PHYS 314 HW7

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<u>Q2 no-cloning theorem</u> a) Consider a qualtum controlled-NOT gate. This gate seems to copy the states for

$$|\psi\rangle = |0\rangle, |1\rangle$$

. Does this gate violate the no-cloning theorem?

<u>Solution</u> No, the no-cloning theorem introduced in Townsend tells us that there does not exist a unitary operator the copies a general quantum state. The c-NOT gate successfully clones the $|0\rangle, |1\rangle$ state, but it fails for an entangled state, for example

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

. An attempt to copy $|\psi\rangle$ through the c-NOT gate results in a state

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

The correct copy must result in a state

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$$

And clearly the two states do not match which leads to a contradiction. $\lightning$

b) By using the method of Quantum Teleportation, Alice can send a quantum state exactly by using entanglement and sending two classical bits. Now, assume Bob recieved a cubit from Alice and Bob made a measurement. How much information about $\{\theta, \phi\}$ can Bob retrieve from this experiemnt?

<u>Solution</u> Suppose Bob recieves a state

$$|\psi\rangle = \begin{bmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{bmatrix}$$

We can retrieve the probability that $|\psi\rangle$ will collapse to either $|0\rangle$ or $|1\rangle$.

$$P(0) = \cos^2(\theta/2) \quad \text{and} \quad P(1) = \sin^2(\theta/2)$$

Depending on Bob's measurement, we can claim that the probability that $|\psi\rangle$ will collapse to the measured state is more likely. If Bob measures 1, then it is likely that

$$\theta \in [\frac{\pi}{4}, \frac{3\pi}{4}] \cup [\frac{5\pi}{4}, \frac{7\pi}{4}]$$

. This method does not allow us to make any claims about the phase ϕ.

c) What if Bob is allowed many duplicates of the same qubit?

Solution It would be possible to narrow down the exact value of θ. Still, it would be impossible to recover the value of ϕ