In[◦]:=

```
(*Problem 1.1:Functions f(x) and g(x)*)(*Define f(x) and g(x)*)
f[x_] := Sin[2 ArcSin[x]] + Tan[3 ArcTan[2 x]]
g[x_] := 3 Sin[Pi / x] + Cos[3 Pi / x]

(*Construct (f[x]/g[x]]+g[x]*)
h[x_] := (f[x] / g[x] + g[x])

(*Evaluate at x=1/2 to 4 dec places*)
mainresult = h[1 / 2]
resultapp = N[h[1 / 2], 4]
{h[x_], resultapp}
```

Out[◦]=

$$\frac{\sqrt{3}}{2}$$

Out[◦]=

0.8660

Out[◦]=

$$\left\{\text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right] + \frac{\text{Sin}[2\,\text{ArcSin}[\text{x\_}]] + \text{Tan}[3\,\text{ArcTan}[2\,\text{x\_}]]}{\text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right]},\ 0.8660\right\}$$

In[◦]:=

$$\text{Sort}\left[\left\{\text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right] + \frac{\text{Sin}[2\,\text{ArcSin}[\text{x\_}]] + \text{Tan}[3\,\text{ArcTan}[2\,\text{x\_}]]}{\text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right]},\ 0.8660\right\}\right]$$

Out[◦]=

$$\left\{0.8660,\ \text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right] + \frac{\text{Sin}[2\,\text{ArcSin}[\text{x\_}]] + \text{Tan}[3\,\text{ArcTan}[2\,\text{x\_}]]}{\text{Cos}\left[\frac{3\pi}{\text{x\_}}\right] + 3\,\text{Sin}\left[\frac{\pi}{\text{x\_}}\right]}\right\}$$

```
(*Problem 1.2:Factorizing the Polynomial and Solving for z*)

(*Define the polynomial*)
poly = 6 x^3 + x^2 y - 11 x y^2 - 6 y^3 - 5 x^2 z + 11 x y z + 11 y^2 z - 2 x z^2 - 6 y z^2 + z^3

(*Factorize the polynomial*)
factoredPoly = Factor[poly]

(*Solve poly=0 for z*)
solutionsPoly = Solve[poly == 0, z]

(*Display data results *)
(factoredPoly, solutionsPoly)
```

Out[4]= $6 x^3 + x^2 y - 11 x y^2 - 6 y^3 - 5 x^2 z + 11 x y z + 11 y^2 z - 2 x z^2 - 6 y z^2 + z^3$

Out[5]= $(x + y - z)(3 x + 2 y - z)(2 x - 3 y + z)$

Out[6]= $\{\{z \to x + y\}, \{z \to 3 x + 2 y\}, \{z \to -2 x + 3 y\}\}$

Syntax: "(" cannot be followed by "factoredPoly, solutionsPoly)".

In[∘]:=
```
(*Problem 1.3:Solving the System of Equations*)

(*Define the system*)
eqns = {
  w - 3 x + y - 2 z == 1,
  w + 2 x - y + 4 z == -3,
  -w + x - y + z == 0
 }

(*Solve the system for {x,y,z}*)
solutionsSys = Solve[eqns, {x, y, z}]

(*Substitute w=0,1,-1 and solve*)
solutions0 = solutionsSys /. w → 0
solutions1 = solutionsSys /. w → 1
solutions_1 = solutionsSys /. w → -1
```

Out[∘]=

$\{w - 3x + y - 2z == 1, w + 2x - y + 4z == -3, -w + x - y + z == 0\}$

Out[∘]=

$\left\{\left\{x \to \frac{2w}{5}, y \to \frac{1}{5}(-5 - 7w), z \to \frac{1}{5}(-5 - 4w)\right\}\right\}$

Out[∘]=

$\{\{x \to 0, y \to -1, z \to -1\}\}$

Out[∘]=

$\left\{\left\{x \to \frac{2}{5}, y \to -\frac{12}{5}, z \to -\frac{9}{5}\right\}\right\}$

••• Set: Tag Times in 1 solutions_ is Protected.

Out[∘]=

$\left\{\left\{x \to -\frac{2}{5}, y \to \frac{2}{5}, z \to -\frac{1}{5}\right\}\right\}$

```
(*Problem 1.4:Analyzing the Function f(x)=x^3-2x^2-3x+1*)
(* 1.4.1*)
(*Define f(x)*)
ClearAll[x, y]
f4[x_] := x^3 - 2 x^2 - 3 x + 1

(*Find critical points*)
Solve[f4'[x] == 0 , x]
(*Numerical values*)
numericalCriticalPoints = NSolve[f4'[x] == 0, x]
Critpoints = {f4''[x]} /. numericalCriticalPoints
{Critpoints}
```

Out[ ]=

$$\left\{\left\{x \to \frac{1}{3}\left(2 - \sqrt{13}\right)\right\}, \left\{x \to \frac{1}{3}\left(2 + \sqrt{13}\right)\right\}\right\}$$

Out[ ]=

{{x → -0.535184}, {x → 1.86852}}

Out[ ]=

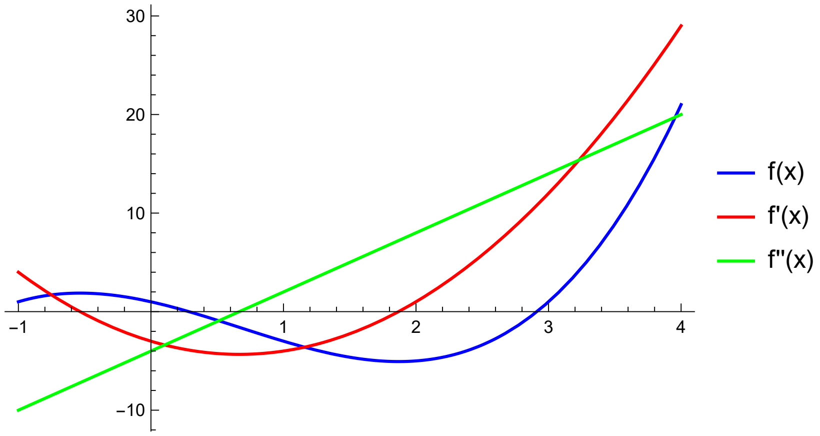
{{-7.2111}, {7.2111}}

Out[ ]=

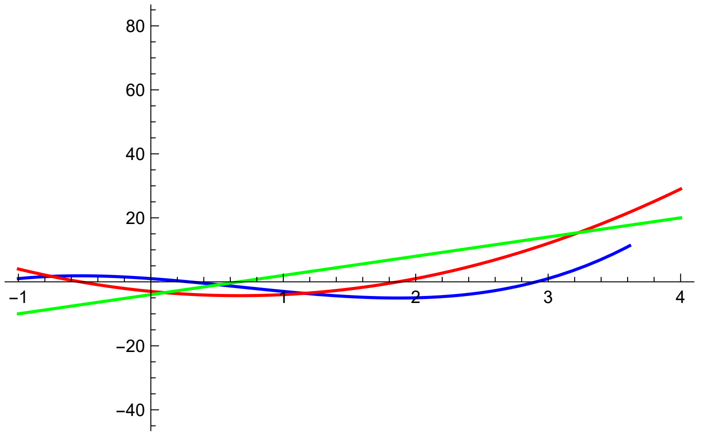
{{{-7.2111}, {7.2111}}}

```
(* 1.4.2*)
(*Plot f(x),f'(x),and f''(x) with different colors*)
Plot[{f4[x], f4'[x], f4''[x]}, {x, -1, 4},
 PlotLegends → {"f(x)", "f'(x)", "f''(x)"}, PlotStyle → {Blue, Red, Green}]

(*Combined graph with specified PlotRange*)
Show[Plot[f4[x], {x, -1, 4}, PlotStyle → Blue],
 Plot[f4'[x], {x, -1, 4}, PlotStyle → Red],
 Plot[f4''[x], {x, -1, 4}, PlotStyle → Green], PlotRange → {-40, 80}]
(*1.4.3*)
(*Calculate the definite integral over the interval[1,4]*)
integral = Integrate[f4[x], {x, 1, 4}]

(*Numerical approximation to 3 significant digits*)
numericalIntegral = N[integral, 3]
```

Out[•]=



Out[•]=



Out[•]=

$$\frac{9}{4}$$

Out[•]=

2.25

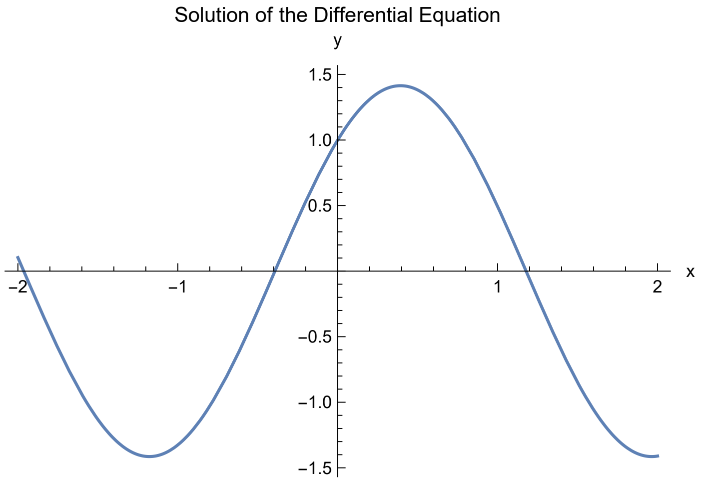
```
(*Problem 2*)

(*2.2*)
```

In[•]:=
```
(*Define the differential equation and initial condition*)
deqn = y'[x] == -2 y[x] + 4 Cos[2 x];
initialCondition = y[Pi / 4] == 1;

(*Solve the differential equation*)
solution = DSolve[{deqn, initialCondition}, y[x], x]

(*Simplify the solution*)
simplifiedSolution = y[x] /. solution[[1]]

(*Plot the solution on the interval[-2,2]*)
Plot[simplifiedSolution, {x, -2, 2},
 PlotLabel → "Solution of the Differential Equation", AxesLabel → {"x", "y"}]
```

Out[•]=
```
{{y[x] → Cos[2 x] + Sin[2 x]}}
```

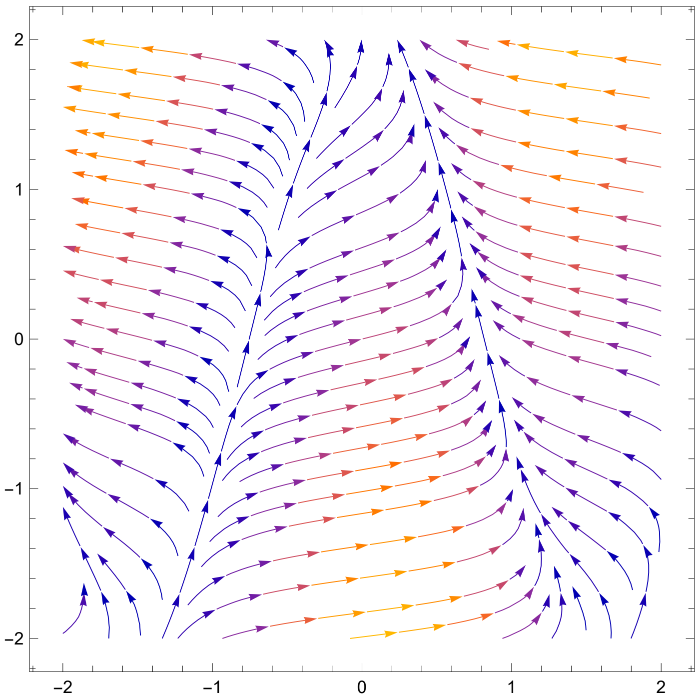
Out[•]=
```
Cos[2 x] + Sin[2 x]
```

Out[•]=

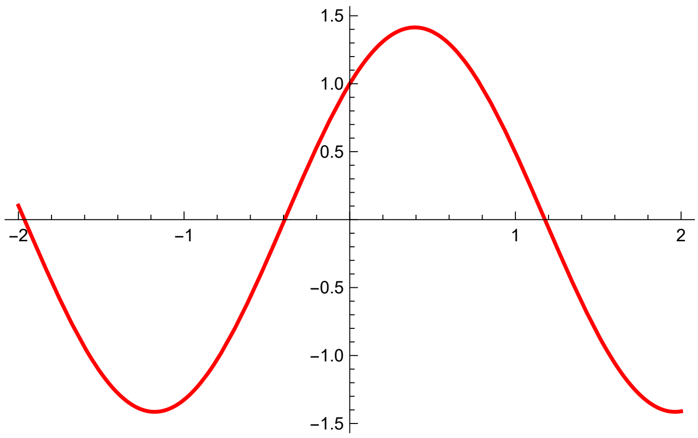


In[ ]:=
```
(*2.3*)
(*Define the vector field*)


vectorField = StreamPlot[{-2 y + 4 Cos[2 x], 1},
  {x, -2, 2}, {y, -2, 2}, StreamStyle → Arrowheads[0.02]]

particularSolution =
 Plot[Evaluate[y[x] /. DSolve[{y'[x] == -2 y[x] + 4 Cos[2 x], y[0] == 1}, y[x], x]],
  {x, -2, 2}, PlotStyle → {Thick, Red}]

Show[vectorField, particularSolution,
 PlotLabel → "Vector Field with Particular Solution"]
```

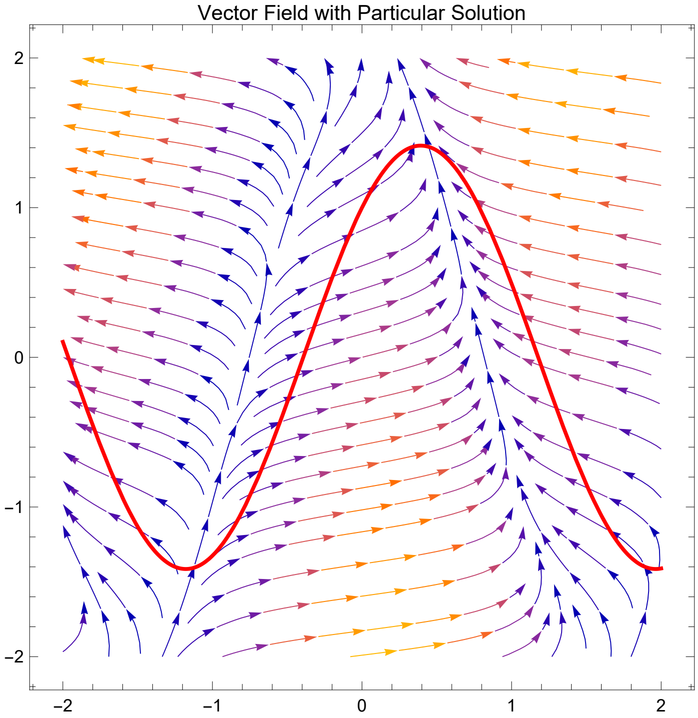
Out[ ]=



Out[ ]=



Out[ ]=



In[ ]:=
```
(*Problem 6.3*)
(*Define the matrix B*)B = {{0, 1}, {-1, 4}};
Eigenvalues[B]
Eigenvectors[B]


(*Define the vector field for the system*)
vectorField = {B[[1, 1]] * p + B[[1, 2]] * q, B[[2, 1]] * p + B[[2, 2]] * q};

(*Visualize the phase portrait*)
StreamPlot[vectorField, {p, -5, 5}, {q, -5, 5}, StreamPoints → Fine, PlotRange → All,
 AxesLabel → {"p", "q"}, Epilog → {Red, PointSize[Large], Point[{0, 0}]}]
```

Out[ ]=

$\{2 + \sqrt{3}, 2 - \sqrt{3}\}$

Out[ ]=

$\{\{2 - \sqrt{3}, 1\}, \{2 + \sqrt{3}, 1\}\}$

Out[ ]=