#### How do we detect collisions?

To check for a collision between two particles, we first compute the vector difference between their positions:

$$diff = \mathbf{p}_i - \mathbf{p}_i$$

Next, we calculate the distance between the particles:

$$distance = \sqrt{diff_x^2 + diff_y^2}$$

To determine if a collision occurs, we compare the distance to the scaled sum of their radii:

(We added a collision\_radius\_scale to make the collision detection more responsive. If we wouldn't have this, the particles would react later than I want them to)

collision\_distance = 
$$(r_i + r_j)$$
 · collision\_radius\_scale

A collision is detected if:

$$distance \leq collision \quad distance$$

### How do we handle particle collisions?

When two particles collide, we need to resolve their positions and velocities. The following steps outline the mathematical process involved:

1. Calculate the current position difference between the particles:

We compute the vector difference between the positions of the two particles:

$$diff = \mathbf{p}_1 - \mathbf{p}_2$$

2. Determine the current distance between the particles:

The distance can be calculated using the Euclidean formula:

current\_distance = 
$$\sqrt{dx^2 + dy^2}$$

where (dx) and (dy) are the differences in the x and y coordinates of the particles, respectively.

## 3. Resolve overlap if a collision has occurred:

If the current distance is less than the collision distance, we have an overlap, which can be defined as:

$$overlap = collision\_distance - current\_distance$$

We then normalize the overlap vector and adjust the positions of both particles:

$$overlap\_vec = normalize(diff) \cdot overlap$$

Finally, we update the positions:

$$\mathbf{p}_1 = \mathbf{p}_1 + \frac{\mathrm{overlap\_vec}}{2}$$

$$\mathbf{p}_2 = \mathbf{p}_2 - \frac{\mathrm{overlap\_vec}}{2}$$

#### 4. Calculate the collision normals:

The normal vector, which indicates the direction of the collision, is calculated as:

$$\mathbf{n} = \left(\frac{dx}{\text{distance}}, \frac{dy}{\text{distance}}\right)$$

The tangent vector can be computed as:

$$\mathbf{t} = (-n_u, n_x)$$

#### 5. Determine the normal and tangent velocities:

We can find the velocities in the normal and tangent directions for both particles:

$$v_{1,\text{normal}} = \mathbf{v}_1 \cdot \mathbf{n}$$

$$v_{1,\mathrm{tangent}} = \mathbf{v}_1 \cdot \mathbf{t}$$

$$v_{2,\text{normal}} = \mathbf{v}_2 \cdot \mathbf{n}$$

$$v_{2,\mathrm{tangent}} = \mathbf{v}_2 \cdot \mathbf{t}$$

# 6. Calculate the new normal velocities based on the elastic collision equations:

Using the conservation of momentum, we calculate the new normal velocities:

$$v_{1,\text{normal}}^{\text{new}} = \frac{v_{1,\text{normal}} \cdot (r_1 - r_2) + 2 \cdot r_2 \cdot v_{2,\text{normal}}}{r_1 + r_2}$$

$$v_{2,\text{normal}}^{\text{new}} = \frac{v_{2,\text{normal}} \cdot (r_2 - r_1) + 2 \cdot r_1 \cdot v_{1,\text{normal}}}{r_1 + r_2}$$

Each normal velocity is then scaled by the coefficient of restitution:

$$v_{1,\text{normal}}^{\text{new}} \leftarrow v_{1,\text{normal}}^{\text{new}} \cdot \text{COEFFICIENT\_OF\_RESTITUTION}$$

$$v_{2.\text{normal}}^{\text{new}} \leftarrow v_{2.\text{normal}}^{\text{new}} \cdot \text{COEFFICIENT\_OF\_RESTITUTION}$$

#### 7. Update the velocities of both particles:

Finally, we compute the new velocity vectors by combining the normal and tangent components:

$$\mathbf{v}_1 = \mathbf{v}_{1,\text{normal}}^{\text{new}} \cdot \mathbf{n} + v_{1,\text{tangent}} \cdot \mathbf{t}$$

$$\mathbf{v}_2 = \mathbf{v}_{2,\mathrm{normal}}^{\mathrm{new}} \cdot \mathbf{n} + v_{2,\mathrm{tangent}} \cdot \mathbf{t}$$

By following these steps, we ensure that particles respond appropriately to collisions while conserving momentum and energy according to the principles of physics.