Deep Learning

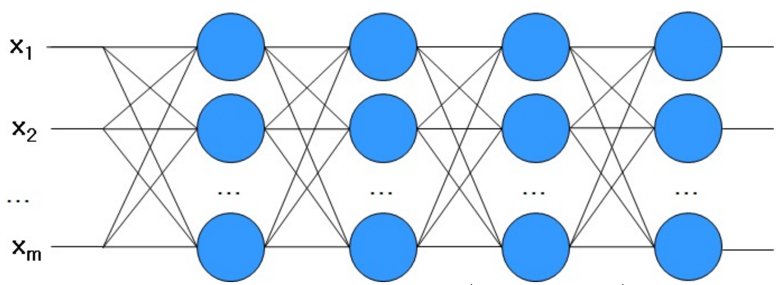
Content

- Vanishing Gradient & Activation Functions
- Dropout
- Batch Normalization

Gradient Vanishing & Activation Functions

Gradient Vanishing & Exploding

- **Gradient is easy to vanish or explode**
  - To many terms are multiplied.
  - If some are small numbers, gradient becomes very small.
  - If some are large numbers, gradient becomes very large.



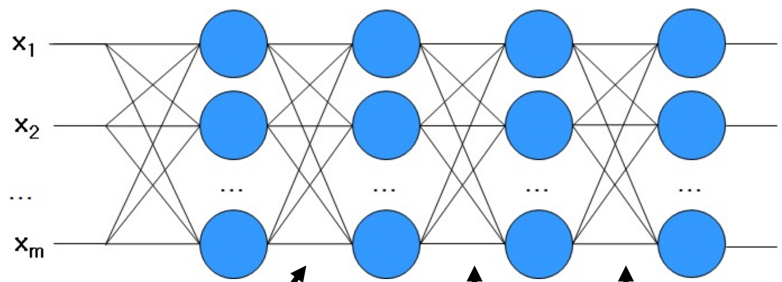
$$\frac{\partial E_n}{\partial w_{kj}} = -(t_{nk} - o_{nk}) o_{nk} (1 - o_{nk}) h_{nj}$$

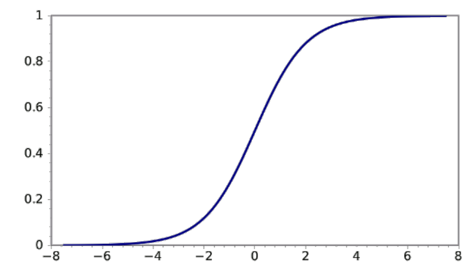
$$\frac{\partial E_n}{\partial w_{ji}} = -h_{nj}(1-h_{nj}) x_{ni} \sum_{k=1}^{m} w_{kj} (t_{nk} - o_{nk}) o_{nk} (1 - o_{nk})$$

$$\frac{\partial E}{\partial w_{ip}} = \left(\sum_{j=1}^{J} \left(\sum_{k=1}^{K} -(t_n - o_{nk}) o_{nk} (1 - o_{nk}) w_{kj} \right) h_{nj} (1 - h_{nj}) w_{ji} \right) h_{ni} (1 - h_{ni}) h_{np}$$

Activation Function

- **Vanishing Gradient**
  - The major terms are the derivatives of the activation function





$$\frac{\partial \textbf{Sigmoid}}{\partial w} \le \frac{1}{4}$$

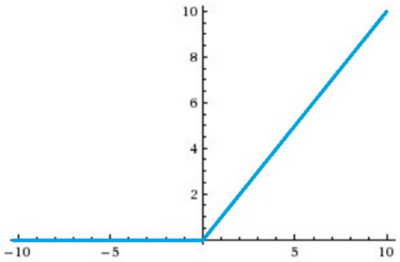
$$\frac{\partial E_n}{\partial w_{kj}} = -(t_{nk} - o_{nk})o_{nk}(1-o_{nk})h_{nj}$$

$$\frac{\partial E_n}{\partial w_{ji}} = -h_{nj}(1-h_{nj})x_{ni}\sum_{k=1}^{m} w_{kj}(t_{nk}-o_{nk})o_{nk}(1-o_{nk})$$

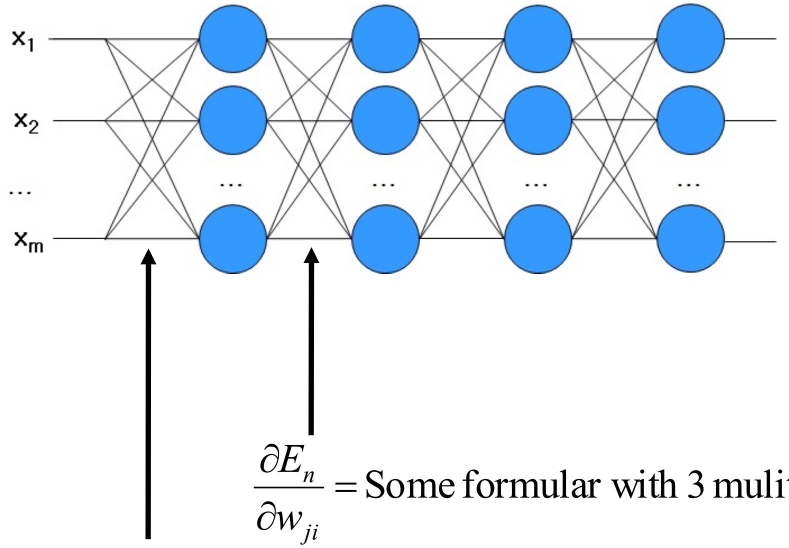
$$\frac{\partial E}{\partial w_{ip}} = \left(\sum_{j=1}^{J}\left(\sum_{k=1}^{K} -(t_n - o_{nk})o_{nk}(1-o_{nk})w_{kj}\right)h_{nj}(1-h_{nj})w_{ji}\right)h_{ni}(1-h_{ni})h_{np}$$

Activation Function

- **Using another functions instead of sigmoid**
  - Rectified Linear Unit (ReLU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$



$$\frac{\partial E_n}{\partial w_{ji}} = \text{Some formular with 3 mulitiplications of } \frac{\partial f}{\partial w}$$

$$\frac{\partial E_n}{\partial w_{hg}} = \text{Some formular with 4 mulitiplications of } \frac{\partial f}{\partial w}$$

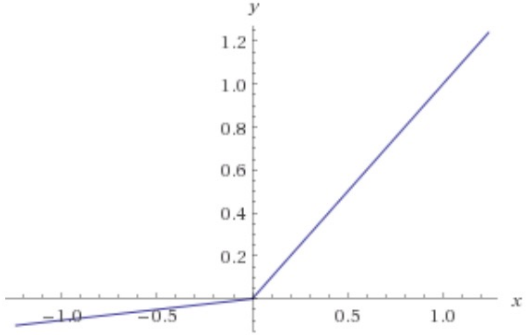
Activation Function

- **Advantage**
  - No vanishing gradient problems.
    - Deep networks can be trained without pre-training
  - Sparse activation
    - In a randomly initialized network, only about 50% of hidden units are activated
  - Fast computation:
    - 6 times faster than sigmoid function
- **Disadvantage**
  - Knockout Problem

Activation Function

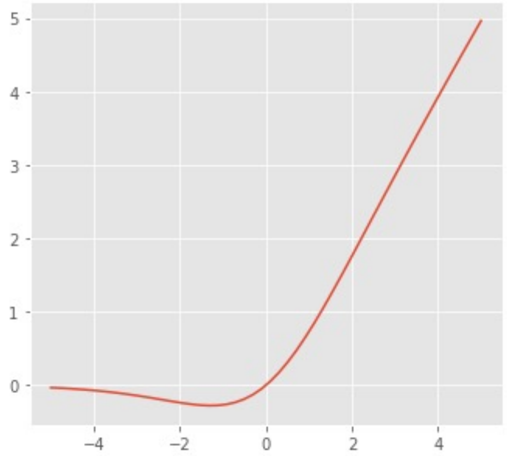
- **You may use another**
  - Leaky ReLU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ 0.01x & \text{otherwise} \end{cases}$$

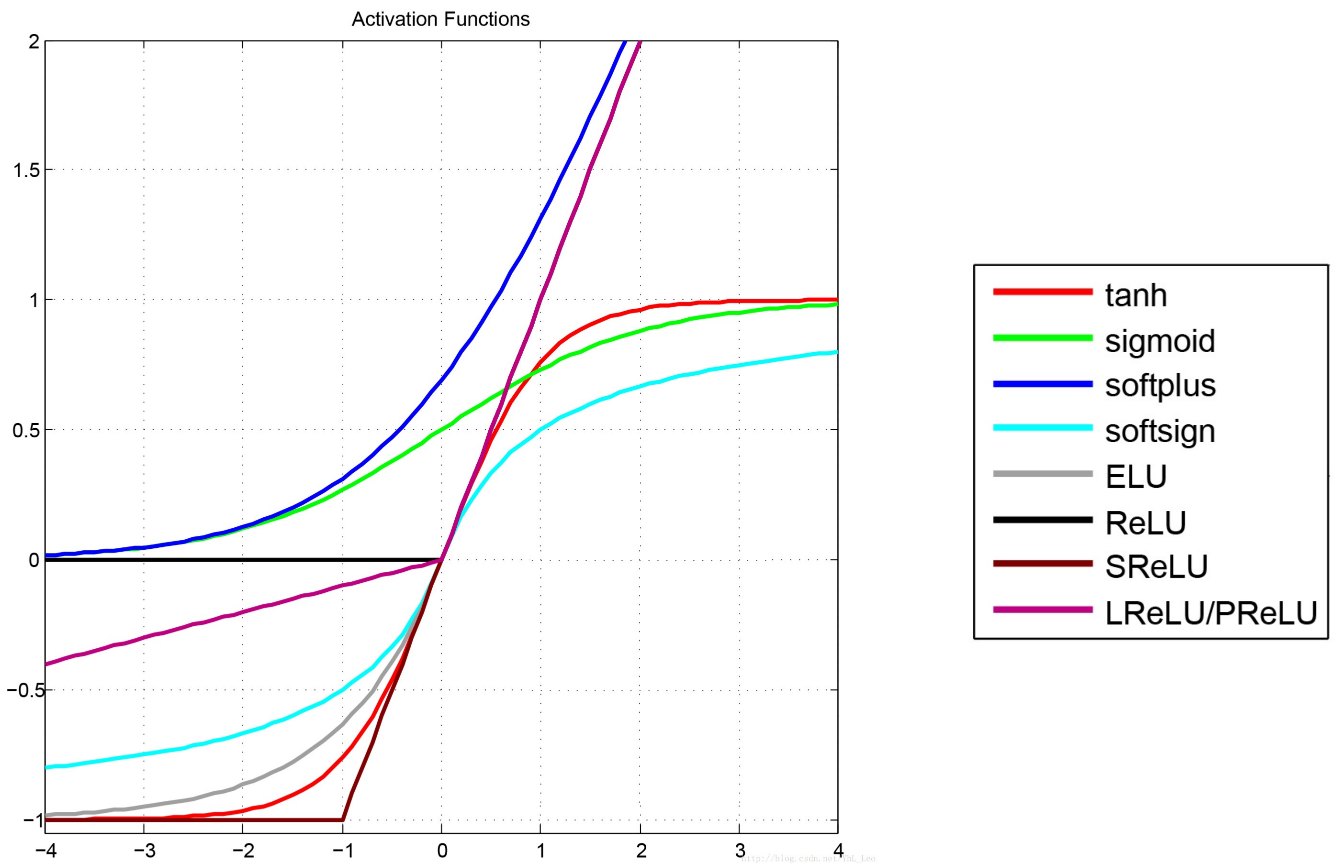


  - Swish (or SiLU-Sigmoid Linear Unit)

$$f(x) = \frac{x}{1 + e^{-x}}$$



Other Activation Functions



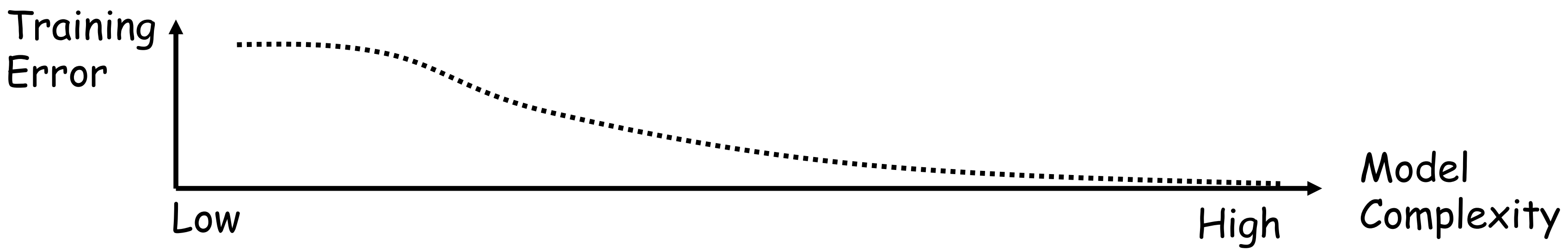
Activation Function

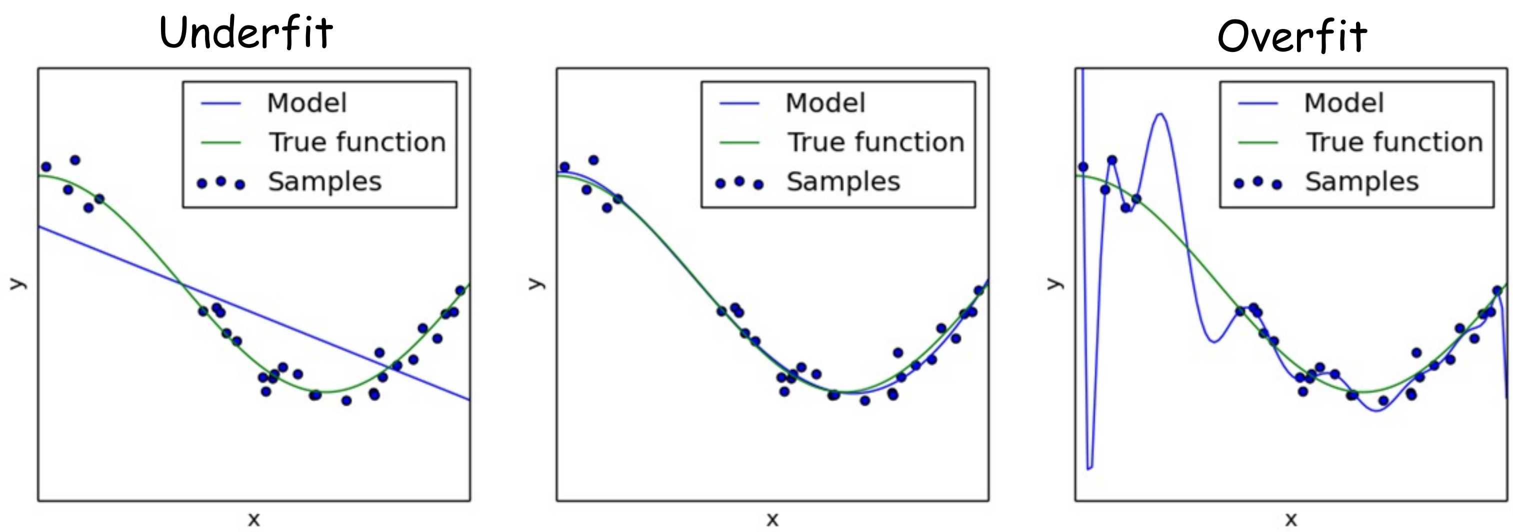
- **Summary**
  - Sigmoid functions and their combinations generally work better but are sometimes avoided due to the vanishing gradient problem
  - ReLU function is a general activation function and is used in most cases these days
  - If we encounter a case of dead neurons in our networks the leaky ReLU function is the best choice
  - ReLU function is usually used in the hidden layers
  - As a rule of thumb, you can begin with using ReLU function and then move over to other activation functions in case ReLU doesn’t provide with optimum results

Regularization

Overfitting

- **Overfitting**





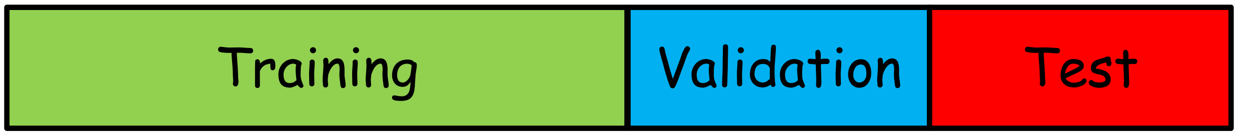
Regularization

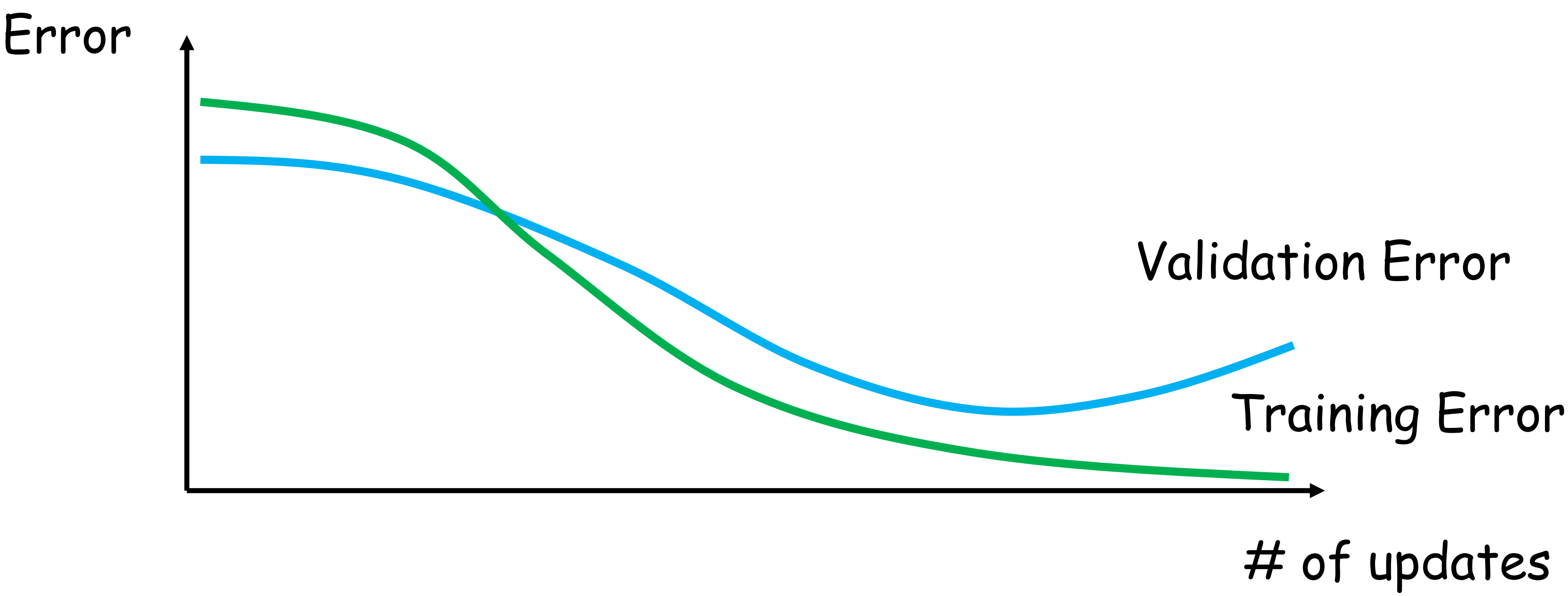
- **What is Regularization**
  - Introducing additional information to prevent over-fitting

- **Approaches**
  - Proper Learning: Early stopping
  - Proper Structure: Weight decay, Dropout,
    DropConnect, Stochastic pooling

Early Stopping

- **Split data into 3 groups**





Weight Decay

- **L1 Regularization**
  - Leading most weights very close to zero
  - Choosing a small subset of most important inputs
  - Resistant to noise in the inputs.

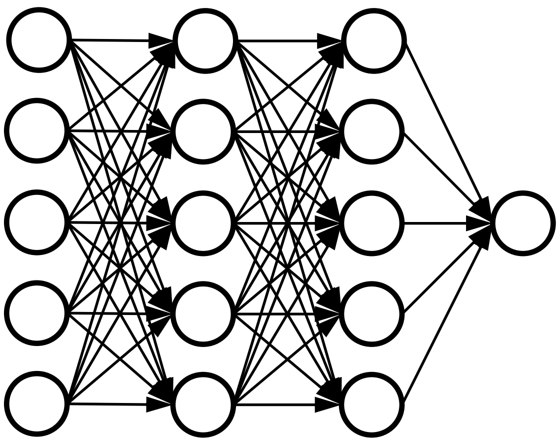
$$\widetilde{E}(\mathbf{w}) = E(\mathbf{w}) + \frac{\lambda}{2}|\mathbf{w}|$$

- **L2 Regularization**
  - Penalizing peaky weights
  - Encouraging to use all of its inputs a little rather than using only some of its inputs a lot.

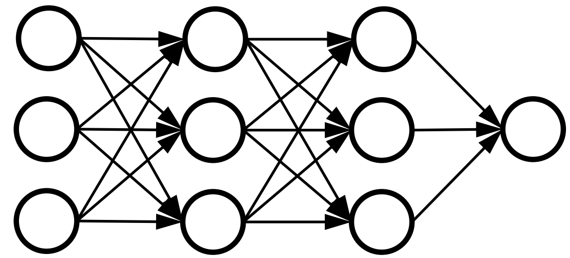
$$\widetilde{E}(\mathbf{w}) = E(\mathbf{w}) + \frac{\lambda}{2}\mathbf{w}^T\mathbf{w}$$

Weight Decay

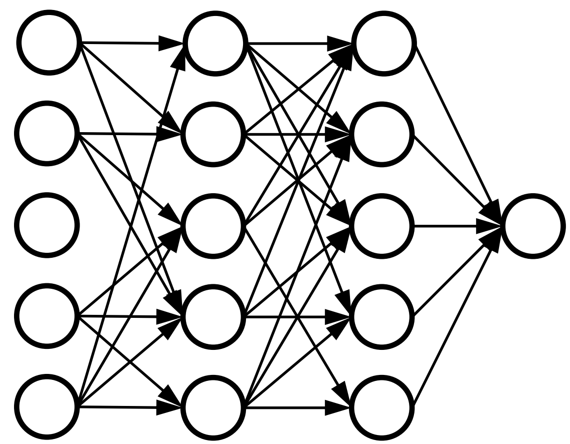
- **Complex Structure vs Simple Structure**



Node
Pruning

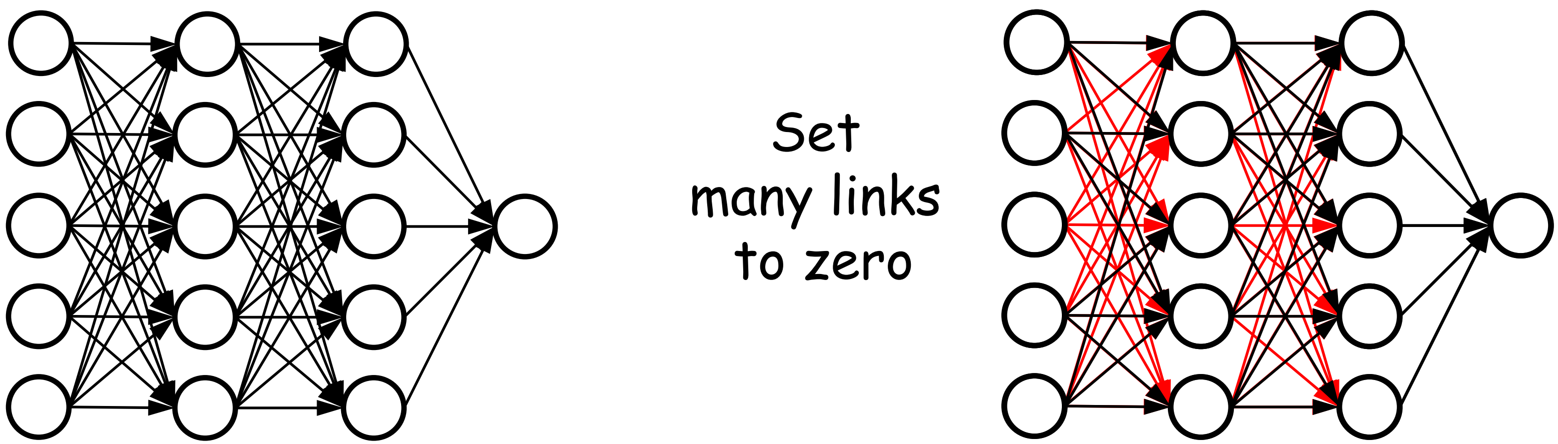


Link
Pruning



Weight Decay

- **Complex Structure vs Simple Structure**

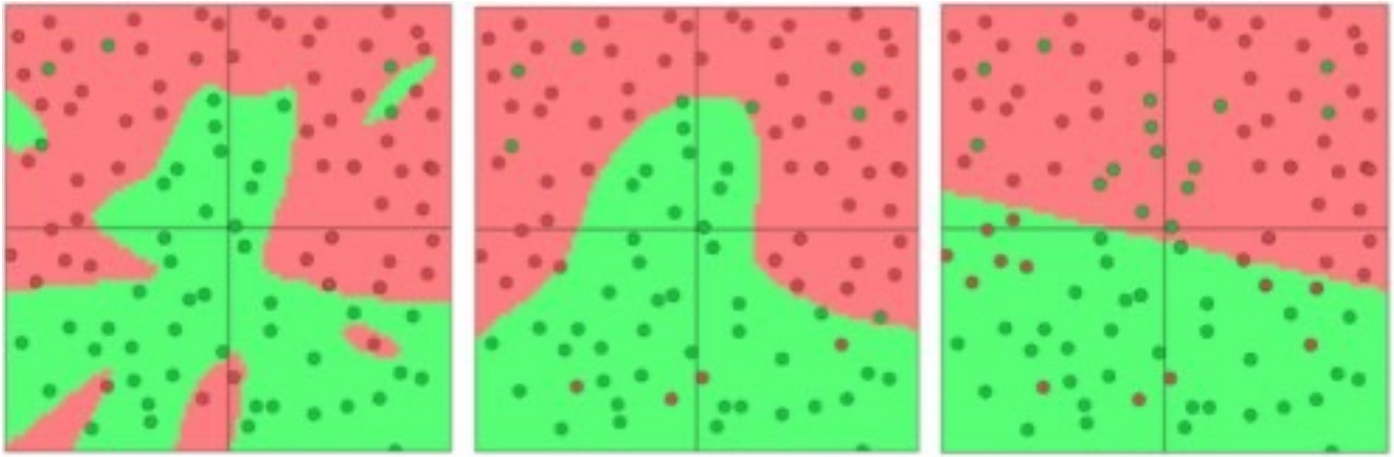


|w| is large <-> NN is Complex

|w| is small <-> NN is Simple

Weight Decay

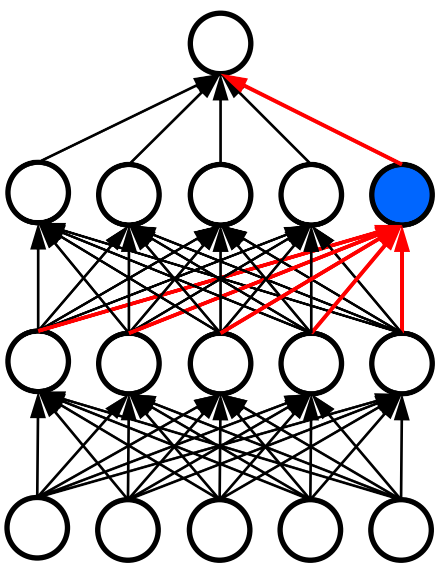
- **Example: Separating green and red**



L2 regularization strengths of 0.01, 0.1, and 1

Dropout

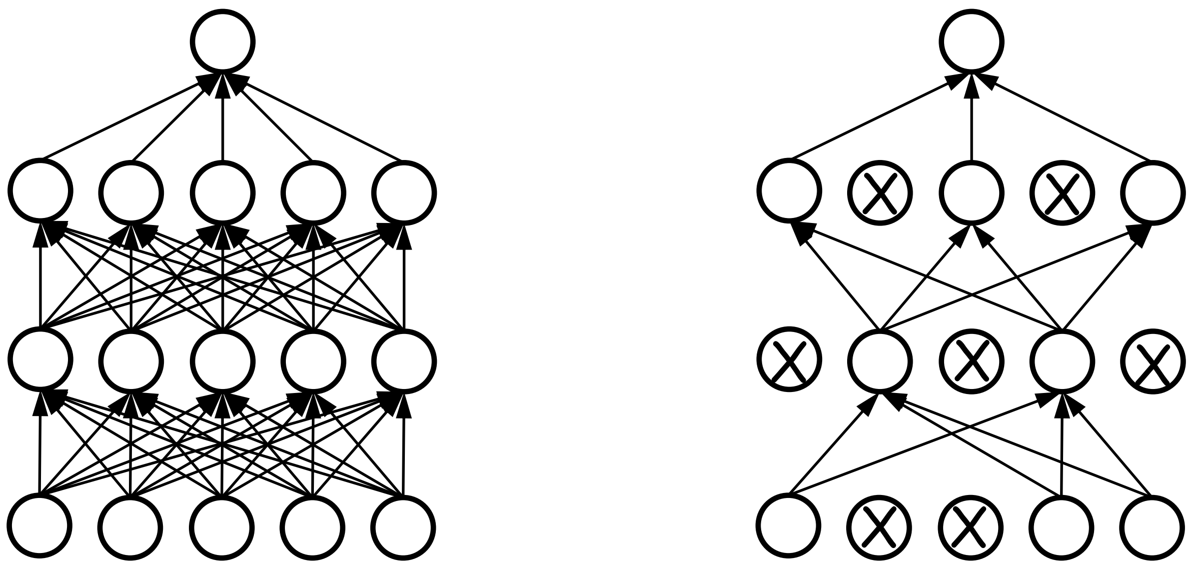
- **In a complex Neural Network**
  - All nodes do not take the same amount of responsibility
    - While training, some nodes are correlated
  - All nodes are not equally trained
    - Some nodes are trained much, but some are not



- If the output of the node is bad, the connection weight will decrease.
- If connection weight is close to 0, precedent connection weights are hardly trained.

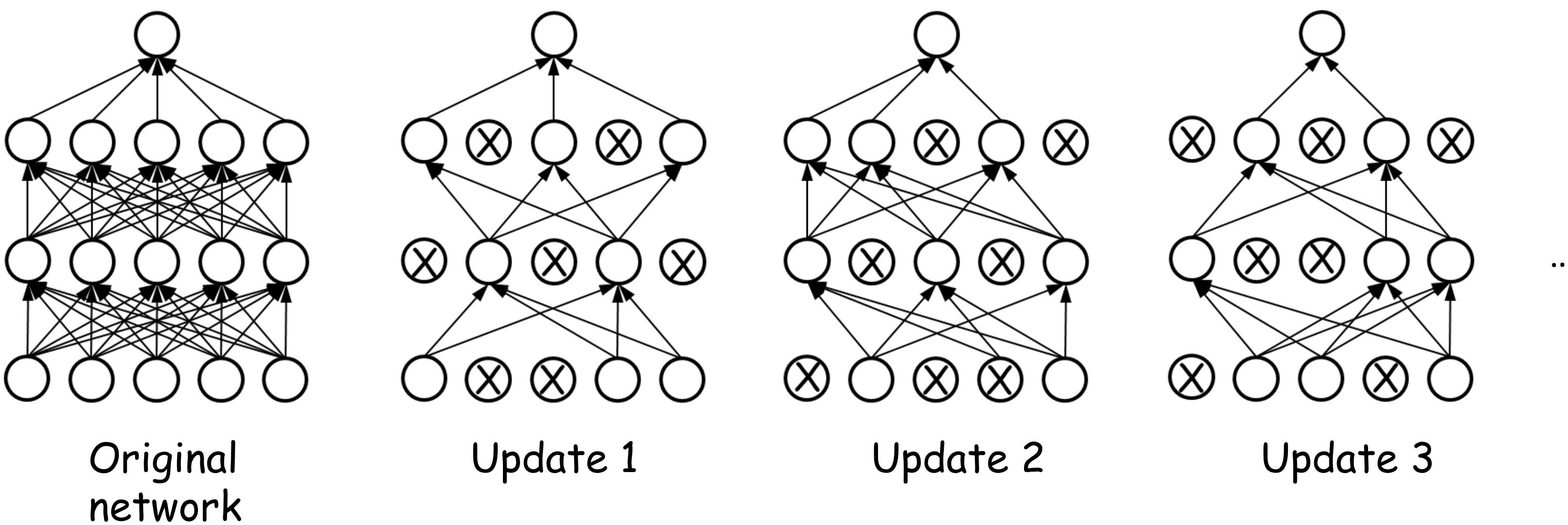
Dropout

- **How can we reduce the structural complexity?**
  - Let's simply remove some nodes, and
  - Train the simplified neural network
  - Hmm??



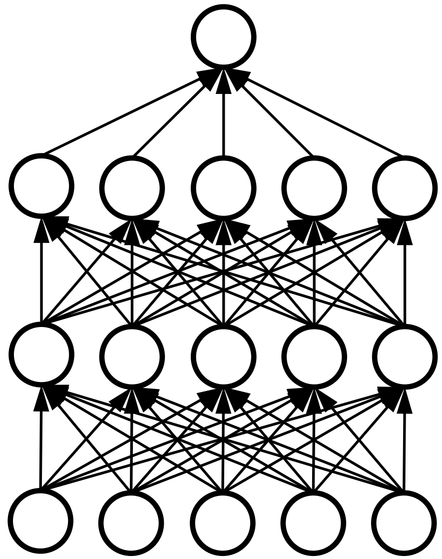
Dropout

- **Do this at every epoch**
  - Randomly choose nodes with a probability of *p*
    - Usually p = 0.5
  - Train the simplified neural network
    - At every epoch, we train different neural network which share connection weight each other



Dropout

- **Testing**
  - Use all the nodes without dropout

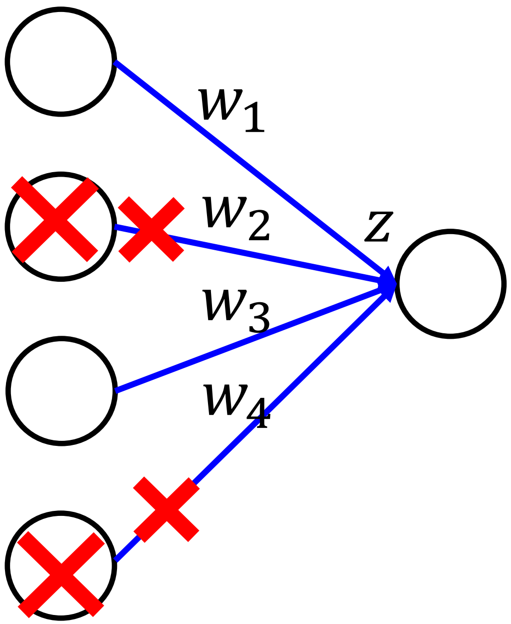


Dropout

- **Testing**

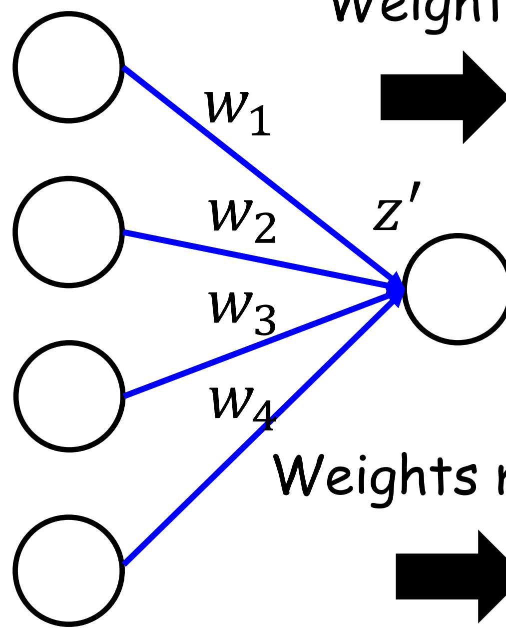
Training of Dropout

Assume dropout rate is 50%



Testing of Dropout

No dropout



Weights from training

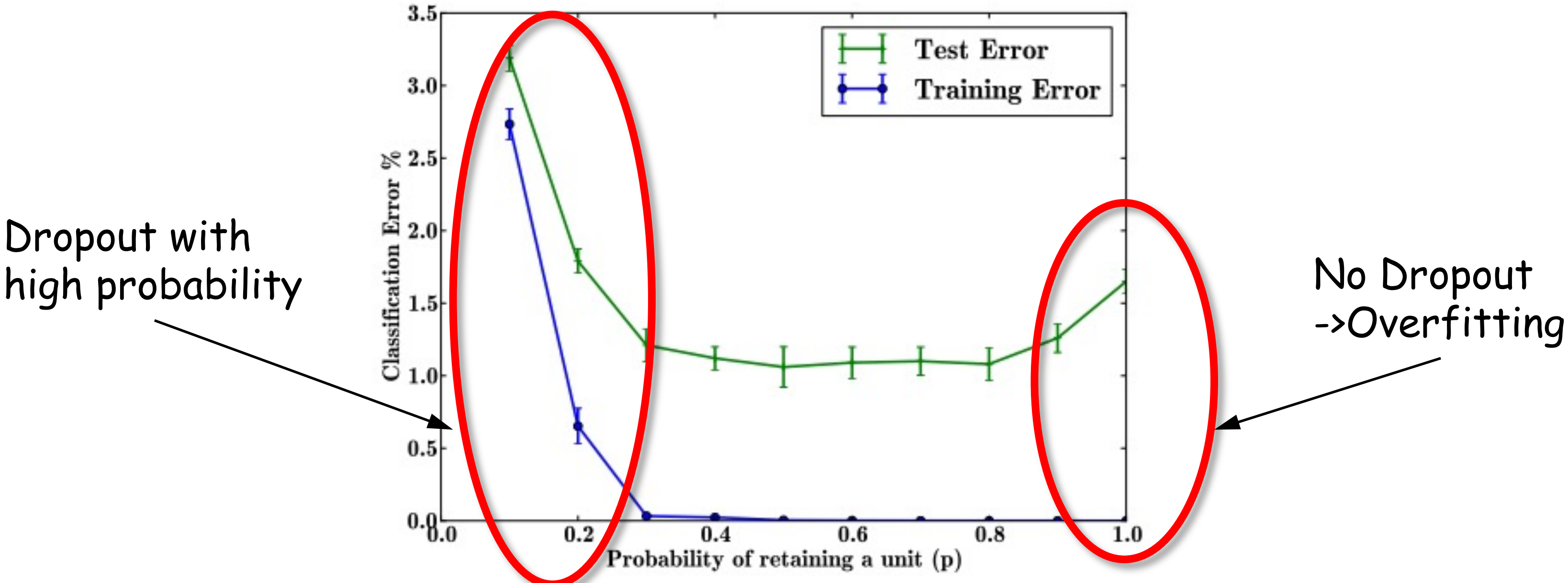
$z' \approx 2z$

Weights multiply (1-p)%

$z' \approx z$

Dropout

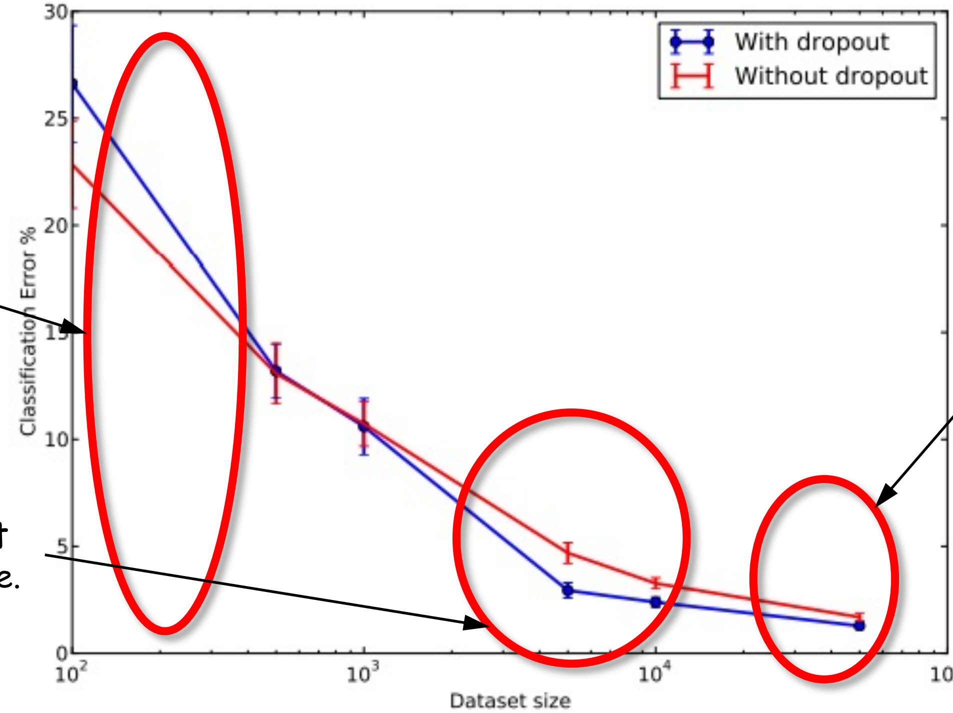
- **The effect of the dropout rate p:**
  - An architecture of 784-2048-2048-2048-10 is used on the MNIST dataset.
  - The dropout rate p is changed from small numbers (most units are dropped out) to 1.0 (no dropout).



Dropout

- **The effect of data set size:**
  - An architecture of 784-1024-1024-2048-10 is used on the MNIST dataset.

Extremely small data set
- Dropout does not improve error rate, and even makes it worse.



Huge data set
- Dropout barely improves the error rate. The data set is big enough, so that overfitting is not an issue.

Average to large data set
- Dropout improves error rate.

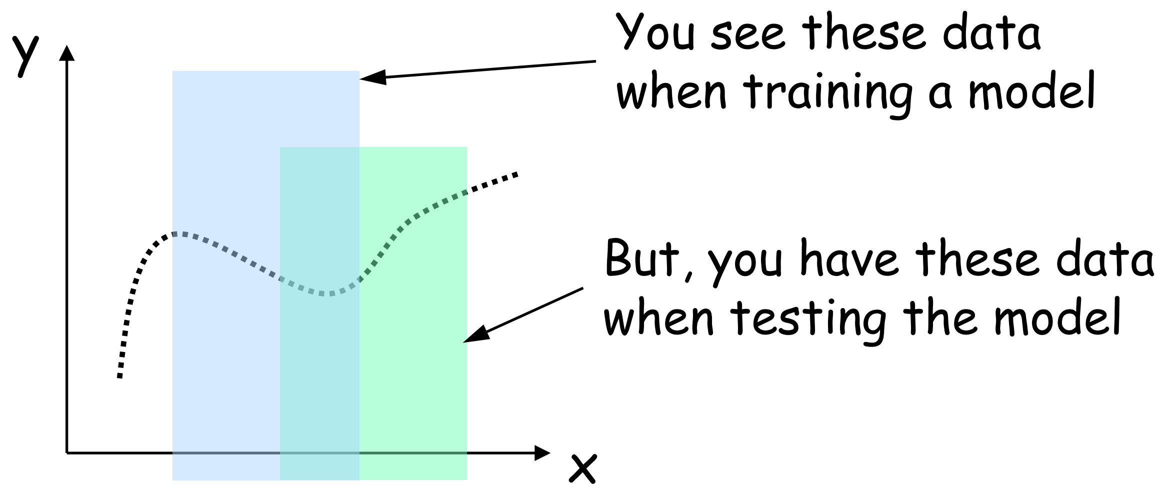
Dropout

- **Summary**
  - Dropout is a very good and fast regularization method.
  - Dropout is a bit slow to train (2-3 times slower than without dropout).
  - If the amount of data is average-large – dropout excels. When data is big enough, dropout does not help much.
  - Dropout achieves better results than former used regularization methods (Weight Decay).

Batch Normalization

Batch Normalization

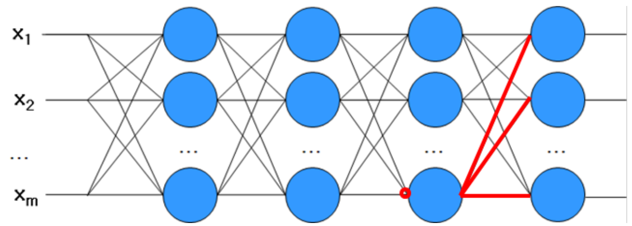
- **Covariate Shift**
  - A change in the distribution of a function's domain.

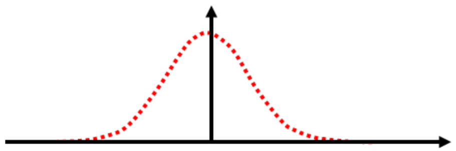


  - Can your model work properly?

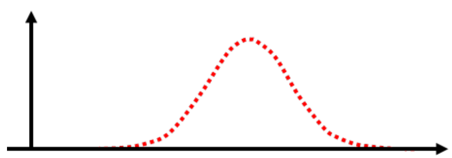
Batch Normalization

- **Internal Covariate Shift**
  - Input distribution of the red node





  - While learning, red connection weights will change based on the input distribution
  - After learning, the whole connection weights changes, which cause the change of the input distribution

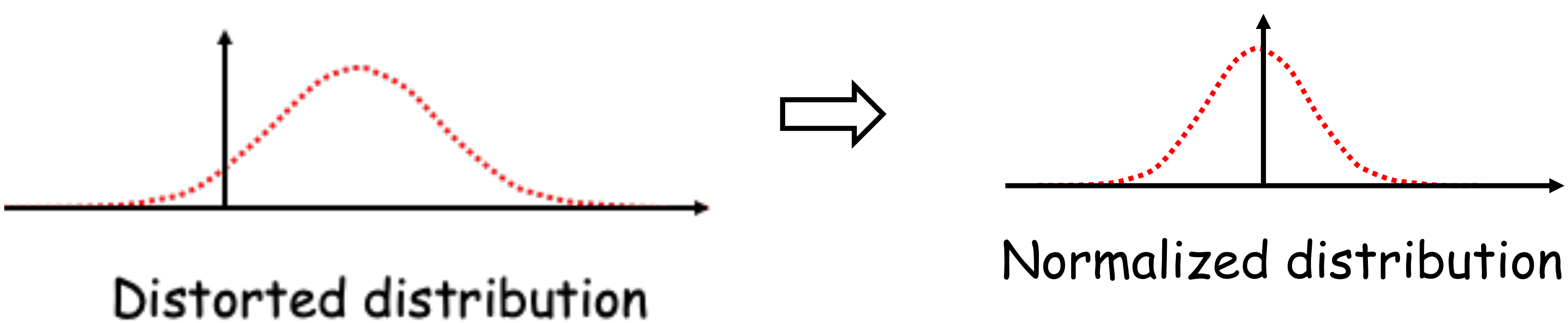


  - The assumption of the learning is broken

Batch Normalization

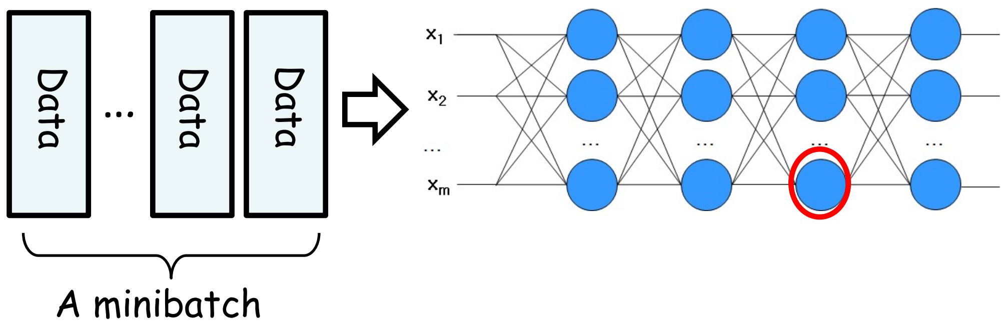
- **Internal Covariate Shift**
  - It disturbs the learning process,
  - Learning is getting slow down

- **What shall we do?**
  - Why don't we normalize the distribution of inputs



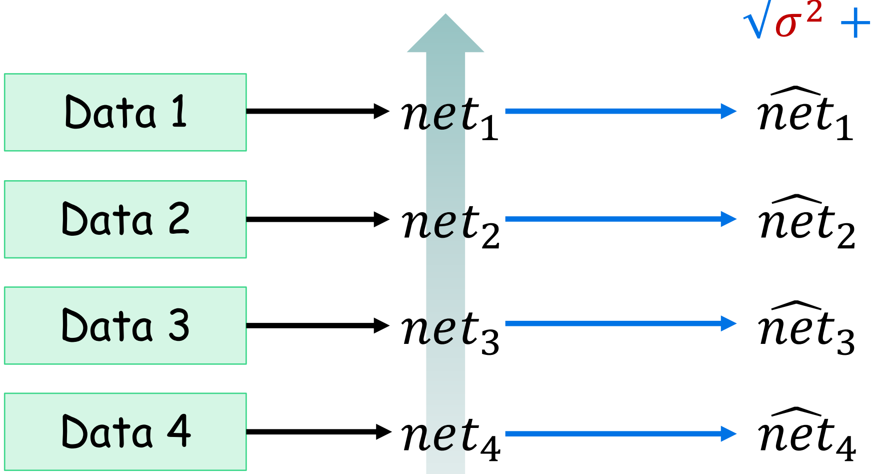
Batch Normalization

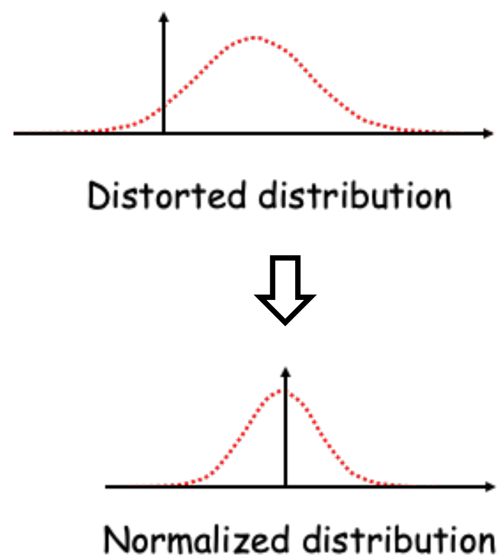
- **Input Normalization**



μ, σ^2

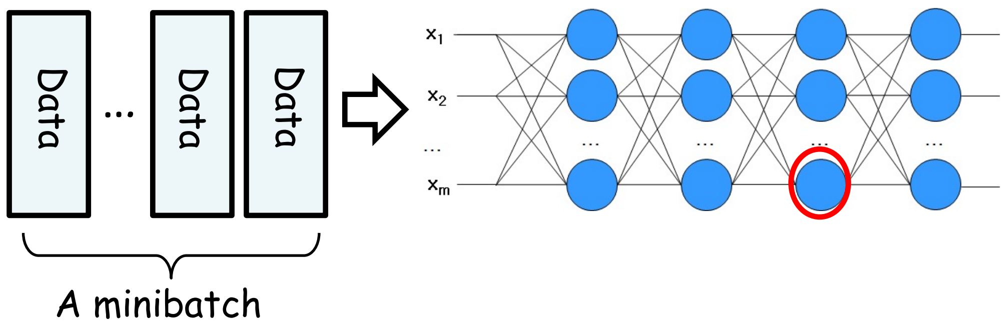
$$\widehat{net} = \frac{net - \mu}{\sqrt{\sigma^2 + \varepsilon}}$$





Batch Normalization

Input Normalization



μ, σ^2

$$\widehat{net} = \frac{net - \mu}{\sqrt{\sigma^2 + \varepsilon}}$$

Data 1 $\rightarrow net_1 \rightarrow \widehat{net}_1 \rightarrow f(\widehat{net}_1)$

Data 2 $\rightarrow net_2 \rightarrow \widehat{net}_2 \rightarrow f(\widehat{net}_2)$

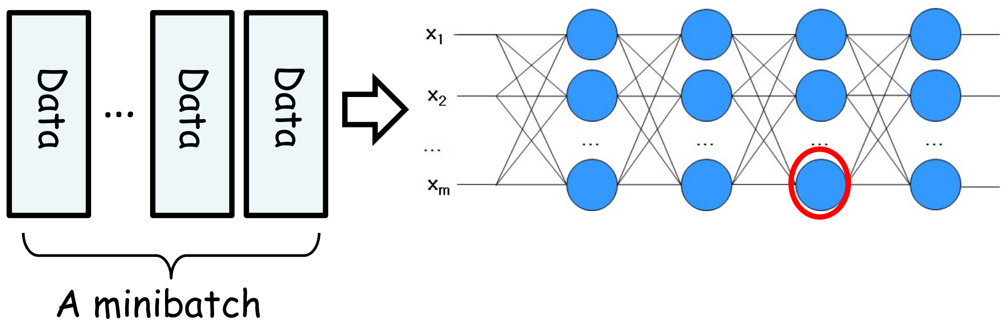
Data 3 $\rightarrow net_3 \rightarrow \widehat{net}_3 \rightarrow f(\widehat{net}_3)$

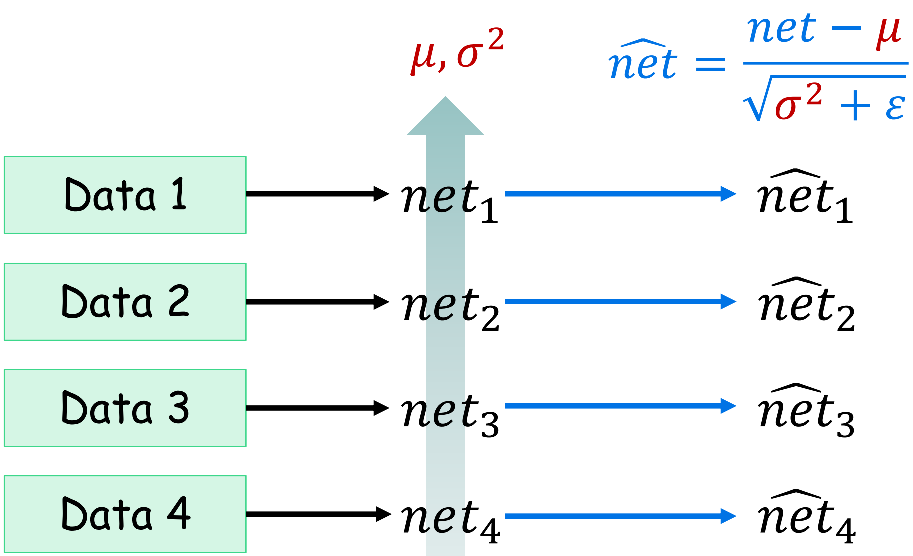
Data 4 $\rightarrow net_4 \rightarrow \widehat{net}_4 \rightarrow f(\widehat{net}_4)$

?

Batch Normalization

- **Input Normalization**





$$net = \mathbf{wx} + b$$

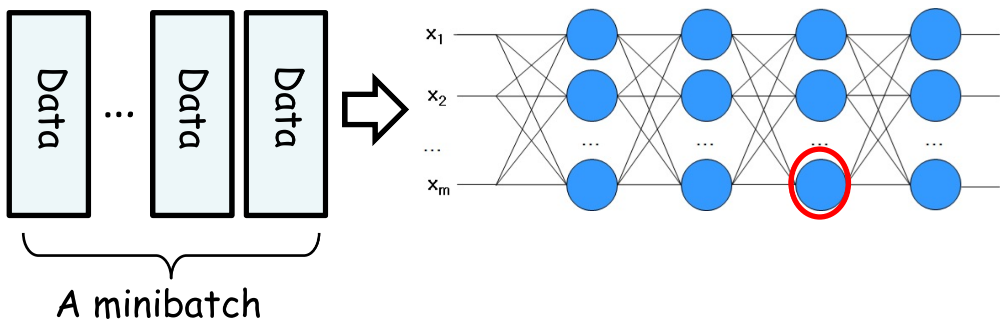
$$E(net) = E(\mathbf{wx}) + b$$

$$\widehat{net} = \frac{\mathbf{wx} + b - (E(\mathbf{wx}) + b)}{\sqrt{\sigma^2 + \varepsilon}}$$

$$\widehat{net} = \frac{\mathbf{wx} - E(\mathbf{wx})}{\sqrt{\sigma^2 + \varepsilon}}$$

Batch Normalization

▪ Input Normalization



μ, σ^2 $\qquad$ $\widehat{net} = \dfrac{net - \mu}{\sqrt{\sigma^2 + \varepsilon}}$ $\qquad$ $\widetilde{net} = \gamma \widehat{net} + \beta$

Data 1 → net_1 → $\widehat{net}_1$ → $\widetilde{net}_1$ → $f(\widetilde{net}_1)$

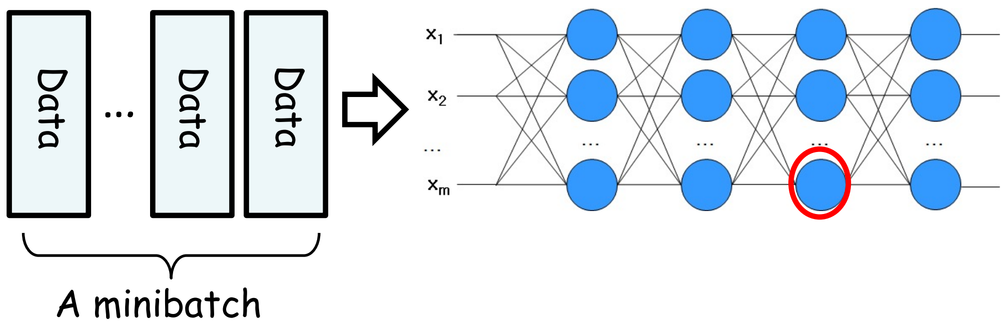
Data 2 → net_2 → $\widehat{net}_2$ → $\widetilde{net}_2$ → $f(\widetilde{net}_2)$

Data 3 → net_3 → $\widehat{net}_3$ → $\widetilde{net}_3$ → $f(\widetilde{net}_3)$

Data 4 → net_4 → $\widehat{net}_4$ → $\widetilde{net}_4$ → $f(\widetilde{net}_4)$

Batch Normalization

- **Input Normalization**



μ, σ^2

$$\widehat{net} = \frac{net - \mu}{\sqrt{\sigma^2 + \varepsilon}}$$

$$\widetilde{net} = \gamma \widehat{net} + \beta$$

Data 1 $\rightarrow net_1 \rightarrow \widehat{net}_1 \rightarrow \widetilde{net}_1 \rightarrow f(\widetilde{net}_1)$

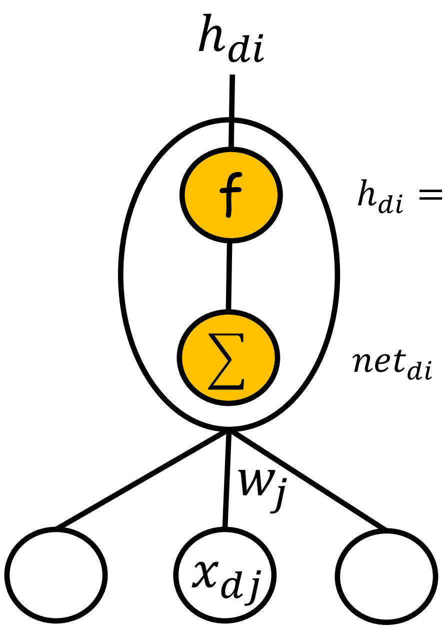
Data 2 $\rightarrow net_2 \rightarrow \widehat{net}_2 \rightarrow \widetilde{net}_2 \rightarrow f(\widetilde{net}_2)$

Data 3 $\rightarrow net_3 \rightarrow \widehat{net}_3 \rightarrow \widetilde{net}_3 \rightarrow f(\widetilde{net}_3)$

Data 4 $\rightarrow net_4 \rightarrow \widehat{net}_4 \rightarrow \widetilde{net}_4 \rightarrow f(\widetilde{net}_4)$

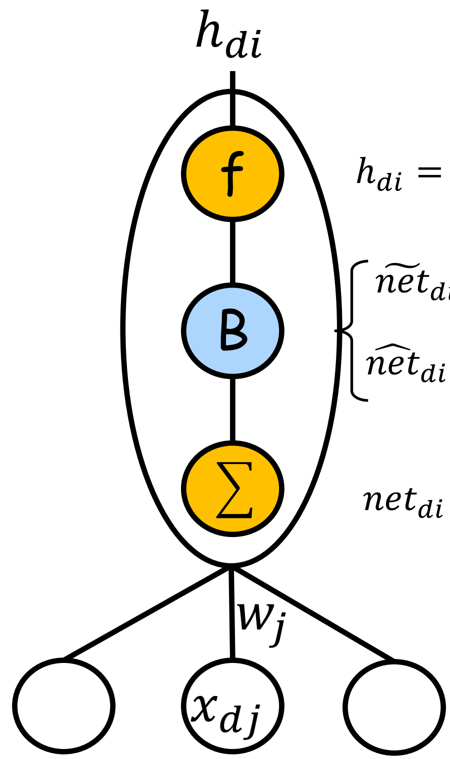
Batch Normalization

- **For a Single Node**



$$h_{di} = Activation(net_{di})$$

$$net_{di} = \sum_{j=1}^{J} w_j x_{dj} + b$$



$$h_{di} = Activation(\widetilde{net}_{di})$$

$$\begin{cases} \widetilde{net}_{di} = \gamma \widehat{net}_{di} + \beta \\ \widehat{net}_{di} = \dfrac{net_{di} - \mu_B}{\sqrt{\sigma_B^2 + \varepsilon}} \end{cases}$$

$$net_{di} = \sum_{j=1}^{J} w_j x_{dj} + b$$

$$\mu_B = \frac{1}{D_B} \sum_{d=1}^{D_B} net_{di}$$

$$\sigma_B^2 = \frac{1}{D_B} \sum_{d=1}^{D_B} (net_{di} - \mu)^2$$

Batch Normalization

- **Testing**
  - For Training, the mean and variance of each batch are used for normalization
  - For Testing, of which data the mean and variance will be used?
    - Estimated with those of batches in the training

Test sample

$$net_{ti} = \sum_{j=1}^{J} w_j x_{tj} + b$$

Transform

$$\widehat{net}_{ti} = \frac{net_{di} - \mu}{\sqrt{\sigma^2 + \varepsilon}}$$

$$\widetilde{net}_{di} = \gamma \widehat{net}_{di} + \beta$$

$$h_{di} = Activation(\widetilde{net}_{di})$$

$$\mu = \frac{1}{B} \sum_{b=1}^{B} \mu_b$$

Unbiased estimator of mean of the population

$$\sigma^2 = \frac{1}{B} \sum_{b=1}^{B} \frac{m}{m-1} \sigma_b^2$$

Mean of unbiased estimator of variance of the population

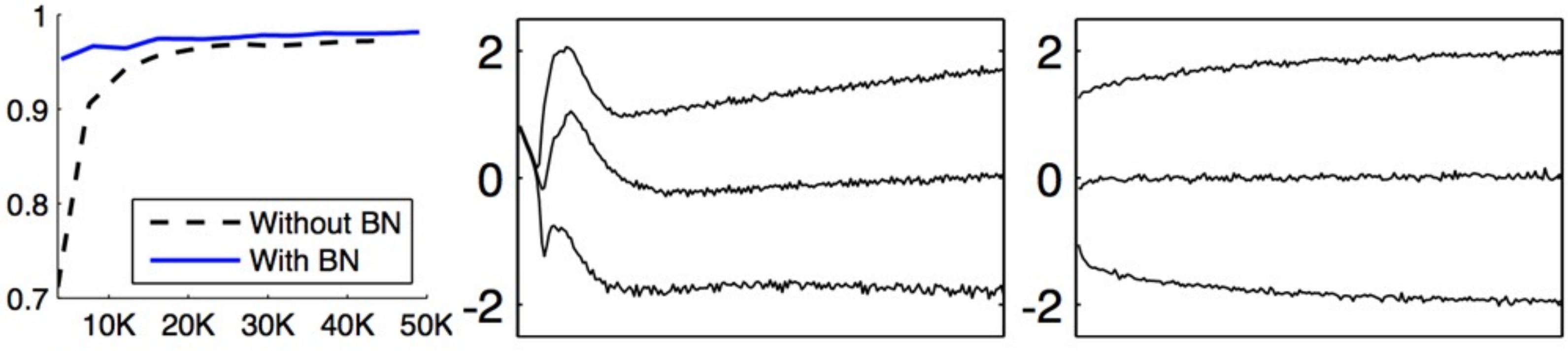
B: # of batches
m: # of samples in each batch

Batch Normalization

- **Advantage**
  - Reduces internal covariant shift.
  - Reduces the dependence of gradients on the scale of the connection weights.
  - Regularizes the model and reduces the need for regularization techniques.
    - It adds some stochastic noise to the activations as a result of using noisy estimates computed on the mini-batches. This has a regularization effect in some applications,

Batch Normalization

- **Performance with BN**



(a) *accuracy* (b) Without BN (c) With BN

input distributions

Batch Normalization

- **Disadvantage**
  - Expensive: Memory and time
    - Must keep interim results of all instances in a batch
    - Especially in CNN, usually an image is large
  - Hard to apply when the batch size is small
    - If batches are small, the means and variances cannot approximate the global ones.
  - Hard to apply to recurrent networks
    - It doesn’t match to structure of recurrent networks
    - Hard to implement with recurrent networks

Layer Normalization

- **Recap: Batch Normalization**
  - Normalization of each node output

Batch normalization

$$\mu_j = \frac{1}{m}\sum_{i=1}^{m} x_{ij}$$
$$\sigma_j^2 = \frac{1}{m}\sum_{i=1}^{m}(x_{ij} - \mu_j)^2$$
$$\hat{x_{ij}} = \frac{x_{ij} - \mu_j}{\sqrt{\sigma_j^2 + \epsilon}}$$

i , j : index of the batch and the node of hidden layers

Layer Normalization

- **Recap: Batch Normalization**
  - Normalization of each node output

Outputs of a layer for a mini-batch

			Avg	Std
1.0	3.0	6.0	3.3	2.5
2.0	2.0	2.0	2.0	0.0
0.0	1.0	5.0	2.0	2.6
4.0	6.0	1.0	3.7	2.5
5.0	2.0	3.0	3.3	1.5
1.0	0.0	1.0	0.7	0.6

Normalized outputs

-0.9	-0.1	1.1
0.0	0.0	0.0
-0.8	-0.4	1.1
0.1	0.9	-1.1
1.1	-0.9	-0.2
0.6	-1.2	0.6

Layer Normalization

- **Proposed as an alternative to Batch Normalization**
  - Works regardless of batch size (batch size = 1)
  - Performs well with RNNs

Layer normalization:

$$\mu_i = \frac{1}{m}\sum_{j=1}^{m} x_{ij}$$
$$\sigma_i^2 = \frac{1}{m}\sum_{j=1}^{m}(x_{ij} - \mu_i)^2$$
$$\hat{x_{ij}} = \frac{x_{ij} - \mu_i}{\sqrt{\sigma_i^2 + \epsilon}}$$

i , j : index of the batch and the node of hidden layers

Layer Normalization

Outputs of a layer for a mini-batch

	1.0	3.0	6.0
	2.0	2.0	2.0
	0.0	1.0	5.0
	4.0	6.0	1.0
	5.0	2.0	3.0
	1.0	0.0	1.0
Avg	2.2	2.3	3.0
Std	1.9	2.1	2.1

Normalized outputs

-0.6	0.3	1.4
-0.1	-0.2	-0.5
-1.1	-0.6	1.0
0.9	1.8	-1.0
1.5	-0.2	0.0
-0.6	-1.1	-1.0

Layer Normalization

- **Group Normalization shows consistent accuracy with smaller batches**
  - Tested on ImageNet (1000 Classes, 1.28M training, 50K validation), ResNet-50