CS4384 : Automata Theory
Homework Assignment 3

Matthew McMillian
mgm160130@utdallas.edu

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1. Given the following CFG G, give the derivations for each string.

   (a.)

   E → T → F → a

   (b.)

   E → E + T → E + T + T → T + T + T → F + T + T → F + F + T → F + F + F → a + F + F → a + a + F → a + a + a

   (c.)

   E → E + T → T + T → F + T → F + F → a + F → a + a

   (d.)

   E → T → F → (E) → (T) → (F) (F) → ((E)) → ((T)) → ((F)) → ((a))

2. Construct a CFG that generates { w | w starts and ends with the same symbol.}, given $\Sigma = \{0, 1\}$.

   S → 0A0 | 1A1

   A → 0A | A0 | 1A | A1 | 1 | 0

3. Construct a CFG that generates { w | w is a palindrome. }, given $\Sigma = \{0, 1\}$.

   S → 0A0 | 1A1

   A → 1 | 0 | S

4. Construct a CFG that generates { $a^i b^j c^k$ | i=j or j=k }

$S \rightarrow A \mid B$

$A \rightarrow aA \mid X$

$B \rightarrow Bb \mid Y$

$X \rightarrow bXc \mid \epsilon$

$Y \rightarrow aYb \mid \epsilon$

5. Show that the following grammars are ambiguous by demonstrating two different left-most derivations.

   a.) $S \rightarrow SS \rightarrow SSS \rightarrow aSS \rightarrow aaS \rightarrow aaa$
   $S \rightarrow SS \rightarrow aS \rightarrow aSS \rightarrow aaS \rightarrow aaa$
   We have found two different left-most derivations that result in the same derivation.

   b.) $S \rightarrow A \rightarrow ab$
   $S \rightarrow B \rightarrow abB \rightarrow ab$
   We have found two different left-most derivations that result in the same derivation.

6. Let G be the CFG $S \rightarrow aS \mid Sb \mid a \mid b$. Show that no string in L(G) contains ba as a substring. (Hint: This can be shown by induction on the length of the string in G.)

   Base Case: Derivation with one step, $S \rightarrow aS \mid Sb \mid a \mid b$. Neither of these initial steps contain the substring 'ba'.
   IH: Assume that for every derivation, $S \rightarrow^* w$ with $n \geq 1$ steps, that 'ba' is not a substring.
   IS: Prove that for every derivation with n+1 steps generates a string in L. Let $S \rightarrow^*$ w be a derivation with n+1 steps. We can see that with any case we are given, it is impossible to end up with a string that has a substring 'ba'. Since the transitions that end up with 'a' and 'b' can only transition once so they can never have a substring 'ba'.

   Case 1: aS: If we start with aS, for any n+1 derivations we will end up with a string of the form aS such that a is the starting character of the string. In this case, there is no way to obtain a string that have ba.

   Case 2: Sb: If we start with Sb, for any n+1 derivations will will end up with a string of the form Sb such that b is the ending character of the string. Similarly to the last case, there is not a string that we can create where b comes before a.

   Thus we have proven that for no matter how many n+1 steps, it is impossible to generate a string that has a substring 'ba'.