Formula Workspace

Let X, A be elements of a lie algebra $\mathfrak{g}$. Recall that the elements of the algebra generates elements in the lie group. This means,

$$\exp(Xt) \in G$$

For any small enough t. We know that conjugation is a group action on G. Take some element $A \in G$.

$$A \exp(Xt) A^{-1} \in G$$

A nice property of this element is that the element evaluates to the identity around $t = 0$. Oh, taking the differential at $t = 0$ must yield a lie algebra. Hence,

$$AXA^{-1} \in \mathfrak{g}$$

All the elements of the algebra and the group are matricies. The operation between the elements are matrix multiplication, which is known to be linear. Hence, this action is indeed a representation. We call this representation as **the adjoint representation** .

By the symmetry of the adjoint, we denote that $\mathfrak{g}$ is stable under the adjoint representation.

Some comment on Operators

$$[a_n, a_n^\dagger] \;=\; a_n a_n^\dagger \varphi_0 - a_n^\dagger a_n \varphi_0 \;=\; \varphi_0 \tag{1}$$