Computer Simulation Problems 2

1. Consider the system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 \\ -2 & -4.5 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

$$\mathrm{y} = [1 \quad 0]\boldsymbol{x}$$

Determine if the system is controllable and observable. Compute the transfer function from u to y.

2. The following model has been proposed to describe the motion of a constant-velocity guided missile:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -0.1 & -0.5 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0.5 & 1 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u,$$

$$\mathrm{y} = [0 \quad 0 \quad 0 \quad 1 \quad 0]\boldsymbol{x}$$

(a) Verify that the system is not controllable by analyzing the controllability matrix using the **ctrb** function.

(b) Develop a controllable state variable model by first computing the transfer function from u to y, then cancel any common factors in the numerator and denominator polynomials of transfer function. With the modified transfer function just obtained, use the **ss** function to determine a modified state variable model for the system.

(c) Verify that the modified state variable model in part (b) is controllable.

(d) Comment on the relationship between controllability and complexity of the state variable model (where complexity is measured by the number of sate variables).

3. A linearized model of a vertical takeoff and landing(VTOL) aircraft is:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B_1}u_1 + \boldsymbol{B_2}u_2,$$

Where

$$\mathbf{A} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

And

$$\boldsymbol{B_1} = \begin{bmatrix} 0.4422 \\ 3.5446 \\ -5.5200 \\ 0 \end{bmatrix}, \boldsymbol{B_2} = \begin{bmatrix} 0.1761 \\ -7.5922 \\ 4.4900 \\ 0 \end{bmatrix}$$

The state vector components are

(i) x_1 is the horizontal velocity(knots);

(ii) x_2 is the vertical velocity (knots);

(iii) x_3 is the pitch rate (degrees/second);

(iv) x_4 is the pitch angle(degrees).

The input u_1 is used mainly to control the vertical motion, and u_2 is for the horizontal motion.

(a) Compute the eigenvalues of the system matrix **A**.Is the system stable?

(b) Determine the characteristic polynomial associated with **A** using the **poly** function. Compute the roots of the characteristic equation, and compare with the eigenvalues in part (a).

(c) Is the system controllable from u_1 alone? What about from u_2 alone? Comment on the results.

4. In an effort to open up the far side of the moon to exploration, studies have been conducted to determine the feasibility of operating a communication satellite around the translunar equilibrium point in the earth-sun-moon system. The desired satellite orbit, known as a halo orbit, is shown in Fig. 1. the objective of the controller is to keep the satellite on a halo orbit trajectory that can be seen from the earth so that the lines of communication are accessible at all the times. The communication link is from the earth to the satellite and then to the far side of the moon.

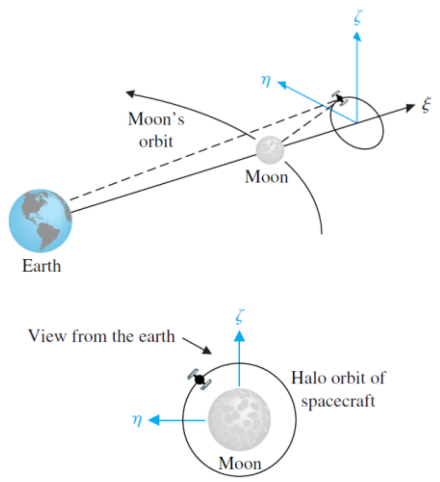

Fig. 1 The translunar satellite Halo orbit

The linearized (and normalized) equations of motion of the satellite around the translunar equilibrium point are:

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.3809 & 0 & 0 & 0 & 2 & 0 \\ 0 & -2.1904 & 0 & -2 & 0 & 0 \\ 0 & 0 & -3.1904 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_3$$

The state vector $\boldsymbol{x}$ is the satellite position and velocity, and the inputs u_i, i=1, 2, 3, are the engine thrust accelerations in the ζ, η and ξ directions respectively.

(a) Is the translunar equilibrium point a stable location?

(b) Is the system controllable from u_1 alone?

(c) Repeat part (b) for u_2.

(d) Repeat part (b) for u_3.

(e) Suppose that we can observe the position in the η direction. Determine the transfer function from u_2 to η .(Hint: Let y=$[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1]\boldsymbol{x}$)

(f) Compute a state-space representation of the transfer function in part (e) using the **ss** function. Verify that the system is controllable.

(g) Using state feedback

$$u_2 = -\boldsymbol{Kx},$$

Design a controller (i.e., find **K**) for the system in part (f) such that the closed-loop system poles are at $s_{1,2} = -1 \pm j \; and \; s_{3,4} = -10$.