CH-232-A

Answers to ICS 2020 Problem Sheet #7

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$$\varphi(A,B,C,D,E) = m_0 + m_2 + m_4 + m_6 + m_9 + m_{10} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{21} + m_{26} + m_{28} + m_{30} + m_{31}$$

a)

Classification and sorting minterms:

Minterm	Pattern	Used
m_0	00000	
m_2	00010	
m_4	00100	
m_{16}	10000	
m_6	00110	
m_9	01001	
m_{10}	01010	
m_{17}	10001	
m_{13}	01101	
m_{14}	01110	
m_{21}	10101	
m_{26}	11010	
m_{28}	11100	
m_{15}	01111	
m_{30}	11110	
m_{31}	11111	

Combination Steps

Minterm	Pattern	Used	Minterm	Pattern	Used	Minterm	Pattern	Used
m_0	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0	
m_2	00010	✓	$m_{0,4}$	00-00	✓	$m_{2,6,10,14}$	0--10	
m_4	00100	✓	$m_{0,16}$	-0000		$m_{10,14,26,30}$	-1-10	
m_{16}	10000	✓	$m_{2,6}$	00-10	✓	$m_{14,15,30,31}$	-111-	
m_6	00110	✓	$m_{2,10}$	0-010	✓			
m_9	01001	✓	$m_{4,6}$	001-0	✓			
m_{10}	01010	✓	$m_{16,17}$	1000-				
m_{17}	10001	✓	$m_{6,14}$	0-110	✓			
m_{13}	01101	✓	$m_{9,13}$	01-01				
m_{14}	01110	✓	$m_{10,14}$	01-10	✓			
m_{21}	10101	✓	$m_{10,26}$	-1010	✓			
m_{26}	11010	✓	$m_{17,21}$	10-01				

m_{28}	11100	✓	$m_{13,15}$	011-1				
m_{15}	01111	✓	$m_{14,15}$	0111-	✓			
m_{30}	11110	✓	$m_{14,30}$	-1110	✓			
m_{31}	11111	✓	$m_{26,30}$	11-10	✓			
			$m_{26,30}$	111-0				
			$m_{15,31}$	-1111	✓			
			$m_{30,31}$	1111-	✓			

This gives us 10 prime implicants:

$m_{0,16} = -000 = \neg B \wedge \neg C \wedge \neg D \wedge \neg E$

$m_{16,17} = 100- = A \wedge \neg B \wedge \neg C \wedge \neg D$

$m_{9,13} = 01 - 01 = \neg A \wedge B \wedge \neg D \wedge E$

$m_{17,21} = 10 - 01 = A \wedge \neg B \wedge \neg D \wedge E$

$m_{13,15} = 011 - 1 = \neg A \wedge B \wedge C \wedge E$

$m_{28,30} = 111 - 0 = A \wedge B \wedge C \wedge \neg E$

$m_{0,2,4,6} = 00 - -0 = \neg A \wedge \neg B \wedge \neg E$

$m_{2,6,10,14} = 0 - -10 = \neg A \wedge D \wedge \neg E$

$m_{10,14,26.30} = -1 - 10 = B \wedge D \wedge \neg E$

$m_{14,15,30,31} = -11- = B \wedge C \wedge D$

b)

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}
$m_{0,16}$	✓									✓						
$m_{16,17}$										✓	✓					
$m_{9,13}$					✓		✓									
$m_{17,21}$											✓	✓				
$m_{13,15}$							✓		✓							
$m_{28,30}$														✓	✓	
$m_{0,2,4,6}$	✓	✓	✓	✓												
$m_{2,6,10,14}$		✓		✓		✓		✓								
$m_{10,14,26,30}$						✓		✓					✓		✓	
$m_{14,15,30,31}$								✓	✓						✓	✓
Used	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Columns that only have a single marked cell indicate essential prime implicants.

Such that, the columns are with respect to the essential prime implicants are:

$\boldsymbol{m_4 : m_{0,2,4,6}}$

$\boldsymbol{m_9 : m_{9,13}}$

$\boldsymbol{m_{21} : m_{17,21}}$

$\boldsymbol{m_{26} : m_{10,14,26,30}}$

$\boldsymbol{m_{28} : m_{28,30}}$

$\boldsymbol{m_{31} : m_{14,15,30,31}}$

To include 16:

$\boldsymbol{m_{16,17}}$

c) The resulting minimal expression is:

$$\boldsymbol{\varphi(A,B,C,D,E)} = (\neg \boldsymbol{A} \wedge \neg \boldsymbol{B} \wedge \neg \boldsymbol{E}) \vee (\neg \boldsymbol{A} \wedge \boldsymbol{B} \wedge \neg \boldsymbol{D} \wedge \boldsymbol{E}) \vee (\boldsymbol{A} \wedge \neg \boldsymbol{B} \wedge \neg \boldsymbol{D} \wedge \boldsymbol{E}) \vee \boldsymbol{B} \wedge \boldsymbol{D} \wedge \neg \boldsymbol{E} \vee (\boldsymbol{A} \wedge \boldsymbol{B} \wedge \boldsymbol{C} \wedge \neg \boldsymbol{E}) \vee (\boldsymbol{B} \wedge \boldsymbol{C} \wedge \boldsymbol{D}) \vee (\boldsymbol{A} \wedge \neg \boldsymbol{B} \wedge \neg \boldsymbol{C} \wedge \neg \boldsymbol{D})$$