CS4384 : Automata Theory
Homework Assignment 2

Matthew McMillian
mgm160130@utdallas.edu

September 25, 2018

1. Let L = { $0^i 1^j 0^k \mid k > i + j$ }. Use the pumping lemma to show that L is not regular.

   Assume L is regular and let p be it's pumping length. Define S such that $S = 0^p 1^p 0^{2p+1}$, such that |S| $> p$ and that $S \in$ L. Let $S = xyz$ be a partitioning satisfying $|xy| \leq$ p, and $|y| > 0$. Since we know $|y| > 0$, we know that y contains only zeros. Since y contains some number of zeros equal to $|y|$ = k for some k $\in$ (0,p], we can conclude that xy contains 0^{p-k} zeros. Let $i = 2$ such that $S' = xy^iz = xy^2z = xyyz = 0^{p-k}0^k0^k1^p0^{2p+1}$. Simplifying we obtain $0^{p+k}1^p0^{2p+1}$. This contradicts the pumping lemma, since we have come up with a string S' with pumping length p that is NOT in the language L since $k < i + j$ for S'. Thus, this language is not regular.

2. Let L = { $w \mid w \in \{0, 1\}^*$ w is not a palindrome }. Use the pumping lemma to show that L is not regular.

   By closure properties of regular languages, if L is regular then $\overline{L}$ is also regular. Take for consideration the compliment of L, such that $\overline{L}$ = { $w \mid w \in \{0, 1\}^*$ w is a palindrome }. Assume $\overline{L}$ is regular and let p be it's pumping length. Define S such that S = $0^p1^p0^p$, such that |S| $> p$ and that $S \in \overline{L}$. Let $S = xyz$ be a partitioning satisfying $|xy| \leq$ p, and $|y| > 0$. Since we know $|y| > 0$, we know that y contains only zeros. Since $y$contains some number of zeros equal to $|y|$ = k for some k $\in$ (0,p], we can conclude that xy contains 0^{p-k} zeros. Let $i = 2$ such that $S' = xy^iz = xy^2z = xyyz = 0^{p-k}0^k0^k1^p0^p$. Simplifying we obtain $0^{p+k}1^p0^p$. This contradicts the pumping lemma, since we have come up with a string S' with pumping length p that is NOT in the language $\overline{L}$ since S' is not a palindrome because k is strictly greater than zero. Thus since $\overline{L}$ is not regular, the language L is not regular.

3. Let L = $\{ \ 0^m 1^n \mid m \neq n \ \}$. Use closure properties of regular languages to prove that L is not regular.

   By closure properties of regular languages, if L is regular then $\overline{L}$ is also regular. Take for consideration the compliment of L, such that $\overline{L} = \{ \ 0^m 1^n \mid m = n \ \}$. Define S such that $S = 0^p 1^p$, such that |S| $> p$ and that $S \in \overline{L}$. Assume $\overline{L}$ is regular and let p be its pumping length. Let $S = xyz$ be a partitioning satisfying $|xy| \leq$ p, and $|y| >$ 0. Since we know $|y| > 0$, we know that y contains only zeros. Since y contains some number of zeros equal to $|y|$ = k for some k $\in$ (0,p], we can conclude that xy contains 0^{p-k} zeros. Let $i = 2$ such that $S^{'} = xy^iz = xy^2z = xyyz = 0^{p-k}0^k0^k1^p$. Simplifying we obtain $0^{p+k}1^p$. This contradicts the pumping lemma, since we have come up with a string $S^{'}$ with pumping length p that is NOT in the language $\overline{L}$ since $m \neq n$ since k is greater than zero. Thus since $\overline{L}$ is not regular, the language L is not regular.

4. Let $\Sigma = \{ \ 0, 1, +, = \ \}$. Let ADD = $\{ \ x = y + z \mid x, y, z$ are binary integers, and x is the sum of y and z $\}$. Use the pumping lemma to prove ADD is not regular.

   Assume ADD is regular and let p be its pumping length. Define S such that $S = 1^{p+1} = 10^p + 1^p$, such that |S| $> p$ and that $S \in$ L. Let $S = xyz$ be a partitioning satisfying $|xy| \leq$ p, and $|y| > 0$. Since we know $|y| > 0$, we know that y contains only ones. Since y contains some number of ones equal to $|y|$ = k for some k $\in$ (0,p], we can conclude that xy contains 1^{p-k} ones. Let $i = 0$ such that $S^{'} = xy^iz = xy^0z = xz = 1^{p+1-k} = 10^p + 1^p$. This contradicts the pumping lemma, since we have come up with a string $S^{'}$ with pumping length p that is NOT in the language L since k is greater than 0; S' will never be true. Thus, we can conclude that ADD is not regular.

5. Let L = $\{ \ 1^k y \mid y \in \{0, 1\}^*$ and y contains at least k 1s for k $\geq 1 \ \}$. Use the pumping lemma to prove L is not regular.

   Assume L is regular and let p be its pumping length. Define S such that $S = 1^p 1^p$, such that |S| $> p$ and that $S \in$ L. Let $S = xyz$ be a partitioning satisfying $|xy| \leq$ p, and $|y|$ > 0. Since we know $|y| > 0$, we know that y contains only ones. Since y contains some number of ones equal to $|y|$ = k for some k $\in$ (0,p], we can conclude that xy contains 1^{p-k} ones. Let $i = 2$ such that $S^{'} = xy^iz = xy^2z = xyyz = 1^{p-k}1^k1^k1^p$. Simplifying we obtain $1^{p+k}1^p$. This contradicts the pumping lemma, since we have come up with a string $S^{'}$ with pumping length p that is NOT in the language L since y does not have at least 'k' ones. Thus, this language is not regular.