E1 - Assignment

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$$
\begin{aligned}
\mathbb{P}(X_2 = x_2 \mid X_4 = \text{thin}) &= \frac{\mathbb{P}(X_4 = \text{thin} \mid X_2 = x_2)\mathbb{P}(X_2 = x_2)}{\mathbb{P}(X_4 = \text{thin})} \\
&\propto \mathbb{P}(X_4 = \text{thin} \mid X_2 = x_2)\mathbb{P}(X_2 = x_2) \\
&= \mathbb{P}(X_4 = \text{thin} \mid X_2 = x_2) \sum_{x_1} \mathbb{P}(X_2 = x_2, X_1 = x_1) \\
&= \mathbb{P}(X_4 = \text{thin} \mid X_2 = x_2) \sum_{x_1} \mathbb{P}(X_2 = x_2 \mid X_1 = x_1)\mathbb{P}(X_1 = x_1)
\end{aligned}
$$

$$\mathbb{P}(X_1 = x_1 | X_3 = \text{medium}, X_4 = \text{thin}) = \frac{\mathbb{P}(X_1 = x_1, X_3 = \text{medium}, X_4 = \text{thin})}{\mathbb{P}(X_3 = \text{medium})\mathbb{P}(X_4 = \text{thin} | X_3 = \text{medium})}$$

$$\propto \mathbb{P}(X_1 = x_1, X_3 = \text{medium}, X_4 = \text{thin})$$

$$= \mathbb{P}(X_1 = x_1)\mathbb{P}(X_3 = \text{medium} | X_1 = x_1)$$

$$\mathbb{P}(X_4 = \text{thin} | X_3 = \text{medium}, X_1 = x_1)$$

$$\mathbb{P}(X_4 = \text{T} | X_3 = \text{M}, X_1 = x_1) = \sum_{x_2} \mathbb{P}(X_4 = \text{T}, X_2 = x_2 | X_3 = \text{M}, X_1 = x_1)$$

$$= \sum_{x_2} \mathbb{P}(X_4 = \text{T}, X_2 = x_2 | X_3 = \text{M}, X_1 = x_1)$$