

Aufgaben - Woche 1

Aufgabe 1.1

$$p = \frac{nRT}{V}; n = \frac{m}{M}$$

$$p = \frac{mRT}{MV}$$

$$M = \frac{mRT}{pV} = \frac{2.55\text{g} \cdot 373.15\text{K} \cdot 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}}{101325\text{kPa}} = 78.08 \frac{\text{g}}{\text{mol}}$$

Für den Stoff mit der Formel C_6H_6 ergibt die Molmasse

$$M = 6 \cdot M(\text{C}) + 6 \cdot M(\text{H}) = 78.08 \frac{\text{g}}{\text{mol}}$$

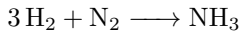
Aufgabe 1.2

Für den Druck gilt:

$$p_H = \frac{nRT}{V} = \frac{2\text{mol} \cdot 273.1\text{K} \cdot 8.135 \frac{\text{J}}{\text{mol} \cdot \text{K}}}{0.0224\text{m}^3} = 99187 \frac{\text{N}}{\text{m}^2}$$

$$p_N = 198375, 2 \frac{\text{N}}{\text{m}^2}$$

Die Reaktion läuft nach folgender Gleichung ab:



Somit ergibt der Druck nach der vollständigen Umsetzung:

$$p_{Ges} = \frac{3}{4}p_H + \frac{1}{4}p_N = 198375, 2 \frac{\text{N}}{\text{m}^2}$$

Korrekte Lösung:

-	n_H	n_N	n_{NH_3}
vor:	2 mol	1 mol	-
Reakt:	2 mol	$\frac{2}{3}$ mol	$\frac{4}{3}$ mol
nach:	-	$\frac{1}{3}$ mol	$\frac{4}{3}$ mol

$$n_{ges} = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}(\text{mol})$$

$$p_{ges} = 1,69 \cdot 10^5 \text{Pa}$$

$$x_i = \frac{n_i}{n_{ges}}$$

$$x_H = 0; x_N = \frac{\frac{1}{3}}{\frac{5}{3}}; x_{\text{NH}_3} = 0,8$$

$$p_i = x_i \cdot p_{ges}$$

Damit berechnen.

Aufgabe 1.3

a)

$$dp = \left(\frac{\partial p}{\partial V} \right)_T dV + \left(\frac{\partial p}{\partial T} \right)_V dT$$

b)

$$d^2p = \frac{2nR}{V^3} d^2V - 2 \frac{nR}{V^2} dT dV$$

Schwartzschen Satz beweisen:

$$\frac{\partial^2 p}{\partial V \partial T} = \frac{\partial^2 p}{\partial T \partial V}$$
$$-\frac{nr}{V^2} = -\frac{nr}{V^2}$$

Aufgabe 1.4

a)

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \frac{1}{\frac{nRT}{p}} \cdot \frac{nR}{p} = \frac{1}{T}$$
$$\beta = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_V = \frac{1}{\frac{nRT}{V}} \cdot \frac{nR}{V} = \frac{1}{T}$$
$$K = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T = -\frac{1}{\frac{nRT}{V}} \cdot -\left(\frac{nRT}{p^2} \right) = \frac{1}{p}$$

b)

$$\alpha = \beta K p$$
$$\frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = \left(\frac{\partial p}{\partial T} \right)_V \cdot \left(-\frac{1}{V} \right) \left(\frac{\partial V}{\partial p} \right)_T$$
$$\left(\frac{\partial V}{\partial T} \right)_p = - \left(\frac{\partial p}{\partial T} \right)_V \left(\frac{\partial V}{\partial p} \right)_T \cdot \left(\frac{\partial T}{\partial V} \right)_p$$
$$\left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial T}{\partial V} \right)_p = - \left(\frac{\partial p}{\partial T} \right)_V \left(\frac{\partial V}{\partial p} \right)_T \left(\frac{\partial T}{\partial V} \right)_p$$

Da gilt:

$$\left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_V \left(\frac{\partial V}{\partial p} \right)_T = -1; \left(\frac{\partial V}{\partial T} \right)_p \left(\frac{\partial T}{\partial V} \right)_p = 1$$

Ergibt dies:

$$1 = 1$$

Aufgabe 1.5

$$\lambda = \frac{\langle v \rangle}{z_1} = \frac{1}{\sqrt{2} \sigma \frac{N}{V}} = \frac{1}{\sqrt{2} \sigma \frac{p}{K_B T}}$$

Auf diese Gleichung kommt man mit folgenden Umformungen:

$$z_1 = \sqrt{2} \langle v \rangle \sigma \frac{N}{V}$$
$$pV = nRT | n = \frac{N}{N_A}$$
$$pV = \frac{N}{N_A} RT = N K_B T$$
$$p = \frac{N}{V} K_B T$$
$$\frac{N}{V} = \frac{p}{K_B T}$$

Somit:

$$\lambda_N = 6.76 \cdot 10^{-5} \text{ m}$$

Aufgabe 1.6

$$T_1 = 273.15 \text{ K}, T_2 = 373.15 \text{ K}$$

$$p_1 = p_2$$

$$\frac{n_1 R T_1}{V} = \frac{n_2 R T_2}{V}$$

$$n_1 T_1 = n_2 T_2$$

$$n_1 + n_2 = n = 2 \text{ mol}$$

$$n_1 T_1 = (2 - n_1) T_2$$

$$n_1 T_1 = 2 T_2 - n_1 T_2$$

$$n_1 (T_1 + T_2) = 2 T_2$$

$$n_1 \frac{2 T_2}{T_1 + T_2} = 0.845 \text{ mol}$$

$$n_2 = 2 - n_1 = 1.155 \text{ mol}$$

$$p = \frac{n_1 R T_1}{V} = 1.072 \cdot 10^5 \text{ Pa}$$

Aufgabe 1.7

$$E_{pot} = 4\varepsilon \left(\left(\frac{r_0}{r} \right)^{12} - \left(\frac{r_0}{r} \right)^6 \right)$$

$$F = \frac{dE_{pot}}{dr} = ((-12 \cdot 4\varepsilon r_0^{12} \cdot r^{-13}) - (-6 \cdot 4\varepsilon r_0^6 r^{-7}))$$

$$\frac{48\varepsilon r_0^{12}}{r^{13}} - \frac{24\varepsilon r_0^6}{r^7} = 0$$

damit:

$$0 = \frac{2r_0^{12}}{r^{13}} - \frac{r_0^6}{r^7}$$

Damit:

$$r_0^6 r^6 = 2 r_0^{12}$$

$$r = \sqrt[6]{2} r_0$$

$$r^6 = \frac{2 r_0^{12}}{r_0^6} = 2 r_0^6$$

$$r = \sqrt[6]{2} r_0$$

$$E_{pot} = -\varepsilon$$

Aufgaben - Woche 2

Aufgabe 2.1

Wird die Virialgleichung nach dem zweiten Glied abgebrochen lautet diese:

$$\frac{p V_m}{R T} = 1 + B_p p$$

Mit den ersten Werten $p = 1.013 \text{ bar}$ und $p V_m = 22.693 \frac{\text{bar}}{\text{mol}}$, ergibt sich:

$$\frac{22.693 \frac{\text{bar}}{\text{mol}}}{8.3145 \frac{\text{J}}{\text{molK}} \cdot 273 \text{ K}} = 1 + B_p \cdot 1.013 \text{ bar} \rightarrow B_p = -0.977 \text{ J}$$

p [bar]	pV_m [$\frac{\text{bar}}{\text{mol}}$]	B_p [J]
1.013	22.693	-0.9772
3.039	22.673	-0.3256
5.065	22.652	-0.1955

Mit der Gleichung

$$T_B \approx \frac{a}{bR}$$

mit den Van-der-Waals-Koeffizienten $a(\text{N}_2) = 140.8 \cdot 10^{-3} \frac{\text{Jm}^3}{\text{mol}^2}$ und $b(\text{N}_2) = 39.1 \cdot 10^{-6} \frac{\text{m}^3}{\text{mol}}$

$$T_B = 433.10 \text{ K} < 273 \text{ K}$$

Somit liegt die Messtemperatur unter der Boyletemperatur T_B .

Aufgabe 2.2

$$p = \frac{nRT}{V - nb} - a \left(\frac{n}{V} \right)^2 = \frac{1 \cdot 8.3145 \cdot 200}{0.005 - 1 \cdot 39.13 \cdot 10^{-6}} - 140.8 \cdot 10^{-3} \left(\frac{1}{0.005} \right)^2 \text{ bar} = 329.5740 \text{ kbar}$$

$$p = \frac{nRT}{V} \rightarrow p = \frac{1 \cdot 8.3145 \cdot 200}{0.005} \text{ bar} = 332.5800 \text{ kbar}$$

Da der Druck nach der VdW Gleichung kleiner ist als nach der idealen Gasgleichung ist davon auszugehen, dass die anziehenden Kräfte zwischen den Molekülen überwiegt, dafür spricht auch das typische Verhalten bei kleinem T und $B_p < 0$

Aufgabe 2.3

a)

$$p = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

$$\frac{\partial p}{\partial V_m} = 0, \frac{\partial^2 p}{\partial^2 V_m} = 0$$

$$\frac{\partial p}{\partial V_m} = \frac{RT}{(V_m - b)^2} + \frac{2a}{V_m^3} = 0 \Rightarrow \frac{2}{(V_m - b)} \frac{2a}{V_m^3} = \frac{6a}{V_m^4}$$

$$\frac{\partial^2 p}{\partial V_m^2} = \frac{2RT}{(V_m - b)^3} - \frac{6a}{V_m^4} = 0 \Rightarrow \frac{2}{V_m - b} = \frac{3}{V_m}$$

$$V_{m,krit} = 3b$$

Daraus folgt:

$$T_{krit} = \frac{2a(V_m - b)^2}{V_m^3 \cdot R} = \frac{8a}{27Rb}$$

$$p_{krit} = \frac{\frac{8a}{b}}{3b - b} - \frac{a}{9b^2} = \frac{a}{27b^2}$$

b)

$$T_{krit} = 304,01 \text{ K}$$

c)

$$\left(p_r \frac{3}{V_r^2} \right) = \frac{\frac{8T_r}{3}}{\left(V_r - \frac{1}{3} \right)}$$

$$\left(p_r \frac{3}{V_r^2} \right) \left(V_r - \frac{1}{3} \right) = \frac{8}{3} T_r$$

Aufgabe 2.4

$$p = \frac{RT}{V_m} - \frac{B}{V_m^2} + \frac{C}{V_m^3}$$

$$\begin{aligned} p' &= -RTV_m^{-2} + 2BV_m^{-3} - 3CV_m^{-4} = 0 \\ p'' &= 2RTV_m^{-3} - 6BV_m^{-4} + 12CV_m^{-5} = 0 \end{aligned}$$

Die beiden miteinander verrechnet ergibt:

$$\begin{aligned} (4B - 6B)V_m + 12C - 6C &= 0 \\ -2BV_m + 6C &= 0 \\ V_{m,krit} &= \frac{3C}{B} \end{aligned}$$

Aufgabe 2.5

$$\begin{aligned} \frac{pV_m}{RT} &= 1 + B_p p + C_p p^2 \\ \frac{pV_m}{RT} &= 1 + \frac{B_V}{V_m} + \frac{C_V}{V_m^2} \end{aligned}$$

Die zweite Gleichung nach p umgestellt ergibt:

$$\begin{aligned} p &= \frac{RT}{V_m} + \frac{B_V RT}{V_m} + \frac{C_V RT}{V_m^3} \\ \frac{pV_m}{RT} &= 1 + \frac{B_p RT}{V_m} + \frac{B_p B_V RT}{V_m^2} + \frac{B_p C_V RT}{V_m^3} + C_p \left(\frac{RT}{V_m} + \frac{B_V RT}{V_m^2} + \frac{C_V RT}{V_m^3} \right) \cdot \left(\frac{RT}{V_m} + \frac{B_V RT}{V_m^2} + \frac{C_V RT}{V_m^3} \right) \\ \frac{pV_m}{RT} &\approx 1 + \frac{B_p RT}{V_m} + \frac{B_p B_V RT}{V_m^2} + \frac{C_p (RT)^2}{V_m^2} \\ &= 1 + \frac{B_p RT}{V_m} + \frac{RT B_p B_V + C_p (RT)^2}{V_m^2} \end{aligned}$$

Somit ist $\frac{B_p RT}{V_m} = B_V$ und $\frac{RT B_p B_V + C_p (RT)^2}{V_m^2} = C_V$

$$\begin{aligned} &= B_V^2 + C_p (RT)^2 \\ C_V - B_V^2 &= C_p (RT)^2 \\ C_p &= \frac{C_V - B_V^2}{(RT)^2} \end{aligned}$$

Aufgabe 2.6

a)

$$w = - \int_{V_A}^{V_E} p dV = -nRT \ln \left(\frac{V_E}{V_A} \right) = -1 \cdot 8.3145 \cdot 273 \cdot \ln \left(\frac{0.0448}{0.0224} \right) \text{ J} = -1573,3460 \text{ J}$$

b)

$$-p_{ex} \Delta V = -\frac{RT}{V} \cdot 0.0224 \text{ m}^3 = -1135.5528 \text{ J}$$

c)

$$w = 0$$

Aufgabe 2.7

$$\begin{aligned}
 \partial w &= -p dV \\
 dV &= \left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_T dp, \quad V = \frac{nRT}{p} \\
 \left(\frac{\partial V}{\partial T}\right)_p &= \frac{nR}{p} \\
 \left(\frac{\partial V}{\partial p}\right)_T &= \frac{-nRT}{p^2} \\
 \partial w &= -p \frac{nR}{p} dT + p \frac{nRT}{p^2} dp = -nR dT + \frac{nRT}{p} dp \\
 \left(\frac{\partial w}{\partial T}\right)_p &= -nR, \quad \left(\frac{\partial w}{\partial p}\right)_T = \frac{nRT}{p} \\
 \left(\frac{\partial^2 w}{\partial T \partial p}\right) &= \frac{\partial}{\partial p} \left(\frac{nRT}{p}\right) = \frac{nR}{p} \\
 \frac{\partial^2 w}{\partial p \partial T} &= \frac{\partial}{\partial p}(-nR) = 0 \\
 \frac{\partial^2 w}{\partial T \partial p} &= \frac{\partial^2 w}{\partial p \partial T} \Rightarrow \partial w
 \end{aligned}$$

w ist keine Zustandsgröße

Woche 3

Aufgabe 3.1

a)

$$\begin{aligned}
 V &= \text{const.}, \quad w = 0 \\
 q_v &= nc_{m,v} \Delta T \\
 c_p &= c_v + nR \rightarrow c_{v,m} = c_{p,m} - R \\
 q_V &= (c_{p,m} - R)n\Delta T = 124.76 \text{ J}
 \end{aligned}$$

Es wird keine Arbeit verrichtet

b)

$$\begin{aligned}
 p &= \text{const.} \\
 q_p &= nc_{p,m} \Delta T = 207.9 \text{ J}
 \end{aligned}$$

$$w = -p \Delta V \xrightarrow{pV=nRT} -p \left(\frac{nRT_E}{p} - \frac{nRT_A}{p} \right) = -nr \Delta T = -83.14 \text{ J}$$

Aufgabe 3.2

$$\begin{aligned}
 q_K &= C \Delta T, \quad V = \text{const.} \\
 q_V &= \Delta U = -C \Delta T = n \Delta U_m = \frac{m}{M} \Delta U_m \\
 \Delta U_m &= -\frac{C \Delta T M}{m} = -3261.96 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
H &= U + pV \\
\Delta H &= \Delta U + \Delta(pV) = \Delta U + p\Delta V \rightarrow [p\Delta V = \Delta n_g RT] \rightarrow \Delta U + \Delta n_g RT \\
\Delta n_g &= (6 - 7.5) \text{ mol} = -1.5 \text{ mol} \\
\Delta H_m &= -3265.7 \cdot 10^3 \frac{\text{J}}{\text{mol}}
\end{aligned}$$

Aufgabe 3.3

$$\begin{aligned}
\mu &= \left(\frac{\partial T}{\partial p} \right)_H dH_m = \left(\frac{\partial H_m}{\partial T} \right)_p dT + \left(\frac{\partial H_m}{\partial p} \right)_T dp = c_{p,m} dT + \left(\frac{\partial H_m}{\partial p} \right)_T dp \\
\text{mit: } \left(\frac{\partial H_m}{\partial p} \right)_T &= -T \left(\frac{\partial V_m}{\partial T} \right)_p + V_m \\
c_{p,m} dT + \left[-T \left(\frac{\partial V_m}{\partial T} \right)_p + V_m \right] dp &= 0 \\
\mu &= \left(\frac{\partial T}{\partial p} \right)_H = \frac{1}{c_{p,m}} \left[T \left(\frac{\partial V_m}{\partial T} \right)_p - V_m \right]
\end{aligned}$$

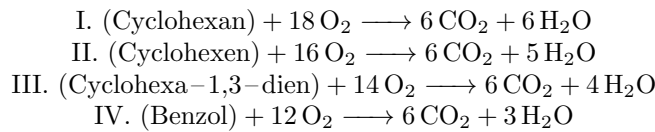
a)

$$\begin{aligned}
pV_m - pb &= RT \\
V_m &= \frac{RT}{p} + b \\
\left(\frac{\partial V_m}{\partial T} \right)_p &= \frac{R}{p} \\
\mu &= \frac{1}{c_{p,m}} \left[T \frac{R}{p} - \left(\frac{RT}{p} + b \right) \right] = -\frac{b}{c_{p,m}} < 0
\end{aligned}$$

b)

$$\begin{aligned}
pV_m &= RT + \left(b - \frac{a}{RT} \right)_p \\
V_m &= \frac{RT}{p} + b - \frac{a}{RT} \\
\left(\frac{\partial V_m}{\partial T} \right)_p &= \frac{R}{p} + \frac{a}{RT^2} \\
\mu &= \frac{1}{c_{p,m}} \left[- \left(\frac{R}{p} + \frac{a}{RT^2} \right) - \left(\frac{RT}{p} + b - \frac{a}{RT} \right) \right] \\
&= \frac{1}{c_{p,m}} \left(\frac{2a}{RT} - b \right)
\end{aligned}$$

Aufgabe 3.4



I.

$$\begin{aligned}
6 \cdot \Delta_B H_m^0(\text{CO}_2) + 6 \cdot \Delta_B H_m^0(\text{H}_2\text{O}) - \Delta_B H_m^0(\text{Cyclohexan}) \\
6 \cdot -395.5 + 6 \cdot -285.9 + 156.2 \frac{\text{kJ}}{\text{mol}} &= -3932.2 \frac{\text{kJ}}{\text{mol}}
\end{aligned}$$

Analog:

II. gegeben: $-3739.0 \frac{\text{kJ}}{\text{mol}}$

III $-3623.6 \frac{\text{kJ}}{\text{mol}}$

IV $-3279.74 \frac{\text{kJ}}{\text{mol}}$

Hydrierungsenthalpien:

II:

$$-3739 + 3932.2 \frac{\text{kJ}}{\text{mol}} = 193.2 \frac{\text{kJ}}{\text{mol}}$$

III:

$$-3623.6 + 3739 \frac{\text{kJ}}{\text{mol}} = 116 \frac{\text{kJ}}{\text{mol}}$$

IV:

$$-3279.74 + 3623.6 \frac{\text{kJ}}{\text{mol}} = 343.86 \frac{\text{kJ}}{\text{mol}}$$

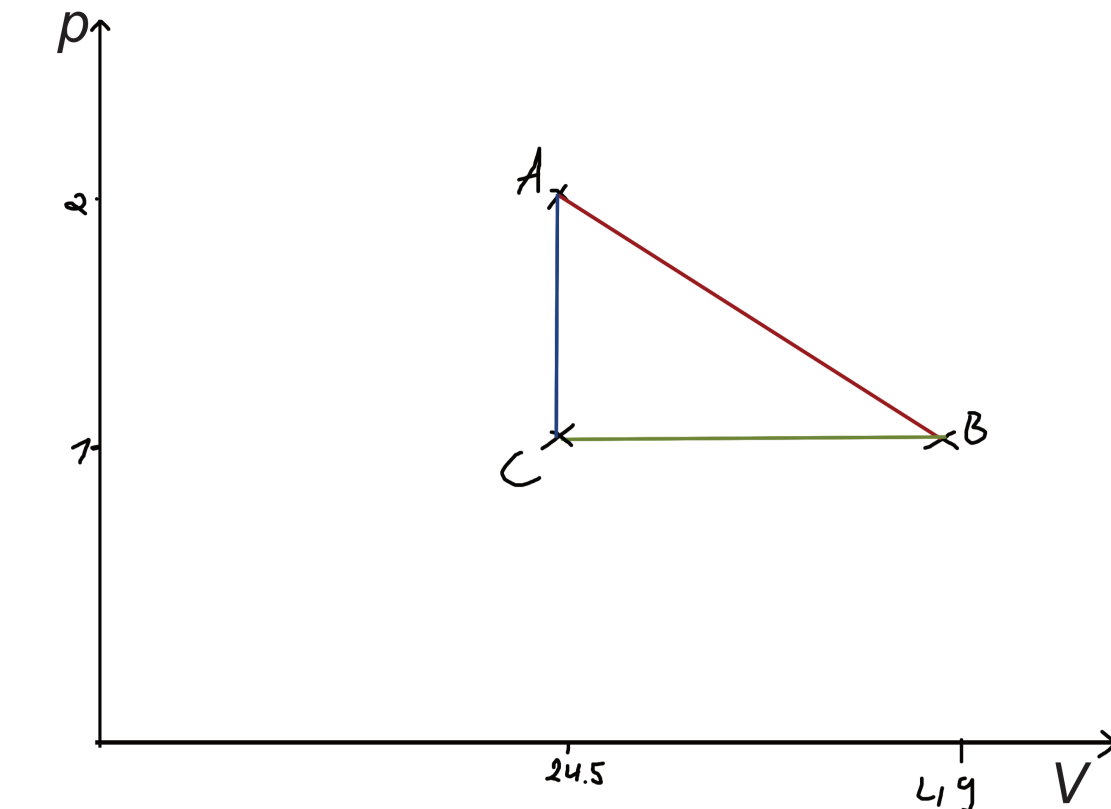
Aufgabe 3.5

$$\Delta_R H^0(600) = \Delta_R H^0(298.15) + (0.001(6 \cdot 36 + 4 \cdot 29 - 5 \cdot 29 - 4 \cdot 42)) \Delta T \frac{\text{kJ}}{\text{mol}} = -904.6 \cdot 0.019 \cdot 301.85 \frac{\text{kJ}}{\text{mol}} = -5188.0167 \frac{\text{kJ}}{\text{mol}}$$

Woche 4

Aufgabe 4.1

a)



b)

$$V_A = 24.51$$

$$V_B = 491$$

$$T_A = 298 \text{ K}$$

$$T_B = 298 \text{ K}$$

$$p_A V_A = p_B V_B (T = \text{const.})$$

$$p_B = \frac{p_A V_A}{V_B} = \frac{p_A}{2} = \frac{2 \text{ bar}}{2} = 1 \text{ bar}$$

$$B \rightarrow C$$

$$p = \text{const} p_C = 1 \text{ bar}$$

$$C \rightarrow A (q = 0)$$

$$p_A V_A^\gamma = p_C V_C^\gamma$$

$$\gamma = \frac{C_{p,m}}{C_{v,m}} = \frac{C_{V,m} + R}{C_{v,m}} = \frac{\frac{5R}{2} + R}{\frac{5R}{2}} = \frac{7}{5} = 1.4, V_C = V_A \left(\frac{p_A}{p_C} \right)^{\frac{1}{\gamma}} = 24.51 \cdot 2^{\frac{1}{1.4}} = 40.21$$

$$B \rightarrow C$$

$$\frac{V_C}{V_B} = \frac{T_C}{T_B}$$

$$T_C = \frac{T_B V_C}{V_B} = 244.5 \text{ K}$$

$$T_V = \frac{p_C V_C}{nR}$$

$$n = \frac{p_A V_A}{RT_A} = 1.978 \text{ mol}$$

c)

$$A \rightarrow B$$

$$T = \text{const}, \Delta U_1 = 0 = q_1 + w_1 \Rightarrow w_1 = -q_1$$

$$w_1 = -nRT_A \ln \frac{V_B}{V_A} = -p_A V_A \ln 2 = -2 \cdot 10^5 \text{ Pa} \cdot 24.5 \cdot 10^{-3} \text{ m}^3 \ln 2 = -3396.42 \text{ J}$$

$$q_1 = 3396.42 \text{ J}$$

$$B \rightarrow C$$

$$p = \text{const.}$$

$$w_2 = -p_B (V_C - V_B) = -10^5 \text{ Pa} (40.2 - 24.5) \cdot 10^{-3} \text{ m}^3 = 880.4 \text{ J}$$

$$q_2 = nC_{p,m}(T_C - T_B) = \frac{7}{2} \cdot 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \cdot 1.978 \text{ mol} (244.5 - 298) \text{ K} = -3081.6 \text{ J}$$

$$C_{p,m} = C_{v,m} + R = \frac{7R}{2}$$

$$C \rightarrow A$$

$$q_3 = 0, \Delta U_3 = w_3 = nC_{v,m}(T_A - T_C) = 2201.18 \text{ J}$$

d)

$$\eta = \frac{|w_{Ges}|}{q_1}$$

$$w_{Ges} = w_1 + w_2 + w_3 = -314.8 \text{ J}$$

$$\eta = \frac{314.8 \text{ J}}{3396.42 \text{ J}} = 0.093 = 9.3 \%$$

Aufgabe 4.2

$$\begin{aligned}
 dS &= \frac{\delta q_{rev}}{T}, \delta q = \frac{nRT}{V} dV \\
 dS &= \frac{1}{T} \left(\frac{nRT}{V} dV \right) \\
 \Delta S &= nR \int_{V_A}^{V_E} \frac{1}{V} dV \\
 &= nR \ln \frac{V_E}{V_A} \\
 \Delta S &= \Delta S_A + \Delta S_B \\
 &= n_A R \ln \left(\frac{V_{ges}}{V_A} \right) + n_B R \ln \left(\frac{V_{ges}}{V_B} \right), p_V = nRT \Rightarrow V = \frac{nRT}{p} \\
 &= n_A R \ln \left(\frac{n_{ges}}{n_A} \right) + n_B R \ln \left(\frac{n_{ges}}{n_B} \right) \\
 B : \Delta S &= 4.16 \frac{\text{J}}{\text{K}} \\
 &\text{Wenn nicht gleich 0, irreversibel}
 \end{aligned}$$

Aufgabe 4.3

$$\begin{aligned}
 T_W &= \frac{T_1 + T_2}{2} \\
 \Delta S_{Sys} &= \Delta S_1 + \Delta S_2 \\
 &= \int_{T_1}^{T_E} C_p dT + \int_{T_1}^{T_E} C_p dT \\
 C_P \ln \frac{T_2}{T_1} + C_p \ln \frac{T_E}{T_2} &\text{ für } T_E \text{ in vorheriger Gleichung einsetzen} \\
 C_p \ln \frac{T_E^2}{T_1 T_2} &= C_p \ln \frac{(T_1 + T_2)^2}{4 T_1 T_2} \\
 \Delta S_{Umg} &= 0 \\
 \frac{\ln (T_1 + T_2)^2}{4 T_1 T_2}
 \end{aligned}$$

Der obere Teil im ln muss größer 1 sein, damit der gesamte obere Teil des Bruches größer als 1 wird

$$\begin{aligned}
 (T_1 + T_2)^2 &> 4 T_1 T_2 \\
 T_1^2 + 2 T_1 T_2 + T_2^2 &> 4 T_1 T_2 \\
 (T_1 - T_2)^2 &> 0 \\
 T_1 &\neq T_2
 \end{aligned}$$

Aufgabe 4.4

a)

$$\begin{aligned}
 \Delta S_m &= -\Delta_{S,m} H_m^{273} = \frac{-6000 \frac{\text{J}}{\text{mol}}}{273.15 \text{ K}} = -21.97 \frac{\text{J}}{\text{mol} \cdot \text{K}} \\
 \Delta_{\text{einfrieren}} H &= -\Delta_{s,m} H \\
 \Delta S_{Umg} &= 21.97 \frac{\text{J}}{\text{mol} \cdot \text{K}} \\
 \Delta S_{Ges} &= 0
 \end{aligned}$$

b)

$$\Delta S = C_p \ln \frac{T_E}{T_A}$$

$$\begin{aligned} \Delta S &= \Delta S_{1,m} + \Delta S_{2,m} + \Delta S_{3,m} = C_{p,m} \ln \frac{T_2}{T_1} - \frac{\Delta_{s,m} H^{273}}{T_2} + C_{p,m} \ln \frac{T_1}{T_2} \\ &= 75 \frac{\text{J}}{\text{mol} \cdot \text{K}} \ln \frac{273.15}{263.15} - 21.97 \frac{\text{J}}{\text{mol} \cdot \text{K}} + 38 \frac{\text{J}}{\text{mol} \cdot \text{K}} \ln \frac{263.15}{273.15} = -20.59 \frac{\text{J}}{\text{mol} \cdot \text{K}} \end{aligned}$$

Umgebung :

$$\Delta S_{Umg} = \frac{\Delta_{s,m} H^{263}}{T_1} = \frac{5630 \frac{\text{J}}{\text{mol} \cdot \text{K}}}{263.15 \text{ K}} = 21.395 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$\Delta S_{Ges} = 0.8 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

Woche 5

Aufgabe 5.1

1) Eis -10 °C auf 0 °C

$$\begin{aligned} q_1 &= n C_{p,m} \Delta T \\ n &= \frac{m}{M} = \frac{1000}{18} = 55.556 \text{ mol} \\ q_1 &= 55.556 \text{ mol} \cdot 38 \frac{\text{J}}{\text{mol} \cdot \text{K}} \cdot 10 \text{ K} = 21.1 \text{ kJ} \end{aligned}$$

2)

$$q_2 = n \Delta H_{schmelz,s}^{Eis} = 333,3 \text{ kJ}$$

3) Wasser 0 °C auf 100 °C

$$q_3 = n C_{p,m}^{Wasser} \Delta T = 419.4 \text{ kJ}$$

4) Verdampfen

$$q_4 = n \Delta H_{verd,m}^{Wasser} = 2277.8 \text{ kJ}$$

5) Dampf 100 °C auf 150 °C

$$\begin{aligned} q_5 &= n C_{p,m}^{Dampf} \Delta T = \\ q_{\Sigma} &= q_1 + q_2 + q_3 + q_4 + q_5 = 3146 \text{ kJ} \end{aligned}$$

$$\Delta S = n C_{p,m} \ln \frac{T_E}{T_A} \text{ Erwärmen}$$

$$\Delta S = \frac{n \Delta H}{T} = \frac{q}{T} \text{ Phasenübergang}$$

$$\Delta S_1 = 78.8 \frac{\text{J}}{\text{K}}$$

$$\Delta S_2 = 1221 \frac{\text{J}}{\text{K}}$$

$$\Delta S_3 = 1309.1 \frac{\text{J}}{\text{K}}$$

$$\Delta S_4 = 6106.6 \frac{\text{J}}{\text{K}}$$

$$\Delta S_5 = 237.6 \frac{\text{J}}{\text{K}}$$

$$\Delta S_{\Sigma} = 8953.1 \frac{\text{J}}{\text{K}}$$

Aufgabe 5.2

$$C = a + bT$$

$$T_1 = 373 \text{ K}$$

$$T_2 = 573 \text{ K}$$

a)

$$\Delta H = a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2) = 22.996 \frac{\text{kJ}}{\text{mol}}$$

b)

$$\Delta S = a \ln \frac{T_2}{T_1} + b(T_2 - T_1) = 48.734 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

Aufgabe 5.3

$$\Delta S = \int_{T_1}^{T_2} \frac{\delta q}{T} = \int_{T_1}^{T_2} \frac{C_p}{T} dT$$

$$\Delta S = \frac{\Delta H}{T}$$

$$\Delta S = S_{T_S} - S_{T=0 \text{ K}} = S_{T_S}$$

$$= \int_0^{T_t} \frac{c_p dT}{T} + \frac{\Delta H_{trans}}{T_t} + \int_{T_t}^{T_i} \frac{c_p dT}{T} + \frac{\Delta H_{Schm}}{T_f} + \int_{T_f}^{T_s} \frac{c_p dT}{T} + \frac{\Delta H_{Verd.}}{T_s} = 27.2 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

$$= 27.2 \frac{\text{J}}{\text{mol} \cdot \text{K}} + \frac{229 \frac{\text{J}}{\text{mol}}}{35.61 \text{ K}} + 23.4 \frac{\text{J}}{\text{mol} \cdot \text{K}} + \frac{721 \frac{\text{J}}{\text{mol}}}{63.14 \text{ K}} + 11.4 \frac{\text{J}}{\text{mol} \cdot \text{K}} + \frac{5580 \frac{\text{J}}{\text{mol}}}{77.32 \text{ K}}$$

$$= 152 \frac{\text{J}}{\text{mol} \cdot \text{K}}$$

Aufgabe 5.4

a)

$$-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p$$

$$dG = -S dT + V dp$$

$$dG = \left(\frac{\partial G}{\partial T}\right)_p dT + \left(\frac{\partial G}{\partial p}\right)_T dp$$

$$\Rightarrow \left(\frac{\partial G}{\partial T}\right)_p = -S$$

$$\Rightarrow \left(\frac{\partial G}{\partial p}\right)_T = V$$

$$\frac{\partial}{\partial p} \left(\frac{\partial G}{\partial T}\right) = \frac{\partial}{\partial T} \left(\frac{\partial G}{\partial p}\right)$$

$$-\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p$$

b)

$$\begin{aligned}dH &= T dS + V dp \\ \frac{\partial H}{\partial p} &= T \frac{\partial S}{\partial p} + V \\ \left(\frac{\partial H}{\partial p} \right)_T &= -T \left(\frac{\partial V}{\partial T} \right)_p + V\end{aligned}$$

Aufgabe 5.5

$$G(p) = ap + b \ln p + c$$

a)

$$\begin{aligned}V(p) &= \left(\frac{\partial G}{\partial p} \right)_T \\ &= \frac{\partial}{\partial p} (ap + b \ln p + c) \\ &= a + \frac{b}{p}\end{aligned}$$

b)

$$\begin{aligned}K &= -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T \\ \left(\frac{\partial V}{\partial p} \right)_T &= \frac{\partial}{\partial p} \left(a + \frac{b}{p} \right) \\ &= -\frac{b}{p^2} \\ K &= -\frac{1}{a + \frac{b}{p}} \left(-\frac{b}{p^2} \right) \\ &= \frac{b}{ap^2 + bp}\end{aligned}$$

c)

$$\begin{aligned}A(p) \\ A &= U - TS \\ H &= U + pV \\ U &= H - pV \\ A &= H - pV - TS \\ G &= H - TS \\ A &= G - pV \\ &= ap + b \ln p + c - p \left(a + \frac{b}{p} \right) \\ &= b \ln p + c - b\end{aligned}$$

Woche 6

6.1

a) 3 Phasen - fest, flüssig, gasförmig. 1 Komponente - Natriumsulfat.

$$\begin{aligned}F &= K - P + 2 \\ F &= 1 - 3 + 2 \\ &= 0\end{aligned}$$

b) 2 Phasen - gasförmig, flüssig. 1 Komponente

$$\begin{aligned} F &= 1 - 2 + 2 \\ &= 1 \end{aligned}$$

6.2

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$$\begin{aligned} \frac{V}{\text{cm}^3} &= 1001.21 + 34.69 \left(\frac{b}{1, \frac{\text{mol}}{\text{kg}}} - 0.07 \right)^2 \\ &= 1001.21 + 34.69 \left(\frac{0.050 \frac{\text{mol}}{\text{kg}}}{1, \frac{\text{mol}}{\text{kg}}} - 0.07 \right)^2 \\ &= 1001.1961 \text{ cm}^3 \end{aligned}$$

0.050 mol MgSO₄ in 1kg Wasser haben ein Volumen von 1001.1961 cm³

$$\begin{aligned} 0.050 &\hat{=} 1001.1961 \\ 1 &\hat{=} 20023.922 \end{aligned}$$

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6.3

$$\begin{aligned} \frac{dp}{dT} &= \frac{\Delta_V H_m}{T_V \Delta_V V_m} \\ \frac{0.68 \text{ bar}}{30 \text{ K}} &= \frac{\Delta_V H_m}{373.15 \text{ K} \cdot \frac{R \cdot 373.15 \text{ K}}{1.0 \text{ bar}}} \\ \Delta_V H_m &= \frac{0.68 \text{ bar}}{30 \text{ K}} \cdot 373.15 \text{ K} \cdot \frac{8.3145 \frac{\text{J}}{\text{K} \cdot \text{mol}} \cdot 373.15 \text{ K}}{1.0 \text{ bar}} \\ \Delta_V H_m &= 26241.6227 \frac{\text{J}}{\text{mol}} \end{aligned}$$

6.4

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$$\begin{aligned} p_2 &= p_1 + \frac{\Delta_{sm} H_m}{\Delta_{sm} V_m} \left(\frac{T_2 - T_1}{T_1} \right) \\ (p_2 - p_1) \cdot \frac{\Delta_{sm} V_m}{\Delta_{sm} H_m} &= \frac{T_2 - T_1}{T_1} \\ T_2 &= (p_2 - p_1) \cdot \frac{T_1 \Delta_{sm} V_m}{\Delta_{sm} H_m} + T_1 \\ T_2 &= (1000 \text{ bar} - 1 \text{ bar}) \cdot \frac{273.15 \text{ K} \cdot 0.083 \frac{\text{kg}}{\text{dm}^3}}{6.01 \frac{\text{kJ}}{\text{mol}}} + 273.15 \text{ K} \end{aligned}$$

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