Name - Blen Daniel Assefa

ICS 2020 Problem Sheet #3 Answers

Problem 3.1: Cartesian products

a) (A n B) x (C n D) = (A x C) n (B x D)

- ⇨ (x, y) ∈ (A n B) and (C n D)
- ⇨ x ∈ (A n B) and y ∈ (C n D)
- ⇨ This would imply x ∈ A and x ∈ B, and also y ∈ C and y ∈ D

To show if (A x C) n (B x D):

- ⇨ (x ∈ A and y ∈ C) and (x ∈ B and y ∈ D)
- ⇨ (x, y) ∈ (A x C) n (x, y) ∈ (B x D)
- ⇨ (x, y) ∈ (A x C) n (B x D)

Therefore (A n B) x (C n D) = (A x C) n (B x D)

b) (A u B) x (C u D) = (A x C) u (B x D)

(A u B) x (C u D)

- ⇨ {(x ,y) | x ∈ (A u B) and y ∈ (C u D) }
- ⇨ {(x ,y) | x ∈ A or x ∈ B and y ∈ C or y ∈ D) }

Four alternatives to what this means:

1st:

- ⇨ {(x ,y) | (x ,y) (A x C) or (x ,y) (B x D) }

  This would proof that the statement is actually correct. But we also have the other possibility since its and Or statement.

2nd:

- ⇨ {(x ,y) | (x ,y) (B x C) or (x ,y) (A x D) }

3rd:

- ⇨ {(x ,y) | (x ,y) (B x D) or (x ,y) (A x C) }

  This would proof that the statement is actually correct.

4th:

- ⇨ {(x ,y) | (x ,y) (A x D) or (x ,y) (B x C) }

This proofs that it is not in fact the same.

Therefore: (A u B) x (C u D) != (A x C) u (B x D)

Problem 3.2: Reflexive, symmetric, transitive

a) $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) | \mathrm{a}, \mathrm{b} \in \mathbb{Z} \wedge |a - b| \leq 3\}$

i. **R is reflexive**

This is because there could exist an element in the relation that could have equal values. Since the condition for the relation is for a and b to be an integer and their absolute difference to be less than or equal to 3, then the condition will always be satisfied in the case of a and b having the same value.

ii. **R is Symmetric**

R is symmetric because since the difference is in absolute value, (a, b) could easily be (b, a). i.e. the value would be the same.

iii. **R is not Transitive**

Transitive by definition is when $(\mathrm{a}, \mathrm{b}) \in R\ and\ (\mathrm{b}, \mathrm{c}) \in R, then\ (\mathrm{a}, \mathrm{c}) \in R$.

This could not happen for this relation as the absolute difference of a and b if it exists in the relation and the absolute difference of b and c, if it exists in the relation, then this does not necessarily mean the absolute difference between a and c might exist in the relation.

b) $\mathrm{R} = \{(\mathrm{a}, \mathrm{b}) | \mathrm{a}, \mathrm{b} \in \mathbb{Z} \wedge (a\ mod\ 10) = (b\ mod\ 10)\}$

i. **R is reflexive**

R is reflexive because there could exist an element in the relation that could have equal values.

ii. **R is Symmetric**

R is symmetric because for every a and b the relation could also have values with (b, a) that satisfy the condition.

iii. **R is Transitive**

R is transitive as a mod 10 = b mod 10 will mean that for whatever value b mod 10 is equal to, a mod 10 will also be equal to that number.

Problem 3.2: Proof by induction

Note: S and t are lists.
Proof of cnt x (con s t) == (cnt x s) + (cnt x t)

First lets just take the smallest possible value for s which is an empty list.
S = []
For the left side equation:
cnt x (con [] t)

    Con [] t = t --- from the Haskell code

    Therefore:

        cnt x (con [] t) = cnt x t

For the right-side equation:
cnt x [] + cnt x t
cnt x [] is 0 --- from the Haskell code
Therefore:

    (cnt x s) + (cnt x t) = cnt x t

The proof holds for the smallest possible value.
The next smallest value I will take for s is a 1 item list and I will call it n.

For the left side equation:
cnt x (con [n] t)

    con [n] t = (n : t) --- from the Haskell code

    Therefore:

        cnt x (con [n] t) = cnt x (n: t)

    if x happens to be equal to n, then:

        1 + cnt x t

    Else:

        0 + cnt x t

For the right-side equation:
cnt x [n] + cnt x t
if x happens to be equal to n, then:

        1 + cnt x t

    Else:

0 + cnt x t

This led me to conclude that cnt x (con s t) == (cnt x s) + (cnt x t) is true.