Quantum Ladder Operators for Predator Prey Model

Benevolent Tomato

0 Preliminary

0.1 Setting up the space

$B(\mathcal{H})$ is defined as the space of bounded operators in the Hilbert space $\mathcal{H}$. $B(\mathcal{H})$ can be considered as a group representation of an abstract $C*$-algebra. A $C*$-algebra is a algebra that satisfies

$$\|A\|^2 \; = \; \|A * A\| \qquad \forall a \in B(\mathcal{H}) \tag{0.1}$$

Note that unbounded operators can be bounded by the exponential map. For example, suppose X is an operator with unbounded operator norm. The following function maps X to a bounded operator.

$$X \mapsto e^{iX} \tag{0.2}$$

0.2 Canonical Commutation Relation(CCR)

We choose $2L$ operators from the space $B(\mathcal{H})$.

$$\{\hat{a}_l, \hat{a}_l^\dagger |\; l \in [L]\;\} \tag{0.3}$$

Also, set this set of operators to satisfy CCR.

Definition 1 (CCR)**.** *The set of operators satisfy CCR if* $\forall l, m \in [N]$

1. $[a_l, a_m^\dagger] = \delta_{l,m} I$
2. $[a_l, a_m] = [a_l^\dagger, a_m^\dagger] = 0$

. This means the operators a_l commute with each other and so does $a_l^\dagger$. Also,

$$a_l a_l^\dagger = a_l^\dagger a_l + I \tag{0.4}$$

so pushing a a_l to the right costs an additional identity matirx. Moreover, if the indicies of the operators do not match, the just commpute.

We also define two operators, $\hat{n}_l, \hat{N}$

$$\begin{aligned} \hat{n}_l \;&=\; a_l^\dagger a_l \\ \hat{N} \;&=\; \sum_{l\in[N]} \hat{n}_l \end{aligned} \tag{0.5}$$

Here is a motivating example. Suppose φ_0 is the vaccum whcih gets anhilated by any of the operator a. e.g. $a_1\varphi_0 = 0$.

$$\begin{aligned} \hat{n}_1(a_1^\dagger)^3\varphi_0 &= (a_1^\dagger a_1)(a_1^\dagger)^3\varphi_0 = (a_1^\dagger)(a_1^\dagger a_1 + I)(a_1^\dagger)^2\varphi_0 \\ &= \cdots = 3(a_1^\dagger)^3 \end{aligned} \tag{0.6}$$

We call the a_l operators as the anhilation operator, and $a_l^\dagger$ as the creation operator.[1]

It is possible to create an orthonormal set of basis in $\mathcal{H}$ by the vaccum φ_0.

$$\varphi_{n_1,\ldots,n_L} = \frac{1}{\sqrt{n_1!\cdots n_L!}}(a_1^\dagger)^{n_1}\cdots(a_L^\dagger)^{n_L}\varphi_0 \tag{0.7}$$

where $n_1, \ldots, n_L \in \mathbb{N}$.

[1] anhilator starts with an a so the anti-anhilator is the creator