

Imaginary time Schrödinger equation

$$\partial_t \Psi = -\hat{H}\Psi$$

$$\psi(x, t) = \rho(x, t) / \sqrt{\rho_0(x)}$$

(caveat)

Feynman path \int

Holland Cost

$$C[v] = \int dx \left[\frac{1}{2} (\nabla \psi)^2 + V(X_t) \psi^2 \right]$$

Feynman-Kac formula

$$\psi(x, t) = \mathbb{E}_{\mathbb{P}} \left[e^{-\int_0^t V(X_\tau, \tau) d\tau} \psi(X_0, 0) \right]$$

Minimized for drift
 $v(x) = \nabla \psi / \psi$ and
 $\pi = \psi^2$

Fokker-Planck

$$\frac{\partial \rho}{\partial t} = \mathcal{L}_{\text{FP}} \rho = \frac{\partial}{\partial x} \left[\frac{\partial \rho}{\partial x} + U'(x) \rho \right]$$

Defines new
path measure \mathbb{P}_{FK}

Minimal cost
achieved when sam-
pling from optimal
drift \equiv FK paths

Defines SDE

SDE

$$dX_t = dW_t + v(X_t)dt$$

Radon-Nikodyn derivative

$$\frac{d\mathbb{P}_{\text{FK}}}{d\mathbb{P}_0} = \mathcal{N} \exp \left(- \int V(X_t) dt \right)$$