

Klausur

A

a)

$$\frac{(2+i)^2}{1-i} = \frac{3+4i}{1-i} = \frac{(3+4i)(1+i)}{(1-i)(1+i)} = -\frac{1+7i}{2}$$

$$\begin{aligned}|z| &= 2 = r^2 \text{ und } \arg(z) = \pi \\ 2e^{i\pi} &= 2(\cos \pi + i \sin \pi) \\ &= -2\end{aligned}$$

b)

n	0	1	2	3	4	5
a_n	$-\frac{1}{1}$	$\frac{0}{3}$	$-\frac{1}{5}$	$\frac{2}{7}$	$-\frac{3}{9}$	$\frac{4}{11}$

$$a_n = \frac{n-1}{2n+1} \cdot (-1)^{n+1}$$

c)

$$P_1(-1, 4), P_2(1, 4), P_3(2, 13)$$

$$P_2(x) = c_0 + c_1(x - x_0) + c_2(x_1 - x_1)(x_1 - x_0)$$

$$P_2(x_0) = P_2(-1) = c_0 = 4$$

$$\begin{aligned}P_2(x_1) &= P_2(1) = 4 + c_1(x_1 - x_0) + c_2(x_1 - x_1)(x_1 - x_0) \\ &= 4 + c_1(1 - (-1))\end{aligned}$$

$$4 = 4 + 2c_1$$

$$c_1 = 0$$

$$P_2(2) = 4 + c_2(2 - 1)(2 - (-1))$$

$$13 = 4 + c_2 \cdot 1 \cdot 3$$

$$9 = 3c_2$$

$$c_2 = 3$$

$$\begin{aligned}\Rightarrow P_2(x) &= 4 + 0(x + 1) + 3(x - 1)(x + 1) \\ &= 4 + 3(x^2 - x + x - 1) \\ &= 4 + 3x^2 - 3 \\ &= 3x^2 + 1\end{aligned}$$

d)

$$\begin{aligned}
f(x) &= 3x^2 + 1[-\pi, \pi] \\
c_0 &= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} (3x^2 + 1) r^{-i \cdot 0 \cdot x} dx \\
&= \frac{1}{\sqrt{2\pi}} [x^3 + x]_{-\pi}^{\pi} \\
&= \frac{1}{\sqrt{2\pi}} [\pi^3 + \pi - (-\pi^3 - \pi)] \\
&= \frac{1}{\sqrt{2\pi}} (2\pi^3 + 2\pi) \\
&= \sqrt{2\pi} (\pi^2 + 1)
\end{aligned}$$

$$\begin{aligned}
c_k &= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} (3x^2 + 1) 3^{-ikx} dx \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{-ikx} dx + \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 3x^2 e^{-ikx} dx
\end{aligned}$$

$$\begin{aligned}
f(x) &= 3x^2, f'(x) = 6x \\
g'(x) &= e^{-ikx}, g(x) = \frac{e^{-ikx}}{-ik} \\
&= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{-ikx}}{-ik} \right]_{-\pi}^{\pi} + \frac{1}{\sqrt{2\pi}} \left[3x^2 \frac{e^{-ikx}}{-ik} \right]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{e^{-ikx}}{-ik} dx \\
&= 0 + 0 - \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{3^{-ikx}}{-ik} dx \\
c_k &= -\frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} 6x \frac{e^{-ikx}}{-ik} dx
\end{aligned}$$

$$\begin{aligned}
f(x) &= 3x^2, f'(x) = 6x \\
g'(x) &= e^{-ikx}, g(x) = \frac{e^{-ikx}}{-ik} \\
&= \frac{1}{\sqrt{2\pi}ik} \left[6x \frac{3^{-ikx}}{-ik} \right]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi}ik} \int_{-\pi}^{\pi} 6 \frac{e^{-ikx}}{-ik} dx \\
&= \frac{1}{\sqrt{2\pi}(-i)ik^2} [6xe^{-ikx}]_{-\pi}^{\pi} - \frac{1}{\sqrt{2\pi}(-i)ik^2} \left[6 \frac{e^{-ikx}}{-ik} \right]_{-\pi}^{\pi} \\
&= \frac{6}{\sqrt{2\pi}k^2} [xe^{-ikx}]_{-\pi}^{\pi} - \frac{6}{\sqrt{2\pi}k^2} \left[\frac{e^{-ikx}}{-ik} \right]_{-\pi}^{\pi} \\
c_k &= \frac{6}{\sqrt{2\pi}k^2} (2\pi) e^{-ikx} \\
&= \frac{6\sqrt{2\pi}}{k^2} (-1)^k
\end{aligned}$$

$$\begin{aligned}
f(x) &= \frac{1}{\sqrt{2\pi}} \sqrt{2\pi} (\pi^2 + 1) + \frac{1}{\sqrt{2\pi}} \sum_{-\infty}^{\infty} \frac{6\sqrt{2\pi}(-1)^k}{k^2} e^{ikx} \\
&= \pi^2 + 1 + \sum_{-\infty}^{\infty} \frac{6(-1)^k}{k^2} e^{ikx}
\end{aligned}$$