**Proposition** The following are equivalent:

1. (Division Algorithm) For any $\alpha, \beta \in \mathcal{O}_K$ there exists a unique integer $q, \gamma$ that satisfies $\alpha = q\beta + \gamma$
2. (Approximation) For any $\theta \in K$, there exists a K-integer $\kappa \in \mathcal{O}_K$ that satisfies $|N(\theta - \kappa)| < 1$

**Proof** $(2 \Leftarrow 1)$ The idea is to approximate the field element $\alpha/\beta$. Let $\theta$ be this fraction, and obtain $\kappa$ accordingly. By the precision of the approximation:

$$|N(\alpha/\beta - \kappa)| < 1$$

With some manipulation:

$$|N(\alpha - \kappa\beta)/N(\beta)| < 1$$

Which in turn, yields:

$$|N(\alpha - \kappa\beta)| < |N(\beta)|$$

$(\alpha - \kappa\beta)$ is in $\mathcal{O}_K$. Call it $\gamma$. By the definition of $\gamma$, we also have:

$$\alpha = \kappa\beta + \gamma$$

as desired. ✓

$(2 \Rightarrow 1)$ The idea is to multiply by some integer c to guarantee $c\theta$ is in the ring of integers. □