CS 3305 HW-3

Question 1:

(a.) $a_0 = 0$

$a_1 = 0$

$$\boxed{a_n = a_{n-1} + a_{n-2} + 2^{n-2} \text{ for } n \geq 2}$$

$\underline{00} = 1$

$--- = \cancel{3}\,2 \; > 1$

$----- = 3 \; > 1$

$------ = 4 \; > 1$

(b.) $\boxed{a_0 = a_1 = 0 \quad a_2 = 1 \quad \text{Base Case}}$

(c.) $a_7 = a_6 + a_5 + 2^5 = 43 + 19 + 32 = \boxed{94}$

$a_6 = a_5 + a_4 + 2^4 = 19 + 8 + 16 = 43$

$a_5 \quad a_3 + a_4 + 2^3 = 3 + 8 + 8 = 19$

$a_4 = a_3 + a_2 + 2^2 = 3 + 1 + 4 = 8$

$a_3 = a_1 + a_2 + 2^1 = 1 + 0 + 2 = 3$

$a_2 = a_0 + a_1 + 2^0 = 1$

Question 2:

(a) using same logic as Q1↑

$$\boxed{a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}}$$

(b) $\boxed{a_0 = a_1 = a_2 = 0 \quad a_3 = 1}$

(c) $a_7 = a_6 + a_5 + a_4 + 2^4 = \boxed{47}$

$a_6 = a_5 + a_4 + a_3 + 2^3 = 20$

$a_5 = a_4 + a_3 + a_2 + 2^2 = 8$

$a_4 = a_3 + a_2 + a_1 + 2^1 = 3$

$a_3 = a_2 + a_1 + a_0 + 2^0 = 1$

Question 3:

(a)

$$\boxed{a_n = 2a_{n-1} + 2a_{n-2} + 3^{n-2}}$$

(b) $\boxed{a_0 = a_1 = 0 \quad a_2 = 1}$

(c) $a_6 = 2a_5 + 2a_4 + 3^4 = \boxed{281}$

$a_5 = 2a_4 + 2a_3 + 3^3 = 79$

$a_4 = 2a_3 + 2a_2 + 3^2 = 21$

$a_3 = 2a_2 + 2a_1 + 3^1 = 5$

$a_2 = 2a_1 + 2a_0 + 3^0 = 1$

$a_0 = 0$,

$a_1 = 0,1,2 = 3$

$a_2 = 0,1,2 \,,\, 0,1,2 = 9$

$a_3 = (0,1,2)\cdot(0,1,2)\cdot(0,1,2) = 27$

$\left\{ \begin{matrix} 00 \\ 01 \\ 02 \\ 10 \\ 11 \\ 12 \\ 20 \\ 21 \\ 22 \end{matrix} \right\} \Rightarrow a_0 = a_1 = 0$

$a_2 =$

$$\begin{array}{r} 42 \\ +10 \\ +27 \\ \hline 79 \end{array}$$

Question 4:

(a.) Linear Homogeneous, constant coefficients 3, 4, 5. Degree of 7.

(c) Linear Homogeneous, constant coefficients of 1, Degree 4.

(e.) Non-Linear

(g.) Linear, but not homogeneous because of 'n'

Question 5:

(a) $a_n = 2a_{n-1}$ for $n \geq 1$, $a_0 = 3$

$r - 2$ = character equ

☆ K distinct roots $\Rightarrow a_n = \beta_1 r_1^n$

$3 = \beta 2^0$, $3 = \beta \rightarrow \boxed{a_n = 3 \cdot 2^n}$

(c) $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

$r^2 - 5r + 6 = 0$ (characteristic equ)

$(r-2)(r-3) \to r = 2, 3$

$b^2 - 4ac = 5^2 - 4(1)(6) = 25 - 24 = 1$

det is greater than $0 \to r_1 \neq r_2$

$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$

- for $a_0 = 1$

$1 = \alpha_1 r_1^n + \alpha_2 r_2^n = a_0$

$1 = \alpha_1 r_1^0 + \alpha_2 r_2^0$

$1 = \alpha_1 2^0 + \alpha_2 r_3^0$

$1 = \alpha_1 + \alpha_2$

$\alpha_1 = 1 - \alpha_2$

$\alpha_1 = 1 - (-2)$

$\alpha_1 = 3$

- for $a_1 = 0$

$a_1 = \alpha_1 r_1^n + \alpha_2 r_2^n = 0$

$0 = \alpha_1 2 + \alpha_2 3$

$0 = (1 - \alpha_2)^2 + \alpha_2 3$

$0 = 2 - 2\alpha_2 + 3\alpha_2$

$0 = 2 + \alpha_2 \Rightarrow \alpha_2 = -2$

$\boxed{a_n = 3 \cdot 2^n + -2 \cdot 3^n}$

(e.) $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2$, $a_0 = 0$, $a_1 = 1$

$r^2 + 4r + 4$ (characteristic equ)

$(r+2)(r+2) \Rightarrow r = -2$, multiplicity 2

$\det(r^2 + 4r + 4) = B^2 - 4AC = 4^2 - 4(1)(4) = 0 \to a_n = \alpha_1 r_0^n + \alpha_2 r_0^n$

• $a_0 = 0$

$a_0 = \alpha_1 r_0^n + \alpha_2 r_0^n$

$0 = \alpha_1(-2)^0 + \alpha_2(-2)^0$

$0 = \alpha_1 + \alpha_2$

$\alpha_1 = -\alpha_2$

• $a_1 = 1$

$a_1 = \alpha_1 r_0^n + \alpha_2 r_0^n$

$1 = \alpha_1(-2)^1 + \alpha_2(-2)^1$

$1 = -2\alpha_1 + -2\alpha_2$

$1 = -2(\alpha_1 + \alpha_2)$

$-1/2 = \alpha_1 + \alpha_2$

$-1/2 = -\alpha_2 + \alpha_2$

• $a_0 = 0$

$a_0 = \alpha_1 r_0^n + \alpha_2 n r_0^n$

$0 = \alpha_1(-2)^0 + \alpha_2(0)(-2)^0$

$0 = \alpha_1$

• $a_1 = 1$

$a_1 = \alpha_1 r_0^n + \alpha_2 n r_0^n$

$1 = \alpha_1(-2)^1 + \alpha_2(1)(-2)^1$

$1 = -2\alpha_1 + -2\alpha_2$

$1 = -2(0) + -2\alpha_2$

$1 = -2\alpha_2$

$\alpha_2 = -1/2$

$a_n = 0(-2)^n + (-1/2)(-2)^n(n) = \boxed{-\frac{n}{2}(-2)^n}$

$\boxed{a_n = -\frac{n}{2}(-2)^n}$

(9.) $\frac{a_{n-2}}{4}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$

$a_n = \frac{1}{4}(a_{n-2})$

$a_n = r^2 - \frac{1}{4}$

$(r - \frac{1}{2})(r + \frac{1}{2}) = 0$

$r = 1/2, -1/2$

$a_n = \alpha_1 (r_0)^n + \alpha_2 (r_1)^n$

$a_n = \alpha_1 (\frac{1}{2})^n + \alpha_2 (-1/2)^n$

• $a_0 = 1$

$1 = \alpha_1 + \alpha_2$

• $a_1 = 0$

$0 = \frac{1}{2}\alpha_1 - 1/2\,\alpha_2 \Rightarrow \alpha_1 = 1/2 = \alpha_2$

$\boxed{a_n = (\frac{1}{2})(1/2)^n + (1/2)(-1/2)^n}$