Title

Benevolent Tomato

1 Abstract

This paper presents some useful linear algebra tools when analyzing migration in terms of matricies. For simplicity, we assume the Leslie matricies of each population are either identical or a constant multiple of each other. Given a positive fertility rate, the leslie matrix is always invertible. All terms that show up in the difference equations are a memeber of the algebra formed by the leslie matrix, and they commute.

We first present a cyclic migration model, then present some lemmas that simplify the computation.

2 The N-Migration Model

We consider N populations of the same species, each denoted by $p_1, p_2, \ldots, p_N$. Each population p_i has an influx from p_{i-1} and an outflux to p_{i+1}. For convinience, we denote $p_0 = p_N$. Each population grows by a Leslie growth, and there is a constant infulx of population M coming from some unknown source.

Definition 1. *Define a square matrix J of order N as follows.*

$$(J)_{ij} \; := \; \begin{cases} 1 & i = j \\ -1 & j - i = 1 \mod n \end{cases}$$

For example, if the degree of the matrix is 4,

$$J = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

The population obeys the following difference equation.

$$\vec{P}^{(t)} = k^t J^t \vec{P} + \frac{(kJ)^t - I}{kJ - I} \vec{M}$$

3 Linear Algebra Tools

Theorem 1 (Power of matrix J)**.** *Given $p < N$, we can explicitly compute each entry of J^p.*

$$(J^p)_{ij} \; = \; (-1)^t \binom{p}{t}$$

The value t is defined as

$$t \; = \; j - i \mod n$$

Theorem 2. *Inverse of $J - kI$ Each entry of $(J - kI)^{-1}$ can be computed as a fraction of binomial sums.*

$$(J - kI)^{-1}_{ij} \;=\; -\frac{\sum_{i=0}^{t} \binom{t}{i} (-k)^i}{\sum_{i=1}^{n} \binom{n}{i} (-k)^i} \;=\; \frac{-(1-k)^t}{1-(1-k)^n}$$