Klausur

A

a)

$$\frac{(2+i)^2}{1-i} = \frac{3+4i}{1-i} = \frac{(3+4i)(1+i)}{(1-i)(1+i)} = -\frac{1+7i}{2}$$

$$\begin{aligned} |z| &= 2 = r^2 \text{und} \arg(z) = \pi \\ 2e^{i\pi} &= 2\left(\cos\pi + i\sin\pi\right) \\ &= -2 \end{aligned}$$

b)

n	0	1	2	3	4	5
a_n	$-\frac{-1}{1}$	$\frac{0}{3}$	$-\frac{1}{5}$	$\frac{2}{7}$	$-\frac{3}{9}$	$\frac{4}{11}$

$$a_n = \frac{n-1}{2n+1} \cdot (-1)^{n+1}$$

c)

$$\begin{aligned} P_1(-1,4),\ P_2(1,4),\ P_3(2,13) \\ P_2(x) &= c_0 + c_1(x-x_0) + c_2(x_1-x_1)(x_1-x_0) \\ P_2(x_0) &= P_2(-1) = c_0 = 4 \\ P_2(x_1) &= P_2(1) = 4 + c_1(x_1-x_0) + c_2(x_1-x_1)(x_1-x_0) \\ &= 4 + c_1(1-(-1)) \\ 4 &= 4 + 2c_1 \\ c_1 &= 0 \\ \\ P_2(2) &= 4 + c_2(2-1)(2-(-1)) \\ 13 &= 4 + c_2 \cdot 1 \cdot 3 \\ 9 &= 3c_2 \\ c_2 &= 3 \\ \\ \Rightarrow P_2(x) &= 4 + 0(x+1) + 3(x-1)(x+1) \\ &= 4 + 3(x^2 - x + x - 1) \\ &= 4 + 3x^2 - 3 \\ &= 3x^2 + 1 \end{aligned}$$

d)

$$
\begin{aligned}
f(x) &= 3x^2 + 1[-\pi,\ \pi] \\
c_0 &= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \left(3x^2+1\right) r^{-i\cdot 0\cdot x}\, dx \\
&= \frac{1}{\sqrt{2\pi}} \left[x^3 + x\right]_{-\pi}^{\pi} \\
&= \frac{1}{\sqrt{2\pi}} \left[\pi^3 + \pi - \left(-\pi^3 - \pi\right)\right] \\
&= \frac{1}{\sqrt{2\pi}} \left(2\pi^3 + 2\pi\right) \\
&= \sqrt{2\pi}\left(\pi^2+1\right)
\end{aligned}
$$

$$
\begin{aligned}
c_k &= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} \left(3x^2+1\right) 3^{-ikx}\, dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-ikx}\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 3x^2 e^{-ikx}\, dx \\
f(x) = 3x^2,\ f'(x) &= 6x \\
g'(x) = e^{-ikx},\ g(x) &= \frac{e^{-}ikx}{-ik} \\
&= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-ikx}}{-ik}\right]_{-\pi}^{\pi} + \frac{1}{\sqrt{2\pi}} \left[3x^2 \frac{e^{-ikx}}{-ik}\right]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{e^{-ikx}}{-ik}\, dx \\
&= 0 + 0 - \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{3^{-ikx}}{-ik}\, dx \\
c_k &= -\frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{e^{-ikx}}{-ik}\, dx \\
f(x) = 3x^2,\ f'(x) &= 6x \\
g'(x) = e^{-ikx},\ g(x) &= \frac{e^{-}ikx}{-ik} \\
&= \frac{1}{\sqrt{2\pi} ik} \left[6x \frac{3^{-ikx}}{-ik}\right]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi} ik} \int_{-\pi}^{\pi} 6 \frac{e^{-ikx}}{-ik}\, dx \\
&= \frac{1}{\sqrt{2\pi}(-i)ik^2} \left[6xe^{-ikx}\right]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi}(-i)ik^2} \left[6 \frac{e^{-ikx}}{-ik}\right]_{-\pi}^{\pi} \\
&= \frac{6}{\sqrt{2\pi}k^2} \left[xe^{-ikx}\right]_{-\pi}^{\pi} - \frac{6}{\sqrt{2\pi}k^2} \left[\frac{e^{-ikx}}{-ik}\right]_{-\pi}^{\pi} \\
c_k &= \frac{6}{\sqrt{2\pi}k^2}(2\pi)e^{-ikx} \\
&= \frac{6\sqrt{2\pi}}{k^2}(-1)^k
\end{aligned}
$$

$$
\begin{aligned}
f(x) &= \frac{1}{\sqrt{2\pi}}\sqrt{2\pi}(\pi^2+1) + \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} \frac{6\sqrt{2\pi}(-1)^k}{k^2} e^{ikx} \\
&= \pi^2 + 1 + \sum_{-\infty}^{\infty} \frac{6(-1)^k}{k^2} e^{ikx}
\end{aligned}
$$

B

$$
\begin{aligned}
A_k &= \begin{pmatrix} k & -1 & -3 \\ 0 & -k & 2 \\ k & 0 & k \end{pmatrix} \\
&= \det(A_k) \\
&= -k(k+1)(k+2)
\end{aligned}
$$

$$
\begin{aligned}
\det(A) &\neq 0 \\
k &= 0,\ -1,\ -2
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= \begin{pmatrix} 1 & -1 & -3 \\ 0 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix} \\
&= A_1 A_1^{-1} = E
\end{aligned}
$$