Foundations of Artificial Intelligence
Wintersemester 2025/26
Assignment 3

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1 Adversarial Search and Games

1.1 Minimax Search

1.2 Games of Chance

1.3 Correctness of $\alpha - \beta$-pruning

2 Propositional Logic

2.1 Who is lying?

We define the variables:

- J: John tells the truth.

- P: Peter tells the truth.

- E: Emma tells the truth.

(a)

The statements:
John says: 'Peter always lies.'
This means J is true if and only if P is false. Formula: $(J) \leftrightarrow \neg(P)$
Peter says: 'Either John is a liar or Emma is a liar, but not both.'
There are two possibilities for this statement to be true:

- Possibility 1: John lies ($\neg J$) AND Emma tells the truth (E).

- Possibility 2: John tells the truth (J) AND Emma lies ($\neg E$).

- We connect these two possibilities with an OR.

Formula: $P \leftrightarrow ((\neg J \wedge E) \vee (J \wedge \neg E))$
Emma says: 'If John is a liar, then Peter is also a liar.'
This is a classical implication.
Formula: $E \leftrightarrow (\neg J \rightarrow \neg P)$

(b)

The set of formulae is:

$$1. \quad J \leftrightarrow \neg P \tag{1}$$
$$2. \quad P \leftrightarrow ((\neg J \vee \neg E) \wedge \neg(\neg J \wedge \neg E)) \tag{2}$$
$$3. \quad E \leftrightarrow (\neg J \rightarrow \neg P) \tag{3}$$

Goal: Find a truth assignment that satisfies all three.

Step 1: Analysis of John's formula

Formula: $J \leftrightarrow \neg P$
    Using the definition $A \leftrightarrow B \equiv (A \to B) \land (B \to A)$, we have:

$$(J \leftrightarrow \neg P) \equiv (J \to \neg P) \land (\neg P \to J)$$

Now rewrite the implications:

$$J \to \neg P \equiv \neg J \lor \neg P$$
$$\neg P \to J \equiv P \lor J$$

So John's statement becomes:

$$(\neg J \lor \neg P) \land (P \lor J)$$

From this, we see the required relation:

$$\boxed{J = \neg P}$$

Step 2: Analysis of Emma's formula

Formula: $E \leftrightarrow (\neg J \to \neg P)$
    Rewrite the implication using $A \to B \equiv \neg A \lor B$:

$$\neg J \to \neg P \equiv J \lor \neg P$$

Now substitute John's relation $J = \neg P$:

$$J \lor \neg P \equiv (\neg P) \lor \neg P \equiv \neg P$$

$$E \leftrightarrow \neg P$$

We now know:

$$J = \neg P \quad \text{and} \quad E = \neg P$$

Therefore:

$$\boxed{J = E}$$

Interim result: John and Emma must have the same truth value.

Step 3: Analysis of Peter's formula

Peter states his 'one lies but not both' condition:

$$P \leftrightarrow ((\neg J \lor \neg E) \land \neg(\neg J \land \neg E))$$

   3.1 Substitute $J = E$
Since we know $J = E$, then $\neg J = \neg E$. Thus the formula becomes:

$$P \leftrightarrow ((\neg J \lor \neg J) \land \neg(\neg J \land \neg J))$$

Simplify step by step:

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(a) $\neg J \vee \neg J = \neg J$

(b) $\neg J \wedge \neg J = \neg J$

(c) $\neg(\neg J) = J$

Therefore Peter's statement reduces to:

$$P \leftrightarrow (\neg J \wedge J)$$

3.2 Simplify the contradiction
The term $(\neg J \wedge J)$ is a contradiction, meaning it is always false:

$$\neg J \wedge J \equiv \text{false}$$

So:
$$P \leftrightarrow \text{false}$$

Thus:
$$\boxed{P = \text{false}}$$

Step 4: Determine J and E

We previously established:
$$J = \neg P \quad \text{and} \quad E = \neg P$$

Since $P = \text{false}$, we conclude:

$$J = \neg\text{false} = \text{true}$$
$$E = \neg\text{false} = \text{true}$$

Final answer

- **John tells the truth**

- **Peter lies**

- **Emma tells the truth**

2.2 Knowledge Bases

We are given the knowledge base:

$$K = \{\, A \vee (B \vee \neg C),\ A \Leftrightarrow B,\ (C \wedge A) \Rightarrow D \,\}.$$

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(a)

1. The formula $A \vee (B \vee \neg C)$ simplifies to:

$$A \vee B \vee \neg C.$$

2. The equivalence $A \Leftrightarrow B$ is rewritten as:

$$(A \rightarrow B) \wedge (B \rightarrow A),$$

which becomes:

$$(\neg A \vee B) \wedge (\neg B \vee A).$$

3. The implication $(C \wedge A) \Rightarrow D$ becomes:

$$\neg(C \wedge A) \vee D = \neg C \vee \neg A \vee D.$$

Thus, the CNF of the entire knowledge base is:

$$(A \vee B \vee \neg C) \wedge (\neg A \vee B) \wedge (\neg B \vee A) \wedge (\neg C \vee \neg A \vee D).$$

(b)

The clauses in the CNF are:

$$\{A \vee B \vee \neg C, \ \neg A \vee B, \ \neg B \vee A, \ \neg C \vee \neg A \vee D\}.$$

As sets of literals:

$$\Big\{\{A, B, \neg C\}, \ \{\neg A, B\}, \ \{\neg B, A\}, \ \{\neg C, \neg A, D\}\Big\}.$$

(c)

A definite clause contains exactly one positive literal.
Checking each clause:

- $A \vee B \vee \neg C$: two positive literals $\rightarrow$ not definite.

- $\neg A \vee B$: one positive literal $B \rightarrow$ definite.

- $\neg B \vee A$: one positive literal $A \rightarrow$ definite.

- $\neg C \vee \neg A \vee D$: one positive literal $D \rightarrow$ definite.

Thus, the definite clauses are:

$$\neg A \vee B, \qquad \neg B \vee A, \qquad \neg C \vee \neg A \vee D.$$

2.3 Models

Consider the four propositions $A, B, C,$ and D. The total number of possible truth assignments (interpretations) for 4 binary variables is $2^4 = 16$. We need to find how many of these 16 assignments make the following formulae true.

(a) $B \vee \neg C$

This formula depends only on B and C. The variables A and D are 'don't cares' (irrelevant to the truth value, but must be counted).

- **Analyze $B \vee \neg C$:** A disjunction is FALSE only if both parts are false.
    - B is False (0)
    - $\neg C$ is False $\Rightarrow C$ is True (1)

  Combinations for (B, C):
    - $(0, 0) \to 0 \vee 1 = 1$ (True)
    - $(0, 1) \to 0 \vee 0 = 0$ (False)
    - $(1, 0) \to 1 \vee 1 = 1$ (True)
    - $(1, 1) \to 1 \vee 0 = 1$ (True)

  So, there are **3** valid assignments for the pair (B, C).

- **Account for A and D:** For each valid combination of B and C, the variables A and D can be either True or False ($2 \times 2 = 4$ variations).

- **Calculation:**
$$\text{Models} = 3 \times 2^2 = 3 \times 4 = 12$$

Answer: 12 models.

(b) $A \wedge \neg(\neg B \vee C) \wedge D$

This is a conjunction. For the formula to be True, **every** part must be True.

- **Analyze the components:**
    - Part 1: A must be **True** (1).
    - Part 2: D must be **True** (1).
    - Part 3: $\neg(\neg B \vee C)$ must be **True**.

- **Simplify Part 3 (De Morgan's Law):**
$$\neg(\neg B \vee C) \equiv \neg(\neg B) \wedge \neg C \equiv B \wedge \neg C$$

  For this to be True:
    - B must be **True** (1).
    - C must be **False** (0).

- **Calculation:** We have fixed values for all four variables:
$$A = 1, \quad B = 1, \quad C = 0, \quad D = 1$$

  There is only 1 specific assignment that satisfies this.

Answer: 1 model.

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(c) $((B \rightarrow D) \vee (A \rightarrow D)) \wedge C$

This is a conjunction of a complex term and C.

- **Constraint on C:** Since it is a conjunction $(\cdots \wedge C)$, C must be **True** (1). This reduces our search space. We effectively assume $C = 1$ and look for satisfying assignments of A, B, D ($2^3 = 8$ possibilities).

- **Analyze the complex term:** Formula: $(B \rightarrow D) \vee (A \rightarrow D)$

  Rewrite implication ($X \rightarrow Y \equiv \neg X \vee Y$):

  $$(\neg B \vee D) \vee (\neg A \vee D)$$

  Simplify (Associativity and Idempotence of $\vee$):

  $$\neg A \vee \neg B \vee D \vee D \equiv \neg A \vee \neg B \vee D$$

  This formula is a disjunction. It is **True** in all cases *except* when all literals are False.

- **Find the failing case:** The expression $\neg A \vee \neg B \vee D$ is False if:
  - $\neg A$ is False $\Rightarrow A = 1$
  - $\neg B$ is False $\Rightarrow B = 1$
  - D is False $\Rightarrow D = 0$

  There is exactly **1** failing combination for (A, B, D) out of 8 possibilities.

- **Calculation:**
  - Total combinations for (A, B, D): 8
  - Failing combinations: 1
  - Valid combinations: $8 - 1 = 7$

  Since C is fixed to True, we do not multiply further.

Answer: 7 models.