6.96. $\dfrac{\sqrt{1+x}}{1+\sqrt{1+x}} \geq \dfrac{\sqrt{1-x}}{1-\sqrt{1-x}}$

Огран.

$\begin{cases} 1+x \geq 0 \\ 1-x \geq 0 \\ 1-\sqrt{1-x} \neq 0 \end{cases}$

$\begin{cases} x \geq -1 \\ x \leq 1 \\ \sqrt{1-x} \neq 1 \end{cases}$

$1-x \neq 1$

$x \neq 0$

$\begin{cases} x \in [-1;1] \\ x \neq 0 \end{cases}$

$$\frac{\sqrt{1+x}(1-\sqrt{1-x})-\sqrt{1-x}(1+\sqrt{1-x})}{\underbrace{(1+\sqrt{1+x})}_{>0}(1-\sqrt{1-x})} \geq 0$$

$f(x) \geq 0$

$$\frac{\sqrt{1+x}-2\sqrt{1-x^2}-\sqrt{1-x}}{1-\sqrt{1-x}} \geq 0$$

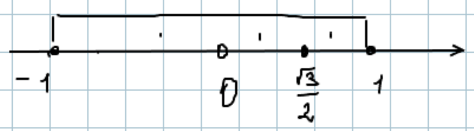
$t = \sqrt{1+x}-\sqrt{1-x}$, $t^2 = 1+\cancel{x}+1-\cancel{x}-2\sqrt{1-x^2}$

$-2\sqrt{1-x^2} = t^2-2$

$t+t^2-2=0$ $\quad t^2+t-2=0$

$(t+2)(t-1)=0$

$(\sqrt{1+x}-\sqrt{1-x}+2)(\sqrt{1+x}-\sqrt{1-x}-1)=0$



1) $\sqrt{1+x}-\sqrt{1-x}+2=0$

$\sqrt{1+x}+2=\sqrt{1-x}$

$\cancel{1}+x+4+4\sqrt{1+x}=\cancel{1}-x$

$4\sqrt{1+x}=-2x-4$

$2\sqrt{1+x}=-x-2$

$-x-2\geq 0$

$-x\geq 2$

$x\leq -2$

$x\in\varnothing$

2) $\sqrt{1+x}-\sqrt{1-x}-1=0$

$\sqrt{1+x}=1+\sqrt{1-x}$

$\cancel{1}+x=1+1-x+2\sqrt{1-x}$

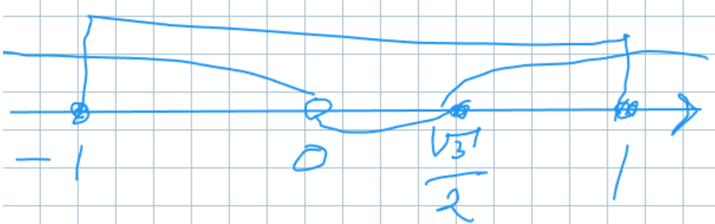
$2x-1=2\sqrt{1-x}$, $2x-1\geq 0$, $x\geq\frac{1}{2}$

$4x^2-\cancel{4x}+1=4-\cancel{4x}$

$4x^2=3$

$x^2=\frac{3}{4}$

$x=\left[\begin{array}{l}\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} \text{ — постор.}\end{array}\right.$



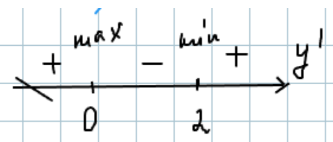
$x\in[-1;0)\cup\left[\frac{\sqrt{3}}{2};1\right]$

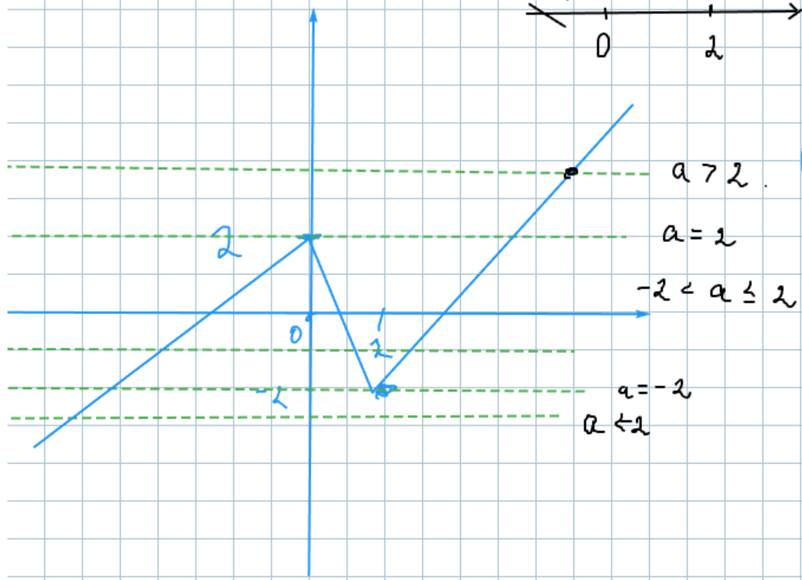
9. $x^3-3x^2+2=a$. ск. корней при $a>2$.

$y=x^3-3x+2$

$y=a$.

$y'=3x^2-6x=3x(x-2)=0$





Ответ: 1

10. $y = \ln x - ax^2$ $\quad x>0 \quad x \in (2;+\infty) \quad (\ln x)' = \frac{1}{x}$

$y' = \frac{1}{x} - 2ax < 0$

$\frac{1-2ax^2}{\underbrace{x}_{>0}} < 0$;

$1-2ax^2 < 0$

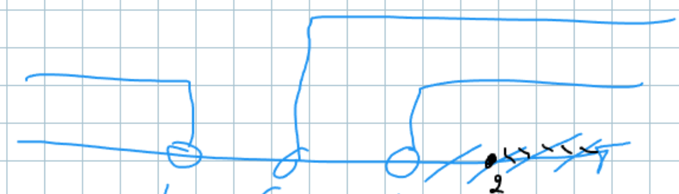
$\boxed{ax^2 > \frac{1}{2}}$

1). $a = 0 \quad 0 > \frac{1}{2}$ неверно

2) $a > 0; x^2 > \frac{1}{2a} \to x \in \left(-\infty; -\frac{1}{\sqrt{2a}}\right) \cup \left(\frac{1}{\sqrt{2a}}; +\infty\right)$

3). $a < 0; x^2 < \frac{1}{2a} \quad \varnothing$

2)



$-\frac{1}{\sqrt{2a}} \quad 0 \quad \frac{1}{\sqrt{2a}} \le 2$

$\frac{1}{\sqrt{2a}} \le 2$

$1 \le 2\sqrt{2a}; \; 1 \le 8a; \; a \ge \frac{1}{8}$

Ответ: $[\frac{1}{8}; +\infty)$

1. $\log_5 20 \cdot \log_{20} 0{,}04$ $\quad \frac{1}{25}$

$\cancel{\log_5 20} \left((-1)\frac{\log_5 25}{\cancel{\log_5 20}}\right) = -2$

2. $2\,tg\frac{11\pi}{12}\sin\frac{\pi}{6} = 2\cdot\frac{\sin\frac{\pi}{12}}{(-\cos\frac{\pi}{12})}\cdot 2\sin\frac{\pi}{12}\cos\frac{\pi}{12} =$ $\quad$ co

-1

$\sin\frac{\pi}{6} = 2\sin\frac{\pi}{12}\cos\frac{\pi}{12}$ $\quad = -4\sin^2\frac{\pi}{12} =$

$\cos\frac{11\pi}{12} = \cos(\pi - \frac{\pi}{12}) = -\cos\frac{\pi}{12}$ $\quad = -2(1-\cos\frac{\pi}{6}) =$

$= -2(1 - \frac{\sqrt{3}}{2})$

3. $\quad$ I, II, III $\quad$ 4 $\quad$ 216

$\mathrm{I}+\mathrm{II}+\mathrm{III}$

$\begin{cases} (v_1+v_2+v_3)\cdot 4 = 216 \\ v_3 - v_2 = v_2 - v_1 \\ 5v_3 = 7v_1 \end{cases}$ $\quad$ $216|2$, $108|2$, $54|$

$\begin{cases} v_1+v_2+v_3 = 54 \\ v_3 + v_1 = 2v_2 \\ 5v_3 = 7v_1 \end{cases}$

$3v_2 = 54$

$v_2 = 18$

$\begin{cases} v_3 + v_1 = 36 \\ 5v_3 = 7v_1 \end{cases}$

$v_3 = \frac{7v_1}{5}$

$\frac{7v_1}{5} + v_1 = 36$

$7v_1 + 5v_1 = 36\cdot 5$

$12 v_1 = 36\cdot 5$

$v_1 = 3\cdot 5 = 15$

Ответ: 15

$\frac{6}{x-1} \geq \frac{1}{\sqrt{x}-1}$ $\quad$ $\begin{cases} x \geq 0 \\ x \neq 1 \end{cases}$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha =$$
$$= 1 - 2\sin^2\alpha$$
$$2\sin^2\alpha = 1 - \cos 2\alpha$$

$$= -2\cdot\frac{2-\sqrt{3}}{2} = \sqrt{3}-2$$

$$\frac{6}{x-1}-\frac{1}{\sqrt{x}-1}\geq 0$$

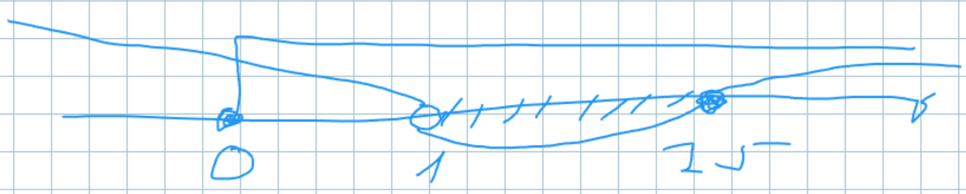
$$\frac{6-(\sqrt{x}+1)}{x-1}\geq 0;\quad \frac{-\sqrt{x}+5}{x-1}\geq 0$$

$$\frac{25-x}{x-1}\geq 0;\quad \frac{x-25}{x-1}\leq 0$$

$5-\sqrt{x}$

$\sqrt{25}-\sqrt{x}$

$f(x)=\sqrt{x}$ – возр



$(1;25]$

2 25

$$\frac{2+25}{2}\cdot 24^{12}=27\cdot 12$$

$$\begin{array}{r} 27\\ \times\ 12\\ \hline 54\\ 27\\ \hline 324 \end{array}$$

Ответ: 324

$$\begin{cases} x+6y-xy=9\\ 2x-3y+xy=6 \end{cases}$$

$3x+3y=15$

$x+y=5$

$x=5-y$

$5-y+6y-y(5-y)=9$

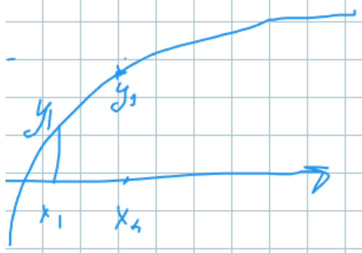
$5+\cancel{5y}-\cancel{5y}+y^2=9$

$y^2=4$

$y=\pm 2$

$y=2$, $x=3$

$y=-2$, $x=7$



y_1
y_1
x_1
x_2

$2 \cdot 3 \cdot (-2) \cdot 7 = -12 \cdot 7 = -84$

Ответ: -84

$p \neq 0 \quad q \neq 0$

6. $x^2 - 4p\,x + q^3 = 0$

$x_1^{-1} + x_2^{-1}$

$x_1 = p \quad x_2 = q$

$\begin{cases} x_1 + x_2 = 4p \\ x_1 \cdot x_2 = q^3 \end{cases}$

$\begin{cases} p + q = 4p \\ p \cdot q = q^3 \end{cases}$

$\begin{cases} p = q^2 \\ 3p = q \end{cases}$

$p = 9p^2$

$p(9p - 1) = 0$

$\cancel{p \neq 0}$ $\quad p = \frac{1}{9}$

$q \neq 0$ $\quad q = \frac{1}{3}$

$\left(\frac{1}{9}\right)^{-1} + \left(\frac{1}{3}\right)^{-1} = 9 + 3 = 12$

Ответ: 12

$\left(2\pi; \frac{7\pi}{2}\right)$

$\cos 2x = \sin\left(\frac{\pi}{2} + x\right) = +\cos x$

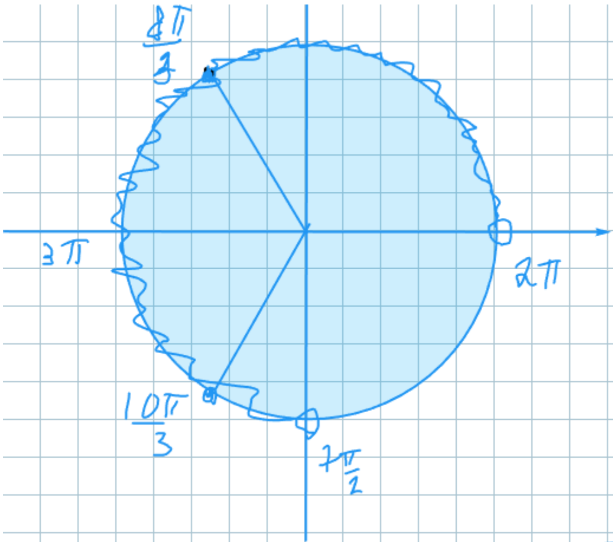
$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = 1 \qquad \cos x = -\frac{1}{2}$

$x = 2\pi k, k \in \mathbb{Z} \qquad x = \pm\frac{2\pi}{3} + 2\pi k$



$\frac{8\pi}{3} + \frac{10\pi}{3} =$

$= \frac{18\pi}{3} =$

$= 6\pi$

$\log_{2x}(7-x) + \log_{2x}(x+6) \geq 1$

$\begin{cases} x > 0 \\ 2x \neq 1 \rightarrow x \neq \frac{1}{2} \\ 7-x > 0 \quad x < 7 \\ x+6 > 0 \quad x > -6 \end{cases}$

$\begin{cases} x \in (0; 7) \\ x \neq \frac{1}{2} \end{cases}$

$\log_{2x}\left(\frac{(7-x)(x+6)}{2x}\right) \geq 0$

$\frac{\log_2\left(\frac{(7-x)(x+6)}{2x}\right) - \log_2 1}{\log_2 2x - \log_2 1} \geq 0$

$F(x) = \log_2 t$ - возр. $\Rightarrow$

$\Rightarrow \frac{\frac{(7-x)(x+6)}{2x} - 1}{2x-1} \geq 0$

$\frac{-1 - \frac{(x-7)(x+6)}{2x}}{2x-1} \geq 0$

$\frac{2x + (x^2 - x - 42)}{2x(2x-1)} \leq 0$

$\log_{2x}(7-x)(x+6) \geqslant \log_{2x} 2x$

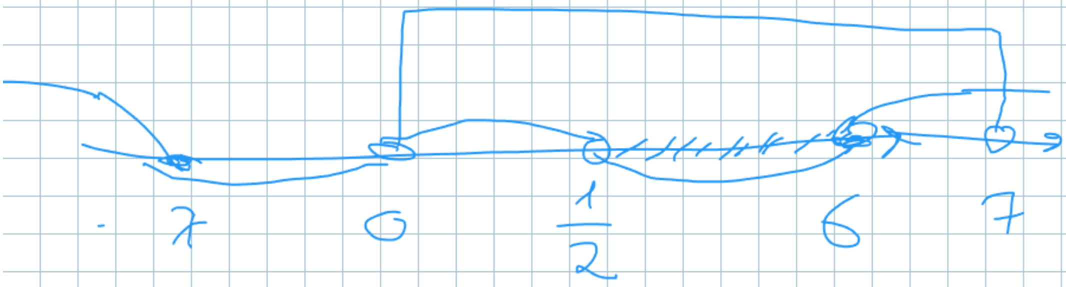
1) $\begin{cases} 2x > 1 \\ (7-x)(x+6) \geqslant 2x \end{cases}$ $\quad \begin{cases} 0 < 2x < \\ (7-x)(x+6) \leq 2 \end{cases}$

$9 - 3\sqrt{3}$ 2

A

$$\frac{x^2+x-42}{x(2x-1)} \le 0$$

$$\frac{(x+7)(x-6)}{x(2x-1)} \le 0$$



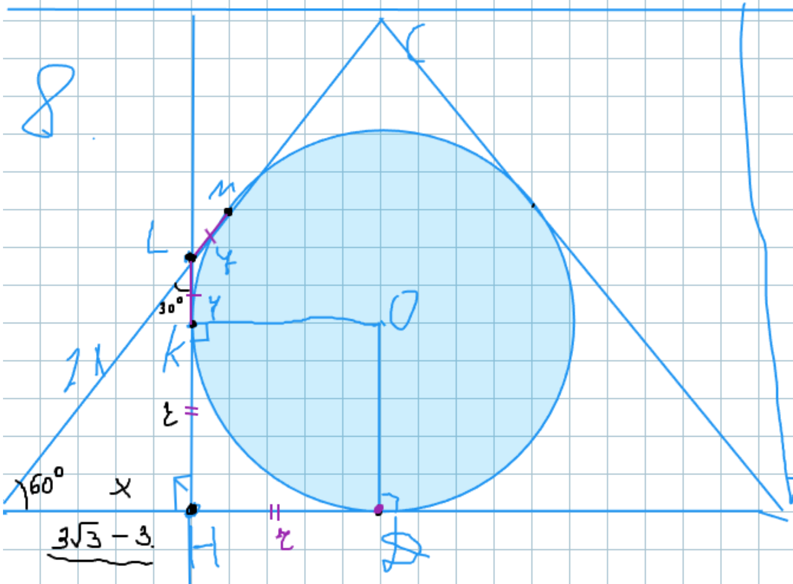
$\left(\frac{1}{2}; 6\right]$

$[1; 6]$

$\frac{1+6}{2} \cdot 6 = 7 \cdot 3 = 21$

Ответ: 21

8



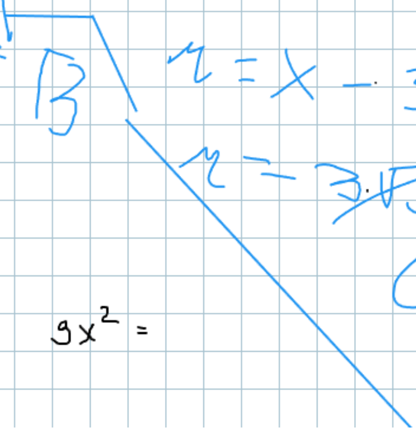
$AM = 1 D$

$2x + y = 3\sqrt{3} - 3 + r$

$2x + 9 - 3\sqrt{3} - r = 3\sqrt{\ }$

$2x - 2r = 6\sqrt{3} -$

$x - r = 3\sqrt{3} - \cancel{3}$



$r = x - 3\sqrt{3} +$

$r = \cancel{3\sqrt{3} - 3} -$

Отве

$HL = 9 - 3\sqrt{3}$

$4x^2 = x^2 + (9 - 3\sqrt{3})^2$

$9x^2 =$

$\overline{3} - 3 + 2$

12

6

$\cancel{3\sqrt{3}} + 6 = 3$

Отв: 3

$x^2 = \frac{(9-3\sqrt{3})^2}{3}$, $x = \frac{9-3\sqrt{3}}{\sqrt{3}} = 3\sqrt{3} - 3$

$a < 0$

$|x^2-1| + ax = 2a+1$

1) $x \in (-\infty; -1] \cup [1; +\infty)$ 2)

$x^2 - 1 + ax - 2a - 1 = 0$

$x^2 + ax - 2a - 2 = 0$

$D = a^2 - 4(-2a-2) = a^2 + 8a + 8 > 0$

$(a+4)^2 - 8 > 0$ $\frac{D}{4} = 16 + 8 = 24$

$(a+4+2\sqrt{2})(a+4-2\sqrt{2}) > 0$

$-4-2\sqrt{2}$ $-4+2\sqrt{2}$ 0 a

10.

$\underbrace{|x^2-1|}_{y} = -ax + 2a + 1.$

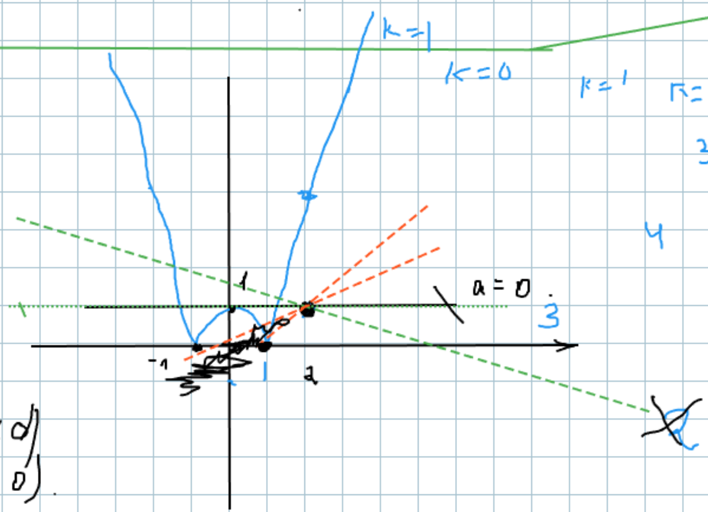
$y = -ax + 2a + 1.$

$y = (-a)(x-2) + 1$ $(2;1)$

$a = 0$. П^{ри} реш.

$a < 0 \Rightarrow -a > 0$.

$(-1;0)$
$(1;0)$



k=1 k=2 3 4

$-a(-1-2)+1 = 0$ $-a(1-2)+1 = 0$

$3a = -1$ $a + 1 = 0$

$a = -\frac{1}{3}$ $a = -1$

$a < -\frac{1}{3}$ $\qquad$ $a > -1$

$$-1 < a < -\frac{1}{3}$$

Ответ: $(-1; \frac{1}{3})$