Exponnential Regression

I – General expression of the model:

$$\boxed{f(x)=a\,e^{-bx} \qquad \forall\, x\in IR}$$

Here we want to find a, b . Our data is vector: $Y=[y_0, y_1, \cdots y_{n-1}]$ with n values.

II – Average:

So the average is: $\overline{Y}=\frac{1}{n}\sum_{i=0}^{n-1} y_i=\frac{1}{n-1}\int_{x=0}^{n-1} f(x)\,dx$

$$\frac{1}{n}\sum_{i=0}^{n-1} y_i = \frac{1}{n-1}\int_{x=0}^{n-1} a\,e^{-bx}\,dx = \frac{1}{n-1}\left[\frac{-a}{b}e^{-bx}\right]_0^{n-1} = \frac{a}{b(n-1)}\left(1-e^{-b(n-1)}\right)$$

So:

$$\boxed{\mu = \frac{1}{n}\sum_{i=0}^{n-1} y_i = \frac{a}{b(n-1)}\left(1-e^{-b(n-1)}\right)}$$

III – Variance not centered:

The momentum of order 2 is: $m_2=\frac{1}{n}\sum_{i=0}^{n-1} y_i^2=\frac{1}{n-1}\int_{x=0}^{n-1} (f(x))^2\,dx$

$$m_2 = \frac{1}{n-1}\int_{x=0}^{n-1} a^2 e^{-2bx}\,dx = \frac{1}{n-1}\left[\frac{-a^2}{2b}e^{-2bx}\right]_0^{n-1} = \frac{a^2}{2b(n-1)}\left(1-e^{-2b(n-1)}\right)$$

So:

$$\boxed{m_2 = \frac{1}{n}\sum_{i=0}^{n-1} y_i^2 = \frac{a^2}{2b(n-1)}\left(1-e^{-2b(n-1)}\right)}$$

IV – Solve the system:

$$\mu = \frac{a}{b(n-1)}\left(1-e^{-b(n-1)}\right) \quad \text{and} \quad m_2 = \frac{a^2}{2b(n-1)}\left(1-e^{-2b(n-1)}\right)$$

So:

$$\boxed{a = \frac{\mu\, b(n-1)}{1-e^{-b(n-1)}}} \quad \text{and} \quad \boxed{\frac{\mu^2 b(n-1)\left(1-e^{-2b(n-1)}\right)}{2m_2\left(1-e^{-b(n-1)}\right)^2}-1 = 0}$$

We need to find a b that satisfied the second equation. A solution to solve this problem is to use the Newton-Raphson algorithm. When the b is found, we have the a parameter with the first equation.