INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

Department of Computational Science and Humanities

Indian Institute of Information Technology Kottayam

FIRST MID TERM EXAMINATION- DECEMBER, 2022

COURSE TITLE: IMA 111 (Discrete Mathematics)

Time: 02:30 PM-4:00 PM **Max. Marks: 50**

Answer all questions.

Part A:

Each question carries 2 marks. **5×2=10**

1: The truth value of the proposition: If 8 is a prime number, then $5^2 = 16$.

(a) True
(b) False
(c) Cannot be determined
(d) None of these

2: If p and q are two propositional variables. Then

(a) $p \to q$ is a tautology
(b) $p \to q \equiv \neg q \to p$
(c) $p \wedge q$ is a contingency
(d) $p \to q$ is a contradiction

3: Which one of the following is the logical translation of the statement "None of my friends are perfect"?

(a) $\exists x(F(x) \wedge \neg P(x))$
(b) $\exists x(\neg F(x) \wedge \neg P(x))$
(c)) $\exists x(\neg F(x) \wedge P(x))$
(d) $\neg \exists x(F(x) \wedge P(x))$

Note: $F(x)$: "x is my friend"
$P(x)$: "x is perfect"

4: Modus ponens implies

(a) $(p \wedge (p \to q)) \to q$ is tautology
(b) $(p \vee (p \to q)) \to q$ is tautology
(c) $(p \wedge (p \leftrightarrow q)) \to q$ is tautology
(d) None of the above.

5: If $P(x, y, z): (x + y = z), \forall x, y, z \in \mathbb{R}$ is a propositional function. Then

(a) $\forall x \forall y \exists z P(x, y, z)$ is true.
(b) $\exists z \forall x \forall y P(x, y, z)$ is true
(c) $\forall x \forall y \forall z P(x, y, z)$ is true
(d) None of the above

PTO

Part B:

Each question carries 5 marks. 4×5=20

1: (a) Is $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$? Justify your answer.

(b) Is $(p \rightarrow q) \rightarrow [(p \rightarrow q) \rightarrow q]$ a tautology? Why or why not?

2: Find the negation of $\forall x\big((x > 2) \rightarrow (x^2 < 3)\big)$, where $x \in \mathbb{R}$. Also determine the truth value of the negated statement.

3: If x and y are integers and $x^2 + y^2$ is even, prove that $x + y$ is even.

4: Use basic inference rules to establish the validity of the following argument:

$$\boxed{\begin{array}{c} p \wedge q \\ p \rightarrow \neg(q \wedge r) \\ \underline{s \rightarrow r} \\ \therefore \neg s \end{array}}$$

Part C:

Each question carries 10 marks. 2×10=20

1: Let $C(x, y)$ be "x has chatted with y", where the domain of x and y consist of all students in your class. Use quantifiers to express each of the following statements:

(a) There is a student in your class who has not chatted with anyone in the class.

(b) There are at least two students in your class who have not chatted with the same person in your class.

2: Check the validity of the following argument using rule of inferences:
"Everyone in Kochi lives within 50 miles of the ocean". "Someone in Kochi has never seen the ocean". Therefore, "someone who lives within 50 miles of the ocean has never seen the ocean."

*Hint: You can use the following predicates:

$L(x)$: x lives in Kochi

$O(x)$: x lives within 50 miles of the ocean

$S(x)$: x has seen the ocean