PHYS 201 Problemset 9

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1. Find the electromotive force induced on a square wire when the wire is passing through the center.

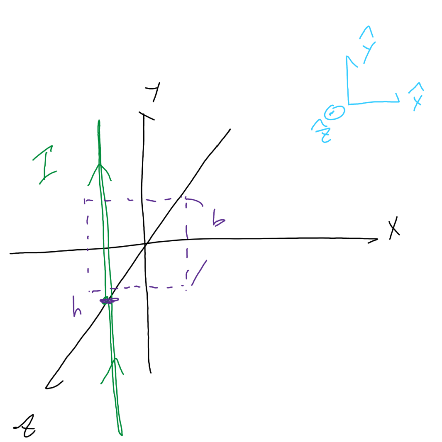


Figure 1: setup

To begin with, we determine the magnetic force along the xy-plane. Notice that by symmetry, the magnetic field is independent of the y axis. Thus, fix $y = z = 0$. We want to derive an equation for $\vec{B}(x)$.

Recall:

$$B = \frac{\mu_0 I}{2\pi r}$$

And that the direction of B is determined by the right hand rule. We write:

$$\vec{B}(x) = \frac{\mu_0 I}{2\pi\sqrt{x^2 + h^2}} - \overline{\langle h, 0, x\rangle}$$

Where the bar denotes the unit vector. As to understand how the directional vetor is derived refer to the following figure:

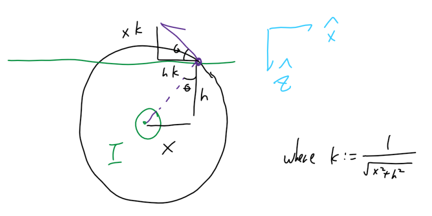


Figure 2: horizontal cut

Moreover, note that:

$$\vec{B} \cdot \vec{z} = \frac{\mu_0 I}{2\pi(x^2 + h^2)}(-x)$$

Now, compute the electromotive force by Faraday's Law. Write:

$$\mathcal{E} = -\frac{d\Phi}{dt}\bigg|_{x=0}$$

We can compute:

$$\frac{d}{dx}\oint_\gamma \vec{B} d\vec{a}\bigg|_{x=0} = b(\vec{B} \cdot \vec{z}\bigg|_{x=b/2} - \vec{B} \cdot \vec{z}\bigg|_{x=-b/2})$$

As we have established earlier, we write:

$$= b\frac{\mu_0 I}{2\pi(b^2/4 + h^2)}\left[-\frac{b}{2} - \frac{b}{2}\right] = -b^2\frac{\mu_0 I}{2\pi(b^2/4 + h^2)}$$

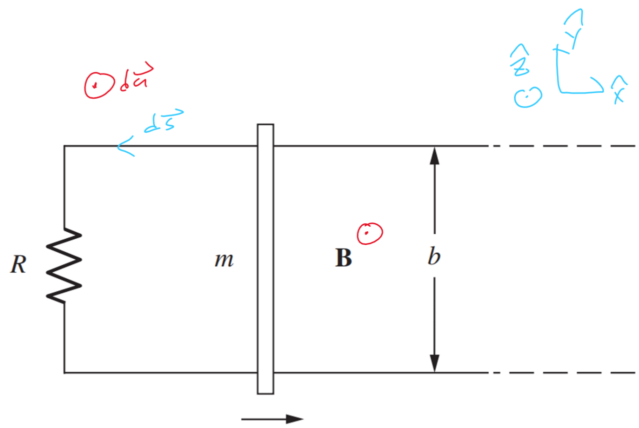
By the chain rule:

$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}\bigg|_{x=0} = -\frac{dx}{dt}\frac{d\Phi}{dx}\bigg|_{x=0} = \frac{2vb^2\mu_0 I}{\pi(b^2 + 4h^2)}}$$

$\square$

2. A rod connected to a circuit is passing through a uniform magnetic field. Answer the following problems:

1. When does the rod stop?
2. How much has the rod traveled?
3. What can be said about the energy of the system?



Solution It seems reasonable to first come up with a time dependent equation of the vertical velocity of the rod. Let $v(t)$ be a scalar function that describes the x direction velocity. Denote the initial velocity as v_0.

We want to compute the current induced on the rod. The induced current will be perpendicular to the direction of the magnetic field. This in turn will create a Lorentz force that is opposite of the direction of movement.

Define the direction of the surface vector $d\vec{a}$ to point out of the surface. Also, let $d\vec{s}$ to be counterclockwise.

Now that all the directions are established move on to compute the integrals. To invoke Faraday's law, compute the magnetic flux. Write:

$$\Phi = \oint_{loop} \vec{B} d\vec{a} = lbB$$

Where l denotes the horizontal distance from the resistor to the rod.

By Faraday:

$$\mathcal{E} = -\frac{d\Phi}{dt} = -vbB$$

This derivative is justified by observing that the time derivative of l is v.

Apply Ohm's law to compute the current.

$$\mathcal{E} = IR$$
$$I = -\frac{vbB}{R}$$

The current flows clockwise. Applying the right hand rule, we deduce that the magnetic field is towards the $-x$ direction.

To compute the magnitude of the force exerted on the rod, consider the following equation:

$$qv_{drift} = Ib$$

where q is the total moving charge inside the rod.

Assuming that there is no electric field, we write:

$$F = qv_{drift}B$$

We consider only the magnitudes, and v_{drift}, B are perpendicular. Thus the equation is justified. Ignore self inductance. Also, $F = ma$. We write:

$$ma = -IbB = -\frac{vb^2B^2}{R}$$

$$a = -\frac{vb^2B^2}{mR}$$

$$\frac{dv}{dt}$$