

1 Adversarial Search and Games

1.1 Minimax Search

1.2 Games of Chance

1.3 Correctness of $\alpha - \beta$ -pruning

2 Propositional Logic

2.1 Who is lying?

We define the variables:

- J : John tells the truth.
- P : Peter tells the truth.
- E : Emma tells the truth.

(a)

The statements:

John says: 'Peter always lies.'

This means J is true if and only if P is false. Formula: $(J) \leftrightarrow \neg(P)$

Peter says: 'Either John is a liar or Emma is a liar, but not both.'

There are two possibilities for this statement to be true:

- Possibility 1: John lies ($\neg J$) AND Emma tells the truth (E).
- Possibility 2: John tells the truth (J) AND Emma lies ($\neg E$).
- We connect these two possibilities with an OR.

Formula: $P \leftrightarrow ((\neg J \wedge E) \vee (J \wedge \neg E))$

Emma says: 'If John is a liar, then Peter is also a liar.'

This is a classical implication.

Formula: $E \leftrightarrow (\neg J \rightarrow \neg P)$

(b)

The set of formulae is:

$$1. \quad J \leftrightarrow \neg P \tag{1}$$

$$2. \quad P \leftrightarrow ((\neg J \vee \neg E) \wedge \neg(\neg J \wedge \neg E)) \tag{2}$$

$$3. \quad E \leftrightarrow (\neg J \rightarrow \neg P) \tag{3}$$

Goal: Find a truth assignment that satisfies all three.

Step 1: Analysis of John's formula

Formula: $J \leftrightarrow \neg P$

Using the definition $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$, we have:

$$(J \leftrightarrow \neg P) \equiv (J \rightarrow \neg P) \wedge (\neg P \rightarrow J)$$

Now rewrite the implications:

$$\begin{aligned} J \rightarrow \neg P &\equiv \neg J \vee \neg P \\ \neg P \rightarrow J &\equiv P \vee J \end{aligned}$$

So John's statement becomes:

$$(\neg J \vee \neg P) \wedge (P \vee J)$$

From this, we see the required relation:

$$J = \neg P$$

Step 2: Analysis of Emma's formula

Formula: $E \leftrightarrow (\neg J \rightarrow \neg P)$

Rewrite the implication using $A \rightarrow B \equiv \neg A \vee B$:

$$\neg J \rightarrow \neg P \equiv J \vee \neg P$$

Now substitute John's relation $J = \neg P$:

$$J \vee \neg P \equiv (\neg P) \vee \neg P \equiv \neg P$$

$$E \leftrightarrow \neg P$$

We now know:

$$J = \neg P \quad \text{and} \quad E = \neg P$$

Therefore:

$$J = E$$

Interim result: John and Emma must have the same truth value.

Step 3: Analysis of Peter's formula

Peter states his 'one lies but not both' condition:

$$P \leftrightarrow ((\neg J \vee \neg E) \wedge \neg(\neg J \wedge \neg E))$$

3.1 Substitute $J = E$

Since we know $J = E$, then $\neg J = \neg E$. Thus the formula becomes:

$$P \leftrightarrow ((\neg J \vee \neg J) \wedge \neg(\neg J \wedge \neg J))$$

Simplify step by step:

(a) $\neg J \vee \neg J = \neg J$

(b) $\neg J \wedge \neg J = \neg J$

(c) $\neg(\neg J) = J$

Therefore Peter's statement reduces to:

$$P \leftrightarrow (\neg J \wedge J)$$

3.2 Simplify the contradiction

The term $(\neg J \wedge J)$ is a contradiction, meaning it is always false:

$$\neg J \wedge J \equiv \text{false}$$

So:

$$P \leftrightarrow \text{false}$$

Thus:

$$P = \text{false}$$

Step 4: Determine J and E

We previously established:

$$J = \neg P \quad \text{and} \quad E = \neg P$$

Since $P = \text{false}$, we conclude:

$$\begin{aligned} J &= \neg \text{false} = \text{true} \\ E &= \neg \text{false} = \text{true} \end{aligned}$$

Final answer

- John tells the truth
- Peter lies
- Emma tells the truth

2.2 Knowledge Bases

We are given the knowledge base:

$$K = \{ A \vee (B \vee \neg C), A \Leftrightarrow B, (C \wedge A) \Rightarrow D \}.$$

(a)

1. The formula $A \vee (B \vee \neg C)$ simplifies to:

$$A \vee B \vee \neg C.$$

2. The equivalence $A \Leftrightarrow B$ is rewritten as:

$$(A \rightarrow B) \wedge (B \rightarrow A),$$

which becomes:

$$(\neg A \vee B) \wedge (\neg B \vee A).$$

3. The implication $(C \wedge A) \Rightarrow D$ becomes:

$$\neg(C \wedge A) \vee D = \neg C \vee \neg A \vee D.$$

Thus, the CNF of the entire knowledge base is:

$$(A \vee B \vee \neg C) \wedge (\neg A \vee B) \wedge (\neg B \vee A) \wedge (\neg C \vee \neg A \vee D).$$

(b)

The clauses in the CNF are:

$$\{A \vee B \vee \neg C, \neg A \vee B, \neg B \vee A, \neg C \vee \neg A \vee D\}.$$

As sets of literals:

$$\{\{A, B, \neg C\}, \{\neg A, B\}, \{\neg B, A\}, \{\neg C, \neg A, D\}\}.$$

(c)

A definite clause contains exactly one positive literal.

Checking each clause:

- $A \vee B \vee \neg C$: two positive literals \rightarrow not definite.
- $\neg A \vee B$: one positive literal $B \rightarrow$ definite.
- $\neg B \vee A$: one positive literal $A \rightarrow$ definite.
- $\neg C \vee \neg A \vee D$: one positive literal $D \rightarrow$ definite.

Thus, the definite clauses are:

$$\neg A \vee B, \quad \neg B \vee A, \quad \neg C \vee \neg A \vee D.$$

2.3 Models

Hier ist der LaTeX-Code für Aufgabe 2.3. Ich habe die Lösungsschritte detailliert aufgeschrieben, damit klar ist, wie die Anzahl der Modelle unter Berücksichtigung aller vier Variablen (A, B, C, D) berechnet wird.

Ich habe die Tippfehler im Originaltext (z. B. „=“) als Standard-Implikationen (\rightarrow) interpretiert.

Code-Snippet

2.3 Models

Consider the four propositions A, B, C , and D . The total number of possible truth assignments (interpretations) for 4 binary variables is $2^4 = 16$. We need to find how many of these 16 assignments make the following formulae true.

a) $B \vee \neg C$

This formula depends only on B and C . The variables A and D are 'don't cares' (irrelevant to the truth value, but must be counted).

- **Analyze $B \vee \neg C$:** A disjunction is FALSE only if both parts are false.

- B is False (0)
- $\neg C$ is False $\Rightarrow C$ is True (1)

Combinations for (B, C) :

- $(0, 0) \rightarrow 0 \vee 1 = 1$ (True)
- $(0, 1) \rightarrow 0 \vee 0 = 0$ (False)
- $(1, 0) \rightarrow 1 \vee 1 = 1$ (True)
- $(1, 1) \rightarrow 1 \vee 0 = 1$ (True)

So, there are **3** valid assignments for the pair (B, C) .

- **Account for A and D :** For each valid combination of B and C , the variables A and D can be either True or False ($2 \times 2 = 4$ variations).

- **Calculation:**

$$\text{Models} = 3 \times 2^2 = 3 \times 4 = 12$$

Answer: 12 models.

b) $A \wedge \neg(\neg B \vee C) \wedge D$

This is a conjunction. For the formula to be True, **every** part must be True.

- **Analyze the components:**

- Part 1: A must be **True** (1).
- Part 2: D must be **True** (1).
- Part 3: $\neg(\neg B \vee C)$ must be **True**.

- **Simplify Part 3 (De Morgan's Law):**

$$\neg(\neg B \vee C) \equiv \neg(\neg B) \wedge \neg C \equiv B \wedge \neg C$$

For this to be True:

- B must be **True** (1).
- C must be **False** (0).

- **Calculation:** We have fixed values for all four variables:

$$A = 1, \quad B = 1, \quad C = 0, \quad D = 1$$

There is only 1 specific assignment that satisfies this.

Answer: 1 model.

(c) $((B \rightarrow D) \vee (A \rightarrow D)) \wedge C$

This is a conjunction of a complex term and C .

- **Constraint on C :** Since it is a conjunction ($\cdots \wedge C$), C must be **True** (1). This reduces our search space. We effectively assume $C = 1$ and look for satisfying assignments of A, B, D ($2^3 = 8$ possibilities).
- **Analyze the complex term:** Formula: $(B \rightarrow D) \vee (A \rightarrow D)$
 Rewrite implication ($X \rightarrow Y \equiv \neg X \vee Y$):

$$(\neg B \vee D) \vee (\neg A \vee D)$$

Simplify (Associativity and Idempotence of \vee):

$$\neg A \vee \neg B \vee D \vee D \equiv \neg A \vee \neg B \vee D$$

This formula is a disjunction. It is **True** in all cases *except* when all literals are False.

- **Find the failing case:** The expression $\neg A \vee \neg B \vee D$ is False if:
 - $\neg A$ is False $\Rightarrow A = 1$
 - $\neg B$ is False $\Rightarrow B = 1$
 - D is False $\Rightarrow D = 0$

There is exactly **1** failing combination for (A, B, D) out of 8 possibilities.

- **Calculation:**
 - Total combinations for (A, B, D) : 8
 - Failing combinations: 1
 - Valid combinations: $8 - 1 = 7$

Since C is fixed to True, we do not multiply further.

Answer: 7 models.