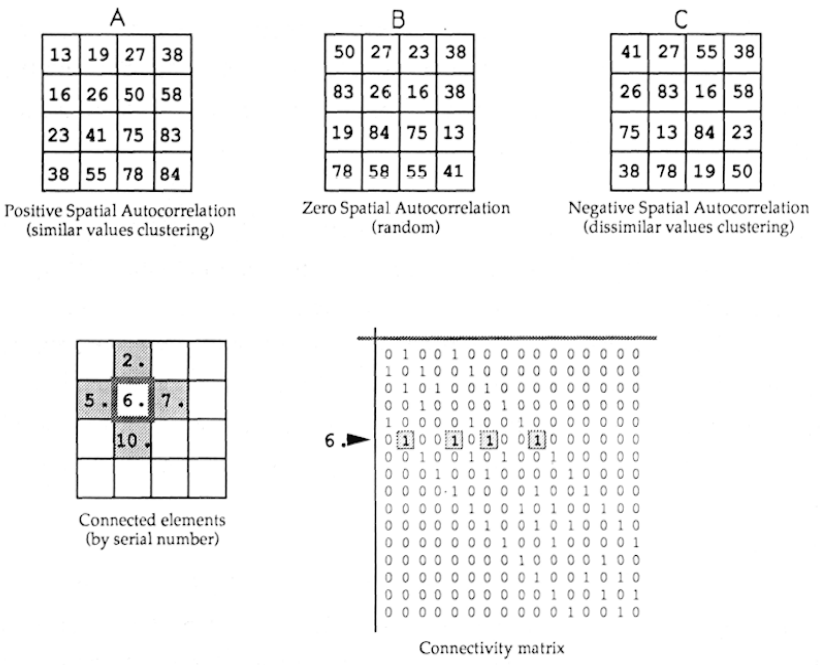


Fig. 1.— *Simulated spatially autocorrelated surfaces (resulting data in table II).*

As an aside, this brief historical overview is not intended to suggest that geographers

have completely ignored spatial autocorrelation research over the years. The first major monograph dealing with this subject was coauthored by a geographer and a statistician (see Cliff and Ord, 1973). Furthermore, the recent spatial autocorrelation literature is replete with contributions by geographers. But the target audience for these essays consists of quantitatively sophisticated geographers, spatial statisticians, regional scientists, and the like.

Unfortunately, a number of geographers believe that the Goodchild-Griffith-Odland trilogy has failed to sufficiently clarify what spatial autocorrelation is all about in a simplified way that is understandable to the average geographer (see reviews by Aitken, Anselin, Boots, Cox, Gatrell, and Haining). The purpose of this paper is to review the spatial autocorrelation literature in a pedagogic fashion so that this recognized void finally is filled. The timeliness of positing this goal lies in achieving improved spatial data analysis, especially with the advent of widespread geographic information systems usage. Common numerical and interpretative problems encountered in the statistical analysis of geo-referenced data, that tend to lie dormant in more conventional categories of data, need to be properly handled, if correct spatial analysis is to accompany the dissemination of GIS technology.

Background

Spatial autocorrelation may be defined in a literal sense by dissecting the phrase. Correlation refers to the relationship prevailing between two variables; pairs of values for individual observations are related. The prefix auto- means self. Hence, autocorrelation may be defined loosely as the relationship among observations of a single variable; pairs of observations are related rather than independent. The adjective spatial describes the source of this self-correlation, which is the two-dimensional ordering of data values. Therefore, spatial autocorrelation refers to the dependencies that exist among observations that are attributable to the relative locations, or underlying two-dimensional ordering, of variable values in geographic space. In turn, these dependencies produce clustering of similar (positive spatial autocorrelation) or dissimilar (negative spatial autocorrelation) values, and hence induce some map pattern.

Haining *et al.* (1983) attempted to clarify this concept by differentiating between the two connotations within which spatial autocorrelation can be cast. Both refer to map pattern. On the one hand, one could ask:
If the set of variable values $\{x_i\}$ had been allocated in a random fashion to the set of n areal units into which a two-dimensional surface had been partitioned, would this observed spatial distribution be a likely outcome of such an allocation procedure?

This randomization perspective is the basis for the computer simulation experiment formulated by Griffith (1987), which gives an experimenter numerical experience with spatial autocorrelation. On the other hand, one could ask:
If the n realizations of the random variables X_i are mutually independent, is the single sample of joint realizations constituting the observed map pattern a representation one?

This perspective places spatial autocorrelation into a multivariate distribution function context, operating as a parameter of the covariance matrix. Although these two questions distinguish between viewpoints on the concept, they do little to help provide an explicit understanding of it.

One purpose of the aforementioned trilogy is to furnish an understanding of spatial autocorrelation. Goodchild (1986, p. 3) defines the concept as:
"in its most general sense, ... [being] concerned with the degree to which objects or activities at some place on the earth's surface are similar to other objects or activities located nearby".

He attempts to clarify this definition by concentrating almost exclusively on measures of spatial autocorrelation, partly in terms of indices and partly in terms of processes (p. 3):
"spatial" autocorrelation can be interpreted as a descriptive index, measuring aspects of the way things are distributed in space, but at the same time it can be seen as a causal process, measuring the degree of influence exerted by something over its neighbours".

Here autocorrelation processes are discussed in terms of simulation experiments, spatial processes, and regression residuals. The notion of information content is raised by Goodchild, too (pp. 3-4):
"Spatial analysis deals with two quite distinct types of

information. On the one hand are the attributes of spatial features, which include measures such as size, value, population or rainfall, as well as qualitative variables such as name, region or soil type. On the other hand each spatial feature has a location, which can be described by its position on a map or by various geographic referencing or coordinate systems... Spatial autocorrelation is one of the relatively small set of techniques which deals simultaneously with both locational and attribute information".

Goodchild further tries to illuminate the notion of spatial autocorrelation by emphasizing why it is important to geographic studies, and by providing BASIC computer programs for the PC; these computer codes calculate spatial autocorrelation indices and significance tests for these indices.

Griffith (1987) begins by describing why autocorrelation is important to geographers and geography, how it fits into scientific investigations concerning the nature of structure, and what some of the salient differences are between time series and spatial series analyses. He devotes considerable attention to defining the term, in a fashion similar to that presented in the preceding section of this paper. He focuses on the generic meaning of autocorrelation, the phenomenon of spatial autocorrelation, sources of spatial autocorrelation, and a simulation experiment employing a BASIC computer program for a PC that may be used to illustrate the notion of spatial autocorrelation. Like both Goodchild and Odland, Griffith articulates relationships between spatial autocorrelation and geographic samples; here his focus is on the two sampling perspectives of randomization and normality, outlined by Haining *et al.* (1983), as well as the assumption of identically distributed variates. In a pedagogic fashion Griffith attempts to both complement and supplement Cliff and Ord's (1981) presentation on the measuring of spatial autocorrelation and the construction of sampling distributions for prominent spatial autocorrelation indices. In addition, he presents a brief overview of spatial autocorrelation as a parameter of statistical models, followed by a treatment of ways in which to rewrite simple classical statistical models for proper geographic analyses.

Odland (1988) introduces spatial autocorrelation in a succinct, readable fashion, commenting on why objects are autocorrelated in space, but never defining the concept in a clear, simple, and concise manner. At best the reader can acquire a nonverbal awareness of this concept from Odland's monograph. Spatial autocorrelation is cast as methods for "... investigating the organization and structuring of phenomena over space..." (p. 7). His strategy is to demonstrate how spatial autocorrelation statistics can be used in diagnosing and correcting for spatial dependency effects that complicate the use of common statisticalmethods, such as regression analysis, when they are applied to spatial data series. As such spatial autocorrelation implicitly is being defined as a nuisance to statistical analyses in geography. But, filtering out spatial autocorrelation effects is equivalent to discarding the geographic content of a spatial data series, restricting attention to nongeographic features and relations latent in the series! Odland makes distinctions between categorical and continuous data. He also discusses autocorrelation statistics as general cross-product statistics, and scale, isotropy, and temporal aspects of spatial autocorrelation.

Besides these elementary introductions to the topic, publications of a more advanced nature are available. Foremost is the update to Cliff and Ord's 1973 monograph (1981), which is pitched at an intermediate level, and outlines much of the statistical theoretical foundations for spatial autocorrelation analysis. Bartlett (1975) and Matern (1986) provide treatments that are couched in terms of stochastic processes theory. Ripley (1988) summarizes many of the advances that have occurred in this field since his earlier book appeared (Ripley, 1981), with many arguments couched in terms of Gibbs states. Haining (1990) supplies an applications oriented book about spatial autocorrelation for the environmental and social sciences. And, Upton and Fingleton (1985) furnish a useful compendium of techniques for handling spatial autocorrelation in statistical analyses. Three additional books merit more detailed discussion here, namely those by Griffith (1988), Anselin (1988), and Arbia (1989). These three publications highlight the major themes to be explicated in this paper. The order of their review has been selected so that it parallels the organization of this paper, in a quest to distill a comprehendible definition of spatial autocorrelation.

Griffith (1988) expands upon his earlier, introductory monograph, and additionally treats the topics of spatial statistics and data handling, development of a theory of spatial statistics, areal unit configuration and locational

information (i. e., the relative locations of areal units for which the data have been collected and information pertaining to the regional mean and the geographic variation of these data), the reformulation of classical linear statistical models, spatial autocorrelation and spectral analysis, the missing data problem of a two-dimensional surface, correcting for edge effects in spatial statistical analyses, multivariate models of spatial dependence, and simulation experimentation in spatial analysis. In doing so, he provides a brief historical background, summarizes the principal problem of spatial statistics, outlines various spatial sampling perspectives, reviews models of spatial autocorrelation and their extensions to statistical model reformulations, and surveys especially his own work devoted to the development of a theory of spatial statistics. With regard to this last theme, Griffith discusses the small sample size problem that is characteristic of much of geographical analysis, differences between finite and infinite surface problems, issues involved with geographic data transformations, multivariate spatial data analysis, and the use of higher order autoregressive models. One strength of this book lies in its treatment of locational information, which is done in a wide variety of contexts.

Anselin (1988) treats three general themes, namely (1) foundations for the econometric analysis of spatial processes, (2) estimation and hypothesis testing, and (3) model validation. His emphasis is on spatial econometrics, going far beyond the seminal treatment of this subject by Paelinck and Klaassen (1979), and he discusses the scope of this topic. Within this context he introduces the formal expression of spatial effects, a topology of spatial econometric models, and an overview of spatial stochastic processes. Next Anselin treats estimation and inference issues, beginning with a discussion of the maximum likelihood approach to spatial process models, presenting alternative approaches to inference with these types of models, and then addressing the problems of spatial dependence in regression residuals, and spatial heterogeneity. These presentations are followed by treatment of models in space and time, operational issues, and empirical applications. Anselin concludes with topics that characterize much of his own research; he first discusses model validation and specification tests in spatial econometric models, and then discusses model selection with this modelling strategy. Among other features, this book gives an excellent presentation of spatial heterogeneity issues affiliated with spatial dependence.

Arbia (1989) portrays spatial autocorrelation under the dual guises of surface partitioning and the ecological fallacy. In the first instance, the measure of an attribute becomes a function of the way in which a surface has been partitioned into areal units. Arbia argues that the spatial partitioning scheme utilized should maintain the closest possible approximation of, while preserving the greatest amount of information in, original ungrouped data. In the second instance, statistical analysis can lead to different findings when the observations are individual objects as opposed to geographically aggregated collections of these objects. He shows theoretically and numerically that spatial autocorrelation can be an artifact of the partitioning scheme employed. He corroborates, in a highly systematic fashion, that spatial autocorrelation distorts information content of geo-referenced data. And, he demonstrates that ecological fallacy effects can arise strictly from the presence of spatial dependency.

The information content perspective

One reason researchers became interested in spatial autocorrelation is because the locational information of geo-referenced data is not captured by classical statistics; traditional estimators are not statistically sufficient (i. e., they do not capture all of the information contained in a sample) when applied to spatial data series. Griffith (1988, pp. 9-10) exemplifies this idea with the randomization sample space illustration presented in figure 2(a). The classical sample mean in this case, namely $x = 2$, is insensitive to changes in geographic configuration; locational information is not captured by x. Accordingly, the sampling distribution is a spike (see fig. 2). Filtering out spatial autocorrelation (see the ensuing analogy with principal components analysis), and hence the locational information, latent in each of these map patterns does allow for some discrimination. The corresponding map patterns, in order to construct artificially independent areal unit values, are given in figure 2(b). The resulting sampling distribution no longer is a spike (see fig. 2).

One apparent question to pose at this time asks whether or not locational information is important. Several empirical findings help to answer this question. Griffith and Jones

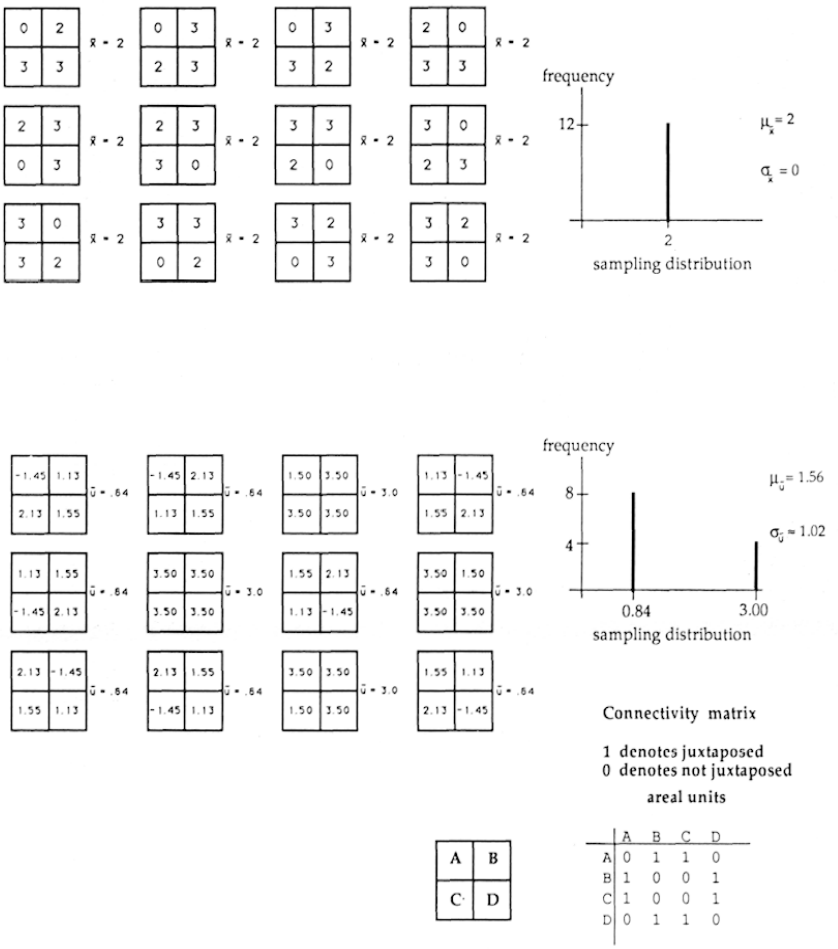


Fig. 2.— *Example sampling distribution for geo-referenced data.*

(1980) studied journey-to-work distance decay parameter estimates, and found that areal unit configuration accounted for approximately 40% of the observed variation across 24 Canadian urban areas. Griffith (1984) investigated the classification of areal units (*municipios*) into the five agricultural administration regions of Puerto Rico, and found that approximately 1/3 of the ability to successfully classify these observations can be attributed to locational information. And, Griffith (1988) uncovered an areal unit configuration dimension spanning journey-to-work flows, land use patterns, and socio-economic geographic distributions for Toronto, finding that this dimension simultaneously accounted for approximately 3%, 19%, and 8%, respectively, of the variation displayed by these three data sets. These conclusions imply that locational information constitutes a nontrivial amount of the total information content of geo-referenced data.

Switzer (1988) has provided a second example of this viewpoint, with reference to the geographic arrangement of air pollution monitoring devices in the San Francisco Bay area. He noted that devices situated too close to one another end up recording essentially the same air pollution measures, rendering spatially autocorrelated readings. In other words, a nearby device adds almost no new information about air pollution gathered by a given device; rather, it furnishes considerable redundant information, and a nearly negligible marginal increment in information gain. Human geographers have encountered this class of problem before in spatial interaction analyses. The extreme case is where n – 1 spatial flows are approximately the same, being small in their observed and expected values, whereas some n^{th} flow is markedly different, being very large in its observed and expected values (see fig. 3). A correlation between the observed and expected values for this situation tends to be nearly perfect ($r \approx 1$). This result is not surprising. The n-1 similar values behave as repeated measures of a single observation, meaning that in practice the correlation is being computed using two points; one always can fit a line through two points without error. This correlation example also can be viewed in terms of a degrees-of-freedom problem; when n = 2, estimating a correlation coefficient means there are no degrees of freedom left. This degrees-of-freedom perspective can be related to the notion of spatial autocorrelation, too, as will be shown shortly.

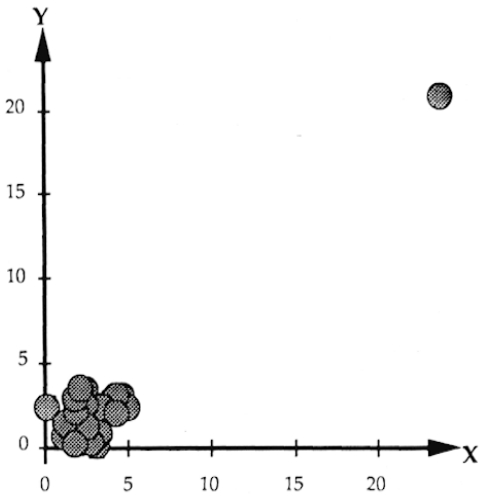
The discussion thus far establishes that the presence of non-zero spatial autocorrelation means information content of geo-referenced data is other than what it appears to be. Sen and Srivaster (1986) briefly described a traditional sampling counterpart that has been more fully developed for presentation here. Consider an independent, random sample of size n from a finite, discrete, uniform distribution,

$$\{x_i | \sum_{i=1}^{i=k} f_i = n; i = 1, 2, \ldots, k\}$$

yielding where n is the sample size, i denotes the observation drawn, x_i is the sample value for observation i, f_i is the frequency of occurrences of value x_i, and k is the number of distinct observations drawn. Suppose that random sampling is done with replacement, order being important; this sampling framework allows a given x_i value to repeat in any sample that is chosen from the population, with the probability of selecting x_i always being equal to 1/N, where N is the number of elements in the population. Now for some unknown set of weights $\{w_i | w_i$ is a non-negative integer$\}$, let

$$\sum_{i=1}^{i=k} w_i f_i = qn$$

This last equation states that if $q = 2 = w_i$ (for all i), then the sample size deliberately is doubled merely by repeating all sample selection entries. Clearly this new sample contains twice as many observations but no more information than does its original counterpart; in other words, complete redundancy exists in the information content provided by the additional n values. In contrast to the original sampling situation, this new sample has forced each x_i value to repeat with probability 1, rather than simply allowing it to repeat at random. Consequently, if an unsuspecting researcher were to study the larger sample, he/she would believe that far more information is available than actually exists. Information redundancy in this contrived example is analogous to what happens conceptually with spatially autocorrelated geo-referenced data, since data collected for two juxtaposed points in space tend to represent essentially the same information (recall the example described by Switzer).



X	Y
22.5779	21.8436
1.3117	1.1844
2.1117	2.6295
2.3598	0.8411
2.2942	1.2009
2.8527	1.3541
3.0173	2.2577
3.2113	2.6337
4.5126	2.5143
0.0717	2.5336
3.1670	0.4957
1.1272	1.1164
2.3731	1.2919
2.1989	0.2773
1.9655	2.0493
2.0549	3.2396
2.4779	1.5317
2.9907	1.0446
4.5326	2.9222
2.5356	2.2610
1.6061	0.8408
1.1803	1.7025
1.8826	2.1617
3.0181	0.1705
1.9816	2.8584
1.9730	2.7362
2.0188	3.4215
2.9785	0.2112
1.7643	1.6894
3.8209	2.7944

Fig. 3.— *Graphical display of an outlier.*

Table I

Variance components for an artificially increased sample size.

Sample Composition Multinomial Count	$\{w_i\}$	Sample Composition n = 3	Sample Composition n = 6	$\mu_{\bar{x}\|\{f_i\}}$	$VAR(\bar{x}\|\{f_i\})$
3	2,0,0	3,0,0	6,0,0	μ	σ^2
36			5,1,0	μ	$7\sigma^2/17$
90	2,2,0 or 1,4,0	2,1,0	4,2,0	μ	$\sigma^2/3$
90			4,1,1	μ	$\sigma^2/4$
360			3,2,1	μ	$\sigma^2/16$
90	2,2,2	1,1,1	2,2,2	μ	0
60			3,3,0	μ	$\sigma^2/4$

$$E[VAR(\bar{x}|n = 3)] = (3/27)\sigma^2 + (18/27)(\sigma^2/3) + (6/27)(0) = \sigma^2/3$$

$$E[VAR(\bar{x}|n = 6)] = (3/729)\sigma^2 + (36/729)(7\sigma^2/12) + (90/729)(\sigma^2/3) + (90/729)(\sigma^2/4) + (360/729)(\sigma^2/12) + (90/729)(0) + (60/729)(\sigma^2/4) = \sigma^2/6$$

$$E[VAR(\bar{x}|\sum_{i=1}^{i=3} w_i f_i = 2 \cdot 3)] = (3/183)\sigma^2 + (90/183)(\sigma^2/3) + (90/183)(0) = (66/61)(\sigma^2/6)$$

$$E[VAR(\bar{x}|\sum_{i=1}^{i=3} w_i f_i = 6) = (3/633)\sigma^2 + (90/633)(\sigma^2/3) + (90/633)(\sigma^2/4) + (360/633)(\sigma^2/12) + (90/633)(0) = (171/211)(\sigma^2/6)$$

Results are reported in Table I for the cases of N = 3, n = 3 and n = 6, for a random sample. A distinction is made in this tabulated information between doubling the frequency of each element appearing in the sample drawn ($w_i = 2$, for all i), doubling the total sample size at random (the sum of the $w_i f_i$ terms equals 6), and pure random sampling. This first case is a special instance of this second case. The expected values of the conditional variances (i. e., variances given the particular types of sample sizes) in each case are calculated in order to illustrate differences between these three cases. The first case of unconstrained random sampling, where n = 6, yields a variance for x of $\sigma^2/6$, where σ^2 is the population variance. Spatial autocorrelation is similar to the third case, where n = 2•3 = 6, which yields a variance for x of (66/61) ($\sigma^2/6$). The accompanying 8.2% increase in sampling variance reflects the information loss attributed to redundancy. One should note that this third case displays a 33.5% increase in variance over the second case, in which individual values, once sampled, are allowed to increase at random.

One additional comparison will be made, with respect to information content, between spatially autocorrelated data and traditional statistical data. Consider the case of a multivariate data set, comprised of p variables. Each of these p variables embraces an independent quantity of information only if variables in the set are mutually orthogonal. If all p of the variables are perfectly correlated with each other, then the information content of any subset containing p–1 variables is perfectly redundant with the remaining single pth variable. Usually the degree of redundancy encountered falls somewhere between these two extremes. For geometric reasons this redundancy is called multicollinearity. Problems arising from the presence of multicollinearity in such data have been the focus of a large part of multivariate statistical methodology. For instance, factor analytic models attempt to transform a set of p variables into q<<p meaningful orthogonal synthetic variates; the principal components model transforms p variables into p orthogonal synthetic variates. Spatial autocorrelation represents the same situation, but where correlations are among observations for a single variable. As with factor analysis, in part spatial autocorrelation analysis has been seeking geographic models for transforming areal unit values into independent synthetic values. Models used to date are summarized by Upton and Fingleton (1985). Consistent with this quest is Q-mode factor analysis (the correlation matrix is n-by-n for observations, using variable values to compute the individual r_{ij} correlation coefficients), to which considerable amounts of geo-referenced data have been subjected to at one time or another for the purpose of attaining regionalizations. In practice, then, spatial autocorrelation is a type of multicollinearity. And, just as eigenvalues of correlation matrices are used in multivariate analysis to index the degree of multicollinearity there, spatial autocorrelation measures can be used to index prevailing spatial multicollinearity.

This perspective is reinforced by returning to an assessment of spatial autocorrelation in terms of degrees of freedom. Three heuristic descriptions of degrees of freedom can be found in the statistics literature. One relates them to constraints placed on the number of values that are free to vary in an analysis, and makes sense in many univariate situations. However, often in multivariate analysis fractional degrees of freedom appear, which dismisses the generality of this description. A second description depicts degrees of freedom as the sample size minus the number of estimates already made in a given calculation. It refers to conditional analysis concerning the substitution of one estimator in the calculation of another. The third view is that degrees of freedom function as an index of information content in sample data, where this index can take on any non-negative real value (hence allowing fractional values) and identifies the correct frequency distribution to be used in an analysis. This is the description that will be pursued here. Richardson (1990) demonstrates degrees of freedom, say υ' for spatially autocorrelated data differ from the number of degrees of freedom, say υ, associated with n independent observations. She reports research findings showing that as positive spatial autocorrelation increases, υ' becomes increasingly less than υ, implying the presence of less information than is apparent. In contrast, as negative spatial autocorrelation increases, she reports that υ' becomes increasingly greater than υ, implying the presence of more information than is apparent. When spatial autocorrelation is zero, $\upsilon' = \upsilon$. Consequently, if similar values tend to cluster on a map, then the information content of a geo-referenced data set is less than it appears to be; if dissimilar values tend to cluster on a map, then the information content is more than it appears to be.

Richardson's findings emphasize that a qualitative difference exits between positive and negative spatial autocorrelation. This difference may be exemplified technically with a measure of information (called the Fisher information) contained in the mean, which is defined as n/σ^2 for independent observations. Let $f(\rho)$ denote the spatial autocorrelation component, where f is some function and ρ is the spatial autocorrelation parameter. If positive spatial autocorrelation is present, then this information measure becomes $[n - f(\rho)]/\sigma^2$, where $f(\rho) > 0$, resulting in a decrease in the information content. If negative spatial autocorrelation is present, then this information measure becomes $[n + f(\rho)]/\sigma^2$, resulting in an increase in the information content. So what does negative spatial autocorrelation actually mean? In continuous space it cannot exist, since it would suggest that moving an infinitesimal distance from some point would result in a variable value that is opposite in nature (negative spatial autocorrelation occurs when dissimilar values cluster on a map), leading to a paradox. Accordingly, negative spatial autocorrelation may well be a consequence of discretizing geographic space (see the ensuing discussion of the modifiable areal unit problem). This inconsistency is illustrated by the often cited checkerboard example, with its marked contrast between neighboring cells, but its implied strong positive spatial autocorrelation within cells. The artificiality of negative spatial autocorrelation is further demonstrated by moving from a square to a hexagonal partitioning of a surface. This latter type of tessellation only allows a tendency for dissimilar values to cluster on a map, and forbids perfectly alternating geographic distributions. Fortunately, to date a preponderance of geographic distributions have been found to display positive spatial autocorrelation. One of the few substantive situations in which negative spatial autocorrelation might make sense conceptually is in the presence of spatial competition (see Haining's, 1983, discussion of gasoline pricing); but, spatially competitive regimes exist that lead to positive spatial autocorrelation, too.

Therefore, spatial autocorrelation may be defined as a measure of the true but masked information content in geo-referenced data. This definition is consistent with the aforementioned treatments given by Goodchild and Griffith.

The model specification error perspective

At times spatial autocorrelation is defined as an artifact of specification error in spatial modelling. This viewpoint underlies the extensive literature linking spatial autocorrelation to residuals in regression analysis. On the one hand, if a single variable is missing from a regression equation, then the spatial distribution of this variable constitutes a communality across regression residuals, causing them to appear to be spatially autocorrelated. This problem is exacerbated by multiple missing variables. Thomas (1968) formalized this aspect into a strategy for spatial regression model building. But how does spatial autocorrelation manifest itself in this context?

Consider the case of a variable having a constant mean over the geographic landscape. Then the corresponding statistical model has a term like $(Y - \mu 1)$, where Y is an n-by 1 data vector, 1 is an n-by-1 vector of ones, and μ is the population mean for the n areal units; in other words, the mean of the geographic data is stationary across the map. The accompanying Moran Coefficient, a popular index of spatial autocorrelation, is defined, using matrix notation, as

$$MC = (n/1^TC1)(Y - \bar{y}1)^TC(Y - \bar{y}1)^T/(Y - \bar{y}1)^T(Y - \bar{y}1)$$

where the configuration of areal units is depicted by the n by-n binary connectivity matrix C (cell c_{ij} equals 1 if row and column areal units are juxtaposed, and 0 otherwise). But what happens to MC if the mean varies from location to location, when actually there is no spatial auto-correlation present in the data? This variation is caused by missing variables, which when introduced result in the term $\mu 1$ being replaced with the regression expression $X\beta$, where matrix X is constructed from the set of missing variables. For a constant, unknown spatial mean, $E(MC) = -1/(n-1)$ under the null hypothesis of zero spatial autocorrelation; if μ is known, then $E(MC) = 0$. For a variable spatial mean that mistakenly has been assumed to be spatially invariant, however, this expected value becomes

$$(n/1^TC1)[(-\sigma^2)(1^TC1/n) + \beta^TX^T(I - 11^T/n)C(I - 11^T/n)X\beta]/[(n-1)\sigma^2 + \beta^TX^T(I - 11^T/n)X\beta].$$

This expression reduces to the previous, simpler one, only if $\beta_1 = \beta_2 = ... = \beta_p = 0$, and $\beta_0 = \mu$ and $X_0 = 1$, which corresponds to a spatially stationary mean. In this latter case the appropriate statistical model has a term like

$(Y - X\beta)$. The corresponding correct expected value is $(n/1^TC1)\{-\text{trace}[XCX(X^TX)^{-1}]\}/(n-p-1)$, where p is the number of missing variables included in the regression model (see Cliff and Ord, 1981, p. 202). If β is known, then again E(MC) = 0 under the null hypothesis of zero spatial autocorrelation. An analysis of MC in this way suggests that if one posits a constant mean when a variable mean is true, then specification error could lead to the detection of spatial autocorrelation when in fact it does not exist in the geo-referenced data. The detection of spatial autocorrelation here is a mistake, an artifact of specification error. Accordingly, spatial autocorrelation functions as a diagnostic concept. This idea constitutes part of the thrust of Odland's (1988) monograph. It also is the notion illustrated by Haining (1978) in his formulation of a model of High Plains agriculture.

Table II

Data for and residuals from simple linear regressions: positive and negative spatial autocorrelation examples.

positive spatial autocorrelation				negative spatial autocorrelation		
x-co-ordinate	y-co-ordinate	attribute value	regression residual	high/low indicator	attribute value	regression residual
4	1	84	-4.525	-1	50	-15.5
4	2	83	8.125	1	23	- 2.0
4	3	58	-3.225	-1	58	- 7.5
4	4	38	-9.575	1	38	13.0
3	1	78	4.675	1	19	- 6.0
3	2	75	15.325	-1	84	18.5
3	3	50	3.975	1	16	- 9.0
3	4	27	-5.375	-1	55	-10.5
2	1	55	-3.125	-1	78	12.5
2	2	41	-3.475	1	13	-12.0
2	3	26	-4.825	-1	83	17.5
2	4	19	1.825	1	27	2.0
1	1	38	-4.925	1	38	13.0
1	2	23	-6.275	-1	75	9.5
1	3	16	0.375	1	26	1.0
1	4	13	11.025	-1	41	-24.5

Consider the first of the three map patterns in figure 1, namely (a). This map pattern displays positive spatial autocorrelation, yielding MC = 0.7188. But this map pattern exhibits a linear trend, in the form of an incline plane, increasing in value as one moves from the lower right-hand corner to the upper left-hand corner of the region. The systematic nature of this map pattern suggests that the geographic mean is not constant over this landscape. In fact, regressing the attribute values, say Z, on the areal unit coordinates, say (U,V), produces

$Z = 41.4 + 15.2\,U - 13.6\,V + E$,

where E is the residual variable. This equation accounts for 92% of the variance displayed by Z. In other words, the mean response surface across this geographic landscape is 41.4 + 15.2 U - 13.6 V. Adjusting the attribute values Z for this mean converts it to the error variate E, which has MC = 0.2130. Moreover, most of the spatial autocorrelation detected in the map pattern under study is attributable to a nonconstant spatial mean. Computational details for this example appear in Table II.

Next, consider the third of the three map patterns in figure 1, namely (c). This map pattern displays negative spatial autocorrelation, yielding MC = - 0.9741. But this map pattern exhibits a cyclical trend, alternating in value as one moves across the region, and is consistent with periodic geographic behavior. The systematic nature of this map pattern suggests that the geographic mean is not consistent over this landscape. Rather, a low mean is interwoven with a high mean. In fact, regressing the attribute values, say Z, on an areal unit differentiating indicator variable, say I (I = 1 for low values, and - 1 for high values), produces

$Z = 45.3 - 20.3\,I + E$,

where, as before, E is the residual variable. This equation accounts for 72% of the variance displayed by Z (and is equivalent to an analysis of variance or difference of means test). In other words, the population mean response surface across this geographic landscape is an alternating 65.6 with a 25.0. Adjusting the attribute values Z for this differential mean converts it to the error variate E, which has MC = - 0.2450. Again, then, most of the spatial autocorrelation detected in the map pattern under study is attributable to a nonconstant spatial mean. Computational details for this example appear in Table II, too.

When missing variables either have been overlooked during data collection or are not observable, though, spatial autocorrelation can be used as a surrogate for them, and as such is left in the model specification. In this circumstance it serves as a correction factor.

This same line of argument can be extended to variances, and has been the theme of much of the spatial econometrics work by Anselin concerned with spatial heterogeneity. If the spatial process under study has a constant variance, then again the standard null hypothesis expected value of MC is $-1/(n-1)$. In the presence of general variance heterogeneity — called heteroscedasticity — (e. g., each areal unit value may have a different variance associated with it), which can be depicted here with the diagonal matrix S_d (Σ_d for the population), the expected value of MC becomes

$(n/1^T C1)\{1^T \Sigma_d[-2C + (1^T C1/n)I]1\}/[(n-1)1^T \Sigma_d 1)$,

which reduces to the preceding simpler result only if $\Sigma_d = \sigma^2 I$. Variance heterogeneity would be expected in, for example, socio-economic data collected for areal units using different sample sizes, say n_i, since $\sigma^2 x = \sigma x^2/n_i$. Once again, if one assumes a constant variance when heteroscedasticity exists, then the accompanying specification error could lead to the detection of spatial autocorrelation when in fact none exists. Here local variation rather than regional trend becomes the critical factor.

Consequently, spatial autocorrelation may be defined as a diagnostic tool, and potential a correction factor, for guiding improvements during spatial model building.

The modifiable areal unit perspective

As was mentioned earlier, Arbia (1989) has produced an impressive, imaginative, and innovative piece discussing the modifiable areal unit problem. In part his treatment is concerned with the detection of spatial autocorrelation merely because of the choice of an inappropriate spatial unit of observation. The importance of recognizing the arbitrary nature of areal unit delimitations has been illustrated with a simulation experiment conducted by Openshaw and Taylor (1979).

Since the geographic landscape over which data are collected is continuous, there will be numerous alternative ways in which this landscape can be partitioned into areal units for reporting data. Although theoretically there appear to be an infinite number of ways in which a study region can be areally divided, the critical question here has to do with determining the number of groups of those elements constituting the geographical distribution under study, not slight changes in areal unit boundaries, which might affect nothing more than the shape or planar surface area of units. The number of ways of sorting r objects into s groups is given by s^r, with the number for each aggregation composition being one of the multinomial coefficients associated with an expansion of this arithmetic operation. The number of ways of sorting r objects into s non-empty groups is a Stirling number of the second kind. If the number of areal units, n, is fixed, then the possible number of distinct groupings yielding distinguishable surface partitionings may be determined. Results for the case of eight objects and either two or four areal units is presented in Table III (these figures are based upon the corrected values appearing in Griffith, 1988). Detailed calculations also are presented in this table for $n = 4$. All individual objects could fall into a single unit, leaving three units empty. Or, all individual objects could be divided between two units, leaving two units empty. Or, all individual objects could be divided among three units, leaving one unit empty. Or, all individual objects could be divided among all four units. The total number of groups is 4^8, which is obtained in Table III by summing the products of the "number of combinations" (the number of ways the items can be combined in order to produce the stated composition — a combinatorial problem) times the "classical model frequencies" (the number of ways the composition can be arranged — a permutation problem). The total number of non-empty groups is shown with the Stirling number calculations.

One apparent question to pose at this time asks whether or not this problem is of practical importance. It becomes meaningful within the context of government censuses. For example, the U.S. decennial census involves drawing a random sample of households to collect extensive information (the long form). Although this sample is drawn without replacement, meaning that the selection probabilities change with each household chosen, the final total sample is an equiprobable one (all samples of the same size are equally likely). This sample information then is aggregated into areal units (e. g., census tracts, townships, counties, SMSAs, states), which are arbitrary with respect to the sampling design used to collect the data. They do not constitute a true stratified random sample, since areal units were not used as strata in the sample design. Their only role is for aiding in tabulating and reporting the data.

Table III

The number of ways eight items can be allocated to a given number of areal units.

Number of Areal Units	Aggregation Composition	Number of Combinations	Frequencies Classical Model	Frequencies Spatial Distributions
2	8,0	1	2	2
	7,1*	8	2	2
	6,2*	28	2	2
	5,3*	56	2	2
	4,4*	70/2	1	2
4	8,0,0,0	1	4	4
	7,1,0,0	8	12	12
	6,2,0,0	28	12	12
	6,1,1,0	56/2	12	24
	5,3,0,0	56	12	12
	5,2,1,0	168	24	24
	5,1,1,1*	336/6	4	24
	4,4,0,0	70/2	6	12
	4,3,1,0	280	24	24
	4,2,2,0	420/2	12	24
	4,2,1,1*	840/2	12	24
	3,3,2,0	560/2	12	24
	3,3,1,1*	1120/4	6	24
	3,2,2,1*	1680/2	12	24
	2,2,2,2*	2520/24	1	24

Stirling numbers of the second kind for n = 4:

(1/1!) $\Sigma_{s=0}^{1}\ (-1)^{1-s}C(1,s)s^8 = 1(0 + 1) = 1$

(1/2!) $\Sigma_{s=0}^{2}\ (-1)^{2-s}C(2,s)s^8 = (1/2)(0 - 2 + 2^8) = 127 = 8 + 28 + 56 + 35$

(1/3!) $\Sigma_{s=0}^{3}\ (-1)^{3-s}C(3,s)s^8 = (1/6)(0 + 3 - 768 + 6561) = 966$
$= 28 + 168 + 280 + 210 + 280$

(1/4!) $\Sigma_{s=0}^{4}\ (-1)^{4-s}C(4,s)s^8 = (1/24)(0 - 4 + 1536 - 26244 + 65536) = 1701$
$= 56 + 420 + 280 + 840 + 105$

* denotes those groups counted by the Stirling numbers of a second kind.

Table IV

Pseudo-random numbers generated for the simulated geographic distribution.

x-coordinate (uniform distribution)	y-coordinate (uniform distribution)	attribute value (normal distribution)
14	48	9
96	45	31
56	85	13
94	55	17
84	32	31
46	69	6
93	35	31
57	56	43

Consider the geographic distribution of individual objects presented in figure 4, together with the triplet of spatial aggregations resulting from the three displayed partitionings. The eight attribute values were sampled from a normal distribution, with a mean of 25 and a variance of 100.

Next, eight Cartesian coordinates were randomly sampled, with both the X and the Y values coming from a uniform distribution lying between 0 and 100. Finally, the eight attribute values were arbitrarily assigned to the eight geo-referenced locations. The selected pseudo-random numbers are presented in Table IV. Moreover, there is no spatial auto-correlation in the process used to generate the underlying geographic distribution of objects or attribute values.

Three different judiciously chosen surface partitionings have been superimposed upon this geographic landscape, and the individual attribute values summed for objects captured by each of the four resulting areal units. Spatial aggregation scheme (a) produces a geographic distribution having MC = – 0.8903, indicating a moderate tendency for dissimilar values to cluster on the map. Spatial aggregation scheme (b) produces a geographic distribution having MC = – 0.3026 (which approximately equals – 1/3, the expected value), indicating a random spatial pattern of values. Spatial aggregation scheme (c) produces a geographic distribution having MC = 0.1724, indicating a moderate tendency for similar values to cluster on the map. Obtaining these highly different descriptions from the same disaggregated database illustrates Arbia's contention. He furnishes many more empirical and numerical examples of this type.

This partitioning problem is further complicated by scale factors, since the n areal units themselves may be combined to form new, larger areal units that embrace a greater degree of aggregation, such as the movement from n = 4 to n = 2 in Table III. This aggregation may be

achieved in a goodly number of ways, even if a contiguity constraint is invoked. Because planar surfaces are being dealt with, a maximum number of 3(n – 2) simultaneous linkages of m individuals plays a critical constraining role in this partitioning problem; once locations of individuals are established, not all possible aspatial combinations of these individuals can materialize on the map. Openshaw and Taylor (1979) illustrate how these two problems may impact upon the calculation of sample correlation coefficients, and find that a very wide range of correlations are possible, depending upon the surface partitioning used to create areal units.

The tabulated results appearing in Table III illustrate a situation in which either two or four areal units are to be formed, using an underlying set of eight individual elements. The counting of possible resulting map patterns has two components: (a) determining the number of ways r objects can be sorted into s non-empty groups, and (b) determining the number of arrangements (permutations) of the resulting n groups on a map. This permutation component affects the frequency of spatial distribution compositions, increasing the number of possible samples over that occurring in classical non-spatial sampling situations, as is illustrated in Table III. Arbia explores impacts of these areal unit changes on sample means, variances, and correlation coefficients; his results both supplement and complement those of Openshaw and Taylor (1979).

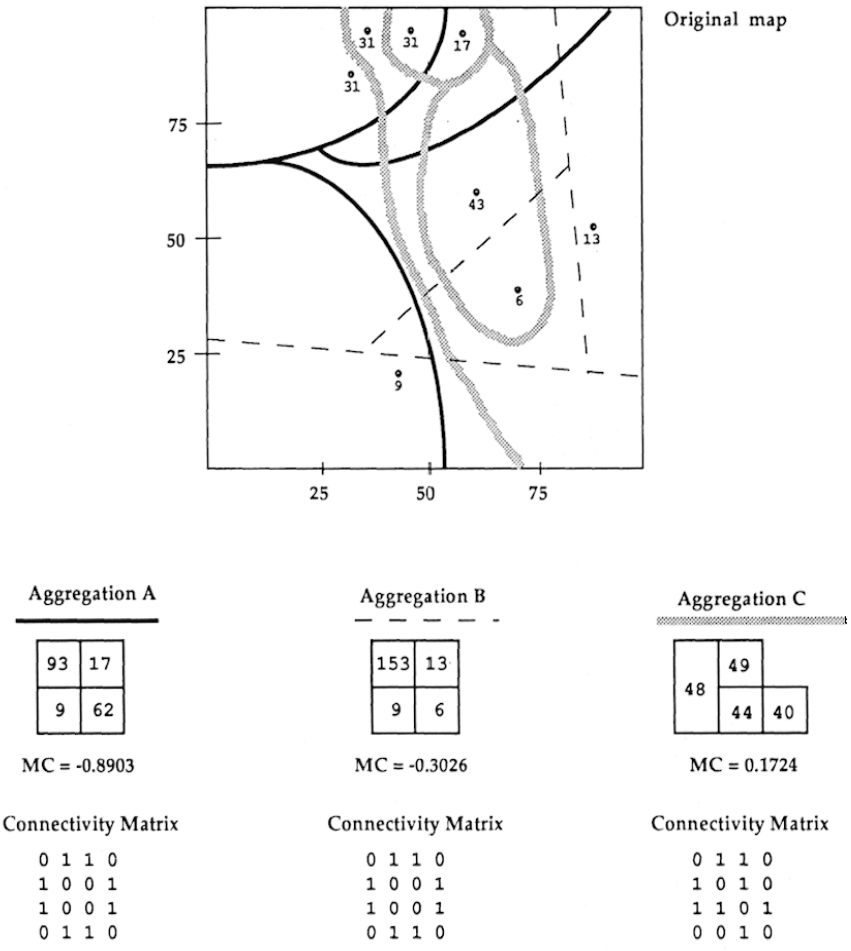


Fig. 4.— Spatial autocorrelation resulting from aggregation (original data in Table IV).

Goodchild (1989) further argues that negative spatial autocorrelation is indicative of a poor surface partitioning. In the continuous case, negative spatial autocorrelation cannot exist. To demonstrate this point, consider the two attribute values x_1 and x_2. Negative spatial autocorrelation would mean that x_1 and x_2 would be dissimilar values if situating an infinitesimal distance apart. Halving this distance would make them similar; halving the distance again would make them dissimilar. And so on. The well known limit of $(1/2)^n$ states that the two values will never be located at the same place (an infinite number of halvings would have to occur). The behavior of what happens to the relationship between x^1 and x^2 as this imperceptible distance converges upon zero indicates the illogical nature of negative spatial autocorrelation in continuous space. This contention is corroborated by the preceding counterintuitive finding that negatively spatially autocorrelated geo-referenced data have hidden information; how can data have more information than they contain?

One last example merits attention here. Spatial autocorrelation frequently is portrayed as a spillover effect. In other words, what belongs in a given areal unit somehow migrates to adjacent areal units. This characterization may well have its origins in agricultural field experiments. One problem originally faced with these plots was that fertilizer,

for instance, would be applied to them in some differential but controlled way (types and levels). Rain then would wash a portion of this fertilizer off some plots and onto their adjacent plots, while experimenters would have recorded this migrating fertilizer as being applied to its original plots. Hence, positive spillover occurred. Similar arguments can be made concerning plant competition for moisture or light, suggesting a negative spillover effect. Spatial autocorrelation techniques attempt to correct for this interaction effect. If the plots were demarcated in a way that eliminated this spillover effect, the problem would vanish. This is the class of problem addressed by Q-mode factor analysis. In either case, this is a principal topic of concern in Arbia's text.

Therefore, spatial autocorrelation may be defined as an artifact of surface partitioning, perhaps suggesting its appropriateness or inappropriateness.

Spatial processes

Spatial autocorrelation often is linked to spatial geographic processes. For example, Curry (1972) has connected it to spatial flows. Cliff *et al.* (1981) have characterized it in terms of the diffusion of disease, and Sokal and Menozzi (1982) have characterized it in terms of the diffusion of blood types. Anselin and Can (1986) have linked it to spatial organizing externalities that generate urban density gradients. Growth pole theory of regional economic development is another example of this externality field effect. Haag and Weidlich (1983) have tied it to self-organizing spatial systems. Haining (1983) has used it to depict gasoline price competition. O'Neill (1987) has coupled it with stream evolution processes (similar special linear cases include digitization of sequences of points, and sequential field measurement). In each of these situations, the phenomena in question need to be represented by nonlinear mathematical formulae, and tend to be stochastic in nature. These exemplars also tend to be consistent with defining spatial autocorrelation as a spillover effect.

The meaning of spatial autocorrelation in this setting can be referenced to the second connotation put forth by Haining *et al.* (1983). A given map is a single realization of some spatial process. If one could obtain p>>n such realizations, that were independent, then each vector of areal unit values could be treated like a variable, and the correlations between these n variables could be computed and studied. Spatial autocorrelation, then, is some common correlation amongst these areal unit variables. Once again this description is reminiscent of Q-mode factor analysis studies, where geographic distributions of various attributes are treated like repeated independent realizations of the same geographic process. Because of change through time, available resources, or feasibility constraints, in practice spatial analysts are unable to acquire bona fide multiple replications of the outcome of a single process, except when conducting simulation experiments. In some sense, then, most of spatial statistical analysis is conducted with a sample of size one.

Therefore, spatial autocorrelation may be defined as an average correlation between observations based upon replicated realizations of the geographic distribution of some attribute.

Summary and conclusions

An additional analogy helps to synthesize the foregoing discussion. One example most academics can relate to pertains to classroom testing. The instructor wishes to measure the amount of knowledge each student has of designated subject matter, in some independent fashion. Because an exhaustive testing of the material is impossible, the instructor draws a sample of questions to administer to students. Because the class is taught by the same instructor and from the same book, similarities will occur across students in test responses. These commonalities must be acknowledged, to avoid specification error, in pairwise comparisons of test answers. Groups of students that study together will have a common kernel of knowledge, which may cause their test answers to be correlated. More than likely the classroom seating choices of these groups of students will cluster. This is similar to the missing variable interpretation of spatial autocorrelation. Students who copy from their neighbors will display certain common knowledge, which will cause their test answers to be correlated. This is the spillover interpretation of spatial autocorrelation. In fact, because this cheating is geo-referenced, it constitutes a spatial autocorrelation mechanism. If two students collaborate on all answers and submit identical responses to the test, then no new information is gained once one of the test papers is graded.

This is the information content interpretation of spatial autocorrelation. Moreover, the two students should be aggregated into a single test unit (resulting in the loss of one degree of freedom).

In conclusion, a clear, simple, and concise definition of spatial autocorrelation can not be found in most of the literature on this subject. It is inherently geographical. But its meaning is contextual, and multifaceted, as treatments by Gatrell (1979) and Haining (1980) attest to. It can be defined as

(1) self-correlation attributable to the geographical ordering of data,

(2) a descriptor of the nature and degree of certain types of map pattern,

(3) an index of the information content latent in geo-referenced data, especially that information overlooked by classical statistical estimators when applied to spatial data series,

(4) a diagnostic tool for spatial model misspecification,

(5) a surrogate for unobserved geographic variables,

(6) a nuisance in applying conventional statistical methodology to spatial data series,

(7) an indicator of the appropriateness of, and possibly an artifact of, areal unit demarcation,

(8) a spatial process mechanism, and

(9) a spatial spillover effect.

This is what the literature implies is the meaning of spatial autocorrelation.

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