| MODULE NAME: | MODULE CODE: |
|---|---|
| MATHEMATICAL PRINCIPLES OF COMPUTER SCIENCE | MAPC5112 |

| | |
|---|---|
| ASSESSMENT TYPE: | EXAMINATION (PAPER ONLY) |
| TOTAL MARK ALLOCATION: | 120 MARKS |
| TOTAL HOURS: | 2 HOURS (+10 minutes reading time) |

**INSTRUCTIONS:**

1. *Please adhere to all instructions in the assessment booklet.*
2. *Independent work is required.*
3. *Five minutes per hour of the assessment to a maximum of 15 minutes is dedicated to reading time before the start of the assessment. You may make notes on your question paper, but not in your answer sheet. Calculators may not be used during reading time.*
4. *You may not leave the assessment venue during reading time, or during the first hour or during the last 15 minutes of the assessment.*
5. *Ensure that your name is on all pieces of paper or books that you will be submitting. Submit all the pages of this assessment's question paper as well as your answer script.*
6. *Answer all the questions on the answer sheets or in answer booklets provided. The phrase 'END OF PAPER' will appear after the final set question of this assessment.*
7. *Remember to work at a steady pace so that you are able to complete the assessment within the allocated time. Use the mark allocation as a guideline as to how much time to spend on each section.*

***Additional instructions:***

1. *This is a CLOSED BOOK assessment.*
2. *Calculators are allowed.*
3. *For multiple-choice questions, give only one response per question. The marker will ignore any question with more than one answer, unless otherwise stated. You should, therefore, be sure of your answer before committing it to paper.*
4. *Answer All Questions.*

| **Question 1** | | | **(Marks: 20)** |
|---|---|---|---|
| Multiple-choice questions: Select one most correct answer for each of the following. In your answer booklet, write down only the number of the question and next to it, the letter of the correct answer. | | | |
| | | | |
| **Q.1.1** | What is the inverse of the statement ‘All mirrors are reflective’? | | (2) |
| | (a) | If it is reflective, it is a mirror; | |
| | (b) | If it is reflective, it is not a mirror; | |
| | (c) | If it is not a mirror, then it is not reflective; | |
| | (d) | If it is not reflective, then it is not a mirror; | |
| | (e) | None of the above. | |
| | | | |
| **Q.1.2** | Consider a propositional language where:<br>• A =“Simi comes to the party”;<br>• B =“Marike comes to the party”;<br>• C =“Elton comes to the party”;<br>• D =“Vuyo comes to the party”.<br>Which of the below mathematical statements/ propositions is the correct equivalent of the English sentence “If Vuyo comes to the party, then Marike and Elton come too”? | | (2) |
| | (a) | $C \rightarrow \neg A \wedge \neg B$; | |
| | (b) | $D \rightarrow (\neg C \rightarrow A)$; | |
| | (c) | $D \leftrightarrow (C \wedge \neg A)$; | |
| | (d) | $D \rightarrow B \wedge C$; | |
| | (e) | None of the above. | |
| | | | |
| **Q.1.3** | What is the rule for the closed sequence 1, 4, 9, 16, 25…? | | (2) |
| | (a) | $x_n = 1+n$; | |
| | (b) | $x_n = 1+(n+n^2)$; | |
| | (c) | $x_n = n^2$; | |
| | (d) | $x_n = n^2 + 1$; | |
| | (e) | None of the above. | |

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|---|---|---|---|
| **Q.1.4** | The $n^{th}$ term of a sequence is given by $x_n = 3n^2 - 1$. Which term of the sequence is equal to 866? | | (2) |
| | (a) | $12^{th}$; | |
| | (b) | $13^{th}$; | |
| | (c) | $15^{th}$; | |
| | (d) | $20^{th}$; | |
| | (e) | $17^{th}$. | |
| | | | |
| **Q.1.5** | What is the range for the function $f(x) = \sqrt{x} - 2$? | | (2) |
| | (a) | R; | |
| | (b) | {y ∈ R \| y > -2}; | |
| | (c) | {y ∈ R \| -4 ≤ y ≤ 4}; | |
| | (d) | {y ∈ R \| y ≥ -2}; | |
| | (e) | None of the above. | |
| | | | |
| **Q.1.6** | What is the domain for the function $f(x)= \sqrt{(x+3)}$ ? | | (2) |
| | (a) | {$x$ ∈ R \| $x$ ≥ -3}; | |
| | (b) | {$x$ ∈ R \| -3 ≤ $x$ ≤ 3}; | |
| | (c) | {$x$ ∈ R \| $x$ ≥ 3}; | |
| | (d) | R; | |
| | (e) | None of the above. | |
| | | | |
| **Q.1.7** | Give the inverse of the function $f(x) = 5x - 4$. | | (2) |
| | (a) | $f^{-1}(y) = (y - 4)/5$; | |
| | (b) | $f^{-1}(y) = (y + 4)/5$; | |
| | (c) | $f^{-1}(y) = (4 - y)/5$; | |
| | (d) | $f^{-1}(y) = 1/(5y - 4)$; | |
| | (e) | None of the above. | |

| | | | |
|---|---|---|---|
| **Q.1.8** | Which of the following functions is NOT injective (one-to-one)? | | (2) |
| | (a) | f( $x$ ) = $x^3 + 4$ from R to R; | |
| | (b) | f( $x$ ) = $x^3 + 4$ from N to N; | |
| | (c) | f( $x$ ) = $x^2 + 4$ from R to R; | |
| | (d) | f( $x$ ) = $x^2 + 4$ from N to N; | |
| | (e) | f( $x$ ) = $x^4 + 4$ from N to N. | |
| | | | |
| **Q.1.9** | The function f:Z→Z defined by f(n) = [n/2] is ___________. | | (2) |
| | (a) | Injective; | |
| | (b) | Surjective; | |
| | (c) | Bijective; | |
| | (d) | Composite; | |
| | (e) | Inverse. | |
| | | | |
| **Q.1.10** | What is the <u>converse</u> of the statement: ‘If something is an orange, then it has pips’? | | (2) |
| | (a) | If something has pips, then it is an orange; | |
| | (b) | If something does not have pips, then it is not an orange; | |
| | (c) | If something is not an orange, then it does not have pips; | |
| | (d) | If something has no pips, then it is an orange; | |
| | (e) | None of the above. | |

**Question 2** **(Marks: 20)**

Match the description in Column A with the most correct term/ phrase from Column B. In your answer booklet, write down only the question number and, next to it, the letter of the correct answer.

| Column A | | Column B | |
|---|---|---|---|
| **Q.2.1** | A particular way of assigning bit patterns to the characters on a keyboard. | a. | Additive Principle (with sets) |
| **Q.2.2** | A scheme that tells us how each number should be represented with a pattern of bits. | b. | Multiplicative Principle |
| **Q.2.3** | If event A can occur in m ways, and event B can occur in n disjoint ways, then the event "A or B" can occur in m + n ways. | c. | Cardinality of a union (two sets) |
| **Q.2.4** | Given sets A and B, we can form the set $A\times B=\{(x,y):\ x\in A \land y\in B\}$ to be the set of all ordered pairs (x,y) where x is an element of A and y is an element of B. We call A×B the Cartesian product of A and B. | d. | Cardinality of a union (three sets) |
| **Q.2.5** | Given two sets A and B, we have $\|A\times B\|=\|A\| x \|B\|$. | e. | Disjoint events |
| **Q.2.6** | For any finite sets A and B, $\|A\cup B\|=\|A\|+\|B\|-\|A\cap B\|$. | f. | Cartesian Product |
| **Q.2.7** | There is no way for A and B to both happen at the same time. | g. | IP addressing |
| **Q.2.8** | If event A can occur in m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur in m x n ways. | h. | Multiplicative Principle (with sets) |
| **Q.2.9** | For any finite sets A, B, and C, $\|A\cup B\cup C\|=\|A\|+\|B\|+\|C\|-\|A\cap B\|-\|A\cap C\|-\|B\cap C\|+\|A\cap B\cap C\|$. | i. | Additive Principle |
| **Q.2.10** | Given two sets A and B, if $A\cap B=\emptyset$ (that is, if there is no element in common to both A and B), then $\|A\cup B\|=\|A\|+\|B\|$. | j. | ASCII |
| | | k. | Unicode encoding |
| | | l. | Text |
| | | m. | Two's complement |

## Question 3 (Marks: 25)

| | | | |
|---|---|---|---|
| **Q.3.1** | Let $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$ and $D = \{7, 8, 9\}$.<br><br>Determine whether each of the following is true, false, or meaningless. Motivate your answer. | | |
| | **Q.3.1.1** | $B \in C$; | (1) |
| | **Q.3.1.2** | $A < D$; | (1) |
| | **Q.3.1.3** | $\{3\} \subset C$; | (1) |
| | **Q.3.1.4** | $B \subset A$; | (1) |
| | **Q.3.1.5** | $\emptyset \in A$. | (1) |
| **Q.3.2** | Find the intersection $A \cap B$, union $A \cup B$ and differences $A$-$B$, $B$-$A$ of sets $A$, $B$ if:<br>$A = \{2,4,5,6,8\}$;<br>$B = \{-3,0,2,3,4,5,6,7\}$.<br><br>Answer:<br>$A \cap B =$;<br>$A \cup B =$;<br>$A$-$B =$;<br>$B$-$A =$. | | (4) |
| **Q.3.3** | Answer the following questions regarding sequences: | | |
| | **Q.3.3.1** | Find $a_6$ in the sequence defined by $a_n = 2a_{n-1} - a_{n-2}$ with $a_0 = 3$ and $a_1 = 4$. | (5) |
| | **Q.3.3.2** | You are given the formulas below:<br><br>$T_n = \frac{n(n+1)}{2}$ and $a_n = 2^n$<br><br>Use the formulas to find closed formula(s) for the following sequence:<br>$(b_n)$: 1, 2, 4, 7, 11, 16, 22, . . .. | (4) |

**Q.3.4** Use a set notation to describe each of the following Venn diagrams: (2)

Diagram A:

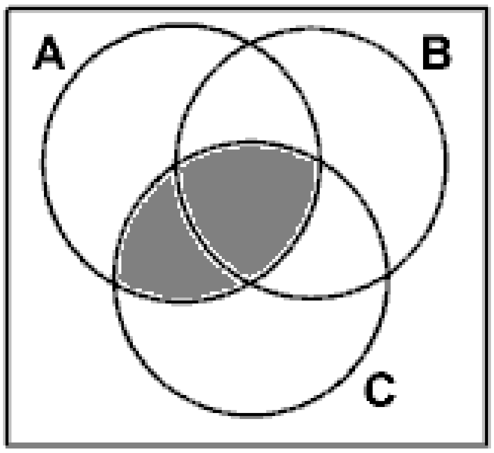
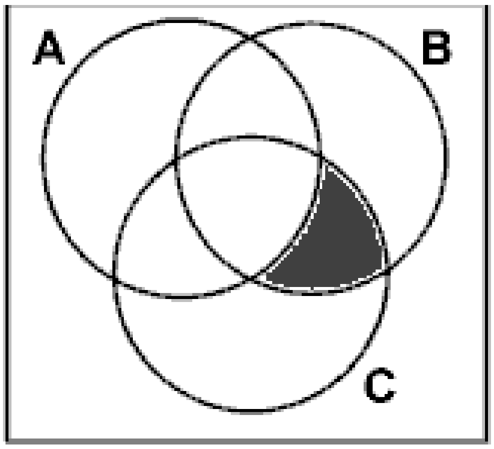



Diagram B:




**Q.3.5** Consider the function $f:$ {1, 2, 3, 4, 5, 6} → {$a$, $b$, $c$, $d$} given by: (2)

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ a & a & b & c & c & c \end{pmatrix}.$$

Find the complete inverse image of each element in the codomain.

**Q.3.6** Find the domain, codomain and range of the function: (3)

*g : {10,20,30} → {vwx,wxy,xyz} defined by g(10) = xyz, g(20) = vwx and g(30) = vwx.*

| **Question 4** | | | **(Marks: 25)** |
|---|---|---|---|
| | | | |
| **Q.4.1** | Use one appropriate example to explain the following counting principles. In your explanation, provide:<br>• The description of the principle (2);<br>• One example using a calculation (3). | | |
| | **Q.4.1.1** | Additive Principle; | (5) |
| | **Q.4.1.2** | Additive Principle (with sets). | (5) |
| | | | |
| **Q.4.2** | How many functions $f : \{1, 2, 3, 4, 5\} \rightarrow \{a, b\}$ are surjective? Explain your answer. | | (5) |
| | | | |
| **Q.4.3** | Answer the following questions regarding data representation and IP addressing: | | |
| | **Q.4.3.1** | Convert the following 0110001 from binary to decimal. | (2) |
| | **Q.4.3.2** | Convert the following 50 from decimal to binary. | (2) |
| | **Q.4.3.3** | Which class does the following IP address 119.18.45.0 belong to? | (1) |
| | **Q.4.3.4** | Identify the network portion and the host portion of the IP address 192.200.15.0. Write down the default subnet mark for each address. | (3) |
| | **Q.4.3.5** | Using the IP address and subnet mask shown write out the <u>network address:</u> | (2) |

| IP Address: | Subnet Mask: |
|---|---|
| 199.20.150.35 | 255.255.255.0 |

| **Question 5** | | **(Marks: 30)** |
|---|---|---|
| | | |
| **Q.5.1** | Consider the statement about a maths test,<br><br><u>"If it's algebra or there will be algebra, then I will pass."</u><br><br>Translate the above statement into symbols. Clearly state which statement is P and which is Q. | (4) |
| | | |
| **Q.5.2** | Make a truth table for the statement in Q.5.1 above. | (12) |

| | | | |
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| **Q.5.3** | Consider the statement:<br><br>"For all integers a and b, if a + b is even, then a and b are even". | | |
| | **Q.5.3.1** | Write the contrapositive of the statement. | (1) |
| | **Q.5.3.2** | Write the converse of the statement. | (1) |
| | **Q.5.3.3** | Write the negation of the statement. | (2) |
| | | | |
| **Q.5.4** | For each of the statements below, say what method of proof you should use to prove them. Then say how the proof starts and how it ends.<br><br>"There are no integers x and y, such that x is a prime greater than 5 and x = 6y + 3." | | (4) |
| | | | |
| **Q.5.5** | Consider the statement:<br><br>"For any integer $n$, the number $(n^3 - n)$ is even."<br><br>Prove the statement using cases *if n is even* or *if n is odd*. | | (6) |

**END OF PAPER**