PHYS 202 Formula Sheet

Daniel Son

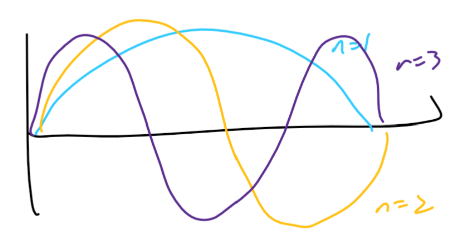
Boundary conditions for N masses The solutions for the n mass oscillators are in the form of:

$$\psi(m,t) = A\text{Ixp}(\omega t + k_m a) + B\text{Ixp}(\omega t - k_m a)$$

Plugging into the 2nd order DE, we derive an expression for the angular frequency.

$$\omega_m = 2\omega_0 \sin(k_m \frac{a}{2})$$

k_m is the mth wave number. Recall that $k := \frac{2\pi}{\lambda}$. λ_n can be computed by drawing diagrams.



With some algebraic hassle, it is possible to derive the expressions for k_m.

For closed-closed and open-open ends:

$$k_m a = \frac{m\pi}{n+1} \quad \text{or} \quad k_m = \frac{m\pi}{a(n+1)} = \frac{m\pi}{L}$$

Where $m \in \mathbb{Z}^+$

For open-open ends:

$$k_m a = \frac{m\pi}{(2n+1)/2} \quad \text{or} \quad k_m = \frac{m\pi}{L}$$

Where $m \in \mathbb{Z}^+$

Transverse waves From the geometry of the springs, we use approximation. Let theta be the angle between the horizontal axis and the string. $\tan(\theta) \approxeq \theta = \Delta y/a$. Consequently, we arrive at:

$$k \mapsto T/a$$

Dispersion Relation The differential equation of the masses in the center oscillators provide a closed form equation for the angular frequency in terms of the wave number k.

$$\omega(k) = 2\omega_0 \sin(ka/2)$$

where a is the distance between the masses.

Dispersion Relation 2 The dispersion relation defines the wave number k.

$$\frac{\partial^2\psi}{\partial t^2} = c^2\frac{\partial^2\psi}{\partial x^2} \quad \text{then} \quad \omega^2 = c^2k^2$$

And c is called the phase velocity of the wave. The wave number depends on the frequency of the wave, which is not necessarily the normal mode frequency