CH-232-A

Answers to ICS 2020 Problem Sheet #8

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1.

a) $(\neg A \uparrow \neg B) \uparrow \big((A \uparrow B) \uparrow C\big)$ $\qquad \uparrow\ is\ NAND$

$= \neg((\neg(\neg A \wedge \neg B)) \wedge (\neg(\neg(A \wedge B) \wedge C)))$

$= \neg\Big((A \vee B) \wedge \big((A \wedge B) \vee \neg C\big)\Big)$

$= \neg(A \vee B) \vee \neg\big((A \wedge B) \vee \neg C\big)$

$\therefore (\neg A \wedge \neg B) \vee (\neg(A \wedge B) \wedge C)$

b)

A	B	C	$\neg A$	$\neg B$	$\neg C$	$(\neg A \uparrow \neg B)$	$(A \uparrow B)$	$\big((A \uparrow B) \uparrow C\big)$	$(\neg A \uparrow \neg B) \uparrow \big((A \uparrow B) \uparrow C\big)$
0	0	0	1	1	1	0	1	1	1
0	0	1	1	1	0	0	1	0	1
0	1	0	1	0	1	1	1	1	0
0	1	1	1	0	0	1	1	0	1
1	0	0	0	1	1	1	1	1	0
1	0	1	0	1	0	1	1	0	1
1	1	0	0	0	1	1	0	1	0
1	1	1	0	0	0	1	0	1	0

minterms are:

$m_0, m_1, m_3\ and\ m_5$

Minterm	Pattern	Used	Minterm	Pattern
m_0	000	✓	$m_{0,1}$	00-
m_1	001	✓	$m_{1,3}$	0-1
m_3	011	✓	$m_{1,5}$	-01
m_5	101	✓		

$Sum\ of\ minterms\ = (\neg A \wedge \neg B) \vee (\neg A \wedge C) \vee (\neg B \wedge C)$

Deriving it algebraically,

$= (\neg A \wedge \neg B) \vee (\neg(A \wedge B) \wedge C)$

$= (\neg A \wedge \neg B) \vee (\neg A \vee \neg B) \wedge C)$

$\therefore (\neg A \wedge \neg B) \vee ((\neg A \wedge C) \vee (\neg B \wedge C))$

2.

$$S = (A \dot{\vee} B \dot{\vee} C)$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

a)

	A	B	C_{IN}	C_{OUT}	S
$\boldsymbol{m_0}$	0	0	0	0	0
$\boldsymbol{m_1}$	0	0	1	0	1
$\boldsymbol{m_2}$	0	1	0	0	1
$\boldsymbol{m_3}$	0	1	1	1	0
$\boldsymbol{m_4}$	1	0	0	0	1
$\boldsymbol{m_5}$	1	0	1	1	0
$\boldsymbol{m_6}$	1	1	0	1	0
$\boldsymbol{m_7}$	1	1	1	1	1

I will consider the output where it is 1 for Disjunctive of Product terms.

Disjunctive of Product terms is:

$Carry_{out}(A, B, C_{in}) = \boldsymbol{m_3} + \boldsymbol{m_5} + \boldsymbol{m_6} + \boldsymbol{m_7}$

$Carry_{out}(A, B, C_{in}) = (\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$Sum\ (A, B, C_{in}) = \boldsymbol{m_1} + \boldsymbol{m_2} + \boldsymbol{m_4} + \boldsymbol{m_7}$

$Sum\ (A, B, C_{in}) = (\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

b)

I will consider the output where it is 0 for Conjunction of Sum terms.

Conjunction of Sum terms is:

$Carry_{out}(A, B, C_{in}) = \boldsymbol{M_0.M_1.M_2.M_4}$

$Carry_{out}(A, B, C_{in}) = (A \vee B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (\neg A \vee B \vee C_{in})$

$Sum\ (A, B, C_{in}) = \boldsymbol{M_0.M_3.M_5.M_6}$

$Sum\ (A, B, C_{in}) = (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in}) \wedge (\neg A \vee \neg B \vee C_{in})$

c)

For Disjunctive of Product terms:

$\boldsymbol{Since}\ \neg x = x \uparrow x\ then,$

$x \wedge y = (x \uparrow y) \uparrow (x \uparrow y) = \neg(x \uparrow y)$

$x \vee y = (x \uparrow x) \uparrow (y \uparrow y) = \neg x \uparrow \neg x$

For the carry:

$Carry_{out}(A, B, C_{in}) = \boldsymbol{m_3} + \boldsymbol{m_5} + \boldsymbol{m_6} + \boldsymbol{m_7}$

$Carry_{out}(A, B, C_{in}) = (\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$\boldsymbol{m_3} = (\neg A \wedge B \wedge C_{in})$

$\boldsymbol{m_5} = (A \wedge \neg B \wedge C_{in})$

$\boldsymbol{m_6} = (A \wedge B \wedge \neg C_{in})$

$\boldsymbol{m_7} = (A \wedge B \wedge C_{in})$

$Carry_{out}(A, B, C_{in}) = (\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$= \neg(\neg((\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})))$

$= \neg(\neg(\neg A \wedge B \wedge C_{in}) \wedge \neg(A \wedge \neg B \wedge C_{in}) \wedge \neg(A \wedge B \wedge \neg C_{in}) \neg(A \wedge B \wedge C_{in}))$

$= \neg(\neg A \uparrow B \uparrow C_{in}) \wedge (A \uparrow \neg B \uparrow C_{in}) \wedge (A \uparrow B \uparrow \neg C_{in}) \wedge (A \uparrow B \uparrow C_{in})$

$= (\neg A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})$

$\therefore\ (\neg A \uparrow B \uparrow C_{in}) \uparrow (\neg A \wedge \neg B \wedge C_{in}) \uparrow (A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})$

For the sum:

$Sum\ (A, B, C_{in}) = \boldsymbol{m_1} + \boldsymbol{m_2} + \boldsymbol{m_4} + \boldsymbol{m_7}$

$Sum\ (A, B, C_{in}) = (\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$\boldsymbol{m_1} = (\neg A \wedge \neg B \wedge C_{in})$

$\boldsymbol{m_2} = (\neg A \wedge B \wedge \neg C_{in})$

$\boldsymbol{m_4} = (A \wedge \neg B \wedge \neg C_{in})$

$\boldsymbol{m_7} = (A \wedge B \wedge C_{in})$

$Sum\ (A, B, C_{in}) = (\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$= \neg(\neg\big((\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})\big))$

$= \neg\Big(\big((\neg A \wedge \neg B \wedge C_{in}) \wedge (\neg A \wedge B \wedge \neg C_{in}) \wedge (A \wedge \neg B \wedge \neg C_{in}) \wedge (A \wedge B \wedge C_{in})\big)\Big)$

$= \neg\big((\neg A \uparrow \neg B \uparrow C_{in}) \wedge (\neg A \uparrow B \uparrow \neg C_{in}) \wedge (A \uparrow \neg B \uparrow \neg C_{in}) \wedge (A \uparrow B \uparrow C_{in})\big)$

$= (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (\neg A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})$

$\therefore\ (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (\neg A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})$

d)

$$S = (A \dot{\vee} B \dot{\vee} C)$$

$$C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \dot{\vee} B))$$

$\boldsymbol{Since}\ \neg x = x \uparrow x\ then,$

$x \wedge y = (x \uparrow y) \uparrow (x \uparrow y) = \neg(x \uparrow y)$

$x \vee y = (x \uparrow x) \uparrow (y \uparrow y) = \neg x \uparrow \neg x$

$x \dot{\vee} \boldsymbol{y} = \big(x \uparrow (x \uparrow y)\big) \uparrow (y \uparrow (x \uparrow y))$

$Sum\ (A, B, C_{in}) = \big((A \dot{\vee} B) \dot{\vee} C\big)$

$= (\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)) \dot{\vee} C$

$\therefore \Big(\big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big) \uparrow ((\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)) \uparrow C)\Big) \uparrow \big(C \uparrow ((\big(A \uparrow (A \uparrow B)\big) \uparrow$
$\big(B \uparrow (A \uparrow B)\big)) \uparrow C)\big)$

$C_{out}(A, B, C_{in}) = (A \wedge B) \vee \big(C_{in} \wedge (A \dot{\vee} B)\big)$

$= \big((A \uparrow B) \uparrow (A \uparrow B)\big) \vee \Big(C \wedge \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big)$

$= \big((A \uparrow B) \uparrow (A \uparrow B)\big) \vee \Big(\Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big) \uparrow \Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big)\Big)$

$= \Big(\big((A \uparrow B) \uparrow (A \uparrow B)\big) \uparrow \big((A \uparrow B) \uparrow (A \uparrow B)\big)\Big) \uparrow \bigg(\Big(\Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big) \uparrow$
$\Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big)\Big) \uparrow \Big(\Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow \big(B \uparrow (A \uparrow B)\big)\big)\Big) \uparrow \Big(C \uparrow \big(\big(A \uparrow (A \uparrow B)\big) \uparrow$
$\big(B \uparrow (A \uparrow B)\big)\big)\Big)\Big)\bigg)$