Foundations of Artificial Intelligence

Wintersemester 2025/26

Assignment 4

Oliver Löhr (3311903) Bachelor Informatik

Lara Aziz (3720604) Bachelor Informatik

Joel Thomas (3814387) Master Computer Science

1 Predicate Logic

1.1 First Order Logic

Definitions

- $P(x)$: x is a person

- $F(x)$: x is an ice cream flavor

- $L(x, y)$: x loves y

- $S(x, y)$: x shaves y

- $R(x)$: x is a reindeer

- $N(x)$: x has a red nose

- $W(x)$: x is weird

- $C(x)$: x is a clown

- b: The Barber (constant)

- r: Rudolph (constant)

Solutions

    a. There is an ice cream flavor loved by everyone.

$$\exists x(F(x) \wedge \forall y(P(y) \rightarrow L(y, x)))$$

    b. It is not true that everyone loves some ice cream flavor.

$$\neg\forall x(P(x) \rightarrow \exists y(F(y) \wedge L(x, y)))$$

    c. Anyone who does not shave himself must be shaved by the barber.

$$\forall x(P(x) \wedge \neg S(x, x) \rightarrow S(b, x))$$

    d. Whomever the barber shaves, must not shave himself.

$$\forall x(P(x) \wedge S(b, x) \rightarrow \neg S(x, x))$$

    e. Rudolph is a reindeer and Rudolph has a red nose.

$$R(r) \wedge N(r)$$

    f. Anyone with a red nose is weird or is a clown.

$$\forall x(N(x) \rightarrow (W(x) \vee C(x)))$$

    g. No reindeer is a clown.

$$\forall x(R(x) \rightarrow \neg C(x))$$

1.2 Clausal Forms

a. $\forall x \forall y (R(x,y) \rightarrow (R(x,y) \wedge Q(y)))$

Step 1: Eliminate implication $(A \rightarrow B \equiv \neg A \vee B)$

$$\forall x \forall y (\neg R(x,y) \vee (R(x,y) \wedge Q(y)))$$

Step 2: Distribute $\vee$ over $\wedge$

$$\forall x \forall y ((\neg R(x,y) \vee R(x,y)) \wedge (\neg R(x,y) \vee Q(y)))$$

Step 3: Simplify $(\neg A \vee A \equiv \text{True})$

$$\forall x \forall y (\text{True} \wedge (\neg R(x,y) \vee Q(y)))$$

$$\forall x \forall y (\neg R(x,y) \vee Q(y))$$

Step 4: Drop universal quantifiers

$$\neg R(x,y) \vee Q(y)$$

Resulting Clause:

$$\{\neg R(x,y), Q(y)\}$$

b. $\forall x \exists y \forall z (P(x,y,z) \rightarrow \exists u R(x,u,z))$

Step 1: Eliminate implication

$$\forall x \exists y \forall z (\neg P(x,y,z) \vee \exists u R(x,u,z))$$

Step 2: Prenex Normal Form (move quantifiers left)

$$\forall x \exists y \forall z \exists u (\neg P(x,y,z) \vee R(x,u,z))$$

Step 3: Skolemization

- Replace y with $f(x)$ (depends on $\forall x$)
- Replace u with $g(x,z)$ (depends on $\forall x$ and $\forall z$)

$$\forall x \forall z (\neg P(x, f(x), z) \vee R(x, g(x,z), z))$$

Step 4: Drop universal quantifiers

$$\neg P(x, f(x), z) \vee R(x, g(x,z), z)$$

Resulting Clause:

$$\{\neg P(x, f(x), z), R(x, g(x,z), z)\}$$

c. $\forall x(\neg \exists y P(x,y) \wedge \neg(Q(x) \wedge \neg R(x)))$

Step 1: Move negations inward

$$\forall x(\forall y \neg P(x,y) \wedge (\neg Q(x) \vee \neg\neg R(x)))$$

$$\forall x(\forall y \neg P(x,y) \wedge (\neg Q(x) \vee R(x)))$$

Step 2: Standardize variables and move quantifiers

$$\forall x \forall y(\neg P(x,y) \wedge (\neg Q(x) \vee R(x)))$$

Step 3: Drop universal quantifiers and split conjunction

$$\neg P(x,y) \wedge (\neg Q(x) \vee R(x))$$

Resulting Clauses:

1. $\{\neg P(x,y)\}$
2. $\{\neg Q(x), R(x)\}$

1.3 Knowledge Extraction via Resolution

Conversion to Clause Form (CNF)

Convert the Knowledge Base sentences $S1 - S4$ and the negated goal into a set of clauses.

- Sentence S1: $\forall t \forall x(Observe(t,x) \wedge Danger(x) \rightarrow Suggestion(t, \text{flee}))$
  Elim. Implication: $\neg Observe(t,x) \vee \neg Danger(x) \vee Suggestion(t, \text{flee})$
  Clause C_1: $\{\neg Observe(t,x), \neg Danger(x), Suggestion(t, \text{flee})\}$

- Sentence S2: $\forall t(\neg \exists x(Observe(t,x) \wedge Danger(x)) \rightarrow Suggestion(t, \text{stay}))$
  Elim. Implication: $\exists x(Observe(t,x) \wedge Danger(x)) \vee Suggestion(t, \text{stay})$
  Skolemization $(x \rightarrow h(t))$: $(Observe(t, h(t)) \wedge Danger(h(t))) \vee Suggestion(t, \text{stay})$
  Clause C_{2a}: $\{Observe(t, h(t)), Suggestion(t, \text{stay})\}$
  Clause C_{2b}: $\{Danger(h(t)), Suggestion(t, \text{stay})\}$

- Sentence S3: $Danger(\text{lion})$
  Clause C_3: $\{Danger(\text{lion})\}$

- Sentence S4: $Observe(\text{now}, \text{lion})$
  Clause C_4: $\{Observe(\text{now}, \text{lion})\}$

- Goal: Prove $\exists x Suggestion(\text{now}, x)$.
  Negated Goal with Answer Literal: $\neg Suggestion(\text{now}, z) \vee Ans(z)$
  Clause C_G: $\{\neg Suggestion(\text{now}, z), Ans(z)\}$

Resolution Trace

- Resolve C_1 and C_4:
  $C_1 : \{\neg Observe(t, x), \neg Danger(x), Suggestion(t, \text{flee})\}$
  $C_4 : \{Observe(\text{now}, \text{lion})\}$
  Substitution: $\theta = \{t/\text{now}, x/\text{lion}\}$
  Resolvent R_1: $\{\neg Danger(\text{lion}), Suggestion(\text{now}, \text{flee})\}$

- Resolve R_1 and C_3:
  $R_1 : \{\neg Danger(\text{lion}), Suggestion(\text{now}, \text{flee})\}$
  $C_3 : \{Danger(\text{lion})\}$
  Substitution: $\theta = \emptyset$
  Resolvent R_2: $\{Suggestion(\text{now}, \text{flee})\}$

- Resolve R_2 and C_G:
  $R_2 : \{Suggestion(\text{now}, \text{flee})\}$
  $C_G : \{\neg Suggestion(\text{now}, z), Ans(z)\}$
  Substitution: $\theta = \{z/\text{flee}\}$
  Resolvent R_3: $\{Ans(\text{flee})\}$

Conclusion

The clause contains only the answer literal. Thus, the Knowledge Base entails the query with the answer:

$$\mathbf{x} = \text{flee}$$

2 Uncertainty

2.1 Probability

2.2 Naive Bayes' Classifier