One-way and two-way ANOVA (excercise)

One-way ANOVA: Compares the means of one continuous dependent variable based on three or more groups of one categorical variable.

Two-way ANOVA: Compares the means of one continuous dependent variable based on three or more groups of two categorical variables.

In [1]:
```
# Import pandas and seaborn packages
import pandas as pd
import seaborn as sns
```

Data exploration

In [2]:
```
df = pd.read_csv("archive (2).zip")
```

In [3]:
```
df.head()
```

Out[3]:

	Unnamed: 0	carat	cut	color	clarity	depth	table	price	x	y	z
0	1	0.23	Ideal	E	SI2	61.5	55.0	326	3.95	3.98	2.43
1	2	0.21	Premium	E	SI1	59.8	61.0	326	3.89	3.84	2.31
2	3	0.23	Good	E	VS1	56.9	65.0	327	4.05	4.07	2.31
3	4	0.29	Premium	I	VS2	62.4	58.0	334	4.20	4.23	2.63
4	5	0.31	Good	J	SI2	63.3	58.0	335	4.34	4.35	2.75

In [5]:
```
df.cut.unique()
```

Out[5]:
```
array(['Ideal', 'Premium', 'Good', 'Very Good', 'Fair'], dtype=object)
```

In [6]:
```
df.color.unique()
```

Out[6]:
```
array(['E', 'I', 'J', 'H', 'F', 'G', 'D'], dtype=object)
```

In [7]:
```
# Check how many diamonds are each color grade
df["color"].value_counts()
```

Out[7]:
```
G    11292
E     9797
F     9542
H     8304
D     6775
I     5422
J     2808
Name: color, dtype: int64
```

In [8]:
```
# Subset for colorless diamonds
colorless = df[df["color"].isin(["E","F","H","D","I"])]

# Select only color and price columns, and reset index
colorless = colorless[["color","price"]].reset_index(drop=True)
```

In [10]:
```


# Check that the dropped categories have been removed
colorless["color"].values
```

Out[10]:
```
array(['E', 'E', 'E', ..., 'D', 'H', 'D'], dtype=object)
```

In [11]:
```
# Import math package
import math

# Take the logarithm of the price, and insert it as the third column
colorless.insert(2, "log_price", [math.log(price) for price in colorless["price"]])
```

In [12]:
```
# Drop rows with missing values
colorless.dropna(inplace=True)

# Reset index
colorless.reset_index(inplace=True, drop=True)
```

In [13]:
```
# Examine first 5 rows of cleaned data set
colorless.head()
```

Out[13]:

	color	price	log_price
0	E	326	5.786897
1	E	326	5.786897
2	E	327	5.789960
3	I	334	5.811141
4	I	336	5.817111

In [14]:
```
# Save to diamonds.csv
colorless.to_csv('diamonds.csv',index=False,header=list(colorless.columns))
```

One-way ANOVA

To run one-way ANOVA, we first load in the data, and save it as a variable called diamonds, and then examine it using the head() function.

In [15]:
```
diamonds = pd.read_csv("diamonds.csv")
```

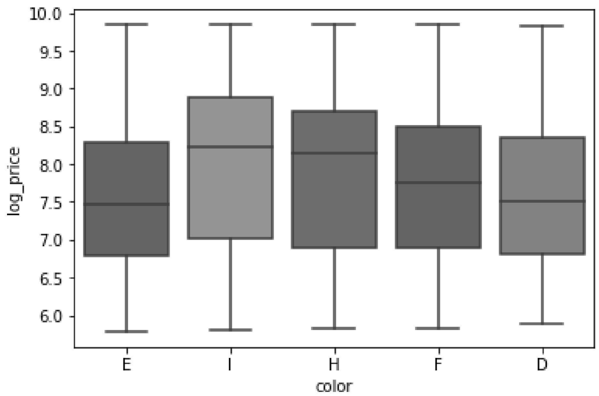
In [16]:
```
# Examine first 5 rows of diamonds data set
diamonds.head()
```

Out[16]:

	color	price	log_price
0	E	326	5.786897
1	E	326	5.786897
2	E	327	5.789960
3	I	334	5.811141
4	I	336	5.817111

In [17]:
```
# Create boxplot to show distribution of price by color grade
sns.boxplot(x = "color", y = "log_price", data = diamonds)
```

Out[17]: <AxesSubplot:xlabel='color', ylabel='log_price'>



In order to run ANOVA, we need to create a regression model. To do this, we'll import the statsmodels.api package and the ols() function. Next, we'll create a simple linear regression model where the X variable is color, which we will code as categorical using C(). Then, we'll fit the model to the data, and generate model summary statistics.

In [18]:
```
# Import statsmodels and ols function
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

In [19]:
```
# Construct simple linear regression model, and fit the model
model = ols(formula = "log_price ~ C(color)", data = diamonds).fit()
```

In [20]:
```
# Get summary statistics
model.summary()
```

Out[20]:

OLS Regression Results

Dep. Variable:	log_price	**R-squared:**	0.026
Model:	OLS	**Adj. R-squared:**	0.026
Method:	Least Squares	**F-statistic:**	265.0
Date:	Fri, 02 Jun 2023	**Prob (F-statistic):**	3.61e-225
Time:	01:38:43	**Log-Likelihood:**	-56182.
No. Observations:	39840	**AIC:**	1.124e+05
Df Residuals:	39835	**BIC:**	1.124e+05
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	7.6169	0.012	632.421	0.000	7.593	7.641
C(color)[T.E]	-0.0375	0.016	-2.394	0.017	-0.068	-0.007
C(color)[T.F]	0.1455	0.016	9.240	0.000	0.115	0.176
C(color)[T.H]	0.3015	0.016	18.579	0.000	0.270	0.333
C(color)[T.I]	0.4061	0.018	22.479	0.000	0.371	0.441

Omnibus:	7112.992	**Durbin-Watson:**	0.065
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1542.881
Skew:	0.079	**Prob(JB):**	0.00
Kurtosis:	2.049	**Cond. No.**	6.32

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Based on the model summary table, the color grades' associated beta coefficients all have a p-value of less than 0.05 (check the P>|t| column). But we can't be sure if there is a significant price difference between the various color grades. This is where one-way ANOVA comes in.

First, we have to state our null and alternative hypotheses:

Null Hypothesis $H0: priceD = priceE = priceF = priceH = priceI$

There is no difference in the price of diamonds based on color grade.

Alternative Hypothesis $H1$:Not $priceD = priceE = priceF = priceH = priceI$

There is a difference in the price of diamonds based on color grade.

In [21]:
```
# Run one-way ANOVA
sm.stats.anova_lm(model, typ = 2)
```

Out[21]:

	sum_sq	df	F	PR(>F)
C(color)	1041.690290	4.0	264.987395	3.609774e-225
Residual	39148.779822	39835.0	NaN	NaN

In [22]:
```
sm.stats.anova_lm(model, typ = 1)
```

Out[22]:

	df	sum_sq	mean_sq	F	PR(>F)
C(color)	4.0	1041.690290	260.422572	264.987395	3.609774e-225
Residual	39835.0	39148.779822	0.982773	NaN	NaN

In [23]:
```
sm.stats.anova_lm(model, typ = 3)
```

Out[23]:

	sum_sq	df	F	PR(>F)
Intercept	393066.804852	1.0	399956.684283	0.000000e+00
C(color)	1041.690290	4.0	264.987395	3.609774e-225
Residual	39148.779822	39835.0	NaN	NaN

since the p values are less than 0.05 we reject the null hypothesis

Two-Way ANOVA

In [24]:
```
# Import diamonds data set from seaborn package
diamonds = sns.load_dataset("diamonds")
```

In [25]:
```
# Examine first 5 rows of data set
diamonds.head()
```

Out[25]:

	carat	cut	color	clarity	depth	table	price	x	y	z
0	0.23	Ideal	E	SI2	61.5	55.0	326	3.95	3.98	2.43
1	0.21	Premium	E	SI1	59.8	61.0	326	3.89	3.84	2.31
2	0.23	Good	E	VS1	56.9	65.0	327	4.05	4.07	2.31
3	0.29	Premium	I	VS2	62.4	58.0	334	4.20	4.23	2.63
4	0.31	Good	J	SI2	63.3	58.0	335	4.34	4.35	2.75

In [26]:
```
# Subset for color, cut, price columns
diamonds2 = diamonds[["color","cut","price"]]

# Only include colorless diamonds
diamonds2 = diamonds2[diamonds2["color"].isin(["E","F","H","D","I"])]

# Drop removed colors, G and J
diamonds2.color = diamonds2.color.cat.remove_categories(["G","J"])

# Only include ideal, premium, and very good diamonds
diamonds2 = diamonds2[diamonds2["cut"].isin(["Ideal","Premium","Very Good"])]

# Drop removed cuts
diamonds2.cut = diamonds2.cut.cat.remove_categories(["Good","Fair"])

# Drop NaNs
diamonds2.dropna(inplace = True)

# Reset index
diamonds2.reset_index(inplace = True, drop = True)

# Add column for logarithm of price
diamonds2.insert(3,"log_price",[math.log(price) for price in diamonds2["price"]])
```

In [32]:
```
# Examine the data set
diamonds2.head()
```

Out[32]:

	color	cut	price	log_price
0	E	Ideal	326	5.786897
1	E	Premium	326	5.786897
2	I	Premium	334	5.811141
3	I	Very Good	336	5.817111
4	H	Very Good	337	5.820083

In [33]:
```
# Save as diamonds2.csv
diamonds2.to_csv('diamonds2.csv',index=False,header=list(diamonds2.columns))
```

In [34]:
```
# Load the data set
diamonds2 = pd.read_csv("diamonds2.csv")
```

In [35]:
```
diamonds2.head()
```

Out[35]:

	color	cut	price	log_price
0	E	Ideal	326	5.786897
1	E	Premium	326	5.786897
2	I	Premium	334	5.811141
3	I	Very Good	336	5.817111
4	H	Very Good	337	5.820083

This regression model includes two categorical X variables: color and cut, and a variable to account for the interaction between color and cut. The interaction is denoted using the : symbol.

In [36]:
```
# Construct a multiple linear regression with an interaction term between color and cut
model2 = ols(formula = "log_price ~ C(color) + C(cut) + C(color):C(cut)", data = diamonds2).fit()
```

In [37]:
```
# Get summary statistics
model2.summary()
```

Out[37]:

OLS Regression Results

Dep. Variable:	log_price	**R-squared:**	0.046
Model:	OLS	**Adj. R-squared:**	0.045
Method:	Least Squares	**F-statistic:**	119.5
Date:	Fri, 02 Jun 2023	**Prob (F-statistic):**	0.00
Time:	02:04:52	**Log-Likelihood:**	-49159.
No. Observations:	34935	**AIC:**	9.835e+04
Df Residuals:	34920	**BIC:**	9.847e+04
Df Model:	14		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	7.4567	0.019	401.583	0.000	7.420	7.493
C(color)[T.E]	-0.0056	0.024	-0.231	0.817	-0.053	0.042
C(color)[T.F]	0.1755	0.024	7.166	0.000	0.128	0.224
C(color)[T.H]	0.2756	0.026	10.739	0.000	0.225	0.326
C(color)[T.I]	0.3787	0.028	13.294	0.000	0.323	0.435
C(cut)[T.Premium]	0.2828	0.031	9.153	0.000	0.222	0.343
C(cut)[T.Very Good]	0.2295	0.031	7.290	0.000	0.168	0.291
C(color)[T.E]:C(cut)[T.Premium]	-0.0322	0.040	-0.800	0.424	-0.111	0.047
C(color)[T.F]:C(cut)[T.Premium]	0.0313	0.040	0.775	0.438	-0.048	0.110
C(color)[T.H]:C(cut)[T.Premium]	0.0947	0.041	2.308	0.021	0.014	0.175
C(color)[T.I]:C(cut)[T.Premium]	0.0841	0.046	1.832	0.067	-0.006	0.174
C(color)[T.E]:C(cut)[T.Very Good]	-0.0931	0.041	-2.294	0.022	-0.173	-0.014
C(color)[T.F]:C(cut)[T.Very Good]	-0.1013	0.041	-2.459	0.014	-0.182	-0.021
C(color)[T.H]:C(cut)[T.Very Good]	-0.0247	0.043	-0.576	0.564	-0.109	0.059
C(color)[T.I]:C(cut)[T.Very Good]	0.0359	0.048	0.753	0.451	-0.057	0.129

Omnibus:	4862.888	**Durbin-Watson:**	0.101
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1246.556
Skew:	0.108	**Prob(JB):**	2.06e-271
Kurtosis:	2.100	**Cond. No.**	20.8

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

Based on the model summary table, many of the color grades' and cuts' associated beta coefficients have a p-value of less than 0.05 (check the P>|t| column). Additionally, some of the interactions also seem statistically signifcant. We'll use a two-way ANOVA to examine further the relationships between price and the two categories of color grade and cut.

First, we have to state our three pairs of null and alternative hypotheses:

Null Hypothesis (Color) $H0: priceD = priceE = priceF = priceH = priceI$

There is no difference in the price of diamonds based on color.

Alternative Hypothesis (Color) $H1$:Not $priceD = priceE = priceF = priceH = priceI$

There is a difference in the price of diamonds based on color.

Null Hypothesis (Cut) $H0: priceIdeal = pricePremium = priceVery\ Good$

There is no difference in the price of diamonds based on cut.

Alternative Hypothesis (Cut) $H1$:Not $priceIdeal$=$pricePremium$=$priceVery\ Good$

There is a difference in the price of diamonds based on cut.

Null Hypothesis (Interaction) $H0$:The effect of color on diamond price is independent of the cut, and vice versa.

Alternative Hypothesis (Interaction) $H1$:There is an interaction effect between color and cut on diamond price.

The syntax for a two-way ANOVA is the same as for a one-way ANOVA. We will continue to use the anova_lm() function from statsmodels.stats.

Run two-wav ANOVA

In [38]:

```
sm.stats.anova_lm(model2, typ = 2)
```

Out[38]:

	sum_sq	df	F	PR(>F)
C(color)	926.361461	4.0	237.014783	3.481145e-201
C(cut)	630.641441	2.0	322.706309	1.348511e-139
C(color):C(cut)	27.478611	8.0	3.515279	4.531734e-04
Residual	34120.806577	34920.0	NaN	NaN

In [39]:

```
sm.stats.anova_lm(model2, typ = 1)
```

Out[39]:

	df	sum_sq	mean_sq	F	PR(>F)
C(color)	4.0	977.195814	244.298954	250.021037	3.747388e-212
C(cut)	2.0	630.641441	315.320721	322.706309	1.348511e-139
C(color):C(cut)	8.0	27.478611	3.434826	3.515279	4.531734e-04
Residual	34920.0	34120.806577	0.977114	NaN	NaN

In [40]:

```
sm.stats.anova_lm(model2, typ = 3)
```

Out[40]:

	sum_sq	df	F	PR(>F)
Intercept	157578.043681	1.0	161268.910012	0.000000e+00
C(color)	319.145817	4.0	81.655250	4.134649e-69
C(cut)	100.144107	2.0	51.244864	5.987341e-23
C(color):C(cut)	27.478611	8.0	3.515279	4.531734e-04
Residual	34120.806577	34920.0	NaN	NaN

Since all of the p-values (column PR(>F)) are very small, we can reject all three null hypotheses.

ANOVA post hoc test

One-way ANOVA: Compares the means of one continuous dependent variable based on three or more groups of one categorical variable.

Post hoc test: Performs a pairwise comparison between all available groups while controlling for the error rate.

In [41]:

```
# Import statsmodels package and ols function
import statsmodels.api as sm
from statsmodels.formula.api import ols
```

In [42]:
```
# Load in the data set from one-way ANOVA
diamonds = pd.read_csv("diamonds.csv")
```

In [43]:
```
diamonds.head()
```

Out[43]:

	color	price	log_price
0	E	326	5.786897
1	E	326	5.786897
2	E	327	5.789960
3	I	334	5.811141
4	I	336	5.817111

In [44]:
```
# Construct simple linear regression model, and fit the model
model = ols(formula = "log_price ~ C(color)", data = diamonds).fit()
```

In [45]:
```
# Get summary statistics
model.summary()
```

Out[45]:

OLS Regression Results

Dep. Variable:	log_price	**R-squared:**	0.026
Model:	OLS	**Adj. R-squared:**	0.026
Method:	Least Squares	**F-statistic:**	265.0
Date:	Fri, 02 Jun 2023	**Prob (F-statistic):**	3.61e-225
Time:	02:14:49	**Log-Likelihood:**	-56182.
No. Observations:	39840	**AIC:**	1.124e+05
Df Residuals:	39835	**BIC:**	1.124e+05
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	7.6169	0.012	632.421	0.000	7.593	7.641
C(color)[T.E]	-0.0375	0.016	-2.394	0.017	-0.068	-0.007
C(color)[T.F]	0.1455	0.016	9.240	0.000	0.115	0.176
C(color)[T.H]	0.3015	0.016	18.579	0.000	0.270	0.333
C(color)[T.I]	0.4061	0.018	22.479	0.000	0.371	0.441

Omnibus:	7112.992	**Durbin-Watson:**	0.065
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1542.881
Skew:	0.079	**Prob(JB):**	0.00
Kurtosis:	2.049	**Cond. No.**	6.32

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [46]:
```
# Run one-way ANOVA
sm.stats.anova_lm(model, typ=2)
```

Out[46]:

	sum_sq	df	F	PR(>F)
C(color)	1041.690290	4.0	264.987395	3.609774e-225
Residual	39148.779822	39835.0	NaN	NaN

Since the p-value is very small and we can reject the null hypothesis that the mean price is the same for all diamond color grades, we can continue on to run a post hoc test. The post hoc test is useful because the one-way ANOVA does not tell us which colors are associated with different prices. The post hoc test will give us more information.

Post hoc test

In [48]:
```
# Import Tukey's HSD function
from statsmodels.stats.multicomp import pairwise_tukeyhsd
```

In [49]:
```
# Run Tukey's HSD post hoc test for one-way ANOVA
tukey_oneway = pairwise_tukeyhsd(endog = diamonds["log_price"], groups = diamonds["color"], alpha = 0.05
```

Then we can run the test. The endog variable specifies which variable is being compared across groups, which is log_price in this case. Then the groups variables indicates which variable holds the groups we're comparing, which is color. alpha tells the function the significance or confidence level, which we'll set to 0.05. We'll aim for the typical 95% confidence level.

In [50]:
```
# Get results (pairwise comparisons)
tukey_oneway.summary()
```

Out[50]:

Multiple Comparison of Means - Tukey HSD, FWER=0.05

group1	group2	meandiff	p-adj	lower	upper	reject
D	E	-0.0375	0.1169	-0.0802	0.0052	False
D	F	0.1455	-0.0	0.1026	0.1885	True
D	H	0.3015	-0.0	0.2573	0.3458	True
D	I	0.4061	-0.0	0.3568	0.4553	True
E	F	0.183	-0.0	0.1441	0.2219	True
E	H	0.339	-0.0	0.2987	0.3794	True
E	I	0.4436	-0.0	0.3978	0.4893	True
F	H	0.156	-0.0	0.1154	0.1966	True
F	I	0.2605	-0.0	0.2145	0.3065	True
H	I	0.1045	0.0	0.0573	0.1517	True

Each row represents a pariwise comparison between the prices of two diamond color grades. The reject column tells us which null hypotheses we can reject. Based on the values in that column, we can reject each null hypothesis, except when comparing D and E color diamonds. We cannot reject the null hypothesis that the diamond price of D and E color diamonds are the same.

result

We can reject the null hypothesis that the price of H and I color grade diamonds are the same.

In [ ]:
```

```