GATE 2026

Complete

Aptitude

Quick Revision

Handwrite Notes

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NUMBER SYSTEM

* Number $= a^p b^q c^r$ — a, b, c → Prime No.; p, q, r → Nat. No.
- Total factor $= (p+1)(q+1)(r+1)$

Ex:- N = 9000

$N = 2^3 \times 3^2 \times 5^3$

Total factor $= (3+1)(2+1)(3+1) = 48$

odd factor $= (2+1)(3+1) = 12$

Even factor = TF − OF = 36

2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

- Prime factor & composite factor :-

1 → neither prime nor composite.

prime factor → not consider higher power.

$TF = CF + PF + 1$

Ex:- $N = 9000 = 2^3 \times 3^2 \times 5^2$

⇒ TF = 48, PF = 3, CF = 48 − 3 − 1 = 44

HCF & LCM

- HCf of fraction $= \frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$
- LCM of fraction $= \frac{\text{LCM of Numerator}}{\text{Hcf of denominator}}$

Ex:- $\frac{1}{2}, \frac{2}{3}, \frac{3}{7}$

⇒ $HCf = \frac{\text{HCf of } (1,2,3)}{\text{LCM of } (2,3,7)} = \frac{1}{42}$

$LCM = \frac{\text{LCM of } (1,2,3)}{\text{Hcf of } (2,3,7)} = 6/1$

BASE SYSTEM

Ex:- 32 + 24 = 100 find Base. Base = 6

Ex:- 127 + 276 = 425 then, Base = 8

731 (?) + 672 = 1623 Ans

CYCLICITY

2 → 2, 4, 8, 6

3 → 3, 9, 7, 1

7 → 7, 9, 3, 1

8 → 8, 4, 2, 6

4 → 4, 6

9 → 9, 1

0, 1, 5, 6 have no cyclicity.

Ex:- (i) 3^{323}

⇒ 4) 323 (8, 32, ×(3)

unit digit $= 3^3$

⇒ 27 = 7 Ans

(ii) 14^{49}

⇒ 2) 49 (24, 48, ×(1)

unit digit $= 14^1$

⇒ $4^1 = 4$ Ans

FACTORIAL

Ex:- $\frac{100!}{3^n}$ find 'n'

⇒ 100! = [1×2×3× ---- 100]

$\frac{100}{3} = 33$

$\frac{33}{3} = 11$

$\frac{11}{3} = 3$

$\frac{3}{3} = 1$

n = 33 + 11 + 3 + 1 = 48 Ans

Ex:- 1! + 2! + ---- 99! unit digit = ?

⇒ 1! = 1, 2! = 1×2 = 2, 3! = 1×2×3 = 6

4! = 1×2×3×4 = 24, 5! = 1×2×3×4×5 = 120

⇒ 1! + 2! + 3! + 4! = 1 + 2 + 6 + 24 = 33

Ex:- 100! end with how many zero.

⇒ 5×2 = 10 (zero generate) ⇒ $\frac{100!}{5^n}$ ⇒ n = 24

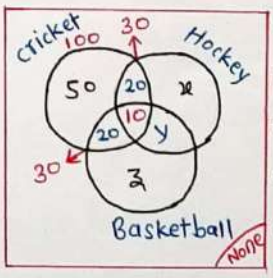
i.e 24 zero

SET THEORY

- $n(A \cup B) = [n(A) + n(B)] - [n(A \cap B)]$
- $n(A \cup B \cup C) = [n(A) + n(B) + n(C)] - [n(A \cap B) + n(B \cap C) + n(A \cup C)] + [n(A \cap B \cap C)]$

A (or) B ⇓ A ∪ B | A 'and' B ⇓ A ∩ B

#



- Exactly 1 of games = 50 + x + z
- Exactly 2 " " = 20 + 20 + y
- Atleast 2 " " = 20 + 20 + y + 10
- Any of 3 games (or) n(A ∪ B ∪ C) (or) Atleast one of the 3 games = 100 + x + y + z

"SUMIT KR"

CALENDAR

- Every century is not leap year.
  Ex:- 100 years → 5 odd day
  200 → 3 odd day
  300 → 1 odd day
- Every 4th century is leap year
  Ex:- 400, 800, 1200, 1600, 2000 ... ⇒ 0 odd day.
- Up to 1900 ⇒ 1 odd day
- 1 normal year ⇒ 1 odd day • 1 leap year ⇒ 2 odd d.

month	odd day
Jan	3
feb	0 normal, 1 LY
Mar	3
Apr	2
May	3
June	2
July	3
Aug	3
Sep	2
Oct	3
Nov	2
Dec	3

Week	odd day
Sun	0
Mon	1
Tues	2
Wed	3
Thur	4
Fri	5
Sat	6

(Q.) 2 oct 1869. days-?

⇒

	odd days
1600	0
200	3
for 68 years { LY	17×2 = 34
for 68 years { Nor. year	51
month up to Sep.	21
days	2

7) 111 (15
105
6 → sat.

CLOCK

- Hrs hand in 1 hr → 30° Angle
  1 min → ½ Angle
- Min hand in 5 min → 30° Angle
  1 min → 6° Angle
- coincide = $5x \times \frac{12}{11}$
- Right angle = $(5x \pm 15) \times \frac{12}{11}$
  +ve → 1st ans
  -ve → 2nd ans
- Opposite = $(5x \pm 30) \times \frac{12}{11}$
  If $x \geq 6$ → Take -ve sign
  $x < 6$ → " +ve "
- Mirror image = 12 - given

(Q.) coincide, right Ls, opposite bln 4 o clock & 5 o clock.
⇒ Put X = 4 then solve.

* General formula

$\left[5x \pm \left(\frac{D^\circ}{6}\right)\right] \times \frac{12}{11}$

Put, D → 0° (coinside)
D → 90° (Right Ls)
D → 180° (opposite)

for Right angle

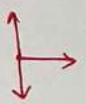
$(5\times4 + 15) \times \frac{12}{11} = \frac{420}{11} = 38\frac{2}{11} \Rightarrow 4:38\frac{2}{11}$ Ans

$(5\times4 - 15) \times \frac{12}{11} = \frac{60}{11} = 11)60(5 = 5\frac{5}{11} \Rightarrow 4:5\frac{5}{11}$ Ans

55
5

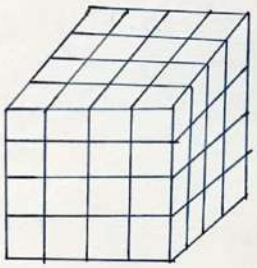
BLOOD RELATION

A^+ → for male
A^- → for female



Don't Judge gender by name
Bhanja, Bhateja ⇒ Nephew
Bhanji, Bhatiji ⇒ Niece
Relation from mother side ⇒ Maternal
Relation from father side ⇒ Paternal

CUBE

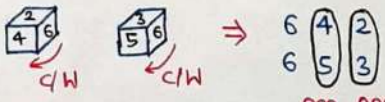


64 equal parts
$\sqrt[3]{64} = 4$ ⇒ It means one side has 4 cube.

- Total Blocks = l×b×h
- 3 side painted = 8
- 2 side painted = $4[(l-2)+(b-2)+(h-2)]$
- 1 side " = $2[(l-2)(b-2)+(b-2)(h-2)+(h-2)(l-2)]$
- 0 side painted = $(l-2)(b-2)(h-2)$

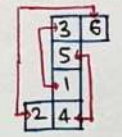
DICE

- When one no. is common on both the dices.



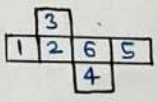
Open dice

(i) General dice



3 ↔ 1 (opp)
5 ↔ 4
6 ↔ 2

(ii) Standard dice :- opposite surface sum should be 7.



CHESS BOARD

n×n board

- No. of squares = Σn^2
- No. of Rectangles = Σn^3
- No. of types of rectangles = Σn

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SYLLOGISM

+ve statement :- (i) All (ii) some
-ve statement :- (i) No (ii) some not

Some statement are :-

** (i) All A is B
** (ii) Some A is B
** (iii) No A is B
(iv) Some A is not B
(v) only A is B
(vi) only a few A is B

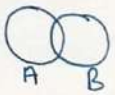
Replacement of [some] :-
many, most of, at least, generally more, few

Replacement of word [All] :-
each, every, 100 %.

(1.) All A is B :-

concⁿ :- 1. Some B are A
2. Some A are B

(2.) Some A is B :-

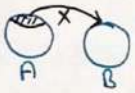


concⁿ :- (i) Some B are A

(3.) No A is B :-

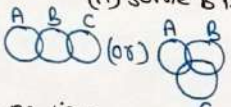
concⁿ :- (i) No B is A
(ii) Some A are not B
(iii) Some B are not A

(4.) Some A is not B :-



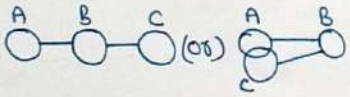
बाकी दोनों भी सकते हैं। नहीं भी।

Ex:- Stat :- (i) Some A is B
(ii) Some B is C



concⁿ :- (i) Some c are B [100 %. True]
(ii) No A is B [100 %. false]
(iii) No A is c [can't say]

Ex:- Stat :- (i) No A is B
(ii) No B is C



concⁿ :- (i) No B are A [100 %. sure]
(ii) Some c are not B [100 %. sure]
(iii) No A is c [CNS]

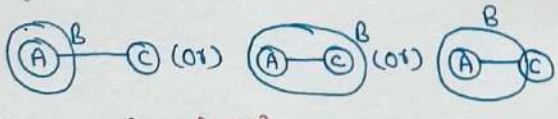
Either, or concept :-

Stat. → concⁿ → option → Either, or

2 concⁿ होना चाहिए
+ve stat. - ve stat.
(All, some) (no, some not)

दोनों stat. CNS वाले concⁿ होना चाहिए।
element common [मुद्दे समान होना चाहिए]

Ex:- Stat :- (i) All A is B (ii) No A is C



concⁿ :- X (i) No B is C [CNS]
X (ii) Some B are C [CNS]

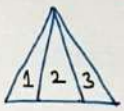
(A) If only concⁿ I follows (B) only concⁿ II follows
(C) If either I (or) II follows

⇒ Point (i) 2 concln → satisfied
(ii) one +ve & -ve → satisfied
(iii) both stat. CNS → satisfied
(iv) common element → satisfied

COUNTING OF FIGURE

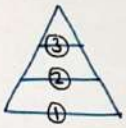
Triangle

Type (i)



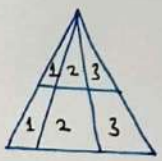
⇒ 1+2+3 ⇒ 6 Ans

Type (ii)



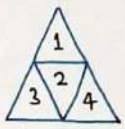
⇒ 3 Ans

Type (iii)

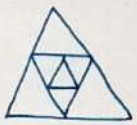


⇒ 12 Ans

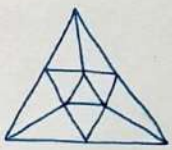
Type (iv)



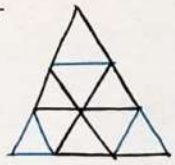
⇒ अंदर - 4
⇒ बाहर - 1
⇒ 5 Ans



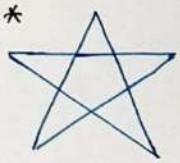
⇒ अंदर - 4
⇒ " - 4
⇒ बाहर - 1
⇒ 9 Ans



⇒ 4+4+1+1 = 6

* 

⇒ 10+3 = 13 Ans

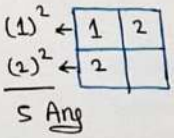
* 

एक एक vertex से 2 triangle बन रहा है।

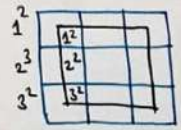
⇒ 5 × 2 = 10 Ans
↓
Vertex

Square

Type-1 [sym fig] Row = column

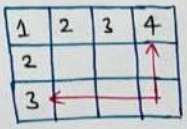


5 Ans

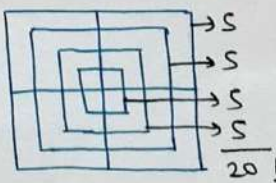


⇒ 28 Ans

Type-2 [Non-sym fig] Row ≠ column



3×4 = 12
2×3 = 6
1×2 = 2
20 Ans

* 

20 Ans

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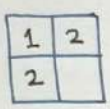
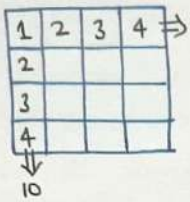
Rectangle



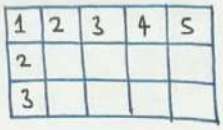
$\Rightarrow (1+2)\times(1+2)$

$\Rightarrow$ 9 Ans

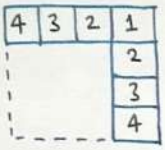


$\Rightarrow (1+2+3+4)\times(1+2+3+4)$

$\Rightarrow$ 100 Ans



$\Rightarrow$ 90 Ans

* 

$\Rightarrow 10+10-1 = 19$ Ans

SURDS, INDICES & LOGARITHMS

Law of surds :-

(i) $(\sqrt[n]{a})^n = (a^{\frac{1}{n}})^n = a$

(ii) $\sqrt[n]{ab} = \sqrt[n]{a}\cdot\sqrt[n]{b}$

Ex:- $\sqrt[3]{2\times5} = \sqrt[3]{2}\times\sqrt[3]{5}$

(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Ex:- $\sqrt[3]{\frac{2}{5}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5}}$

(iv) $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

Ex:- $(\sqrt[3]{2})^5 = \sqrt[3]{2^5} = \sqrt[3]{3^2}$

(v) $\sqrt[m]{\sqrt[n]{\sqrt[p]{a}}} = \sqrt[mnp]{a} = a^{\frac{1}{mnp}}$

Ex:- $\sqrt[2]{\sqrt[3]{\sqrt[4]{10}}} = 10^{\frac{1}{24}}$

law of Indices :-

(i) $a^m \times a^n = a^{m+n}$

Ex:- $2^4\times2^{13} = 2^{17}$

(ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

Ex:- $(3^2)^4 = 3^8$

(iv) $(a\times b)^n = a^n \times b^n$

(v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(vi) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

(vii) $a^{-m} = \frac{1}{a^m}$

Ex:- $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Logarithms :-

$49 \rightarrow 7^2$; $\log_7 7^2 = 2$ — 7^2 is similar as $\log_7 7^2 = 2$

Properties :-

(i) $\log_a(m\times n) = \log_a m + \log_a n$

Ex:- $\log_{10}(15) = \log_{10} 3 + \log_{10} 5$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$

Ex:- $\log_2\left(\frac{6}{5}\right) = \log_2 6 - \log_2 5$

(iii) $\log_a(m^n) = n\log_a m$

Ex:- $\log_5 625 = \log_5(5^4) = 4\log_5 5$

(iv) $\log_{a^n}(m) = \frac{1}{n}\log_a(m)$ Ex:- $\log_{2^4}(8) = \frac{1}{4}\log_2 8$

(v) $\log_a b = \frac{\log_n b}{\log_n a}$ Ex:- $\log_2(5) = \frac{\log_{10}(5)}{\log_{10}(2)}$

$n \rightarrow$ may be nat. nr.

(vi) $\log_a x = \frac{\log_n x}{\log_n a} = \frac{1}{\log_x a}$ Ex:- $\log_2 5 = \frac{1}{\log_5 2}$

(vii) $\log_a b \times \log_b a = \frac{\log_n b}{\log_n a}\times\frac{\log_n a}{\log_n b} = 1$

NOTE:- When Base is not mentioned it is taken as 10

(viii) $\log_a 1 = 0$ Ex:- $\log_5 1 = 0$

(ix) $a^{\log_a x} = x$ Ex:- $5^{\log_5 3} = 3$

(Q.) $\log_2 32 = ?$

$\Rightarrow$ 2 का कितना power करें 32 आ जाए।

$\Rightarrow [2^x = 32] \Rightarrow 5$ Ans

$\Rightarrow [\log_2(2)^5 = 5]$ Ans

By properties

$\Rightarrow \log_2(2)^5 \Rightarrow 5\underbrace{\log_2 2}_{1} \Rightarrow 5$ Ans

(Q.) $\log_5\sqrt{5} \Rightarrow \log_5(5)^{\frac{1}{2}} \Rightarrow \frac{1}{2}$ Ans

(Q.) $\log_{10} 1 \Rightarrow \log_{10}(10)^0 \Rightarrow 0$ Ans

(Q.) $\log_a\sqrt{3} = \frac{1}{6}$ find a $\Rightarrow a^{\frac{1}{6}} = \sqrt{3} \Rightarrow a^{\frac{1}{6}\times6} = 3^{\frac{1}{2}\times6} \Rightarrow a = 27$

(Q.) If $\log_{27} + \log_9 x + \log_3 x = 11$ find x

$\Rightarrow \log_{(3)^3} x + \log_{(3)^2} x + \log_{3^1} x = 11 \Rightarrow x = 729$ Ans

(Q.) $\log_3 n - \log_3 4 = 2$

$\Rightarrow$ hint:- $\log_3\left(\frac{n}{4}\right) = 2$

$\Rightarrow n = 36$ Ans

(Q.) $\log_{10} 2 = 0.3010$, $\log_{10} 5 = ?$

$\Rightarrow$ hint:-

"SUMIT KR"

TIME, SPEED & DISTANCE

- $\text{Speed} = \frac{\text{distance}}{\text{time}}$

$1\ km/hr = \frac{5}{18}\ m/s$

$1\ m/s = \frac{18}{5}\ km/hr$

case(i) Time = const; case(ii) speed = const; case(iii) Dist. = const

case(i)	case(ii)	case(iii)
$S \propto D$	$D \propto T$	$S \propto \frac{1}{T}$
$\frac{S_1}{S_2} = \frac{D_1}{D_2}$	$\frac{D_1}{D_2} = \frac{T_1}{T_2}$	$\frac{S_1}{S_2} = \frac{T_2}{T_1}$

(Q.) A student travels from his home to coaching at 4km/hr & reaches coaching 20min late. If the speed has been 6km/hr he would have reached 10min early.

(i) find the dist. from his home to coaching?

(ii) correct time for student to reach coaching?

⇒ hint:- $S \propto \frac{1}{T}$ [D=C] Ans:- (i) 6km (ii) 70min

Metting point concept:-

case(i) T = const

7:00 7:30 7:00

$S_A \rightarrow$ $\leftarrow S_B$

- $T = \frac{D}{S_A + S_B}$

(Q.) 7:00 — 1000 KM — 7:00

Agra — Jaipur

$S_A = 100\ km/hr$ $S_B = 150\ km/hr$

$\Rightarrow T = \frac{1000}{250} = 4\ hr$ (11 AM)

case(ii) T = const

D — x — metting

$S_A \rightarrow$ $S_B \rightarrow$

- $T = \frac{D}{S_A - S_B}$

(Q.) 200m — x

$S_A = 7\ km/hr$ $S_B = 5\ km/hr$

find x?

$\Rightarrow$ Ans = 500m

A — B

1st meet → D 3rd meet → 5D

2nd meet → 3D

Prob. on Races:-

Type-(i) 100m; A → $S_A > S_B$ 20m; B →

Type-(ii) 100m; A → B → 80m →; → 20m ←

(Q.) In a KM race, A beats B by 200m. In the same race, B beats C by 200m. By what dist.(in m) does A beat C?

$\Rightarrow \frac{S_A}{S_B} = \frac{1000}{800} = \frac{5}{4}$

$\frac{S_B}{S_C} = \frac{1000}{800} = \frac{5}{4}$

$\frac{S_A}{S_B} \times \frac{S_B}{S_C} = \frac{25 \times 40}{16 \times 40} = \frac{1000}{640} \Rightarrow$

A → B → C → C 200 m B

← 640 m → C ← 360 m → A

Problems on train:-

case(i) Moving body & a stat. body of negligible length.

Ex:- ST → ; ← LT →

- Time taken by train to pass the man/pole = $\frac{LT}{ST}$

case(ii) Moving body & a stat. body of length(L)

Ex:- ST → ; ← LT → ; ← L →

- Time taken by train pass the platform = $\frac{LT + L}{ST}$

case(iii) 2 moving body in opposite dirn

$S_1 \rightarrow$ $\leftarrow S_2$

L_1 L_2

- Time taken by train to pass each other = $\frac{L_1 + L_2}{S_1 + S_2}$

case(iv) 2 moving body in same dirn

- Time taken ,, ------- ,, = $\frac{L_1 + L_2}{S_1 - S_2}$

(Q.) A man is standing on Railway Bridge which is 180m long He find that a train crosses the bridge in 20 sec but himself in 8 sec.

(i) find length of train? (ii) speed = ?

⇒ Ans:- (i) 120m (ii) 15 m/s

Boats & Streams:-

upstream; downstream.

Speed of Boat (or) man in still water = 'B'

Speed of stream (or) water = 'S'

case(i) downstream	case(ii) upstream
Boat →	Boat →
Stream →	Stream ←

- Downstream (D) = Speed = B + S
- Upstream (U) = Speed = B − S
- Speed of Boat (B) = $\frac{D + U}{2}$
- Speed of stream (S) = $\frac{D - U}{2}$

(Q.) Ram Row a boat at 15 km/hr upstream & 20 km/hr downstream. find speed with Ram row the boat in still water & also find speed of Stream?

(Q.) A man takes 3 hrs to row a boat 15 km downstream of river & 2.5 hrs to cover 5km upstream. find speed of stream?

⇒ 1.5 km/hr.

"SUMIT KR"

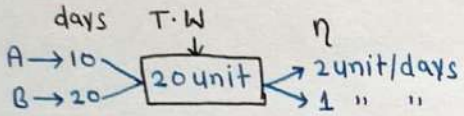
TIME & WORK

- $\left[\eta = \frac{\text{Work}}{\text{Time}}\right]$ Work = const [95% ques]

(Q.) A → 10 days B → 20 days A+B → ? days

⇒ LCM Mtd:-

LCM of 10 & 20 = 20 unit = Total Work

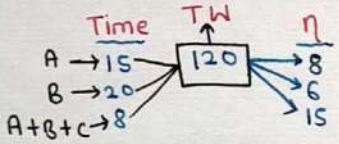


A+B = 3 unit/days

Time = $\frac{20}{3}$ days Ans

(Q.) A can do a work in 15 days. while B alone can do the same work in 20 days while A, B C together can do the same work in 8 days. In how many days 'C' alone will complete the total work?

⇒



A+B+C = 15 unit

8+6+C = 15

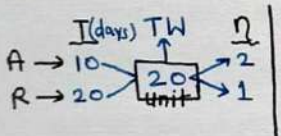
C = 1 unit/day

Time = $\frac{120}{1}$ = 120 days Ans

(Q.) Ajeet can do a job in 10 days. Raman → 20 days

They tog. start doing the job but After 4 days Raman leaves. How many more days will reqd. by Ajeet to complete this job alone?

⇒



A+R → 3 unit/days | A → 2 unit/days

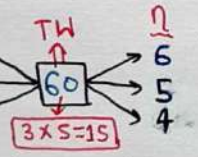
4 days → 12 unit | Remaining → 8 unit

⇒ 4 days

(Q.) Ram → 10 days

deepak → 12 "

parul → 15 "



deepak leaves 3 days before the work is completed Ram leaves after 2 days.

⇒ R+D+P = 15 unit/days | D+P → 9 unit/days

2 days → 30 unit | Rem → 45 unit ⇒ 5 days ⇒ 5+2 = 7 days

Concept of efficiency:-

	A	B
η →	2	1
T →	1	2

* A is 60% more efficient than B

⇒ A : B

η → 16 : 10 let

T → 10 : 16

"SUMIT KR"

[Man × days = Work]

(Q.) 25 → 12 days 30 men → ? days

⇒ 25 men → 12 days

1 " → 12×25 days (or)

30 " → $\frac{12\times25}{30}$ Ans

$M_1D_1 = M_2D_2$

$25\times12 = 30\times x$

$x = \frac{25\times12}{30}$ Ans

[Man × days = Work]

↓

(men + Women) × days = Work

(Q.) 15 men & 20 Women → 10 days

24 " & 32 " → ? days

⇒ Team 1:- 15m + 20W → 10 days (3:4)

Team 2:- 24m + 32W → ? (3:4)

⇒ (15+20) man = 10

1 man = 10×35

24m + 32W = ?

⇒ $\frac{10\times35}{24+32}$ Ans

(Q.) 12 men (or) 18 Wom. → 14 days

8 men (&) 16 Women → ? days

⇒ 12M → 14 days

18W → 14 days

$\frac{M}{W} = \frac{3}{2} \Rightarrow \boxed{M = \frac{3}{2}W}$

$8\times\frac{3}{2}W + 16W = ?$

⇒ $\frac{18\times14}{28}$ = 9 days Ans

(Q.) 2M, 7 boys → 14 days

3M, 8 boys → 11 days

8M, 6 boys → ? days

⇒ $M_1d_1 = m_2d_2$

⇒ (2M+7b)14 = (3M+8b)11

⇒ $\frac{M}{b} = \frac{2}{1} \Rightarrow \boxed{M = 2B}$

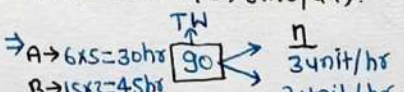
Concept of MDH (Man × Day × hrs):-

(Q.) A → 6 days, 5 hrs a day

B → 15 days, 3 hrs a days

A+B → ? days, 3 hrs/day.

⇒

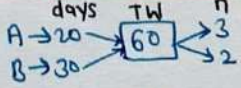


A+B = 5 unit/hr ⇒ $\frac{90}{5}$ = 18 hrs = 6 days.

$\left[\eta = \frac{W}{MDH}\right]$ $\left[\frac{W_1}{M_1D_1H_1} = \frac{W_2}{M_2D_2H_2}\right]$ η = const

(Q.) If 100 cat can kill 100 rat in 100 days in how many days 10 cats can kill 10 rats. ⇒ Ans = 100 days.

Concept of Alternate days:-

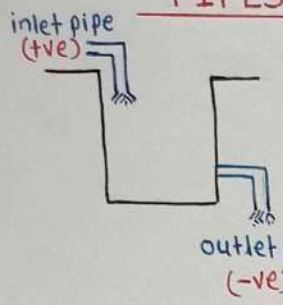


⇒ If A starts:- AB | AB | AB - - - - -

5 unit → 2 days

60 unit → 24 days Ans

PIPES & CISTERNS



Eg:- Time Tc n

$P_1 \uparrow \rightarrow 10hr$, $P_2 \downarrow \rightarrow 40hr$ → 40 → 4 unit/hr, 1 unit/hr

$P_1 + P_2 = 4 - 1 = 3 unit/hrs$

$T = \frac{40}{3}$ hrs

(Q.) 2 pipe can fill a tank in 10 hrs & 20 hrs respectively while a third pipe empty the full tank in 40 hrs. if all the 3 pipes simultaneously, in how much time will the tank be filled?

⇒ I Tc n

10↑, 20↑, 40↓ → 40 → 4, 2, 1

$4 + 2 - 1 = 5 unit/hr$

$T = \frac{40}{5} = 8 hr.$

(Q.) A tap can fill a tank in 12 hrs. but because of a hole in the bottom of the tank, it fills the tank in 15 hrs. det. the time it will take to empty the tank if it is completly filled once & tap is closed.

⇒ A → 12h, A − B → 15h → 60 → 5, 4 ⇒ B = 1 unit/hr ⇒ $\frac{60}{1} = 60 hrs$

SIMPLE INTEREST & COMPOUND INT.

- $\left[SI = \frac{P \times R \times T}{100}\right]$ P → Principal R → Rate of interest T → time period.
- [Amount = Principal + Interest]

Ex:- P = 100, ROI = 10%, T = 2 year

⇒ mtd-(i) By formula → $SI = \frac{100 \times 10 \times 2}{100} = 20$

Amount = 100 + 20 = 120

mtd-(ii) 10% of 100 ⇒ 10 → 1 year interest

⇒ 10 × 2 = 20 → 2 year interest

Amount = 100 + 20 = 120

(Q.) If a sum of money at SI double in 6 year it will be 4 times in − Ans → 18 year

(Q.) A principal becomes ₹900 after 3 year & ₹1200 after 6 year on SI then find principal & ROI.

⇒ Ans → P = 600, ROI = 16.67%

Half yearly / quarterly :-

(Q.) find SI on ₹4800 at rate of 8.5% per annum for period of 2 years 3 months

⇒ ROI = $17\% + \frac{8.5}{4}\%$ ⇒ SI = 918 Ans

Compound Interest (CI)

- $\left[A = P\left(1 + \frac{R}{100}\right)^T\right]$ • $\left[I = A - P = P\left(1 + \frac{R}{100}\right)^T - P\right]$

(Q.) P = ₹100, ROI = 10%, T = 2 years

⇒ $A = 100\left(1 + \frac{10}{100}\right)^2 = 121$ $I = 121 - 100 = 21$ Ans

(or)

$A = 100 \times 1.1 \times 1.1 = 121$ $I = 21$ Ans

(Q.) find CI on Rs 1000 at the rate of 20% p.a for 18 months when intersest is compound half-yearly.

⇒ ROI = 30% → 18 months

ROI = 10% → 6 months

⇒ $1000 \times 1.1 \times 1.1 \times 1.1 = 1331$ Ans

(Q.) A sum of money doubles itself at some rate of compound interest in 15 year. In how many year it become eight times of itself with the same rate?

⇒ $x \xrightarrow{15 year} 2x \xrightarrow{15 year} 4x \xrightarrow{15 year} 8x$ ⇒ 45 year Ans

RATIO & PROPORTION

(Q.) If a:b = 2:3 & b − a = 10 then find a & b?

⇒ 3 − 2 = 1 → 10 ⇒ a = 2 × 10 = 20 b = 3 × 10 = 30

(Q.) $\frac{a}{b} = \frac{2}{3}$ & $\frac{b}{c} = \frac{4}{3}$ find a:b:c

⇒ LCM of 3, 4 = 12

$\frac{a}{b} = \left(\frac{2}{3}\right) \times \frac{4}{4} = \frac{8}{12}$ $\frac{b}{c} = \left(\frac{4}{3}\right) \times \frac{3}{3} = \frac{12}{9}$

a:b:c = 8:12:9

S.cut:-

a : b : c

2 3→3

4←4 3

8 : 12 : 9

(Q.) If m:n = 3:2 then (4m + 5n) : (4m − 5n) = ?

⇒ direct put m = 3, n = 2

(Q.) If m:n = 3:2 then $(4m^2 + 5n^2) : (4m - 5n) = ?$

⇒ Can not determine because power is not same

(Q.) If 3A = 2B = 4C then find A:B:C?

⇒ LCM of 3, 2, 4 = 12 A:B:C = 4:6:3

(Q) If $(a+b):(b+c):(c+a) = 5:7:6$ then find (i) $a:b:c$

(ii) $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$

⇒ (i) $a+b \to 5$ unit, $b+c \to 7$ unit, $c+a \to 6$ unit

$2a+2b+2c \to 18$ unit

$a+b+c \to 9$ unit

$a+b \to 5$ unit

$c = 4$ unit, $a = 2$ unit, $b = 3$ unit

⇒ $\boxed{2:3:4} = a:b:c$

(ii) $\frac{1}{a}:\frac{1}{b}:\frac{1}{c} = \left(\frac{1}{2}:\frac{1}{3}:\frac{1}{4}\right) \times 12 \Rightarrow \boxed{6:4:3}$

(Q) If $(a+b):(b+c):(c+a) = 5:7:6$ & $2a-3b+4c = 66$ then find a, b, c

⇒ $a:b:c = 2:3:4$

$a = 2x$, $b = 3x$, $c = 4x$

$2a-3b+4c = 66$

$4x - 9x + 16x = 66$

$x = 6$

$a = 2 \times 6 = 12$, $b = 3 \times 6 = 18$, $c = 4 \times 6 = 24$

Concept of ratio incr (or) decr by const no:-

(Q) Two no are in the ratio 4:9 If both no incr by 12 the ratio become 11:21. The sum of the original no is-

⇒ $\frac{4x+12}{9x+12} = \frac{11}{21} \Rightarrow x = 8$

$A = 8 \times 4 = 32$

$B = 8 \times 9 = 72$

* A bag contains 50 paisa, 25 paisa, 10 paisa coins in the ratio of 7:8:3 amounting to 87. find the no. of 10 paisa coins?

⇒

	50p	25p	10p
	↓	↓	↓
	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$
	7	8	3
No. of coin	$7x$	$8x$	$3x$

Amount ⇒ $3.5x + 2x + .3x = 87$

$x = 15$

No. of 10 p. coin $= 3 \times 15 = 45$ coin

ALGEBRA

eqn → n, variable → m } When no. of variable are more than no of eqn then all extra variable can be put anything.

(Q) If $x = p + \frac{1}{p}$, $y = p - \frac{1}{p}$, $\frac{(x^2-y^2)(y^2+2)}{x^2-2} = ?$

⇒ eqn → 2

variable → 3 (x, y, p)

✗ put $p=0$ $x=\infty$, $y=\infty$ $\left(\frac{0}{0}, \infty\right)$ form

✓ put $p=1$ $x=2$, $y=0$ ⇒ 4 Ans

(Q) $pq+1=q$, $qr+1=r$. find $r + \frac{1}{p} + 2pqr$

⇒ ✗ $q=1$, $p=0$ ✓ $q=2$, $p=$ ✓ $r=$ ✓ ⇒ −1 Ans

$r+1=r$

$1=0$

(Q) If $abc=1$ then find $\left(\frac{a+1}{ab+a+1} + \frac{b+1}{bc+b+1} + \frac{c+1}{ac+c+1}\right) = ?$

⇒ eqn = 1, variable = 3 ⇒ $a=b=1$ & $c=1$

formula:-

(i) $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

(ii) $(a+b)^2 - (a-b)^2 = 4ab$

(iii) $(a+b)^3 = a^3+b^3+3a^2b+3ab^2$

(iv) $(a-b)^3 = a^3-b^3-3a^2b+3ab^2$

(v) $a^3+b^3 = (a+b)(a^2-ab+b^2)$

(vi) $a^3-b^3 = (a-b)(a^2+ab+b^2)$

(vii) $(a+b+c)^3 = a^3+b^3+c^3+3(a+b)(b+c)(c+a)$

(viii) $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

(ix) If $a+b+c=0 \Rightarrow a^3+b^3+c^3-3abc=0$

(Q) If $x+\frac{1}{x}=a$, then find -

① $x^2+\frac{1}{x^2}$ ② $x-\frac{1}{x}$ ③ $x^3+\frac{1}{x^3}$

$\Rightarrow x+\frac{1}{x} = a$

$\Rightarrow x^2+\frac{1}{x^2} = a^2-2$

$\Rightarrow \left(x+\frac{1}{x}\right)^3 = x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = a^2$

$\Rightarrow x^2+\frac{1}{x^2}-2 = a^2-4$

$\Rightarrow x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$\Rightarrow x^2+\frac{1}{x^2}+2 = a^2$

$\Rightarrow \left(x-\frac{1}{x}\right)^2 = a^2-4$

$\Rightarrow a^3 - 3a$

$\boxed{x^2+\frac{1}{x^2} = a^2-2}$

$\boxed{x-\frac{1}{x} = \sqrt{a^2-4}}$

$\boxed{x^3+\frac{1}{x^3} = a^3-3a}$

AP & GP

Arthematic progression:- $(a, a+d, a+2d, \dots)$

$a \to$ 1st term, $d \to$ common diff.

Ex:- 1, 3, 5, 7, 9 ...

- $T_n = a+(n-1)d$ (T_n = nth term of AP) $\quad T_n = 1+(5-1)2 = 9$ Ans
- $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+T_n] \quad S_n = \frac{5}{2}(1+9) = 25$ Ans
- Sum of 1st 'n' nat. no $(S_n) = \frac{n(n-1)}{2}$
- Sum of sq. of 1st n nat. no $(S_n^2) = \frac{n(n+1)(2n+1)}{6}$
- Sum of cube of 1st n nat. no $(S_n^3) = \left\{\frac{n(n-1)}{2}\right\}^2$ $\boxed{S_n^3 = S_n^2}$

Geometric progression:- $(a, ar, ar^2, ar^3 \dots ar^n)$

Ex:- 2, 4, 6, 16, 32 ....

- $T_n = ar^{n-1}$
- $S_n = \frac{a(r^n-1)}{r-1}$ when $r>1$ $\quad$ • $S_\infty = \infty \quad r>1$
- $S_n = \frac{a(1-r^n)}{1-r}$ when $r<1$ $\quad$ • $S_\infty = \frac{a}{1-r} \quad r<1$