CS4384 : Automata Theory
Homework Assignment 6

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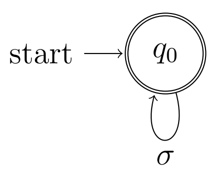
1. Convert the following CFG to a PDA : S $\rightarrow$ aSb | ϵ
   We start by forming transition functions based of the given cfg.

   - δ(q, e, S) $\rightarrow$ { (q, aSb), (q, e) }
   - δ(q, a, a) $\rightarrow$ { (q, e)}
   - δ(q, b, b) $\rightarrow$ { (q, e)}

   - Where σ = the transition functions listed above.

2. Given the PDA below, convert it to a CFG and give the derivation of *aaabbb*
   We can form our transitions as follows:

   - δ(0, e, e) $\rightarrow$ (1 ,$)
   - δ(1, a, e) $\rightarrow$ (1, a)
   - δ(1, b, a) $\rightarrow$ (2, e)
   - δ(2, b, a) $\rightarrow$ (2, e)
   - δ(2, e, $) $\rightarrow$ (3, e)

   We can construct the CFG below:

   - A_{03} = aA_{12}b
   - A_{12} = aA_{12}b | ab

   Our string *aaabbb* derivation is given below:
   $A_{03} \rightarrow aA_{12}b \rightarrow aaA_{12}bb \rightarrow aaabbb$

3. Give a informal description of a Turing machine that decides the language L = { w | w contains an equal number of 0s and 1s }

   Let M be a turing machine <M,w> where:
   m = "on input w
   1. Determine if W is a member of 0^*1^*. Reject if not.
   2. Sweep from left to right and cross off 2 uncrossed positions. Accept if 0 or two positions were crossed off. Reject if only one position was crossed off."

   This turing machine accepts L.

4. Give a informal description of a Turing machine that decides the language L = { w | w contains twice as many 0s as 1s }

   Let M be a turing machine <M,w> where:
   m = "on input w
   1. Determine if W is a member of 0^*1^*. Reject if not.
   2. Sweep from left to right and cross off a single 0 and two 1s. If you cannot cross off exactly a single 0 and two 1s and no other elements are uncrossed, then accept. Otherwise, reject."

   This turing machine accepts L.

5. Show that the collection of decidable languages is closed under the operation of complementation.

   Let M be a turing machine of a decidable language L. Let R be a turing machine that decides the complement of L.

   Define R s.t. where: "On input w
   1. Run M on w. If it accepts, reject.
   2. Otherwise, accept."

   This turing machine shows that a decidable language is closed under complementation.

6. Let INFINITE$_{DFA}$ = { < A > | A is a DFA and L(A) is an infinite language }. Show that INFINITE$_{DFA}$ is decidable.

   Let M be a turing machine s.t. M = "On input $< A >$ where A is a DFA:
   1. Let p be the number of states in A.
   2. Construct a DFA D that accepts all strings of length k or more (meaning there must be some loop).
   3. Construct a DFA Z s.t. L(Z) = L(A) $\cap$ L(D).
   4. Test whether L(B) = $\emptyset$ using E$_{DFA}$ T proved in class.
   5. If T accepts, reject. If T rejects, accept.

   This turing machine shows that INFINITE$_{DFA}$ is decidable.

7. Let T = { < w > | M is a TM that accepts w^r whenever it accepts w }. Show that T is undecidable.

   Let R decide T. Construct a TM S s.t. S = "On input $< M, w >$
   1. Create a TM Q s.t Q = "On input x
      1. If x does not have the form 01 or 10, reject
      2. If x has the form 01, accept
      3. Else run w on M and accept if M accepts"
   2. Run R on Q.
   3. Accepts if R accepts, reject if R rejects."

   Since S decides A$_{TM}$ (since we have reduced it to this) which we have proven to be undecidable, we know that T is undecidable.