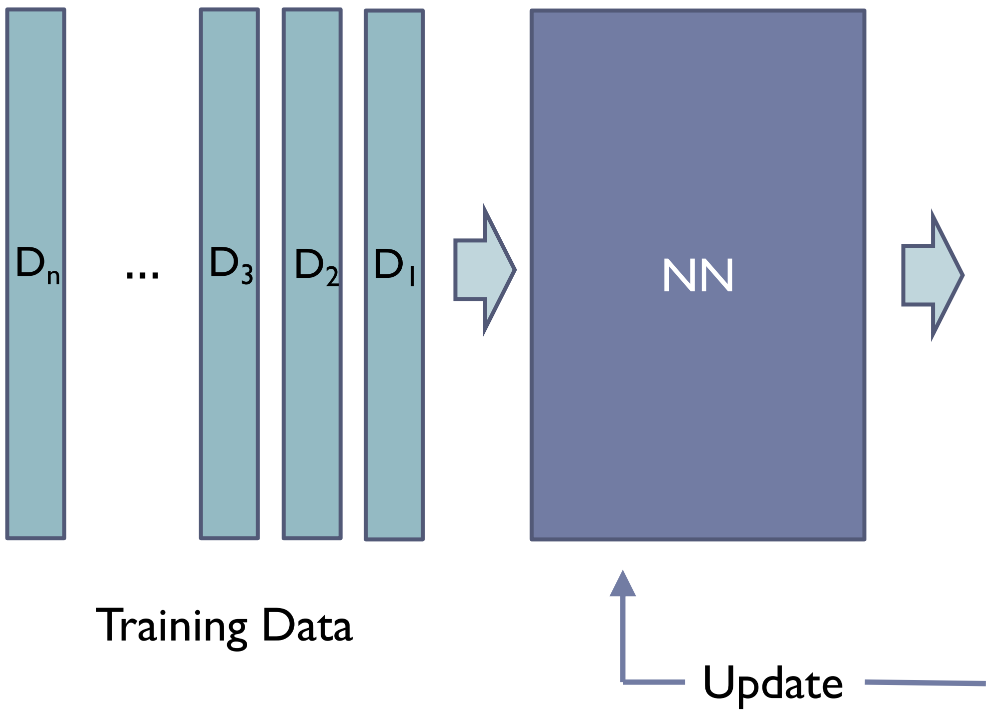
Gradient Descent Methods

차례

- Stochastic Gradient Descent Method
- Momentum
- Adaptive Learning Rates
  - Adagrad
  - RMSprop
  - Adam

Gradient Descent Method

- **Error Back Propagation (Batch mode)**



$$
\begin{array}{ccccc}
\frac{\partial E_1}{\partial w_1} & \frac{\partial E_1}{\partial w_2} & \frac{\partial E_1}{\partial w_3} & \cdots & \frac{\partial E_1}{\partial w_m} \\
\frac{\partial E_2}{\partial w_1} & \frac{\partial E_2}{\partial w_2} & \frac{\partial E_2}{\partial w_3} & \cdots & \frac{\partial E_2}{\partial w_m} \\
\frac{\partial E_3}{\partial w_1} & \frac{\partial E_3}{\partial w_2} & \frac{\partial E_3}{\partial w_3} & \cdots & \frac{\partial E_3}{\partial w_m} \\
\\
\frac{\partial E_n}{\partial w_1} & \frac{\partial E_n}{\partial w_2} & \frac{\partial E_n}{\partial w_3} & \cdots & \frac{\partial E_n}{\partial w_m} \\
\hline
\frac{\partial E}{\partial w_1} & \frac{\partial E}{\partial w_2} & \frac{\partial E}{\partial w_3} & \cdots & \frac{\partial E}{\partial w_m}
\end{array}
$$

Batch Gradient Descent

- **Algorithm**
  - For one update, gradients are calculated for the whole dataset
  - Batch gradient descent is guaranteed to converge to a local minimum
  - Redundant computations for large redundant dataset

Repeat

$$\alpha = 0$$

for n =1 to N (for all training data)

$$\alpha += \frac{\partial E_n}{\partial w}$$

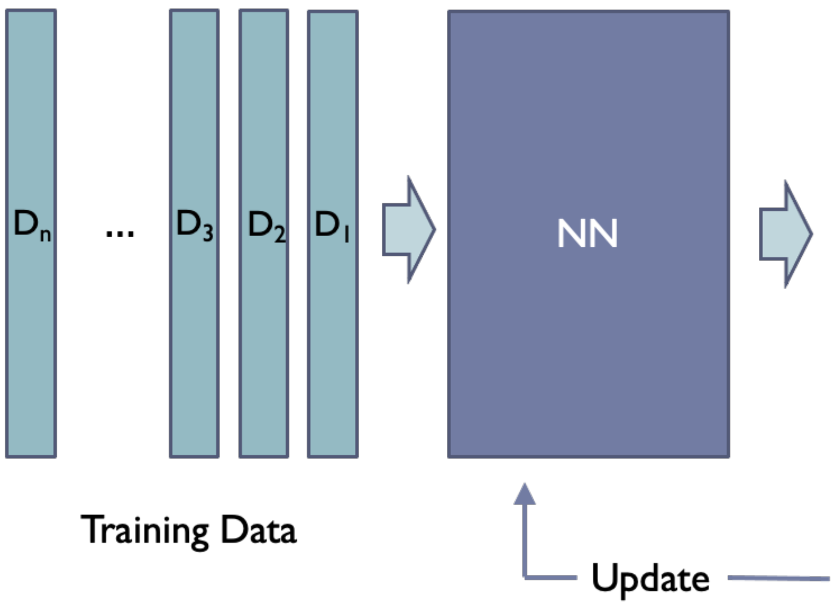
end

$$w = w - \eta\alpha$$

Until end condition satisfied

Gradient Descent Method

- **Error Back Propagation (Batch mode)**



$$
\begin{array}{ccccc}
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\frac{\partial E}{\partial w_1} & \frac{\partial E}{\partial w_2} & \frac{\partial E}{\partial w_3} & \cdots & \frac{\partial E}{\partial w_m}
\end{array}
$$

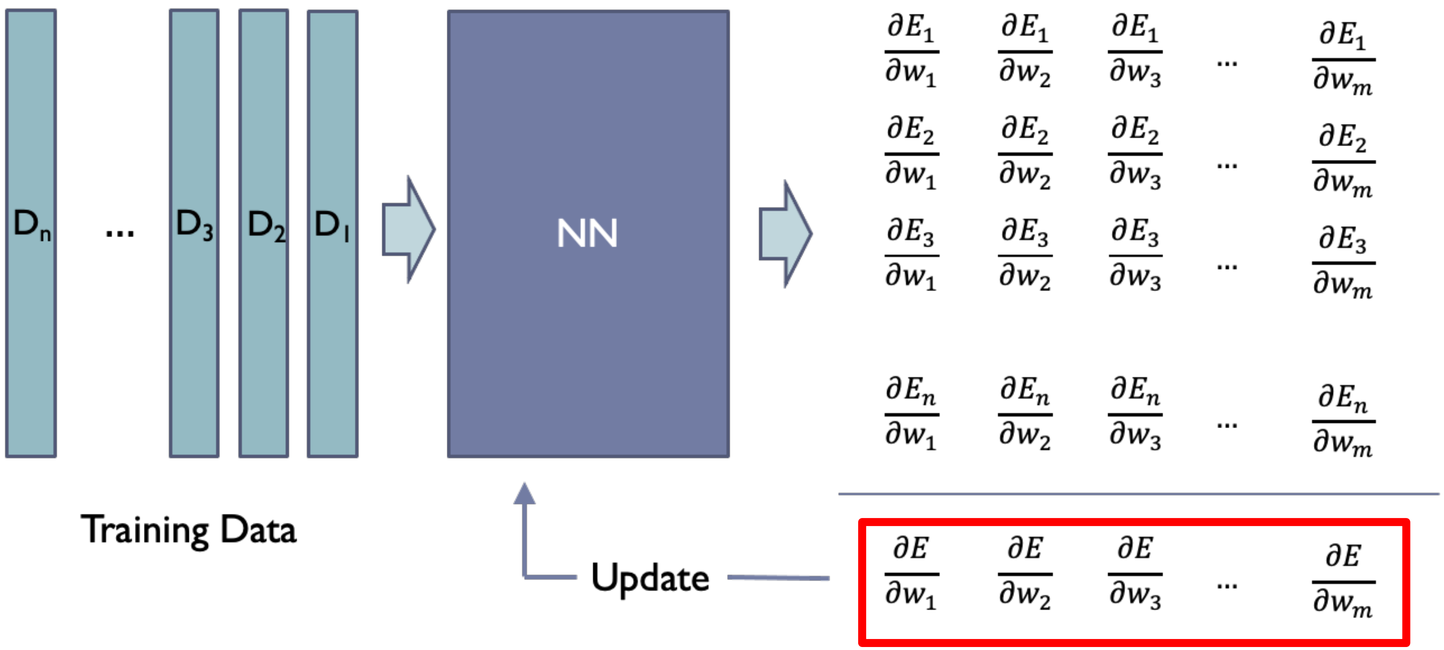
Exact Gradients!! However, too much cost for them !!

Do we need EXACT gradients?

How about ESTIMATING the gradient with cheap cost?

Gradient Descent Method

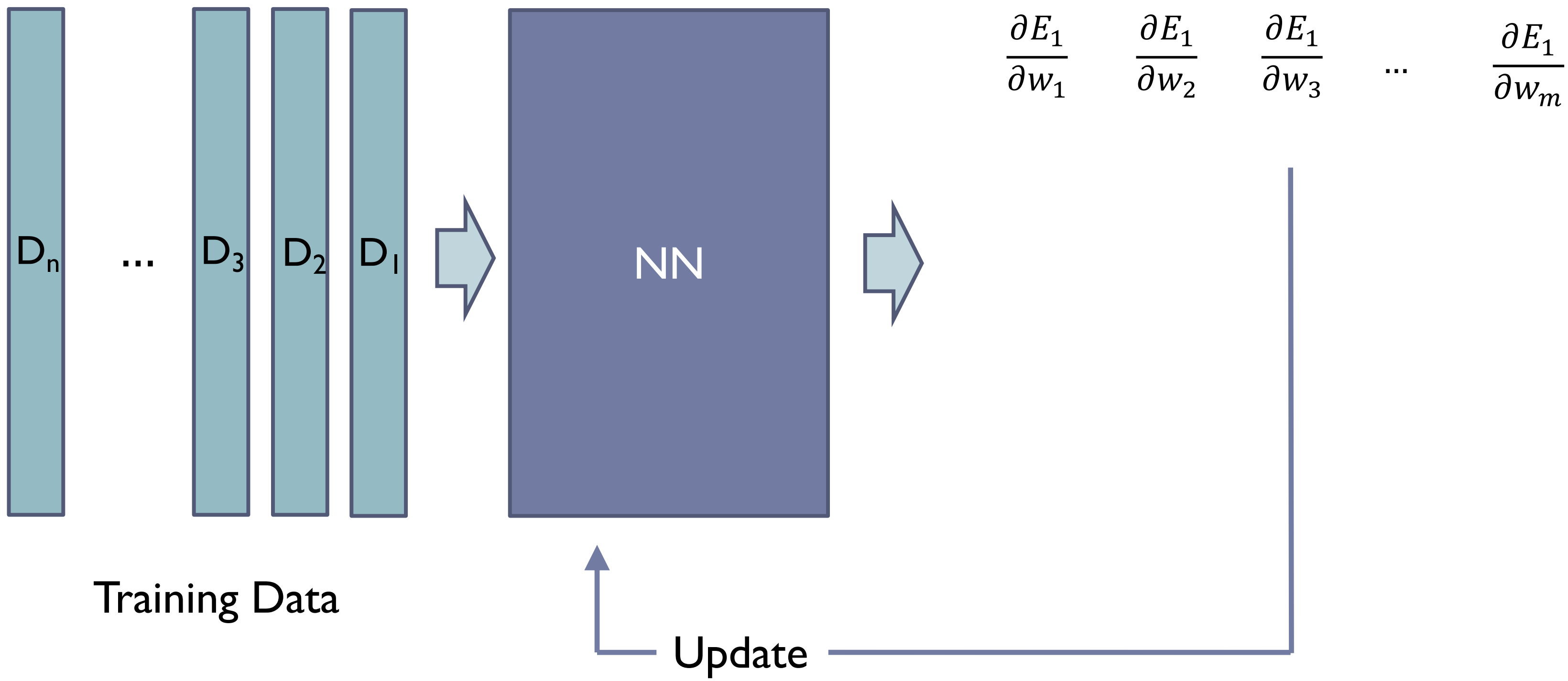
Error Back Propagation (Batch mode)



$$\frac{\partial E}{\partial w_1} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial E_i}{\partial w_1} \qquad \frac{\partial E}{\partial w_1} \approx \frac{\partial E_1}{\partial w_1} \approx \frac{\partial E_2}{\partial w_1} \approx \cdots \approx \frac{\partial E_n}{\partial w_1}$$

Stochastic Gradient Descent

- **Why not?**



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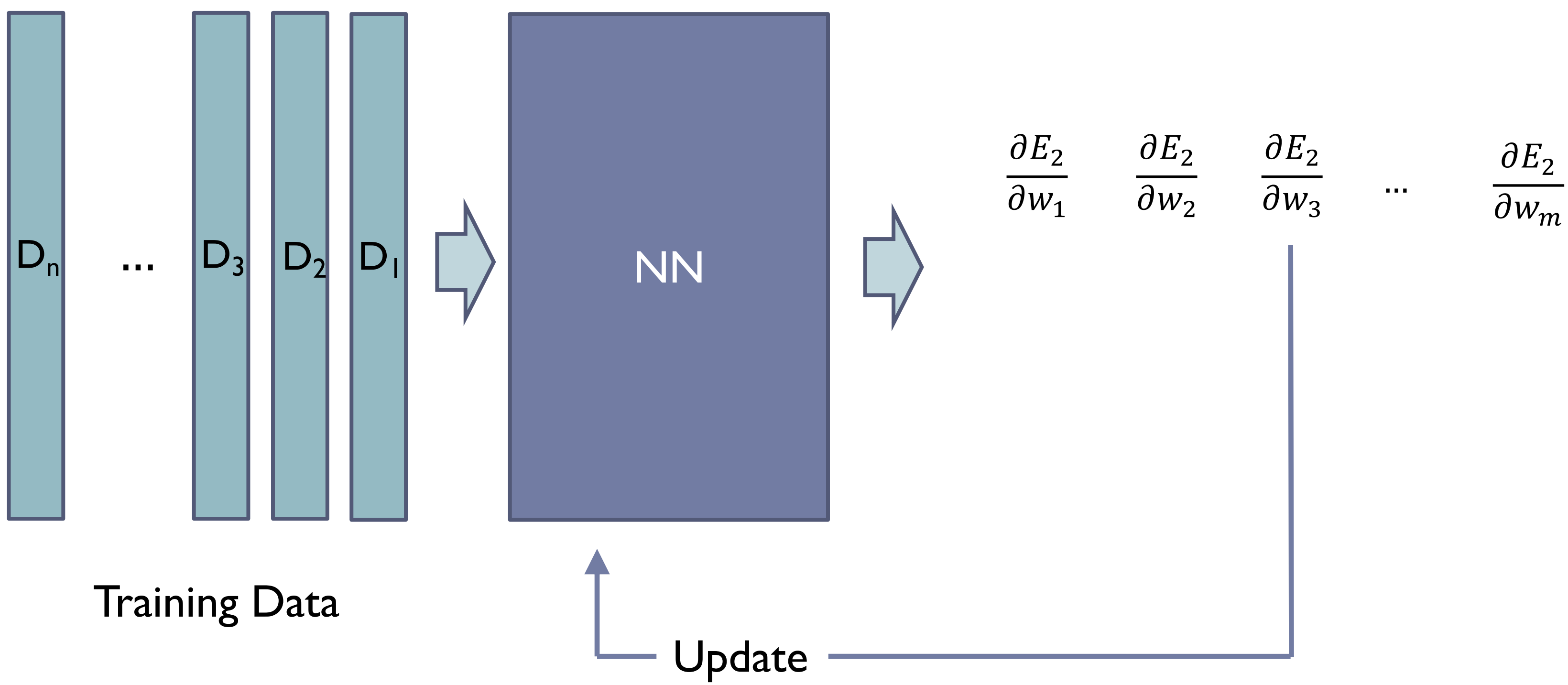
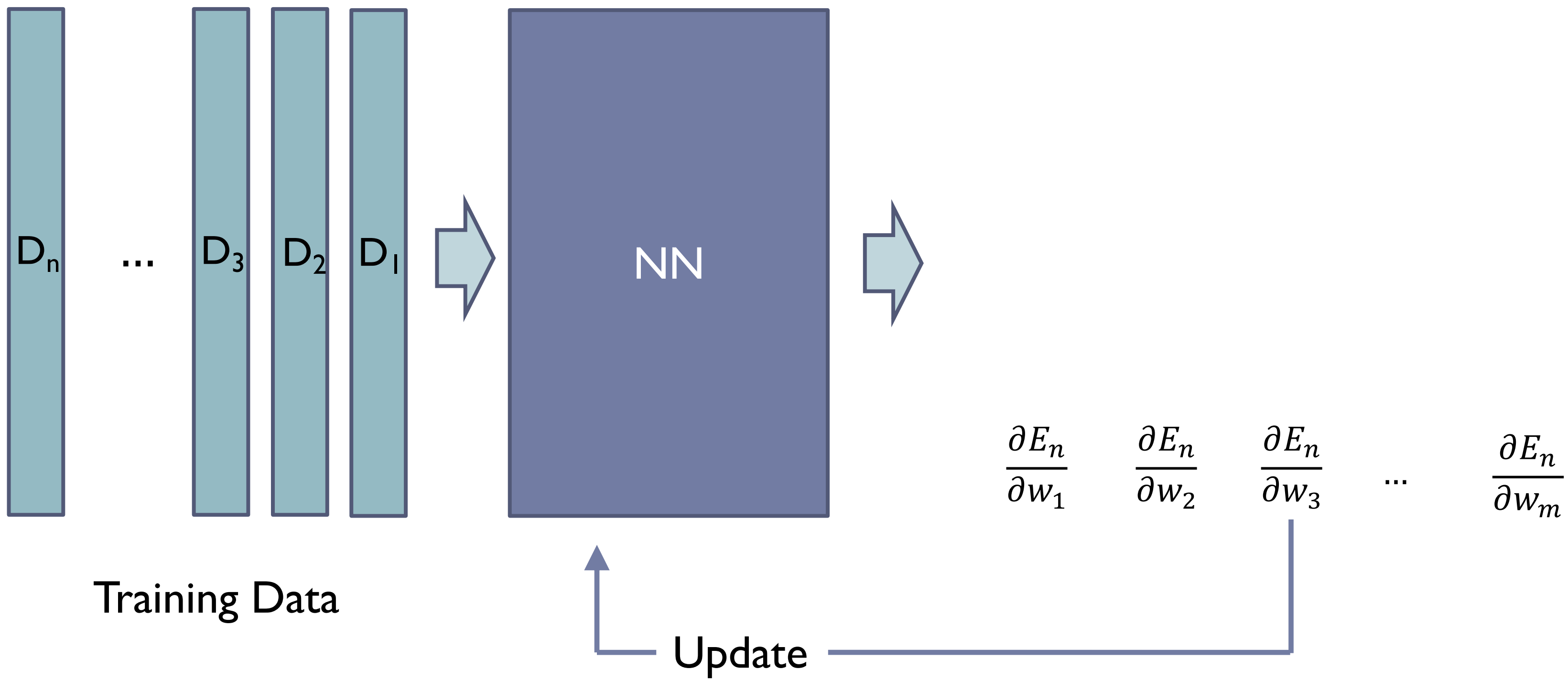
Stochastic Gradient Descent

- **Why not?**



Stochastic Gradient Descent

- **Why not?**



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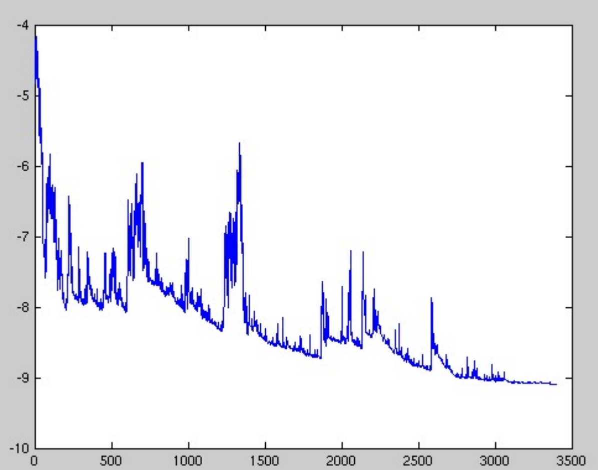
Stochastic Gradient Descent

- **Stochastic Gradient Descent**
  - For one update, gradients are calculated for one sample
  - Usually faster and can be used to learn online
  - Fluctuations: Maybe good or maybe bad
  - With a small learn rate, show similar performance

Repeat
    for n =1 to N (for all training data)

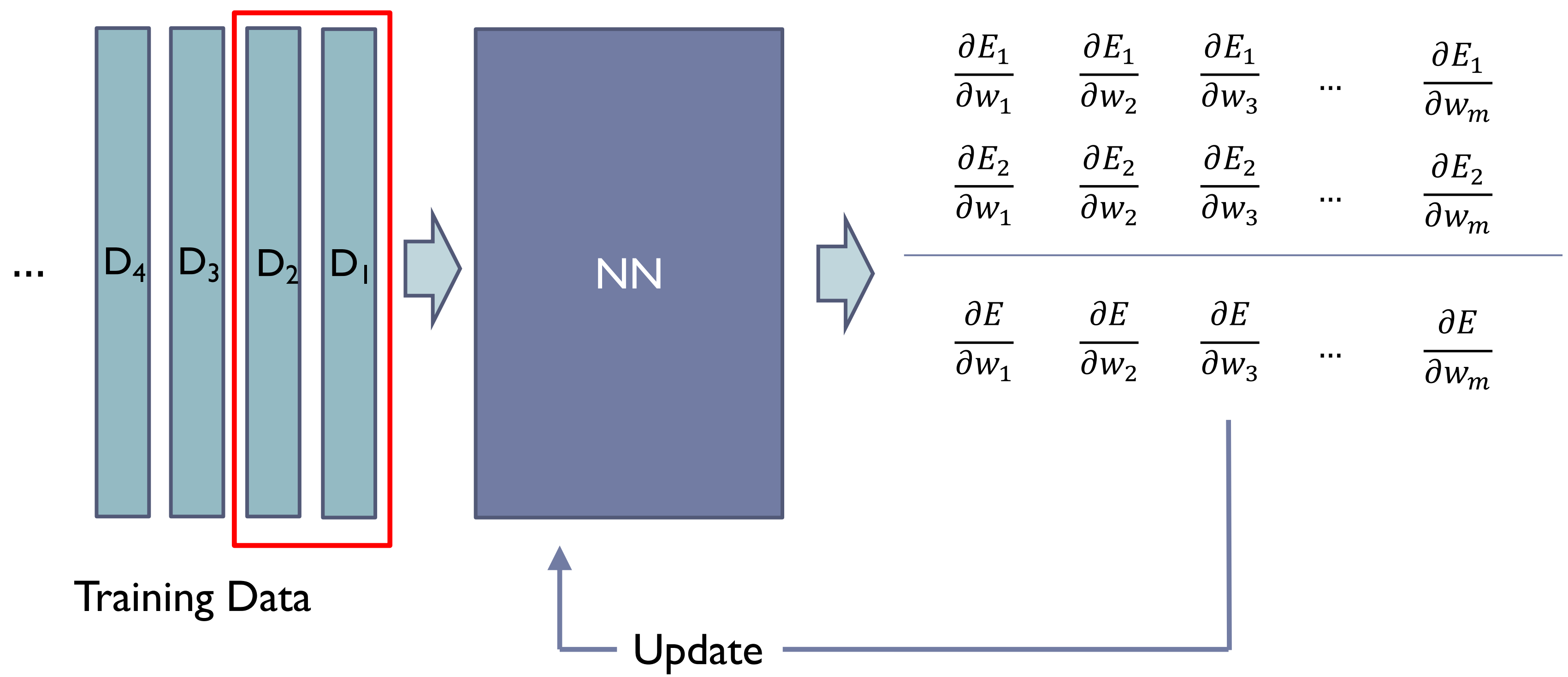
$$w = w - \eta \frac{\partial E_n}{\partial w}$$

    end
Until end condition satisfied



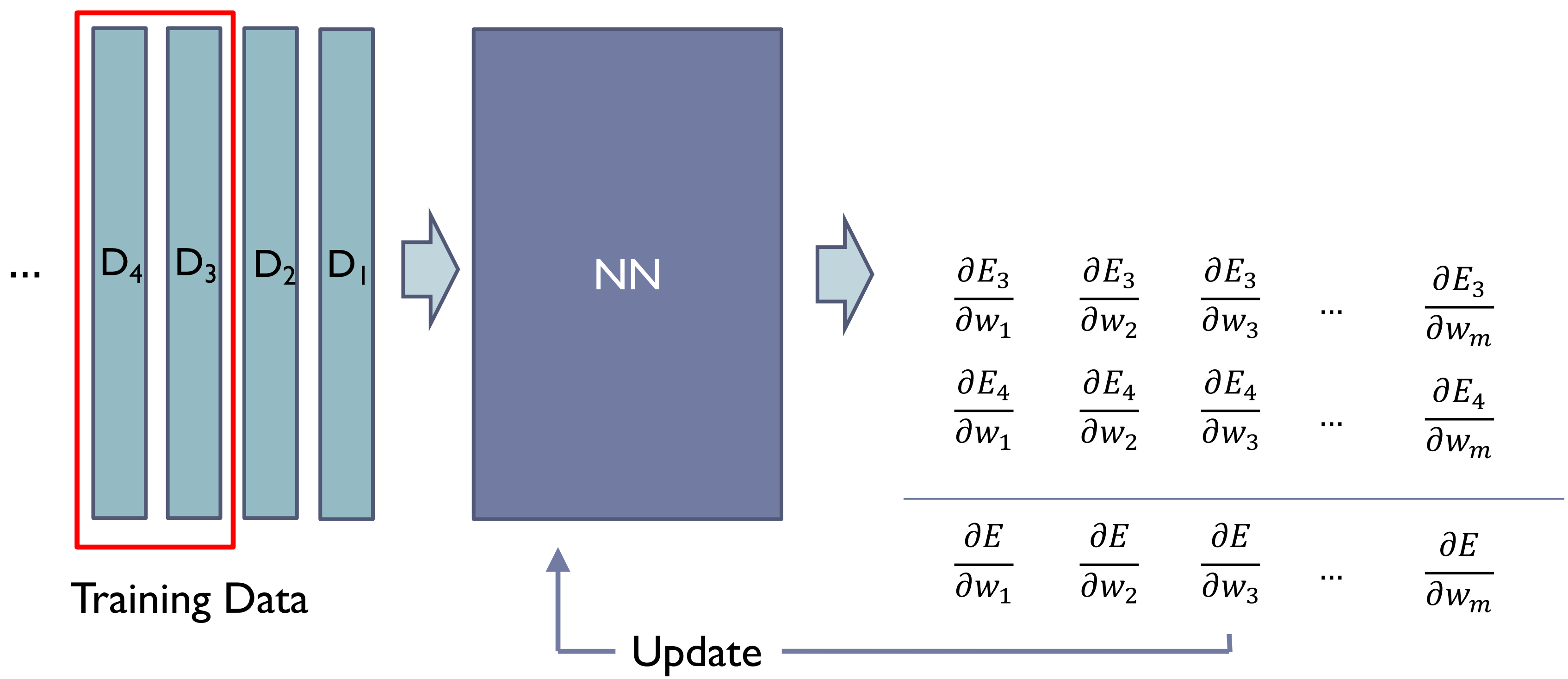
Stochastic Gradient Descent

- **Mini-batch Gradient Descent Method**



Stochastic Gradient Descent

Mini-batch Gradient Descent Method



Stochastic Gradient Descent

- **Mini-batch Gradient Descent**
  - For an update, gradients are calculated for a batch

Repeat
    for b =1 to B (for all batches)

$$\alpha = 0$$

        for n=1 to N_b (for all training data in batch *b*)

$$\alpha += \frac{\partial E_n}{\partial w}$$

        end

$$w = w - \eta\alpha$$

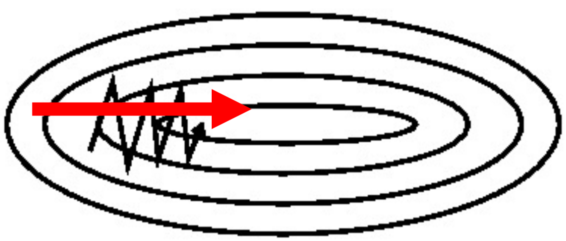
    end
Until end condition satisfied

Stochastic Gradient Descent

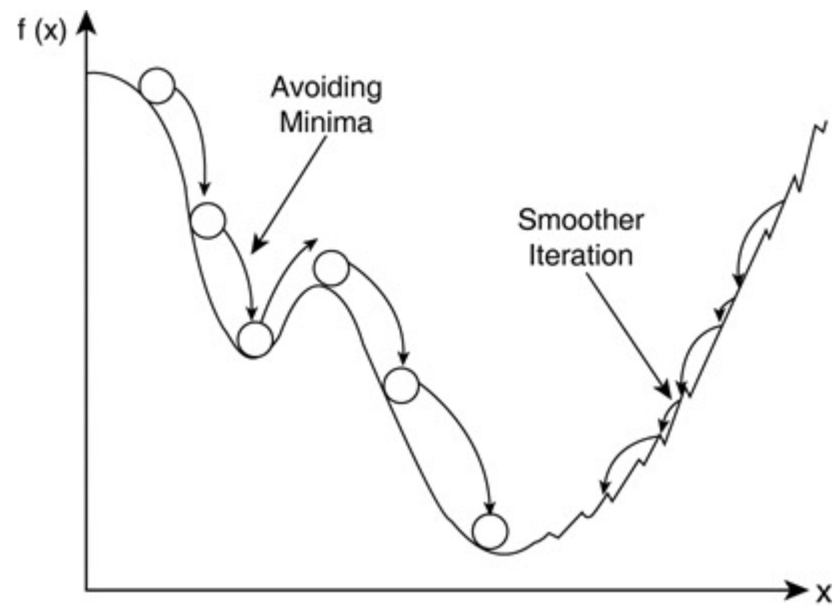
- Usual Batch Size
  - Dependent on datasets from several thousands to several tens
- Advantage
  - Good estimation of real gradient
  - High throughput: may use the large number of cores at once in a GPU.
  - Faster convergence: Good estimation + High throughput
- Disadvantage
  - Inaccurate: dataset with large variances

Momentum

- Simple gradient method depends on the current position
  - Hard to avoid local minimum
  - Gradient can vary much

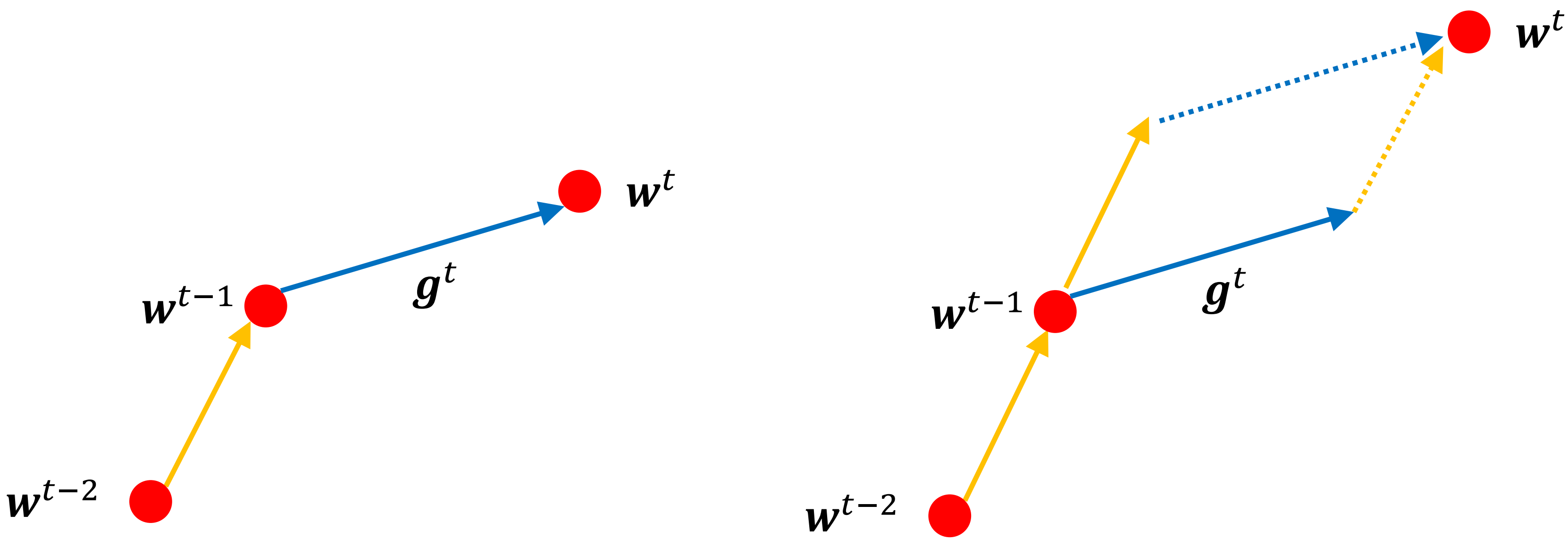


Oscillation



Momentum

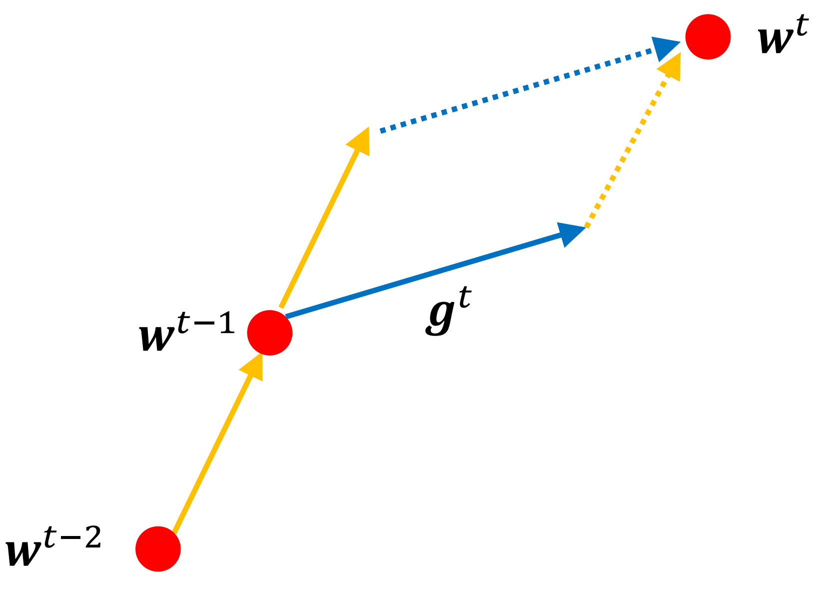
- **How to cross the hill**

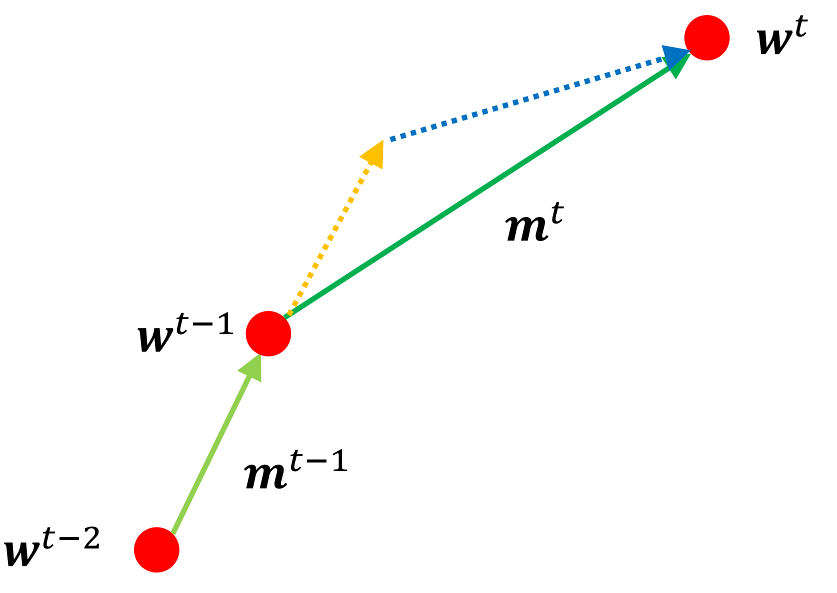


Momentum

Momentum

$$\boldsymbol{m}^t = \boldsymbol{m}^{t-1} + \boldsymbol{g}^t$$





Momentum

- **Momentum**

$\boldsymbol{g}^t$: 시간 t에서의 기울기

Momentum rate

Learning rate

$$\boldsymbol{w}^t = \boldsymbol{w}^{t-1} - \eta \boldsymbol{g}^t$$

표준 방법

$$\boldsymbol{m}^t = \gamma\, \boldsymbol{m}^{t-1} + \eta \boldsymbol{g}^t$$

$$\boldsymbol{w}^t = \boldsymbol{w}^{t-1} - \boldsymbol{m}^t$$

Momentum

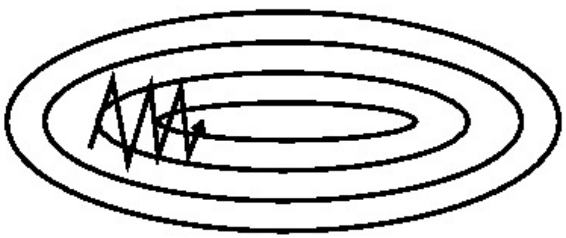
- Update parameters considering both the momentum and the gradient of the current position

Momentum

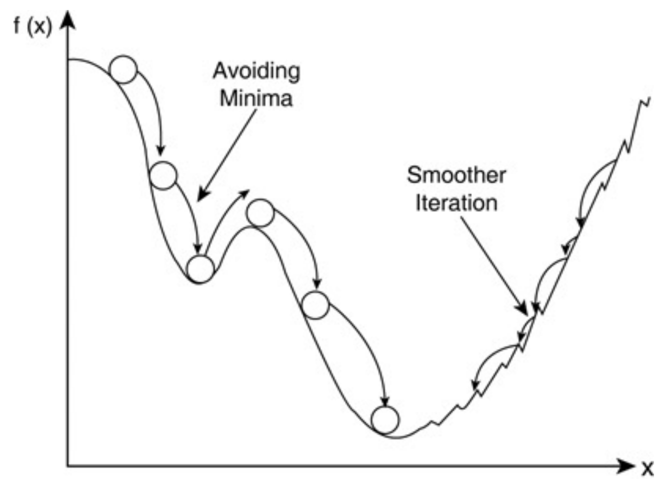
- **Momentum**
  - Momentum is the exponential average of the past gradients

$$\boldsymbol{m}^t = \eta \boldsymbol{g}^t + \gamma\, \eta \boldsymbol{g}^{t-1} + \gamma^2 \eta \boldsymbol{g}^{t-2} + \cdots$$

$$\boldsymbol{m}^t = \gamma\, \boldsymbol{m}^{t-1} + \eta \boldsymbol{g}^t$$
$$\boldsymbol{w}^t = \boldsymbol{w}^{t-1} - \boldsymbol{m}^t$$



Oscillation

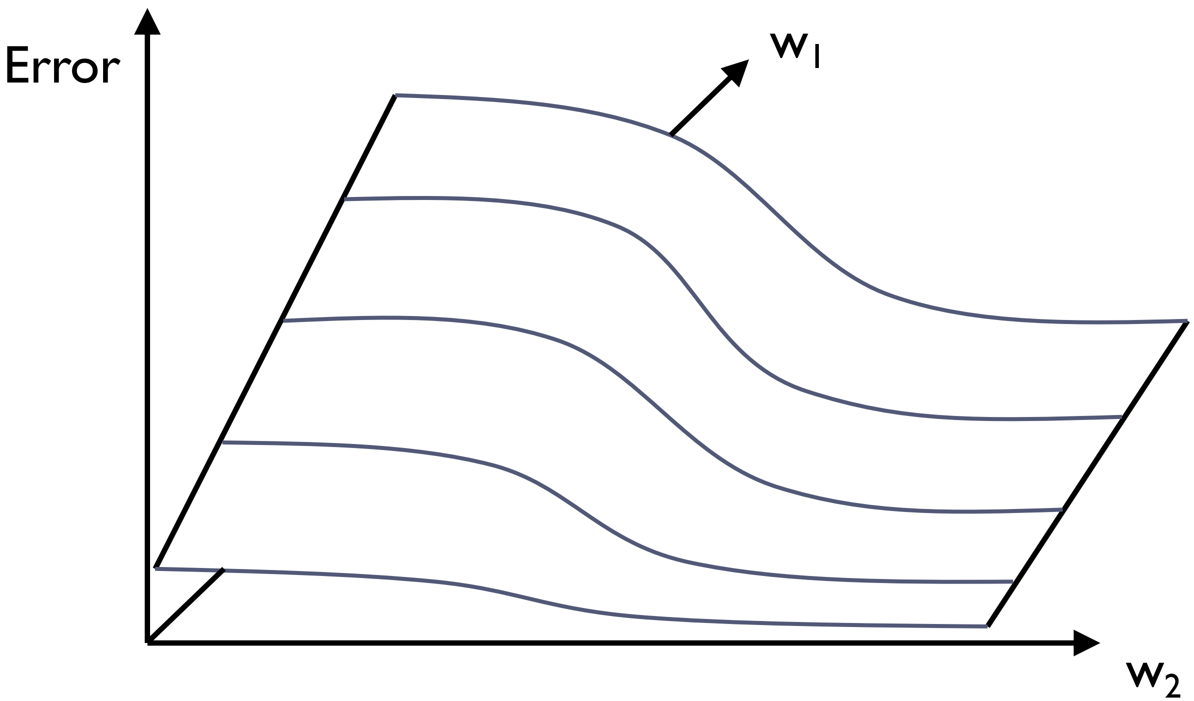


Adaptive Learning Rates

- **Why do we use the SAME learning rate for all the weights?**
  - Can we use different learning rate?
  - Hmm.. Interesting, but why do we need different learning rate?
- **Some weight are updated much and some are not**
  - What if some inputs may be 0 but some may have none-zero values in most of training data
  - OK.. Let's make a large update for the less updated parameters

Adaptive Learning Rates

- **Some weight are updated much and some are not**



$$w_1 = w_1 - \eta \frac{\partial E}{\partial w_1}$$

$$w_2 = w_2 - \eta \frac{\partial E}{\partial w_2}$$

Adaptive Learning Rates

- **Adagrad**
  - Update much less-updated parameters
  - Update less much-updated parameter

g_i^t: parameter i의 시간 t에서의 기울기

G_i^t: parameter i가 지금까지 update된 총량

$$w_i^t = w_i^{t-1} - \eta g_i^t$$

표준 방법

$$G_i^t = G_i^{t-1} + \left(g_i^t\right)^2$$

$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{G_i^t + \epsilon}} g_i^t$$

Adagrad

Very small value

Adaptive Learning Rates

- **Disadvantage of Adagrad**
  - Eventually, G_i^t becomes large as time goes on
  - Parameters are rarely updated at some time

g_i^t: parameter i의 시간 t에서의 기울기

G_i^t: parameter i가 지금까지 update된 총량

$$w_i^t = w_i^{t-1} - \eta g_i^t$$

표준 방법

$$G_i^t = G_i^{t-1} + \left(g_i^t\right)^2$$

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Adagrad

Very small value

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Adaptive Learning Rates

- **RMSProp**
  - Instead of considering the total amount of updates, let's consider the amount of recent updates

g_i^t: parameter *i*의 시간 t에서의 기울기

G_i^t: parameter *i*가 update된 양

$$G_i^t = G_i^{t-1} + \left(g_i^t\right)^2$$

$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{G_i^t + \epsilon}} g_i^t$$

Adagrad

$$G_i^t = \gamma G_i^{t-1} + (1-\gamma)\left(g_i^t\right)^2$$

$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{G_i^t + \epsilon}} g_i^t$$

RMSProp

Very small value

Adaptive Learning Rates

- **Adam**
  - RMSProp + Momentum

g_i^t: parameter i의 시간 t에서의 기울기

G_i^t: parameter i가 update된 양

Momentum

$$m_i^t = \gamma m_i^{t-1} + \eta g_i^t$$
$$w_i^t = w_i^{t-1} - m_i^t$$

$$\rightarrow \quad m_i^t = \beta_1 m_i^{t-1} + (1 - \beta_1) g_i^t$$

RMSProp

$$G_i^t = \gamma G_i^{t-1} + (1 - \gamma)\left(g_i^t\right)^2$$
$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{G_i^t + \epsilon}} g_i^t$$

$$\rightarrow \quad G_i^t = \beta_2 G_i^{t-1} + (1 - \beta_2)\left(g_i^t\right)^2$$

Adaptive Learning Rates

- **Adam**
  - RMSProp + Momentum

Unbiased expectation

$$m_i^t = \beta_1 m_i^{t-1} + (1-\beta_1) g_i^t$$

$$G_i^t = \beta_2 G_i^{t-1} + (1-\beta_2)\left(g_i^t\right)^2$$

$$\widehat{m}_i^t = \frac{m_i^t}{1-(\beta_1)^t} \qquad \hat{G}_i^t = \frac{G_i^t}{1-(\beta_2)^t}$$

$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{\hat{G}_i^t + \epsilon}}\widehat{m}_i^t$$

Adaptive Learning Rates

- **Adam**
  - RMSProp + Momentum

$$m_i^t = \gamma m_i^{t-1} + \eta g_i^t$$
$$w_i^t = w_i^{t-1} - m_i^t$$

Momentum

$$G_i^t = \gamma G_i^{t-1} + (1-\gamma)\left(g_i^t\right)^2$$
$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{G_i^t + \epsilon}} g_i^t$$

RMSProp

$$m_i^t = \beta_1 m_i^{t-1} + (1-\beta_1) g_i^t$$
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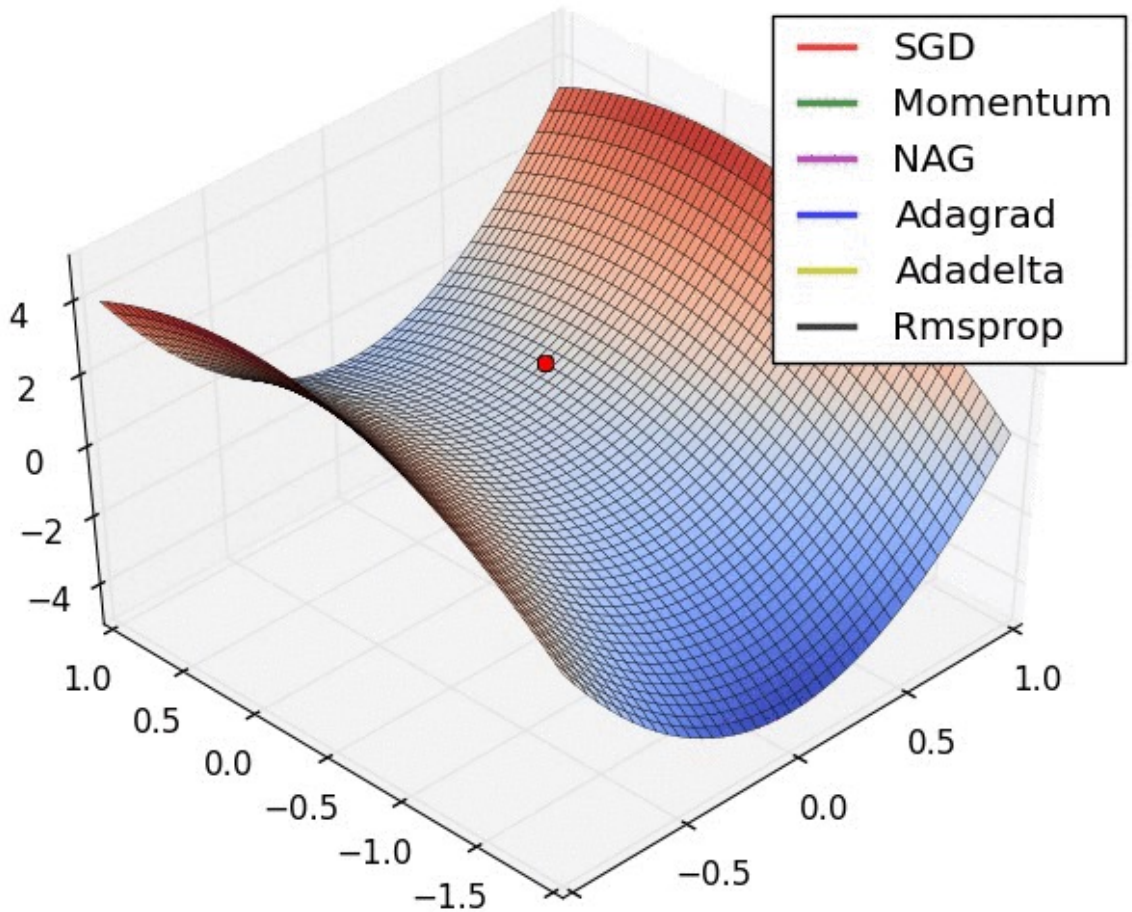
$$w_i^t = w_i^{t-1} - \frac{\eta}{\sqrt{\hat{G}_i^t + \epsilon}} \hat{m}_i^t$$

Adam

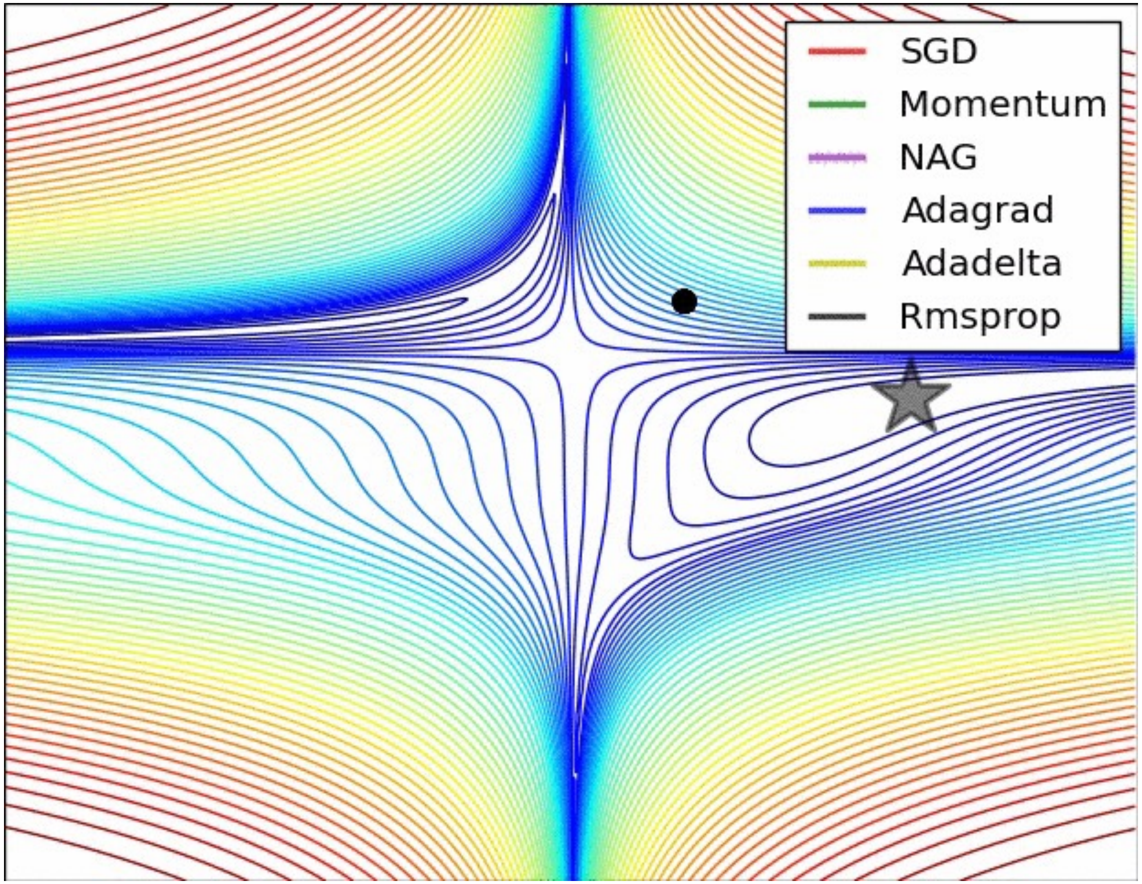
Adaptive Learning Rates

- **Comparison: Long valley**



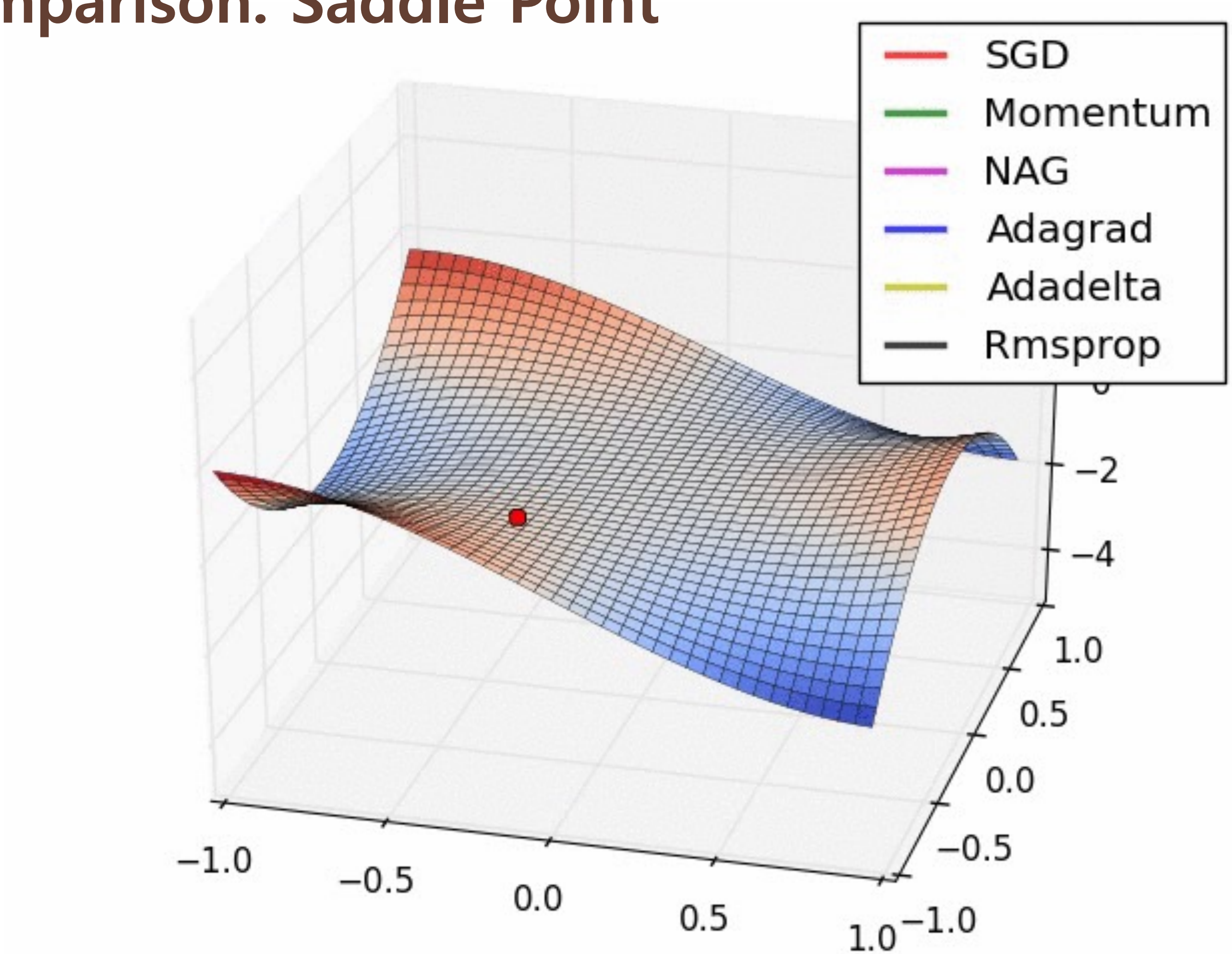
Adaptive Learning Rates

- **Comparison: Beale's Function**



Adaptive Learning Rates

- **Comparison: Saddle Point**



Question and Answer