

Oscillations and Simple Harmonic Motion

$$T = \frac{1}{f}$$
$$x(t) = x_m \cos(\omega t + \phi)$$
$$\omega = \frac{2\pi}{T} = 2\pi f$$
$$v(t) = -\omega x_m \sin(\omega t + \phi)$$
$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$
$$> F = ma = m(-\omega^2 x)$$
$$> F = -kx = -(m\omega^2)x$$
$$\omega = \sqrt{\frac{k}{m}}$$

k : constante da mola
Energy in SHM

$$U(t) = \frac{1}{2} kx^2$$
$$K(t) = \frac{1}{2} mv^2$$
$$E = U + K = \frac{1}{2} kx_m^2$$

Damped SHM
 b : damping constante

$$F_d = -bv$$
$$x(t) = x_m e^{-\frac{bt}{2m}} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

if b small $\omega' \approx \omega$

$$E(t) \approx \frac{1}{2} kx_m^2 e^{-\frac{bt}{m}}$$

Waves

Transverse Waves

Luz vacuo: $c = 299792458 \text{ m/s}$

k : numero de onda

λ : comprimento de onda

$$y(x, t) = y_m \sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}$$

$$> \omega = \frac{2\pi}{T}$$

$$> f = \frac{1}{T} = \frac{\omega}{2\pi}$$

$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

Energy Pow Wave String

$$> dK = \frac{1}{2} dm u^2$$

$$P_{avg} = \frac{1}{2} \mu v \omega^2 y_m^2$$

Wave Equations

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Interference of Waves

$$y'(x, t) = y_1(x, t) + y_2(x, t)$$

$$y'(x, t) = \left[2y_m \cos\left(\frac{1}{2}\phi\right) \right] \sin\left(kx - \omega t + \frac{1}{2}\phi\right)$$

Standing Waves Resonance

$$y'(x, t) = [2y_m \sin(kx)] \cos(\omega t)$$

$$f = \frac{v}{\lambda} = n \frac{v}{2L} \quad n=1, 2, 3, \dots$$

Waves 2

Speed of Sound

$$v = \sqrt{\frac{B}{\rho}}$$

v : som no ar a 20 o C é 343 m/s

Traveling Sound Waves

$$s(x, t) = s_m \cos(kx - \omega t)$$

$$\Delta p(x, t) = \Delta p_m \sin(kx - \omega t)$$

$$\Delta p = (v\rho\omega)s_m$$

Intensity and Sound Level

$$> I = \frac{P}{A}$$

$$I = \frac{1}{2} \rho v \omega^2 s_m^2$$

$$\text{distance } I = \frac{P_s}{4\pi r^2}$$

$$\text{decibel } \beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right)$$

$$I_0 = 10^{-12} \text{ W/m}^2$$

Sources Music

$$\text{dois lados } f = \frac{v}{\lambda} = \frac{nv}{2L}$$

$$\text{um lado } f = \frac{v}{\lambda} = \frac{nv}{4L}$$

Beats

$$f_{beat} = f_1 - f_2$$

Doppler Effect

Detector: aprox +, afast -

Source: aprox -, afast +

$$f' = \frac{v \pm v_D}{v \pm v_S}$$

Supersonic Speed Shock Wave

$$\sin\theta = \frac{v_t}{v_s} = \frac{v}{v_s}$$

Electric Charge

Coulomb's Law

$$> i = \frac{dq}{dt}$$

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

Charge is Quantized

$$q = ne, n = \pm 1, \pm 2, \pm 3, \dots$$

$$e = 1.602 \times 10^{-19} \text{ C}$$

Electric Fields

$$\vec{E} = \frac{\vec{F}}{q_0} \quad (\text{N/C})$$

due to a Point

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \dots =$$

$$\vec{E}_1 + \vec{E}_2 + \dots$$

due to Electric Dipole

$$\vec{p} = qd$$

$$\text{if } z \gg d$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{p}{z^3}$$

Dipole in electric field

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Gauss's Law

Electric Flux

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Gauss's Law

$$\epsilon_0 \Phi = q_{enc}$$

σ : charge per unit area

$$E = \frac{\sigma}{\epsilon_0} \quad \text{Charged Isolated Conductor}$$

ternEF

λ : linear charge density

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad \text{Cylindrical Line}$$

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{Planar Non-conducting Sheet}$$

$$E = \frac{\sigma}{\epsilon_0} \quad \text{Planar Two Conducting Plates}$$

Spherical

Shell

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad r \geq \text{raio Shell}$$

$$E = 0 \quad r < \text{raio Shell}$$

Sphere

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r$$

Electric Potential

$$V = -\frac{W}{q}$$

$$V = \frac{-W_{\infty}}{q_0} = \frac{U}{q_0} \quad (\text{volt=joule per coulomb})$$

Units

$$1 \text{ N/C} = \left(1 \frac{\text{N}}{\text{C}}\right) \left(\frac{1 \text{ V} \cdot \text{C}}{1 \text{ J}}\right) \left(\frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}}\right) =$$

$$1 \text{ V/m} = 1 \text{ eV} = e(1 \text{ V}) =$$

$$(1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) =$$

$$1.60 \times 10^{-19} \text{ J}$$

Work done by Applied Force

$$\Delta K = K_f - K_i = W_{appF} + W_{effF}$$

$$W_{appF} = -W_{effF} \quad \text{if stationary before and after move } K_f = K_i = 0$$

$$\Delta U = U_f - U_i = W_{appF}$$

$$\Delta U = U_f - U_i = W_{appF}$$

Equipotential Surfaces

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$V = -\int_i^f \vec{E} \cdot d\vec{s} \quad \text{se } V_i = 0 \text{ ex:infinity}$$

Potential due to Point Charge

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Potential due to GroupOfPointCharges

Superposition principle, the sum of the values, unlike electric fields that was a sum of vectors

$$V = \sum_{i=1}^n (V_i) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \left(\frac{q_i}{r_i}\right)$$

Potential due to Electric Dipole

$$\vec{p} = qd$$

$$\text{if } r \gg d$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos\theta}{r^2}$$

Potential due to Continous Distribution

$$> V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda dx}{r}$$

Calculating field from potential

$$> E_s = -\frac{\partial V}{\partial s}$$

$$> E_x = -\frac{\partial V}{\partial x}; E_y = -\frac{\partial V}{\partial y}; E_z = -\frac{\partial V}{\partial z}$$

$$> E = -\frac{\Delta V}{\Delta s}$$

Electric Potential Energy System-PointCharges

$$U = W = q_2 V = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Capacitance

$$C = \frac{Q}{V} \quad 1 \text{ farad (1 F)} = 1 \text{ coulomb per volt (1 C/V)}$$

Calculating Capacitance

$$C = \frac{\epsilon_0 A}{d} : \text{placas paralelas}$$

$$C = 2\pi\epsilon_0 \frac{L}{\ln\left(\frac{b}{a}\right)} : \text{cilindros}$$

$$C = 4\pi\epsilon_0 \frac{ab}{b-a} : \text{esferas}$$

$$C = 4\pi\epsilon_0 R : \text{esfera isolada}$$

Capacitors

$$C_{eq} = \sum_{j=1}^n (C_j) : \text{in paralel, mesmo V separa q}$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^n \left(\frac{1}{C_j}\right) : \text{in series, mesmo q separa V}$$

Energy Stored Electric Field

$$U = \frac{q^2}{2C} = \frac{1}{2} CV^2$$

$$u = \frac{1}{2} \epsilon_0 E^2 : \text{energy density}$$

Capacitor with a Dielectric

If dielectric replace ϵ with $k\epsilon$, k dielectric constant

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q : \text{Gauss Law w/ Dielectric}$$

Current And Resistance

Electric Current

$$i = \frac{dq}{dt} \quad 1 \text{ ampere (1A)} = 1 \text{ coulomb per second (1C/s)}$$

Current Density

$$i = \int \vec{j} \cdot d\vec{A}$$

$$\vec{j} = (ne) \vec{v}_e : \vec{v}_e \text{ drift speed}$$

Resistance and Resistivity

$$R = \frac{V}{I} : 1 \text{ ohm (1}\Omega\text{)} = 1 \text{ volt per ampere} = 1 \text{ V/A}$$

$$\rho = \frac{1}{\sigma} = \frac{\vec{E}}{\vec{j}} : \rho \text{ Resistivity, } \sigma \text{ conductivity}$$

$$R = \rho \frac{L}{A}$$

$$p - p_0 = p_0 \alpha (T - T_0)$$

Power Potencia

$$P = iV : \text{rate eletrical energy transfer}$$

$$P = i^2 R = \frac{V^2}{R} : \text{resistive dissipation}$$

$$1 \text{ volt-ampere } 1 \text{ V} \cdot \text{A} =$$

$$\left(1 \frac{\text{J}}{\text{C}}\right) \left(1 \frac{\text{C}}{\text{s}}\right) = 1 \frac{\text{J}}{\text{s}} = 1 \text{ W } 1 \text{ watt}$$

$P = \frac{\Delta t}{\Delta t}$
Circuitos Eletricos
 Amperimetro serie, Voltmetro paralelo
Resistencias
 $R_{eq} = \sum_{j=1}^n (R_j)$: in series, mesmo i separa V
 $\frac{1}{R_{eq}} = \sum_{j=1}^n \left(\frac{1}{R_j}\right)$: in paralel, mesmo V separa i
Circuitos RC
 Reistencia Capacitor em serie
 $RC = \tau$: constante de tempo capacitiva
 $q = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$: carga
 $i = \left(\frac{\mathcal{E}}{R}\right)e^{-\frac{t}{RC}}$: carga
 $q = q_0 e^{-\frac{t}{RC}}$: descarga
 $i = -\left(\frac{q_0}{RC}\right)e^{-\frac{t}{RC}}$: descarga
Campo Magnetico
 $\vec{F}_B = q(\vec{v} \times \vec{B})$
 $F_B = |q|vB\sin(\phi)$
 Regra mao direita(pistola): polegar F_B , indicador v, palma B
 Unidade campo magnetico: Test-la(T) = $T = \frac{Wb}{m^2} = \frac{N}{C \cdot (m/s)} = \frac{N}{A \cdot m}$
 (Wb é weber) (Unidade cgs é um gauss(G) 1T=10⁴G)
ForçaParticCarregMovCircular
 $|q|vB = \frac{mv^2}{r}$
 $r = \frac{mv}{|q|B}$
 $\omega = \frac{v}{r} = \frac{|q|B}{m}$
 $f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{|q|B}{2\pi m}$
MovParticCarregCampoElecMag
 $> \vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})$
SelectorVelocidade
 $> \text{Se } F_B = F_E : v = \frac{E}{B}$ mov reto

CMag Reativo CorrenteRectilinea
 Regra mao direita: polegar corrente eletrica, dedos dobram no sentido de B ; Pontos(sairPag, cima) sao dados pela ponta dos dedos, do lado q aparecem primeiro dps de dar volta fio, outro lado cruz(entrarPag, baixo) ; Fio empurrado po lado do campo contrario ao q esta a receber
CMag Exerc Forca Corrent
 $\vec{F}_B = q(\vec{v} \times \vec{B})$ nAL n numero de cargas, A area, L comprimento
 $\vec{F} = i\vec{L} \times \vec{B}$ força sobre um fio retilineo
 $\vec{F} = i \int_a^b d\vec{L} \times \vec{B}$ força sobre um fio
CMag Exerc MomentForca Espira
 $\tau = iAB\sin(\theta)$, A area, N num espiras, θ angl normalA e B (max qd espira paralelo campo)
 $\vec{\tau} = Ni\vec{A} \times \vec{B}$, A intensidade area, sentido normalDeA
 Regra mao direita espiras (achar sentido de A): Dedos curvam sentido da corrente, polegar sentido de A
Momento Dipolo Magnetico
 $\vec{\mu} = Ni\vec{A}$
 $\vec{\tau} = \vec{\mu} \times \vec{B}$ momento de Forca
 $U(\theta) = -\vec{\mu} \times \vec{B}$ energia potencial magentica
CMag Prodz por Dipolo
 $\vec{B}(z) \cong \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$
 com $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
MagneticFields dueTo Currents
 Permeability constant(constante magnetica):
 $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A} \approx 1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}$
 $d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \vec{r}}{r^2}$ ds = dL (comprimento)

$B = \frac{\mu_0 i}{2\pi R}$ long straight wire, R distancia
 $B = \frac{\mu_0 i}{4\pi R}$ semi-infinite straight wire
 $B = \frac{\mu_0 i \phi}{4\pi R}$ center of circular arc, ϕ arc angle
 $B = \frac{\mu_0 i}{2R}$ full circle
Force between parallel wires
 $F_{ba} = i_b L B_a \sin(90^\circ) = \frac{\mu_0 L i_a i_b}{2\pi d}$
 L length, d distance, mesmo sentido atraí
Amperes Law
 $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{envolvida}$ mao aberta dedos ao longo do loop, pra frente do polegar positivo, atras negativo
 $B = \frac{\mu_0 i}{2\pi r}$ outside straight wire
 $B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$ inside striagh wire
Solenoid and Toroids
 $B = \mu_0 i n$ idela solenoid, n num turns
 $B = \frac{\mu_0 i N}{2\pi} \frac{1}{r}$ toroid, r dist centro
CurrCarryingCoil as magDipole
 $\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$ z dist centro bob
Induction Inductance
Faraday Law of Induction
 $\Phi_B = \int \vec{B} \cdot d\vec{A}$ magnetic flux pela area A
 $\Phi_B = BA$ B prependicular A
 $\mathcal{E} = -N \frac{d\Phi_B}{dt}$ faraday law, N num de espiras
Inducted Eletic Fields
 $\mathcal{E} = \oint \vec{E} \cdot d\vec{s}$
 $\oint \vec{E} \cdot d\vec{s} = -N \frac{d\Phi_B}{dt}$
Inductance
 $L = \frac{N\Phi_B}{i} \left(\text{T m}^2/\text{A}\right)$ inductance
 $\frac{1}{L} = \mu_0 n^2 A$ inductance of Solenoid, A area, l length, n num turns
 $\mathcal{E}_L = -L \frac{di}{dt}$ self-induced eletromotriz

RL Circuits
 $i = \frac{\mathcal{E}}{R} \left(1 - e^{-\frac{t}{\tau_L}}\right)$ rise of current
 $i = \frac{\mathcal{E}}{R} e^{-\frac{t}{\tau_L}} = i_0 e^{-\frac{t}{\tau_L}}$ decay of current
 $\tau_L = \frac{L}{R}$ time constant
EnergyStored MagField
 $U_B = \frac{1}{2} L i^2$ magnetic energy
EnergyDensity MagField
 $u_B = \frac{B^2}{2\mu_0}$ magEnergy density
Mutual Induction
 $\mathcal{E}_2 = -M \frac{di_1}{dt}$
 $\mathcal{E}_1 = -M \frac{di_2}{dt}$
 M medida em henries, induc-tancia mutua
Corrente Alternada
Potencia
 $i_{rms} = \frac{i}{\sqrt{2}}$ corrente rms
 $V_{rms} = \frac{V}{\sqrt{2}}$ tensao rms
 $\mathcal{E}_{rms} = \frac{\mathcal{E}}{\sqrt{2}}$ forca eletomotriz rms
 $P_{med} = i_{rms}^2 R = \mathcal{E}_{rms} i_{rms} \cos(\phi)$ potencia media
Transformadores
 $V_s = V_p \frac{N_s}{N_p}$ transformacao tensao
 $I_s = I_p \frac{N_p}{N_s}$ transformacao corrente
 $R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R_s$
MaxwellEquations
 $\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$ gauss mag field ; $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$
 maxwell law infuction ; $\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$
 ampere-maxwell law
Electrons
 $\mu_s = -\frac{e}{m} \vec{S}$ spin angular momentum, S spin ; $e = 1.60 \times 10^{-19} \text{ C}$; $m = 9.11 \times 10^{-31} \text{ kg}$; ...

Electromagnetic Waves
 $E = E_m \sin(kx - \omega t)$ electric field
 $B = B_m \sin(kx - \omega t)$ magnetic field
 $c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3.0 \times 10^8 \text{ m/s}$
 wave speed vacuum
Energy Transport
 $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ Poyting vector
 $I = \frac{1}{c\mu_0} E_{rms}^2$ intesidade da onda
 $I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}$ onda todos os lado intensidade
Radiation Pressure
 $p_r = \frac{i}{c}$ total absorption
 $p_r = \frac{2i}{c}$ total reflection
 vezes A area para dar a forca feita pela onda
Polarization
 $i = \frac{1}{2} i_0$ passar por filtro polarizador
 $i = i_0 \cos^2 \theta$ bater no filtro a um angulo
Interferencia Difracao
Interferencia
 $\delta = r_2 - r_1 = d \sin \theta$ diferenca caminho ondas, d separacao fendas, θ angulo normal do ecrã/linha q parte na normal ate ao ponto
 $\delta = d \sin \theta_{brilhante} = m\lambda$ m multiplo do comprimento de onda
 $\delta = d \sin \theta_{escura} = \left(m + \frac{1}{2}\right) \lambda$ m multiple de meio
 $y = L \tan \theta \approx L \sin \theta$ L distancia ecrã de visualizacao, L » d » λ
 $y_{brilhante} = \frac{\lambda L}{d}$ m distVert brilhantes
 $y_{escura} = \frac{\lambda L}{d} \left(m + \frac{1}{2}\right)$ distVert escura
Difracao
 $a \sin \theta = m\lambda$ minimos, a largura
 $\theta_{min} = \frac{\lambda}{a}$
Difracao Circular
 $\theta = 1,22 \frac{\lambda}{d}$ d diametro