

# 1 Adversarial Search and Games

## 1.1 Minimax Search

### a) Compute Minimax-Value

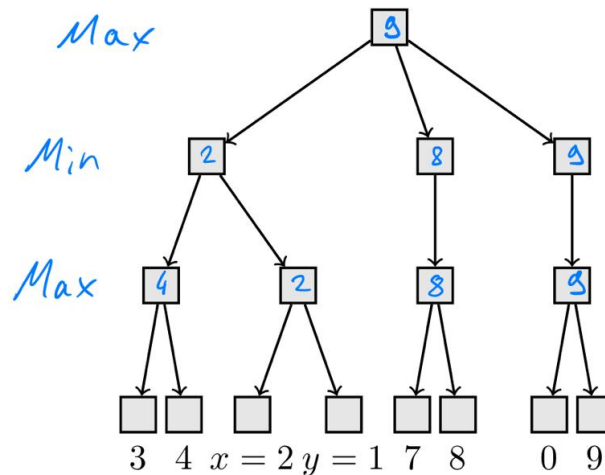
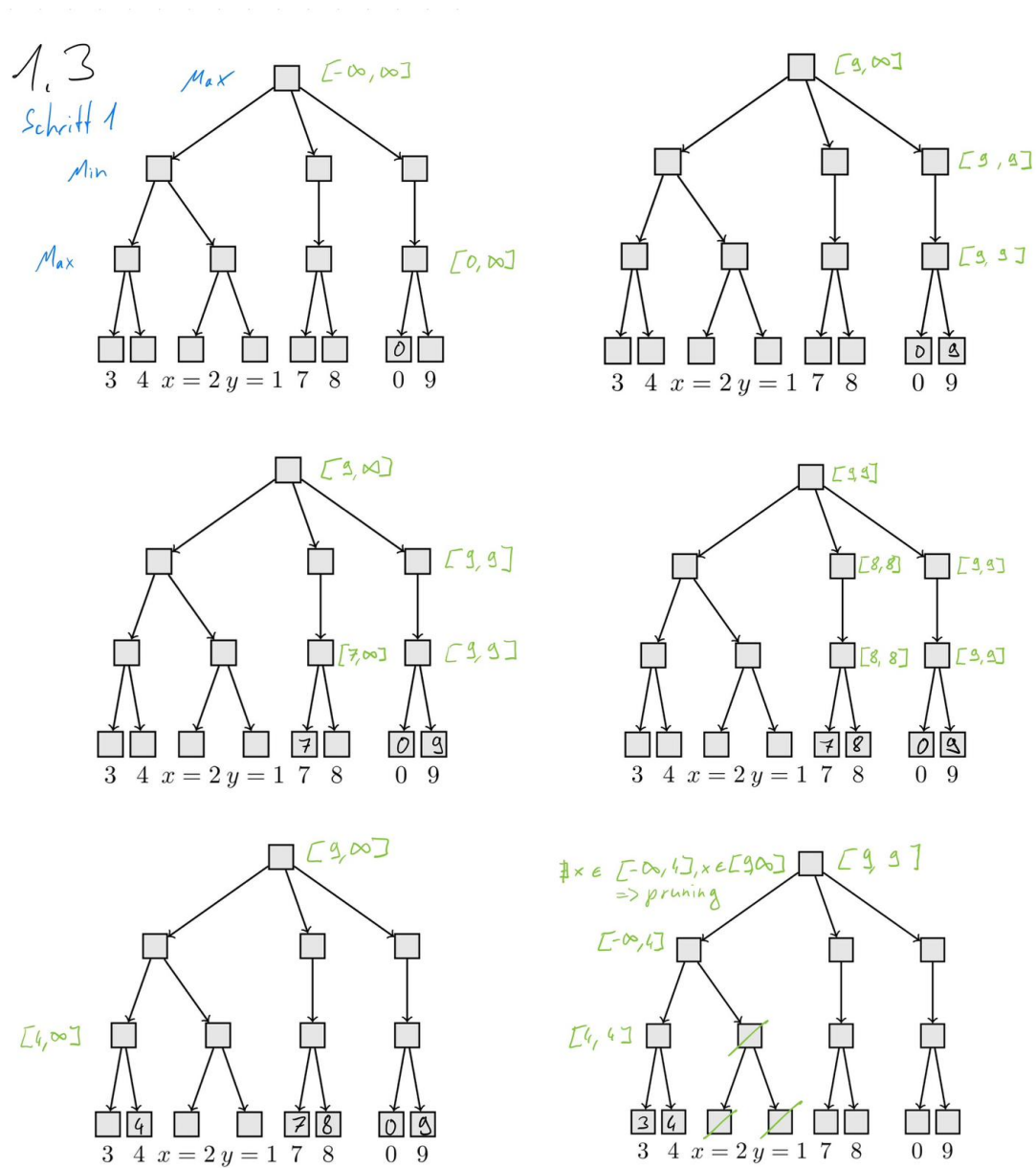


Figure 1: Minimax Search Tree

### b) Independency of value from x and y

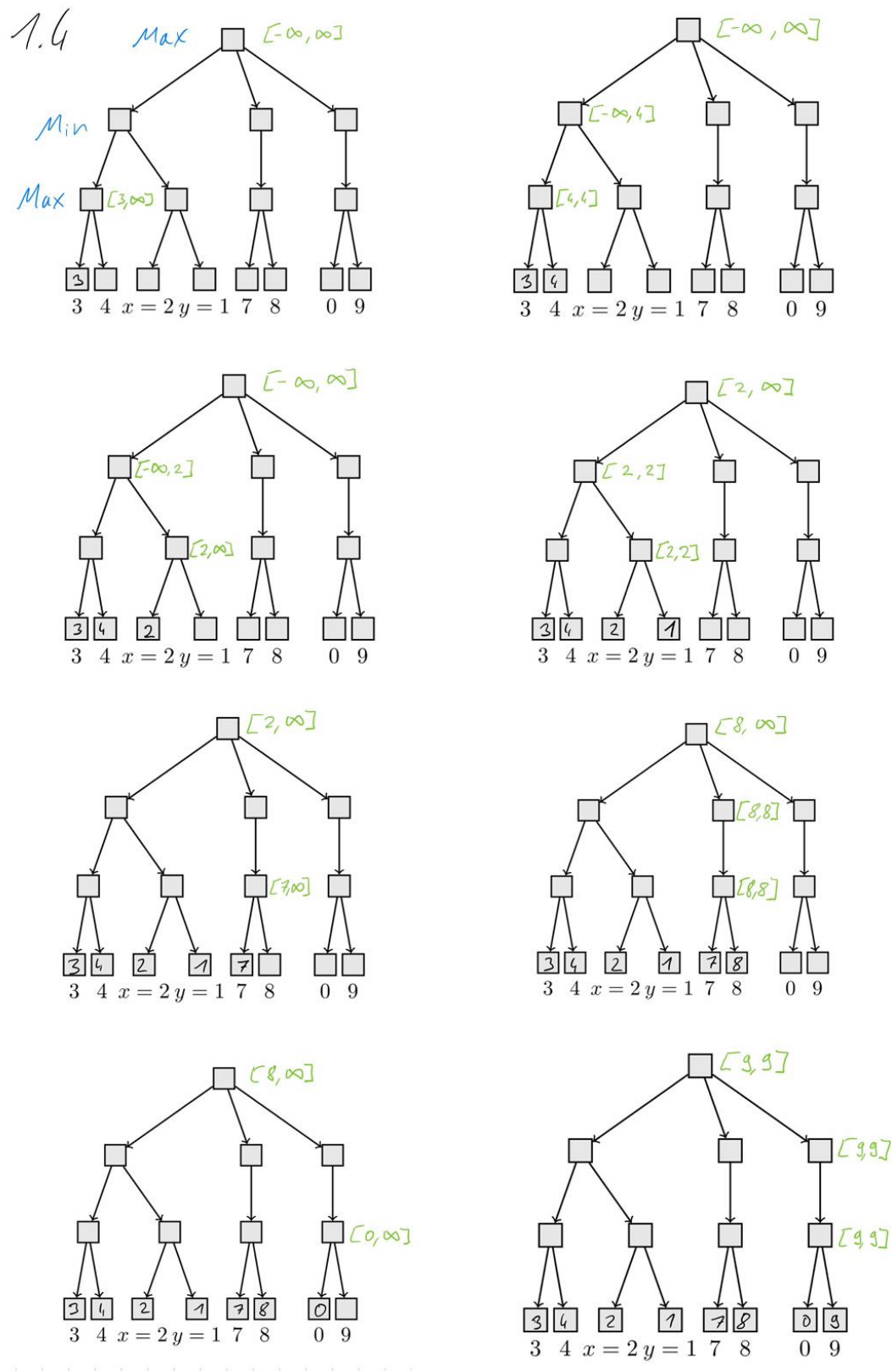
The leafs of x and y don't have any impact on the root node's value, because of the structure of the tree. To change the value of the tree node, a child of the root must have a higher value than 9, otherwise that's the chosen max value. But the left child of the root will always evaluate to 4, if the parent of x and y doesn't have a lower value than that. To have an impact on the root node, x or y would have to be a value which is greater than 9 and lower than 4, which is impossible.

### c) Pruning with optimal move order



Using this order will result in 3 nodes being pruned, with two of them being leaf nodes.

### d) Pruning with worst move order



Using this order, none of the nodes can be pruned.

## 1.2 Games of Chance

## 1.3 Correctness of $\alpha - \beta$ -pruning

# 2 Propositional Logic

## 2.1 Who is lying?

We define the variables:

- $J$ : John tells the truth.
- $P$ : Peter tells the truth.
- $E$ : Emma tells the truth.

### (a)

The statements:

**John says: 'Peter always lies.'**

This means  $J$  is true if and only if  $P$  is false. Formula:  $(J) \leftrightarrow \neg(P)$

**Peter says: 'Either John is a liar or Emma is a liar, but not both.'**

There are two possibilities for this statement to be true:

- Possibility 1: John lies ( $\neg J$ ) AND Emma tells the truth ( $E$ ).
- Possibility 2: John tells the truth ( $J$ ) AND Emma lies ( $\neg E$ ).
- We connect these two possibilities with an OR.

Formula:  $P \leftrightarrow ((\neg J \wedge E) \vee (J \wedge \neg E))$

**Emma says: 'If John is a liar, then Peter is also a liar.'**

This is a classical implication.

Formula:  $E \leftrightarrow (\neg J \rightarrow \neg P)$

### (b)

The set of formulae is:

1.  $J \leftrightarrow \neg P$  (1)
2.  $P \leftrightarrow ((\neg J \vee \neg E) \wedge \neg(\neg J \wedge \neg E))$  (2)
3.  $E \leftrightarrow (\neg J \rightarrow \neg P)$  (3)

**Goal:** Find a truth assignment that satisfies all three.

### Step 1: Analysis of John's formula

Formula:  $J \leftrightarrow \neg P$

Using the definition  $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$ , we have:

$$(J \leftrightarrow \neg P) \equiv (J \rightarrow \neg P) \wedge (\neg P \rightarrow J)$$

Now rewrite the implications:

$$\begin{aligned} J \rightarrow \neg P &\equiv \neg J \vee \neg P \\ \neg P \rightarrow J &\equiv P \vee J \end{aligned}$$

So John's statement becomes:

$$(\neg J \vee \neg P) \wedge (P \vee J)$$

From this, we see the required relation:

$$\boxed{J = \neg P}$$

### Step 2: Analysis of Emma's formula

Formula:  $E \leftrightarrow (\neg J \rightarrow \neg P)$

Rewrite the implication using  $A \rightarrow B \equiv \neg A \vee B$ :

$$\neg J \rightarrow \neg P \equiv J \vee \neg P$$

Now substitute John's relation  $J = \neg P$ :

$$J \vee \neg P \equiv (\neg P) \vee \neg P \equiv \neg P$$

$$E \leftrightarrow \neg P$$

We now know:

$$J = \neg P \quad \text{and} \quad E = \neg P$$

Therefore:

$$\boxed{J = E}$$

**Interim result:** John and Emma must have the same truth value.

### Step 3: Analysis of Peter's formula

Peter states his 'one lies but not both' condition:

$$P \leftrightarrow ((\neg J \vee \neg E) \wedge \neg(\neg J \wedge \neg E))$$

#### 3.1 Substitute $J = E$

Since we know  $J = E$ , then  $\neg J = \neg E$ . Thus the formula becomes:

$$P \leftrightarrow ((\neg J \vee \neg J) \wedge \neg(\neg J \wedge \neg J))$$

Simplify step by step:

(a)  $\neg J \vee \neg J = \neg J$

(b)  $\neg J \wedge \neg J = \neg J$

(c)  $\neg(\neg J) = J$

Therefore Peter's statement reduces to:

$$P \leftrightarrow (\neg J \wedge J)$$

### 3.2 Simplify the contradiction

The term  $(\neg J \wedge J)$  is a contradiction, meaning it is always false:

$$\neg J \wedge J \equiv \text{false}$$

So:

$$P \leftrightarrow \text{false}$$

Thus:

$$\boxed{P = \text{false}}$$

### Step 4: Determine J and E

We previously established:

$$J = \neg P \quad \text{and} \quad E = \neg P$$

Since  $P = \text{false}$ , we conclude:

$$J = \neg \text{false} = \text{true}$$

$$E = \neg \text{false} = \text{true}$$

### Final answer

- John tells the truth
- Peter lies
- Emma tells the truth

## 2.2 Knowledge Bases

We are given the knowledge base:

$$K = \{ A \vee (B \vee \neg C), A \Leftrightarrow B, (C \wedge A) \Rightarrow D \}.$$

**(a)**

1. The formula  $A \vee (B \vee \neg C)$  simplifies to:

$$A \vee B \vee \neg C.$$

2. The equivalence  $A \Leftrightarrow B$  is rewritten as:

$$(A \rightarrow B) \wedge (B \rightarrow A),$$

which becomes:

$$(\neg A \vee B) \wedge (\neg B \vee A).$$

3. The implication  $(C \wedge A) \Rightarrow D$  becomes:

$$\neg(C \wedge A) \vee D = \neg C \vee \neg A \vee D.$$

Thus, the CNF of the entire knowledge base is:

$$(A \vee B \vee \neg C) \wedge (\neg A \vee B) \wedge (\neg B \vee A) \wedge (\neg C \vee \neg A \vee D).$$

**(b)**

The clauses in the CNF are:

$$\{A \vee B \vee \neg C, \neg A \vee B, \neg B \vee A, \neg C \vee \neg A \vee D\}.$$

As sets of literals:

$$\{\{A, B, \neg C\}, \{\neg A, B\}, \{\neg B, A\}, \{\neg C, \neg A, D\}\}.$$

**(c)**

A definite clause contains exactly one positive literal.

Checking each clause:

- $A \vee B \vee \neg C$ : two positive literals  $\rightarrow$  not definite.
- $\neg A \vee B$ : one positive literal  $B \rightarrow$  definite.
- $\neg B \vee A$ : one positive literal  $A \rightarrow$  definite.
- $\neg C \vee \neg A \vee D$ : one positive literal  $D \rightarrow$  definite.

Thus, the definite clauses are:

$$\neg A \vee B, \quad \neg B \vee A, \quad \neg C \vee \neg A \vee D.$$

## 2.3 Models

Consider the four propositions  $A, B, C$ , and  $D$ . The total number of possible truth assignments (interpretations) for 4 binary variables is  $2^4 = 16$ . We need to find how many of these 16 assignments make the following formulae true.

**(a)**  $B \vee \neg C$

This formula depends only on  $B$  and  $C$ . The variables  $A$  and  $D$  are 'don't cares' (irrelevant to the truth value, but must be counted).

- **Analyze  $B \vee \neg C$ :** A disjunction is FALSE only if both parts are false.
  - $B$  is False (0)
  - $\neg C$  is False  $\Rightarrow C$  is True (1)

Combinations for  $(B, C)$ :

- $(0, 0) \rightarrow 0 \vee 1 = 1$  (True)
- $(0, 1) \rightarrow 0 \vee 0 = 0$  (False)
- $(1, 0) \rightarrow 1 \vee 1 = 1$  (True)
- $(1, 1) \rightarrow 1 \vee 0 = 1$  (True)

So, there are **3** valid assignments for the pair  $(B, C)$ .

- **Account for  $A$  and  $D$ :** For each valid combination of  $B$  and  $C$ , the variables  $A$  and  $D$  can be either True or False ( $2 \times 2 = 4$  variations).
- **Calculation:**

$$\text{Models} = 3 \times 2^2 = 3 \times 4 = 12$$

**Answer: 12 models.**

**(b)**  $A \wedge \neg(\neg B \vee C) \wedge D$

This is a conjunction. For the formula to be True, **every** part must be True.

- **Analyze the components:**
  - Part 1:  $A$  must be **True** (1).
  - Part 2:  $D$  must be **True** (1).
  - Part 3:  $\neg(\neg B \vee C)$  must be **True**.
- **Simplify Part 3 (De Morgan's Law):**

$$\neg(\neg B \vee C) \equiv \neg(\neg B) \wedge \neg C \equiv B \wedge \neg C$$

For this to be True:

- $B$  must be **True** (1).
- $C$  must be **False** (0).
- **Calculation:** We have fixed values for all four variables:

$$A = 1, \quad B = 1, \quad C = 0, \quad D = 1$$

There is only 1 specific assignment that satisfies this.

**Answer: 1 model.**

**(c)**  $((B \rightarrow D) \vee (A \rightarrow D)) \wedge C$

This is a conjunction of a complex term and  $C$ .

- **Constraint on  $C$ :** Since it is a conjunction  $(\dots \wedge C)$ ,  $C$  must be **True** (1). This reduces our search space. We effectively assume  $C = 1$  and look for satisfying assignments of  $A, B, D$  ( $2^3 = 8$  possibilities).
- **Analyze the complex term:** Formula:  $(B \rightarrow D) \vee (A \rightarrow D)$   
Rewrite implication ( $X \rightarrow Y \equiv \neg X \vee Y$ ):

$$(\neg B \vee D) \vee (\neg A \vee D)$$

Simplify (Associativity and Idempotence of  $\vee$ ):

$$\neg A \vee \neg B \vee D \vee D \equiv \neg A \vee \neg B \vee D$$

This formula is a disjunction. It is **True** in all cases *except* when all literals are False.

- **Find the failing case:** The expression  $\neg A \vee \neg B \vee D$  is False if:
  - $\neg A$  is False  $\Rightarrow A = 1$
  - $\neg B$  is False  $\Rightarrow B = 1$
  - $D$  is False  $\Rightarrow D = 0$

There is exactly **1** failing combination for  $(A, B, D)$  out of 8 possibilities.

- **Calculation:**
  - Total combinations for  $(A, B, D)$ : 8
  - Failing combinations: 1
  - Valid combinations:  $8 - 1 = 7$

Since  $C$  is fixed to True, we do not multiply further.

**Answer: 7 models.**