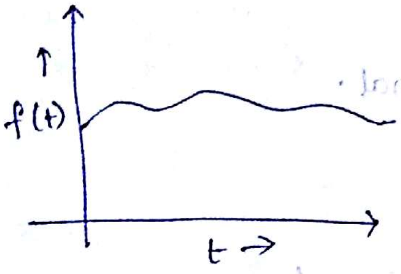
Signal and Systems

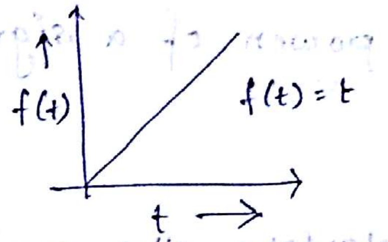
19/7/16

- Signal cannot be any defined function like $\log(1+x)$ or e^{-x} etc.
- signal analysis and signal synthesis :-
- **Signals**: Signal is a func. of one, two or more number of (independent) variables.
  time, space, frequency .....

$f(t) \rightarrow$ independent variable

- a signal can be represented by a waveform, amplitude vs independent variable.

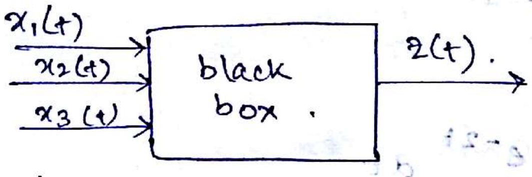




- Information must have randomness

$$f(x,y) = \underbrace{i(x,y)}_{\text{illumination}} \times \overbrace{r(x,y)}^{\text{reflection}}$$

The system is an entity which takes a set of data as input, process it to produce an output signal.



$$z(t) = 5x_1(t) + 10x_2(t) + 15x_3(t).$$

This is the signal where $x_1(t)$, $x_2(t)$, and $x_3(t)$ are the input signal and $z(t)$ the output signal.

- **System**: Mathematical abstraction of a physical process that relates input and output.

(OR) A system is a collection of individual components that do diff jobs but collectively output does a single job.

- **Realisation of signals**:
  - Hardware means
  - Software
  - Hardware + Software

All of them are n dim signals.

Q. How do we measure a signal?

→ Both amplitude and the duration of the signal is considered as a measure of a signal.

$$E_g = \int_{-\alpha}^{\alpha} |g(t)|^2 \, dt \rightarrow \text{correct measure of the energy of a signal as it measures both +ve and -ve area.}$$

Signal Energy.

Then, $P_g = \lim_{T \to \alpha} \left(\frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 \, dt \right)$
(Power)

• Signal energy must be finite for it to be a meaningful measure of the signal size.

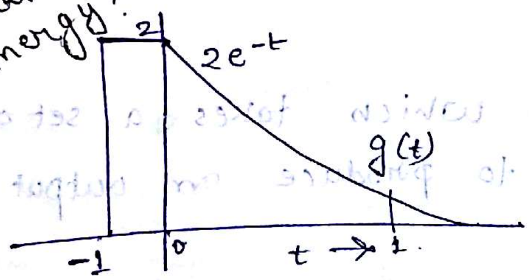
for $g(t) = 2e^{-t}$ $|g(t)| \to 0$ as $t \to \alpha$. Necessary condition: amp → 0 as $t \to \alpha$

∴ P_g is average power of a signal.
→ Signal power

• error signal

Q. When do we calculate the energy of a signal and when do we calculate power?

Q. Calculate its energy?



• Accuracy increases with more no. of points of approximation.

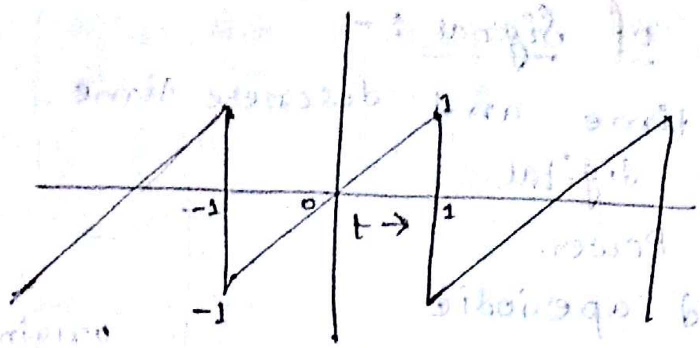
$$E_g = \int_{-1}^{0} (2)^2 \, dt + \int_{0}^{\alpha} 4e^{-2t} \, dt$$

$$= 4 + \left[4 \times \frac{e^{-2t}}{-2} \right]_0^{\alpha}$$

$$= 4 - 2(0 - 1) = 4 + 2 = \underline{\underline{6}}$$

• asymptodic → we get finite value as f^n is converging.
$f(t) \to 0$ when $t \to \alpha$.

Q.



Q. $g(t) = C\cos(\omega_0 t + \theta)$, $\omega_0 = \frac{2\pi}{T_0}$

$$P_g = \lim_{t\to\infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} c^2 \cos^2(\omega_0 t + \theta)\, dt$$

$$= \lim_{t\to\infty} \frac{1}{2T_0} \int_{-T_0/2}^{T_0/2} c^2 [1 + \cos(2\omega_0 t + 2\theta)]\, dt$$

$$= \frac{c^2}{2} \cancel{\lim_{t\to\infty}} \frac{1}{T_0} \left[t + \frac{\sin(2\omega_0 t + 2\theta)}{2\omega_0} \right]_{-T_0/2}^{T_0/2}$$

$$= \frac{c^2}{2} \times \frac{1}{2 \times T_0 \times \frac{2\pi}{T_0}} \times \left[2 \times \frac{2\pi}{T_0} \times t + \sin\left(\frac{4\pi}{T_0} t + 2\theta\right) \right]_{-T_0/2}^{T_0/2}$$

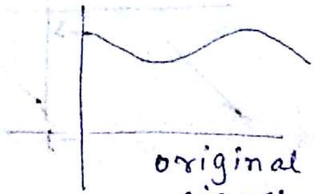
$$= \frac{c^2}{8\pi} \times [2\pi + \sin 2\theta - (-2\pi + \sin 2\theta)]$$

$$= \frac{c^2}{8\pi} [2\pi + \cancel{\sin 2\theta} + 2\pi - \cancel{\sin 2\theta}]$$

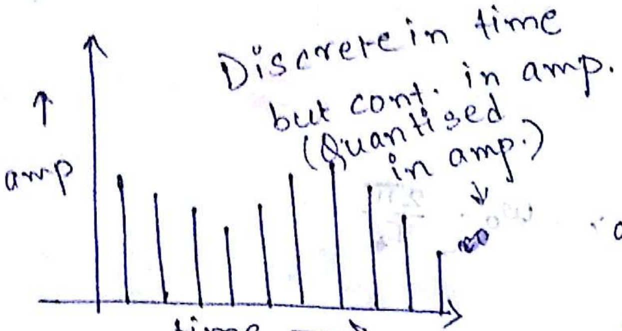
$$= \frac{c^2}{2} (\text{Ans}).$$

Classification of Signal :-

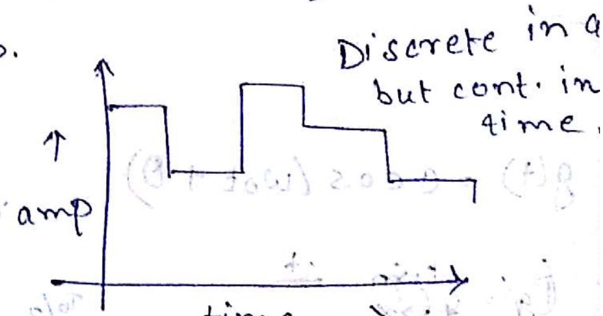
1. Continuous time and discrete time.
2. Analog and digital.
3. Energy and Power.
4. Periodic and aperiodic
5. Deterministic and Random. (or probabilistic).



original signal



(1)



(2)

Analog → cont. in amp (more uncertainty)
Digital → discrete in amp. (easily resolved)

(1) → analog signal with discrete time.
(2) → digital " " cont. time.

26/7

- Signal → scalar, vector

Energy and Power Signal.

- **Energy signal**: A signal having finite energy

Mathematically, $\int_{-\infty}^{\infty} |g(t)|^2 dt < \infty$ (or finite)

has zero power.

- **Power signal**: A signal having finite and non-zero power

$$0 < \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2 dt < \infty.$$

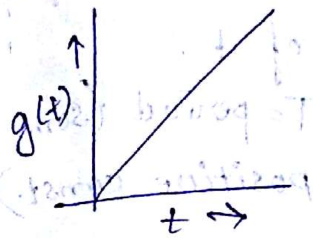
↳ has infinite energy.

$$\boxed{P_g = \lim_{T\to\infty} \frac{1}{T} E_g}$$

For E_g to be finite, P_g will be zero.

$\therefore$ An energy signal has zero power. $\left(P_g = \lim_{t\to\infty} \frac{1}{t} E_g = \frac{1}{\infty} = 0\right)$.

- And ~~an~~ power signal has infinite energy.
- ~~Generally~~ A signal can't be both power and energy signal.
- A power signal must be of infinite duration.



- Ramp signal is neither energy nor power signal.

Neither a power signal nor an energy signal.

$g(t) = t$

- For a signal to be an energy signal, its nature should be asymptodic.

$\because |g(t)| \to 0$ as $t \to \infty$

Analog signal and digital signal:

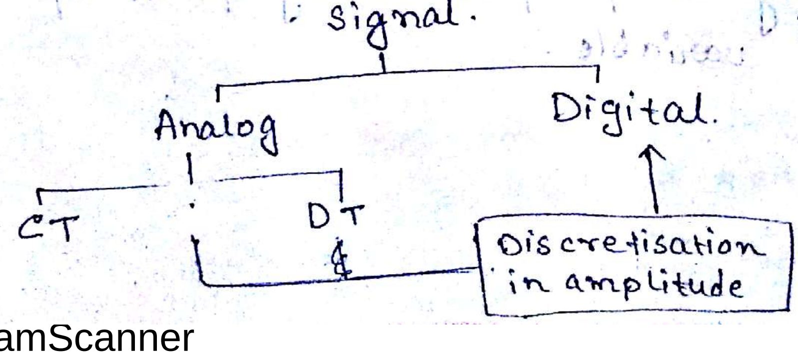
- Analog signal: ~~Both the~~ dependent variable, (amplitude) or the functional value ~~and the independent variable (time) are~~ is continuous in nature.

Analog signal are of two types:

1. Continuous time
2. Discrete time.

Discretisation in the independent variable (time) does not make any diff. ~~in~~ ~~anal~~ It remains analog signal.

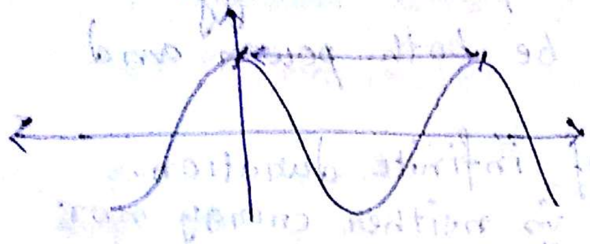
- Discretisation in amplitude creates digital signal.



Digital signal remove uncertainty.

• Digital signal has to have discretisation in amplitude but not necessarily discretisation in time.

Periodic and aperiodic signal :-



$g(t) = 2\cos 50t$

$-\infty < t < \infty$

① $g(t) = g(t+T)$, true for all values of t.
where T = period (some positive const.)

Periodic signal:

$g(t) = g(t+T)$, $-\infty < t < \infty$

then, g(t) is periodic.

period : The smallest duration for which the pattern repeats.

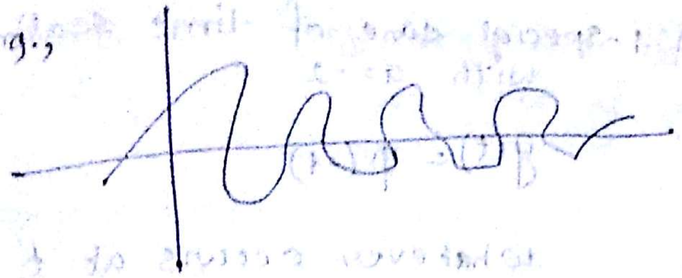
properties:

1. periodic signal starts from $-\infty$ and continues over $+\infty$.
2. A periodic signal must be a power signal but the ~~vic~~ opposite is not true.
3. A periodic signal can be generated from a segment of one time duration i.e., T.
4. Remains unchanged when shifted by one period.

Deterministic and Random signal :

• Deterministic signal is one which can be analytically or graphically be represented without any uncertainty where the value of the signal can precisely be specified at any point of independent variable.

e.g.,



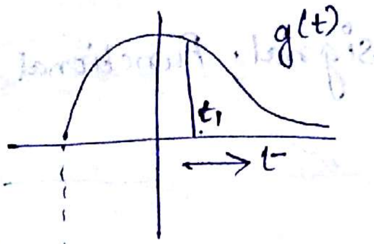
Random signal is one in which you cannot specify the value at any point of independent variable.

e.g., noise signals are random signal (cannot be predicted).

Properties of signal:

1. Time delay → (or Time shifting)

• whatever happens in $g(t)$ at time t_1, the same happens in $\phi(t)$ at time t_1+T.

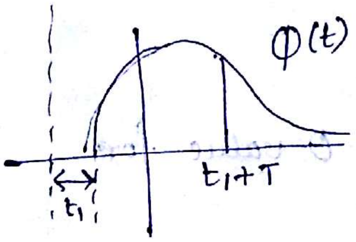


$$g(t_1) = \phi(t_1+T)$$

$$t_1 = t-T$$

$$\phi(t-T+T) = g(t-T)$$

$$\phi(t) = g(t-T)$$



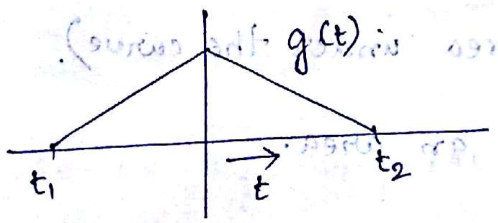
$\phi(t)$ is the delayed version of the signal $g(t)$.

for $T>0$ → (right) delay

$T<0$ → (Left) ~~delay~~ advance.

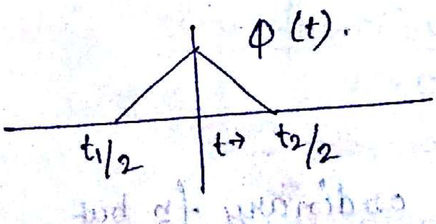
② Time scaling → compression or expansion of a signal.

→ whatever happen at t in $g(t)$, the same happens in $\phi(t)$ at $t/2$.



$$g(t) = \phi(t/2)$$

$$\phi(t) = g(2t)$$



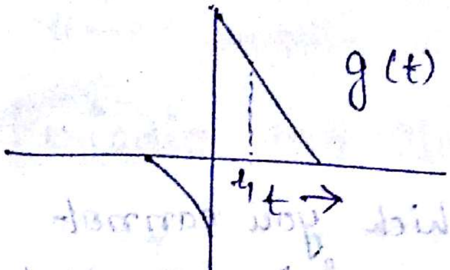
$\phi(t)$ is of higher frequency.

$$\phi(t) = g(t/a)$$

for $a>1$ → time expansion

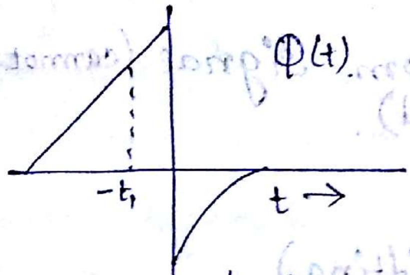
for $a<1$ → time compression/high frequency.

3. Time inversion :- special case of time scaling with $a = -1$.

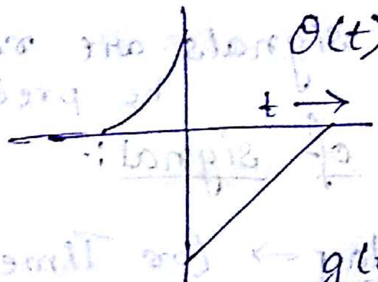


$g(t) = \phi(-t)$

whatever occurs at t in $g(t)$, the same happens in $\phi(t)$ at time $(-t)$.



Rotating about vertical axis



→ Rotating about horizontal axis.

$g(t) = -\theta(t)$.

- $g(t) = -\theta(t)$ is the negation of a signal. Functional value is negated.

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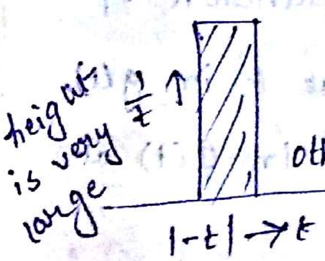
1) Unit impulse function:

$\delta(t) = 0$, $t \neq 0$.

non-zero non-defined value, $t = 0$.

An impulse is a func. which is having 0 value for all values of t except at $t=0$



At $t=0$, height trends to infinite, hence the non-zero value is

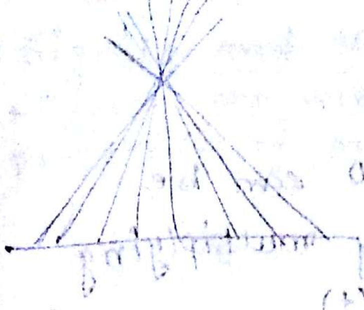
otherwise, $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$ (Area under the curve).

at $t=0$ gives an area.

Every small duration. $\int_{0^-}^{0^+} \delta(t)\,dt = 1$.

infinitesimally small width rectangular curve.

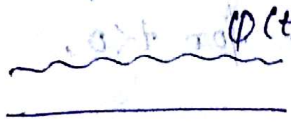
- impulse fn is defined not as an ordinary fn but as a generalised fn.

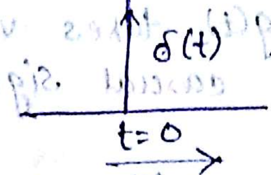


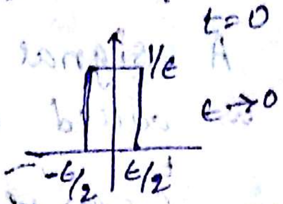
To describe this func, we have to take help of another func.

2) <u>function multiplied by an unit impulse.</u>

$\delta(t)\cdot\phi(t) = \delta(t)\cdot\phi(0)$ (at $t=0$) • $\phi(t)$ is cont. at $t=0$.







$\phi(t)$ should be defined at $t=0$.

$\delta(t-T)\,\phi(t) = \phi(T)\cdot\delta(t-T)$ (meaningful only at t where the impulse exists)

⇒ an impulse located at $t=T$

provided $\phi(t)$ is cont. at $t=T$.

3) <u>Sampling or shifting property of impulse.</u>

$$\int_{-\infty}^{\infty} \delta(t)\,\phi(t)\,dt = \phi(0)\int_{-\infty}^{\infty}\delta(t)\,dt = \phi(0).$$

[provided $\phi(t)$ is cont. at $t=0$].

product of a fn with its impulse

area under the curve.

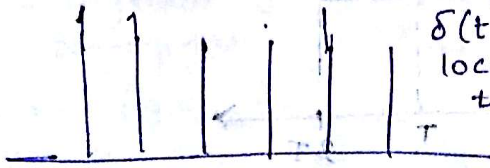
is equal to value of the function at the instant where the unit impulse exists.

Similarly when impulse exists at $t=T$.

$$\int_{-\infty}^{\infty}\delta(t-T)\,\phi(t)\,dt = \phi(T)\int_{-\infty}^{\infty}\delta(t-T)\,dt \Rightarrow \phi(T)$$

• impulse $\delta(t-T)$ is located at $t=T$.

$$\delta(t) = \sum_{n=-\infty}^{\infty}\delta(t-nT)$$



series of impulses multiplied at discrete values.

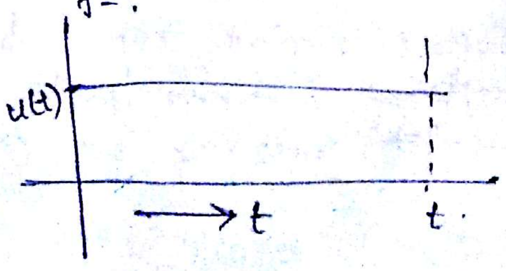
• impulse fn is defined not as an ordinary fn but as a generalized fn.

4) <u>Unit step function.</u>

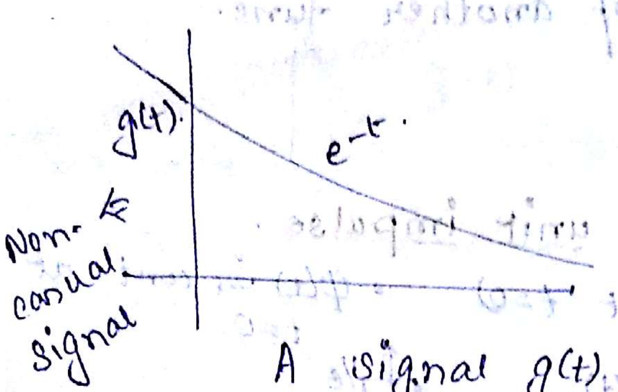
$u(t) = 1$, $t \geq 0$.
$= 0$, otherwise.

⇓

casual signal. is a signal that does not start before $t=0$.



consider $g(t)$ such that:



$g(t)=0$, $t<0$ can be achieved by multiplying it with $u(t)$.

$\therefore$ $g(t)\,e^{-t}$ is a causal signal.

$g(t)\cdot u(t)$ is a causal signal.

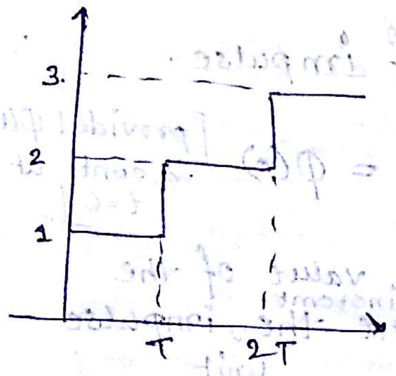
A signal $g(t)$ takes value 0 for $t<0$, is called a causal signal.

$$\int_{-\infty}^{t} \delta(\xi)\, d\xi = u(t) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

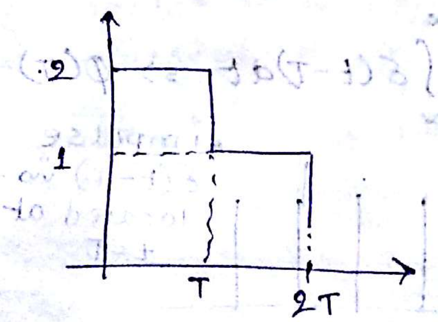
$\Rightarrow$ Relation b/w impulse of step f^n.

$$\delta(t) = \frac{du(t)}{dt}$$

any other variable other than 't'.



$$g(t) = u(t) + u(t-T) + u(t-2T).$$

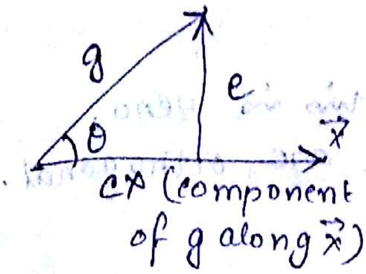


$$g(t) = 2u(t) - u(t-T).$$

Signals and Vectors.

- Signals are vectors.

projection of one vector over another. (g over x).
both real valued.



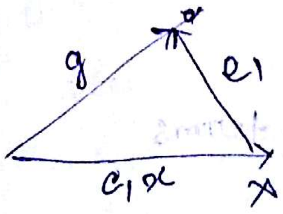
$\vec{g} = \vec{cx} + \vec{e}$

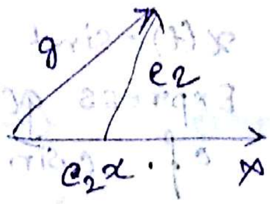
$\vec{g} \simeq \vec{cx}$

$\vec{e} = \vec{g} - \vec{cx}$ $\Rightarrow \vec{g} = \vec{cx} + \vec{e}$

$= \vec{c_1 x} + \vec{e_1}$

$= \vec{c_2 x} + \vec{e_2}$





find c such that e is minimum.

$|g| \cos\theta = c|\vec{x}|$

$|g||x| \cos\theta = c|x|^2$

$c = \dfrac{\vec{g} \cdot \vec{x}}{|x|^2}$

$\{ \vec{x} \cdot \vec{x} = |\vec{x}|^2$

$c = \dfrac{\int_{t_1}^{t_2} g(t)\, x(t)\, dt}{\int_{t_1}^{t_2} x^2(t)\, dt}$

product of $g(t)$ and $x(t)$ are called inner product.

c(projection on x axis) is min when g is $\perp ar$.

Base apx (minimizes loss of info).

$g(t) \rightarrow$ defined over $[t_1, t_2]$.

$g(t) \simeq cx(t)$, $t_1 \leq t \leq t_2$, otherwise.

$g(t) = cx(t) + e(t)$.

E_e = energy of error signal

$= \int_{t_1}^{t_2} e^2(t)\, dt$.

- signals $g(t)$ and $x(t)$ are orthogonal over $[t_1, t_2]$ if $\int_{t_1}^{t_2} g(t)\, x(t)\, dt = 0$

$E_e = \int_{t_1}^{t_2} [g(t) - c(t)]^2 dt$

$= \int_{t_1}^{t_2} g^2(t) dt + \int_{t_1}^{t_2} c^2 x^2(t)\, dt - 2\int_{t_1}^{t_2} g(t) \cdot cx(t)$ (def int)

c is the variable needed for minimization of E_e.
For E_e to be minimum (not w.r.t. t as after integrating t will no longer be present (def. ind).

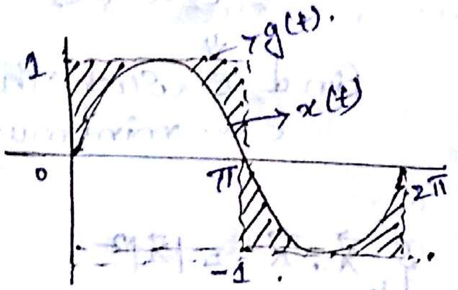
$\dfrac{dE_e}{dc} = 0$.

$$2c\int_{t_1}^{t_2} x^2(t)\,dt = 2\int_{t_1}^{t_2} g(t)\,x(t)\,dt.$$

$$\Rightarrow c = \frac{\int_{t_1}^{t_2} g(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt.}$$

{ when this is zero, g, x are orthogonal.

e.g.)



$x(t) = \sin t$.
Express $g(t)$ in terms of $c\sin t$.
$g(t) \approx c\sin t$.

Shaded portion is the error which has to be minimised.

$$c = \frac{\int_{t_1}^{t_2} g(t)\,x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt} = \frac{\int_0^{2\pi} g(t)\cdot \sin t\,dt}{\int_0^{2\pi} (\sin^2 t)\,dt}$$

$$= \frac{\int_0^{2\pi} g(t)\sin t\,dt}{\pi} \qquad = \frac{1}{2}\int_0^{2\pi}(1-\cos 2t)\,dt.$$

$$\Rightarrow c\pi = \int_0^{2\pi} g(t)\sin t\,dt.$$

$x(t) = \sin t$ & $E_x = \int_0^{2\pi} \sin^2 t\,dt \Rightarrow \left[\frac{1-\cos 2t}{2}\right]_0^{2\pi} = \frac{1}{2}\times 2\pi = \pi$

$$c = \frac{1}{\pi}\int_0^{2\pi} g(t)\sin t\,dt \qquad g(t) = \begin{matrix}1\\-1\end{matrix}.$$

$$= \frac{1}{\pi}\int_0^{\pi}\sin t\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi}(-\sin t)\,dt$$

$$= \frac{1}{\pi}[-\cos t]_0^{\pi} + \frac{1}{\pi}[\cos t]_{\pi}^{2\pi} \Rightarrow \frac{-1}{\pi}(-1-1) + \frac{1}{\pi}(1-(-$$

$$= \frac{2}{\pi} + \frac{2}{\pi} = \frac{4}{\pi} \approx 6.$$

$$g(t) = c\sin t = \frac{4}{\pi}\sin t$$

$g(t)$ is the square signal.

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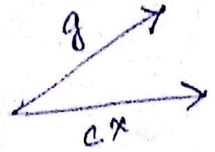


$$c = \frac{g|x|}{x^2} \qquad (g(t)\cdot x(t)). \qquad c = \frac{\int_{t_1}^{t_2} g(t)\cdot x(t)\,dt}{\int_{t_1}^{t_2} x^2(t)\,dt}$$

$c = 0 \Rightarrow gx = 0$

Two signals are orthogonal when → $\int_{t_1}^{t_2} g(t)x(t)\,dt = 0$ (Area under the curve for product signal)

Suppose $g(t)$, $x(t)$ are complex valued signals in time interval $[t_1, t_2]$.

$g(t) \approx cx(t)$.

$e(t) = g(t) - cx(t)$. {error signal}

most often complex in nature.

Signal value will take one real, two imaginary values orthogonal to each other.

$$E_e = \int_{t_1}^{t_2} |g(t) - cx(t)|^2\,dt \quad ; \quad E_x = \int_{t_1}^{t_2} |x(t)|^2\,dt$$

↓

Energy

- Square of the length of vector is analogous to energy of the vector.

$$|u+v|^2 = (u+v)(u^*+v^*)$$
$$= |u|^2 + |v|^2 + u^*v + uv^*$$

$$E_e = \int_{t_1}^{t_2} |g(t)|^2\,dt - \left|\frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)\,x^*(t)\,dt\right|^2 + \left|c\sqrt{E_x} - \frac{1}{\sqrt{E_x}} \int_{t_1}^{t_2} g(t)\,x^*(t)\,dt\right|^2 = 0.$$

1, 2 terms are independent of c

3rd term is so chosen that E_e is min, i.e., 3rd term $= 0$

$$\Rightarrow c = \frac{\int_{t_1}^{t_2} g(t)\,x^*(t)\,dt}{\int_{t_1}^{t_2} |x(t)|^2\,dt} = \frac{\int_{t_1}^{t_2} g(t)\,x^*(t)\,dt}{E_x}.$$

[redacted] two complex value signal will be orthogonal if the area under curve is zero i.e., $c = 0$

if $c=0$ i.e., $\boxed{\int_{t_1}^{t_2} g(t)\, x^*(t)\, dt = 0}$ orthogonal.

and, $\int_{t_1}^{t_2} g^*(t)\, x(t)\, dt = 0$

Energy of the sum of two orthogonal signal.

$$\underbrace{x(t) + g(t)}_{\text{orthogonal signal}} = \underbrace{z(t)}_{\text{composite signal}}$$

E_z : Energy of signal $z(t)$.

$$E_z = \int_{t_1}^{t_2} |z(t)|^2 dt$$

$$= \int_{t_1}^{t_2} |x(t) + g(t)|^2 dt = \int_{t_1}^{t_2} |x(t)|^2 dt + \int_{t_1}^{t_2} |g(t)|^2 dt$$

$$+ \underbrace{\int_{t_1}^{t_2} (x^*(t)\, g(t))dt}_{=0} + \underbrace{\int_{t_1}^{t_2} (x(t)\, g^*(t))\, dt}_{=0}$$

since $g(t)$ and $x(t)$ are complex valued signal orthonal to each other.

$$\therefore E_z = \int_{t_1}^{t_2} |z(t)|^2 dt = \int_{t_1}^{t_2} |(x(t))|^2 dt + \int_{t_1}^{t_2} |g(t)|^2 dt.$$

$$\Rightarrow \boxed{E_z = E_x + E_y}$$

Energy of the sum of two orthogonal signal is the sum of their individual energies.

$\Rightarrow$ Component of one over the other is zero.

$$c = \frac{\vec{g} \cdot \vec{x}}{|x|^2}$$

$$C_n = \cos\theta = \frac{\vec{g} \cdot \vec{x}}{|g||x|}$$

'c' is a measure of similarity.

correlation co-efficient

$$\boxed{-1 \leq C_n \leq 1}$$

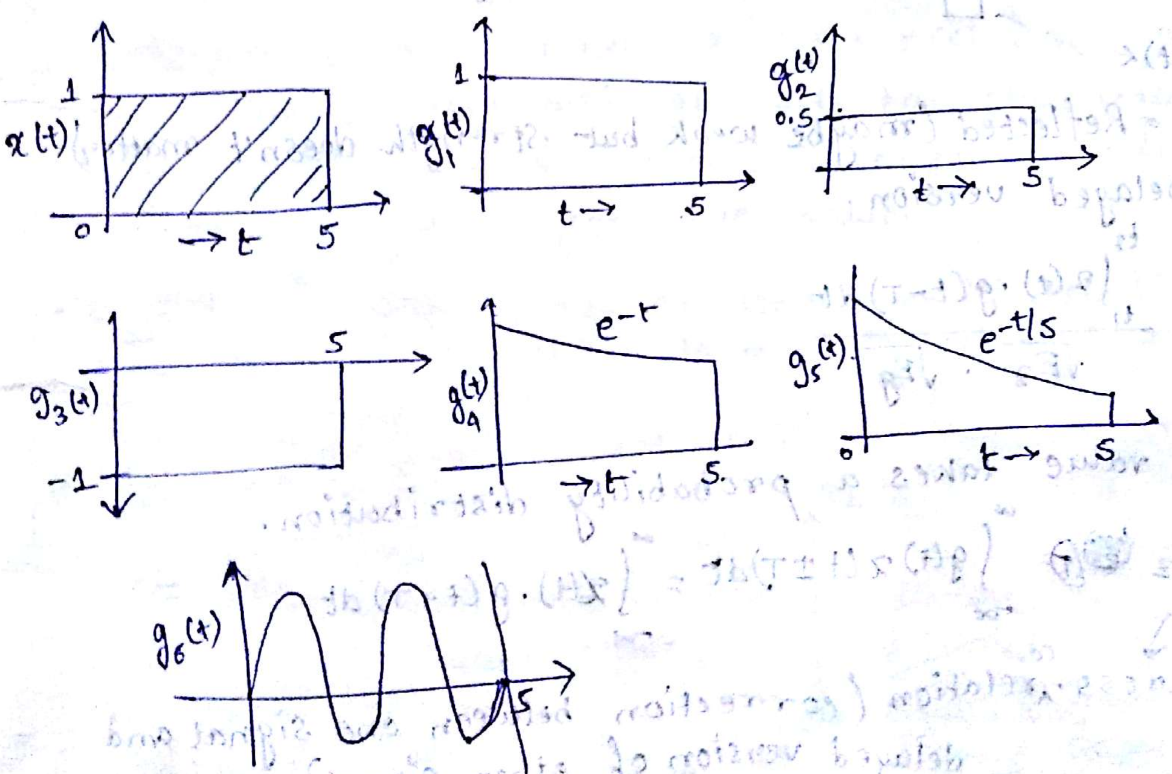
$$C_n = \frac{\int_{t_1}^{t_2} g(t)\, x^*(t)\, dt}{\sqrt{E_g}\, \sqrt{E_x}}$$ /as $E_g = |g(t)|^2$

$$\left\| \sqrt{g(t)\, x(t)} \right\|^2 \qquad -1 \leq C_n \leq 1$$

$$\therefore \int_{t_1}^{t_2} |x(t)\, y(t)|^2\, dt \leq \int_{t_1}^{t_2} |x(t)|^2\, dt \int_{t_1}^{t_2} |y(t)|^2\, dt.$$

- Similarity b/w 2 signals depend on the wave strength.
- 2 signals show max
- changing the length of vectors changes the similarity b/w 2 vectors. But this shouldn't be the case. Similarity should be governed by angle.

☑ Similarity b/w 2 signals:



To calculate similarity, calculate area under the curve or Energy.

i) $C_n = C_{xg_1} = \dfrac{\int_0^5 g(t)\, x(t)\, dt}{\sqrt{E_x} \cdot \sqrt{E_g}} = \dfrac{5}{\sqrt{5} \cdot \sqrt{5}} = 1$ (max similarity).

$\sqrt{E_x} = \sqrt{5} = \sqrt{E_g}$

* Similarity b/w 2 signals depends on their waveforms, not on signal amplitude or strength.

(ii) $C_{xg_2} = \dfrac{\int_0^5 (0.5)(1)}{\sqrt{1.25}\,\sqrt{5}}$

$E_g = 2.5$, $\sqrt{E_g} = \cancel{\sqrt{2.5}}\ \sqrt{1.25}$, $\sqrt{E_x} = \sqrt{5}$.

$\therefore C_{xg_2} = \dfrac{\int_0^5 (0.5)(1)\,dt}{\sqrt{1.25}\ \times \sqrt{5}} = \dfrac{0.5 \times 5}{\sqrt{5} \times \sqrt{1.25}}$

- Two signals show max similarity provided their wave shapes are similar. It does not depend on amplitude or strength.

RADAR

Transmitted Pulse. $C_n = \dfrac{\int_{t_1}^{t_2} z(t) \cdot g(t) \cdot dt}{\sqrt{E_g} \cdot \sqrt{E_z}}$

$z(t)$

$g(t)$

= Reflected (maybe weak but strength doesn't matter) Delayed version.

$$\frac{\int_{t_1}^{t_2} z(t) \cdot g(t-T)\,dt}{\sqrt{E_z} \cdot \sqrt{E_g}}$$

C_n value takes a probability distribution.

$\psi_{gz}(T) = \int_{-\infty}^{\infty} g(t)\, z(t \pm T)\,dt = \int_{-\infty}^{\infty} z(t) \cdot g(t+T)\,dt$

cross-correlation (correction between one signal and delayed version of other signal).

$\psi_{gg}(T) = \int_{-\infty}^{\infty} g(t) \cdot g(t \pm T)\,dt$ → the signal is advanced or delayed by 'T' seconds.

Auto correlation ⇒ corelation b/w signal and (delayed) version of that signal.

4/8/16.

$$C_n = \frac{\int_{-\infty}^{\infty} g^*(t)\, x(t)\, dt}{\sqrt{E_g}\,\sqrt{E_x}}$$

$$\psi_{gx}(\tau) = \int_{-\infty}^{\infty} g(t)\, x(t \pm \tau)\, dt$$

amount of cross-corelation.

$C_1, C_2, C_3, \ldots C_n$ should be mutually orthogonal.

$$\psi_{gg}(\tau) = \int_{-\infty}^{\infty} g(t)\, g(t \pm \tau)\, dt$$

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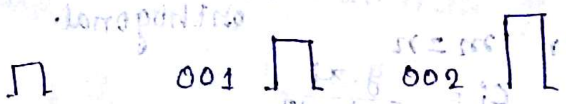
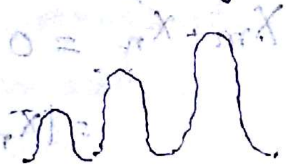




transmitted signal. received signal.

when signal is transmitted through channel it gets contaminated by voice.

- amplitude and width of the received signal remains same with the transmitted signal.
- Detection error → when $x(t) + n(t)$ has $-ve$ value although $x(t)$ has $+ve$ value.

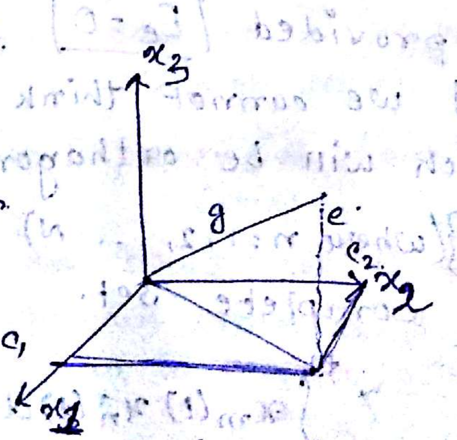
  Bcz of the contamination i.e., huge $-ve$ value of $n(t)$, $x(t) + n(t)$ gets $-ve$ value.

- Signal-corelation plays an imp. role in detection.
- Cont. wave is discretised to minimize error in the far end.

Orthogonal Vector Space

$g = c_1 x_1 + c_2 x_2 + e$

e → error vector



$g \simeq c_1 x_1 + c_2 x_2$

($\because$ e is very small)

↳ Two-dimensional approximation.

$\therefore$ Error $(e) = g - (c_1 x_1 + c_2 x_2)$

- one vector can be represented as a set of orthogonal vector.

For 3-dimensional approximation

$g \simeq c_1 x_1 + c_2 x_2 + c_3 x_3$ (particular values of c_1, c_2, c_3).

'e' will be min if the error vector is $\perp$ar to the plane.

$\{x_1, x_2, x_3\}$ → basis vectors

$\{x_i\}, i=1,2,3$. ($\because x_1, x_2, x_3$ are mutually orthogonal to each other).

$\vec{X_m} \cdot \vec{X_n} = 0$, when $m \neq n$ .. if $\{x_i\}$ is mutually orthogonal.

$= |X_m|^2$, when $m=n$

$c_i = \frac{g \cdot x_i}{|x_i|^2}$

- 3-dimensional vector cannot be represented by a 2-dimensional vector without any error.
- error min. :- Some unique value for which this approximation would be exactly equal.

Orthogonal Signal Space :

$g(t)$ is a signal ~~span~~ over the time span $[t_1, t_2]$

orthogonal basis: $\{x_1(t), x_2(t), \ldots, x_N(t)\}$.

$$g(t) \simeq c_1 x_1(t) + c_2 x_2(t) + \ldots + c_N x_N(t)$$

$$= \sum_{n=1}^{N} c_n x_n(t) \qquad (t_1 \leq t \leq t_2)$$

$$g(t) = \sum_{n=1}^{N} c_n x_n(t) \qquad t_1 \leq t \leq t_2$$

provided $\boxed{E_e = 0}$.

and we cannot think of any $x_{N+1}(t)$ vector which will be orthogonal to all x_N vectors.

$\{x_n(t)\}$(where $n=1,2,\ldots N$) form a basis vector set or complete set.

$$\int_{t_1}^{t_2} x_m(t)\, x_n^*(t)\, dt = 0 \quad \text{for } m \neq n$$

$$= E_n \quad \text{for } m=n.$$

$\{x_i(t)\}$ is called mutually orthogonal if the following relation holds :

where $i = 1, 2, \ldots m, \ldots n, \ldots N$

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when $E_n = 1$ → set $\{x_i(t)\}$ is called orthonormal set or normalized set.

Area under the curve error function is zero but at any instant it may or may not be ~~eq~~ zero.

Parseval's Theorem :-

$z = x + y$ (x, y are orthogonal)

$\Rightarrow |z|^2 = |x+y|^2 = |x|^2 + |y|^2$ [when x and y are orthogonal].

This theorem extends for signal as well.

$$g(t) = \sum_{n=1}^{N} c_n x_n(t).$$

$$E_g = \int_{t_1}^{t_2} \sum_{n=1}^{N} c_n^2 x_n^2(t)\, dt = \int_{t_1}^{t_2} g(t)\, dt$$

total energy of the sum of 'n' orthogonal signals.

$$= \int_{t_1}^{t_2} c_1^2 x_1^2(t)\, dt + \int_{t_1}^{t_2} c_2^2 x_2^2(t) + \ldots + \int_{t_1}^{t_2} c_N^2 x_N^2(t)\, dt.$$

$$\therefore E_g = c_1^2 E x_1 + c_2^2 E x_2 + c_3^2 E x_3 + \ldots + c_N^2 E x_N.$$

- Orthonormal → length of the vector is normalised between 0 to 1 to give rise to an unit vector.

$$\boxed{g(t) = \sum_{n=1}^{N} c_n x_n(t)} \rightarrow \text{Generalised Fourier series.}$$

if x_n is sinosoidal → we call it a trigonometric Fourier series.

if x_n is exponential → Exponential Fourier Series.

- Basis vectors: $\{x_1, x_2, x_3\}$ forms basis set if there are mutually ⊥ar to each other and no other x_4 exists which will be ⊥ar to ~~any~~ all x_1, x_2, x_3.

- signals are vectors in every sense.

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Trigonometric Fourier Series :-

$\cos 0\omega_0 t \rightarrow 0^{th}$ harmonic

$\{1, \cos \omega_0 t, \cos 2\omega_0 t, \ldots, \sin \omega_0 t, \sin 2\omega_0 t, \ldots\}$

$g(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \ldots$

$\cos n\omega_0 t \rightarrow n^{th}$ harmonic.

$\omega_0 = \frac{2\pi}{T_0}$ or $T_0 = \frac{2\pi}{\omega_0}$ - time period.

$\omega_0 \rightarrow$ fundamental frequency (anchor).

Complete and basis orthogonal func.

$\therefore$

$$\int_{T_0} \cos n\omega_0 t \cos m\omega_0 t \, dt = \begin{cases} 0, & m \neq n \\ T_0/2, & m = n \neq 0 \end{cases}$$

Similarly,

$$\int_{T_0} \sin n\omega_0 t \sin m\omega_0 t \, dt = \begin{cases} 0, & m \neq n \\ T_0/2, & m = n \neq 0. \end{cases}$$

$$\int_{T_0} \sin n\omega_0 t \cos m\omega_0 t \, dt = 0 \text{ for all } m \,\&\, n.$$

- anchor = LAN reference pt. $\rightarrow$ fundamental point.

9/8/16

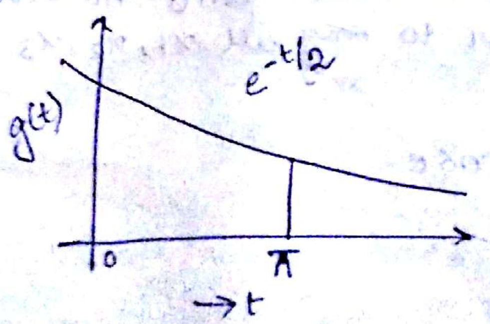
$$g(t) = \sum_{n=1}^{N} C_n x_n(t)$$

$$= \{x_1(t), x_2(t), \ldots, x_N(t)\}.$$

There won't be any $x_{N+1}(t)$ which is orthogonal to the series.

$$g(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \ldots + b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \ldots$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$



consider the func. g(t) from 0 – π.

- Fourier series is a method of breaking a signal into its component in a multidimensional space.

Trigonometric Fourier Series :-

$\cos 0\omega_0 t \rightarrow 0^{th}$ harmonic

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$\omega_0 = \frac{2\pi}{T_0}$ or $T_0 = \frac{2\pi}{\omega_0}$ = time period.

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Complete and basis orthogonal func.

$\therefore$

$$\int_{T_0} \cos n\omega_0 t \cos m\omega_0 t \, dt = \begin{cases} 0, & m \neq n \\ T_0/2, & m = n \neq 0 \end{cases}$$

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$$\int_{T_0} \sin n\omega_0 t \cos m\omega_0 t \, dt = 0 \text{ for all } m \,\&\, n.$$

- anchor = LAN reference pt. $\rightarrow$ fundamental point.

9/8/16

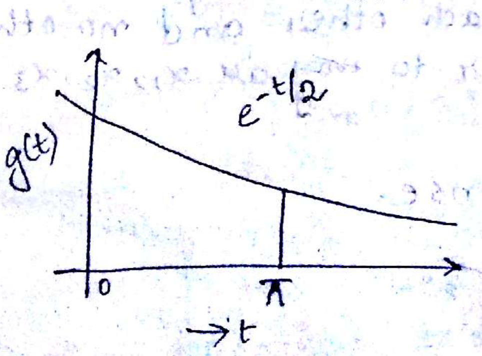
$$g(t) = \sum_{n=1}^{N} C_n x_n(t)$$

$$= \{x_1(t), x_2(t), \ldots, x_N(t)\}$$

There won't be any $x_{N+1}(t)$ which is orthogonal to the series.

$$g(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \ldots + b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + \ldots$$

$$= a_0 + \sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t)$$



consider the func. g(t) from $0 - \pi$.

- Fourier series is a method of breaking a signal into its component in a multidimensional space.

- $a_n \rightarrow$ projection of $g(t)$ on cosine func.
  $b_n \rightarrow$ projection of $g(t)$ on sine func.

$$a_n = \frac{\int_{t_1}^{t_1+T_0} g(t) \cos n\omega_0 t \, dt}{\int_{t_1}^{t_1+T_0} \cos^2 n\omega_0 t \, dt}$$

for $n=0$, $a_n = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} g(t)\, dt$

for $n \neq 0$, $a_n = \frac{\int_{T_0} g(t) \cos n\omega_0 t \, dt}{\int_{T_0} \cos^2 n\omega_0 t \, dt}$

$$\int_{T_0} \cos^2 n\omega_0 t \, dt = \frac{1}{2} \int_{T_0} 2\cos^2 n\omega_0 t \, dt$$

$$= \frac{1}{2} \int_{T_0} (1 + \cos 2n\omega_0 t)\, dt$$

$$= \frac{1}{2} \left[T + \frac{\sin 2n\omega_0 t}{2n\omega_0} \right]_0^{T_0}$$

$$= \frac{1}{2} \left[T_0 + \frac{\sin 4n\pi}{2n\omega_0} - 0 - \frac{\sin 0}{2n\omega_0} \right]$$

$$= T_0/2$$

for $n \neq 0$,

$$\therefore a_n = \frac{2}{T_0} \int_{T_0} g(t) \cos n\omega_0 t \, dt$$

$\therefore a_0 =$ area under the curve.

Similarly, $b_n = \frac{\int_{T_0} g(t) \sin n\omega_0 t \, dt}{\int_{T_0} \sin^2 n\omega_0 t \, dt}$

$$b_n = \frac{2}{T_0} \int_{T_0} g(t) \sin n\omega_0 t \, dt \quad (n \neq 0)$$

$$\boxed{C_n = \sqrt{a_n^2 + b_n^2}}$$

$a_n = r\cos\theta_n$

$$\boxed{\theta_n = \tan^{-1}(-b_n/a_n)}$$

$b_n = -r\sin\theta_n$

$C_0 = a_0$

$t_1 \leq t \leq t_1 + T$

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n) \rightarrow \text{Compact form of T.F.S.}$$

$\therefore C_n = \sqrt{a_n^2 + b_n^2} = r$ and $\boxed{C_0 = a_0}$

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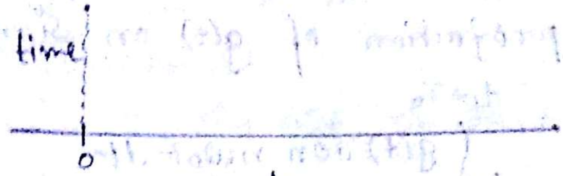
↳ dual representation: C_0/a_0

1. original domain i.e., time

2. freq. domain as in



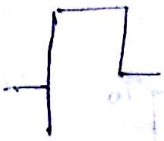
func. of amp vs frequency or phase vs freq.

↓ amplitude spectrum

↳ line spectrum → as it gives non-zero values for integer values only.
⇒ Discrete spectrum.

- Frequency plot is called spectrum. A time f^n can be associated with frequency.

- jump discontinuity:



jump discontinuity (changes value from 0 to 1).

$$\frac{g(t_{0-}) + g(t_{0+})}{2}$$

when there is a jump discontinuity, f^n couldn't converge to fourier series expansion.

- $\psi(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$ ~~expand~~

$\psi(t) = \psi(t + T_0)$ [∵ $\psi(t)$ is periodic with period T_0]

$$\therefore \psi(t+T_0) = C_0 + \sum_{n=1}^{\infty} C_n \cos[n\omega_0 (t+T_0) + \theta_n]$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos[(n\omega_0 t + 2\pi n) + \theta_n]$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$= \psi(t).$$

∴ $\psi(t)$ is periodic.

- Fourier series expansion of a func. $g(t)$ is periodic func. with period T_0.

- If $g(t)$ is periodic, its fourier series expansion will be equal at any point of 't'. or its func. value and fourier series value will be equal.

- For a signal with a jump discontinuity, its value and Fourier Series value do not converge.

Dirichlet's condition :-

1. For the existence of Fourier Series expansion of g(t), the coefficient's a_0, a_n and b_n must be finite.
2. g(t) func. must be absolutely integrable within the duration of T_0.

$$\int_{T_0} |g(t)|\, dt < \infty$$

→ weak Dirichlet's condition.

→ It does not ensure that the func. converge to the Fourier Series.

3. The func. must have finite no. of finite discontinuity and finite no. of maxima and minima within the duration of T_0.

→ Strong Dirichlet's condition (2+3)

$g_{T_0}(t)$

→ 1. $g_{T_0}(t)$ must be single valued within the interval of T_0.

2. $g_{T_0}(t)$ must have finite no. of maxima and minima within T_0.

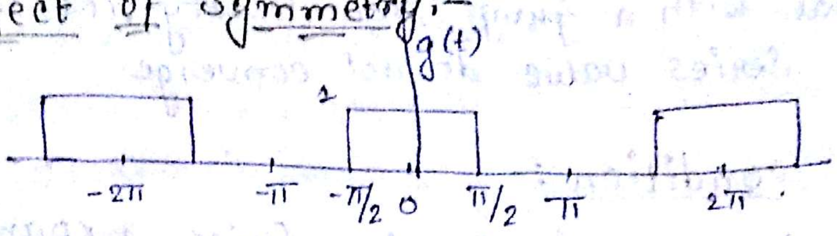
3. $g_{T_0}(t)$ must have finite no. of finite discontinuities within T_0.

4. $g_{T_0}(t)$ must be absolutely integrable within T_0.

- We can readily plot C_n vs ω (amplitude spectrum) and θ_n vs ω (phase spectrum). These two plots together are the frequency spectra of g(t).
- A signal therefore has a dual identity: the time-domain identity φ(t) and the frequency-domain identity (spectra).

Effect of Symmetry :-

11/2/1



even f^n
b^n term ~~varia~~ vanishes

Find the Compact trigonometric Fourier series

(period) $T_0 = 2\pi$. $\therefore$ $T_0/2 = \pi$ $\Rightarrow$ $\pi/2 = T_0/4$

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_0 = \frac{1}{T_0}\int_{T_0} g(t)\,dt = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} g(t)\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} dt = 1/2$$

$$a_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} g(t)\cos n\omega_0 t\,dt \qquad [\because g(t) = 1]$$

$$= \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} \cos n\omega_0 t\,dt = \frac{2}{T_0}\times\frac{1}{n\omega_0}\left[\sin n\omega_0 t\right]_{-T_0/4}^{T_0/4}$$

$$= \frac{2}{T_0}\times\frac{1}{n\omega_0}\left[\sin n\omega_0 T_0/4 + \sin n\omega_0 T_0/4\right]$$

$\omega_0 T_0 = 2\pi$.

$$\therefore a_n = \frac{2}{n\times 2\pi}\times\left[\sin\frac{2n\pi}{4} + \sin\frac{2n\pi}{4}\right].$$

$$\Rightarrow \frac{2}{n\pi}\left[\sin\frac{n\pi}{2}\right]$$

$$a_n = \begin{cases} 0, & n \text{ even.} \\ \dfrac{2}{n\pi}, & n = 1, 5, 9, \ldots \\ \dfrac{-2}{n\pi}, & n = 3, 7, \ldots \end{cases}$$

$$b_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} \sin n\omega_0 t\,dt$$

$$= \frac{-2}{T_0} \times \frac{1}{n\omega_0} \times \left[\cos n\omega_0 t\right]_{-T_0/4}^{T_0/4}$$

$$= \frac{-2}{2n\pi}\left[\cos \cancel{n}\omega_0 T_0/4 - \cos\cancel{n}\omega_0 T_0/4\right]$$

$$= 0.$$

$\therefore$ All sine terms vanishes. $\therefore \theta_n = 0 = \tan^{-1}(b_n/a_n)$

$$\therefore g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$\therefore g(t) = \frac{1}{2} + 2/\pi \cos \omega_0 t - \frac{2}{3\pi}\cos 3\omega_0 t + \frac{2}{5\pi}\cos 5\omega_0 t$$

$$- \cdots$$

The spectrum is valid for all positive 'n' values.

$e^{jn\omega_0 t} \longrightarrow$ complete set of orthogonal f$\underline{n}$s.

$$\int_{T_0} e^{jn\omega_0 t}\, e^{(jm\omega_0 t)^*}\, dt = 0$$

$$= \int_0^{T_0} e^{j(n-m)\omega_0 t}\, dt. = \begin{cases} 0, & m \neq n \\ T_0, & m = n. \end{cases}$$

when $(n \neq m)$ $\Rightarrow \frac{1}{j(n-m)\omega_0}\left[e^{j(n-m)\omega_0 t}\right]_0^{T_0}$

$$= \frac{1}{j(n-m)\omega_0}\left[e^{j(n-m)2\pi} - 1\right]$$

Now, $e^{i\theta} = \cos\theta + i\sin\theta.$

$$\therefore g(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$= D_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} D_n e^{jn\omega_0 t}$$

$$g(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$= C_0 + \frac{1}{2}\sum_{n=1}^{\infty} C_n\left[e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}\right].$$

$$= C_0 + \frac{1}{2}\sum_{n=1}^{\infty}\left[(C_n e^{j\theta_n})\, e^{jn\omega_0 t} + (C_n e^{-j\theta_n})\, e^{-jn\omega_0 t}\right]$$

$$= C_0 + \frac{1}{2}\sum_{n=-\alpha}^{\alpha} D_n e^{jn\omega_0 t} + D_{-n} e^{-jn\omega_0 t}$$

$$= D_0 + \sum_{n=-\alpha}^{\alpha} D_n e^{jn\omega_0 t}$$

where, $D_n = C_n e^{j\theta_n}$

$D_{-n} = C_{-n} e^{-j\theta_n}$

$$g(t) = a_0 + \sum_{n=1}^{\alpha} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$= a_0 + \sum_{n=1}^{\alpha}\left[\left\{ a_n\left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right) + b_n\left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2}\right)\right]\right.$$

$$= a_0 + \sum_{n=1}^{\alpha}\left(\frac{a_n}{2} - j\frac{b_n}{2}\right)e^{jn\omega_0 t} + \sum_{n=1}^{\alpha}\left(\frac{a_n + jb_n}{2}\right)e^{-jn\omega_0 t}$$

$$= D_0 + \sum_{n=1}^{\alpha} D_n e^{jn\omega_0 t}$$

$$g(t) = \sum_{n=-\alpha}^{\alpha} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0}\int_{T_0} g(t) e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\int_{T_0} g(t)\cos n\omega_0 t\, dt. \; - j \times \frac{1}{T_0}\int_{T_0} g(t)\sin n\omega_0 t\, dt.$$

$$= a - jb.$$

$\therefore |D_n| = \sqrt{a^2+b^2}$ $\qquad \angle D_n = \tan^{-1}(-b/a)$

Now, $D_{-n} = a + jb$

$|D_{-n}| = \sqrt{a^2+b^2}$ $\qquad \angle D_{-n} = \tan^{-1}(b/a)$.

$\therefore \angle D_n = -\angle D_{-n}$

- Exponential fourier series $\rightarrow$ n varies from $-\alpha$ to α and coefficients are complex.

Since D_n is complex about spectrum we have,
i) real part vs n and (magnitude spectrum)
ii) imaginary part vs n. (phase spectrum).

even f^n
on $|D_n| = |-D_n|$

odd f^n
on $\angle D_n = -\angle D_{-n}$

$$g(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$= C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n).$$

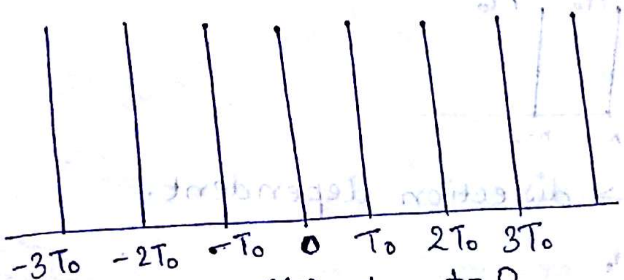
$$= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$D_n = \frac{1}{T_0} \int g(t) e^{-jn\omega_0 t} dt.$$

T.F.S : $n \in [1, \infty)$

E.F.S : $n \in (-\infty, \infty)$

Series of impulses or train of impulse.



$\delta(t) = 1, \; t = 0$
$= 0$, elsewhere.

$$g_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

$$a_0 = \frac{1}{T_0} \int_{T_0} g_{T_0}(t)\, dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t)\, dt = \frac{1}{T_0}$$

(as δ exists only at $T = 0$, So take int over t_{0-} to t_{0+})

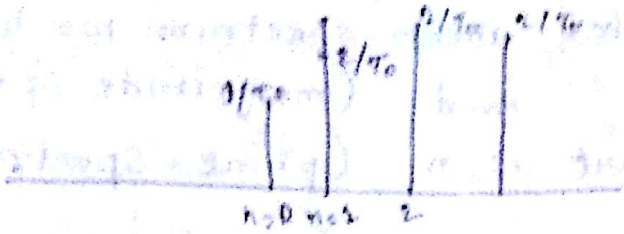
$$a_n = \frac{2}{T_0} \int_{T_0} g_{T_0}(t) \cos n\omega_0 t\, dt$$

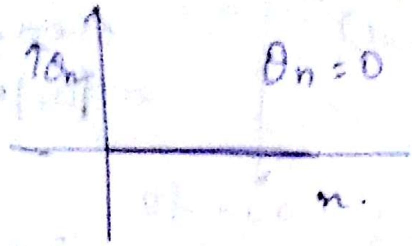
$$= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cos n\omega_0 t\, dt$$

(putting $t=0$). taking value of $f(t)$ at $t=0$.

$$= \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t)\, dt = 2/T_0$$

$\therefore a_0 = \frac{1}{T_0}$, $a_n = 2/T_0$, $b_n = 0$, $\theta_n = 0$
phase spectrum = constant.

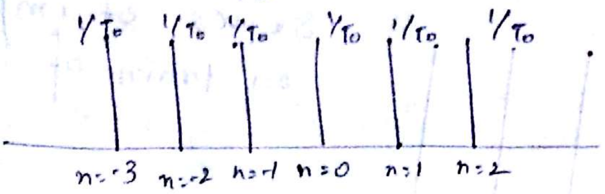




$$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

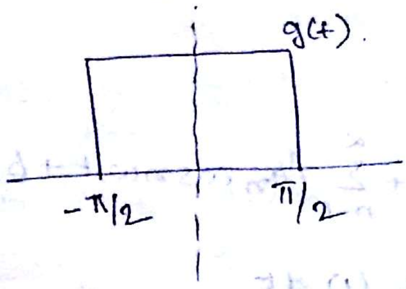
$$D_n = \frac{1}{T_0} \int_{T_0} \delta_{T_0}(t) e^{jn\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) e^{jn\omega_0 t} dt = 1/T_0$$



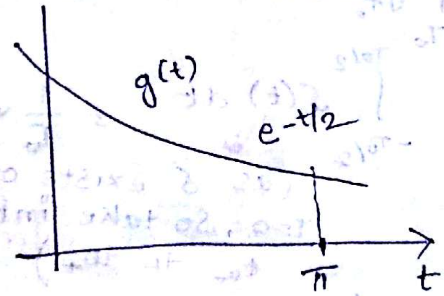
negative freq. → direction dependent.

e.g.)



Express as exponential fourier series.

e.g.)



Express as E.F.S. $0 \le t \le \pi$.

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$a_0 = 1/\pi \int_0^{\pi} g(t)\, dt = 1/\pi \int_0^{\pi} e^{-t/2}\, dt$$

$$= \frac{1}{\pi}\left[\frac{2e^{-t/2}}{-1}\right]_0^{\pi} = \frac{-2}{\pi}(e^{-\pi/2} - e^0)$$

$$\boxed{a_0 = -2/\pi \, (e^{-\pi/2} - 1)}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\pi} = 2$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} \underbrace{(e^{-t/2})}_{\text{II}} \underbrace{\cos 2nt}_{\text{I}} \, dt$$

$=$

Parseval Power Theorem :-

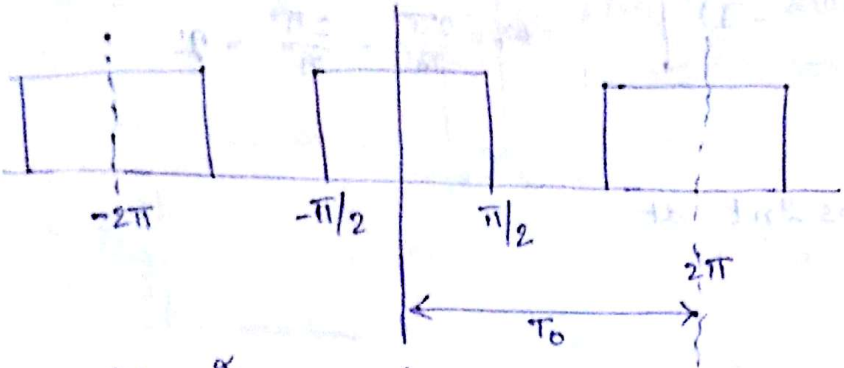
$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$P_g = a_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} |a_n|^2 + \frac{1}{2} \sum_{n=1}^{\infty} |b_n|^2$$

$$= \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t} \Rightarrow D_0 + \sum_{n=-\infty}^{\infty} D_n e^{jn\omega_0 t}$$

$$P_g = D_0^2 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} |D_n|.$$

16/8/16



$$g(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega_0 t}$$

where $C_n = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} g(t) \cdot e^{-jn\omega_0 t}$ [not $-\pi/2$ to $\pi/2$ as its θ, but indirect variable t]

$T_0 = 2\pi$

$T_0/4 = \pi/2$

$$\Rightarrow C_n = \frac{1}{T_0}\int_{-\pi/2}^{\pi/2} (1) \cdot e^{-jn\omega_0 t} \, dt$$

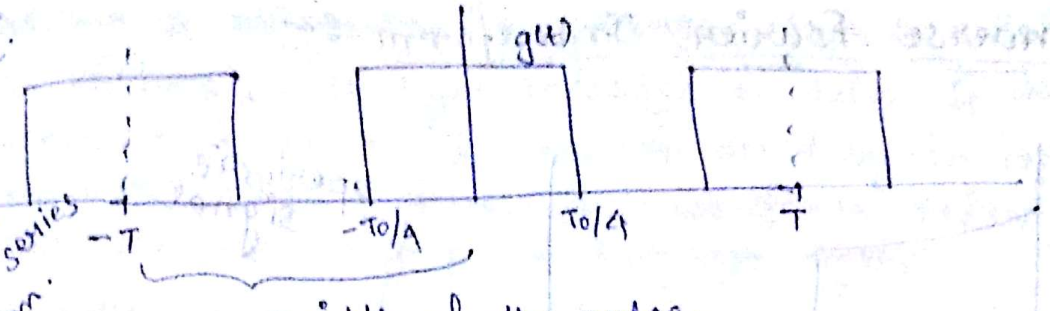
$$= \frac{1}{T_0}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-T_0/4}^{T_0/4}$$

$$= \frac{-1}{nj\omega_0 T_0}\left[e^{-jn\omega_0 T_0/4} - e^{+jn\omega_0 T_0/4}\right]$$

$$= \frac{-2j\sin \omega_0 T_0/4}{-jn \cdot \omega_0 T_0}$$

$$= \frac{2}{2n\pi} = \frac{1}{n\pi}$$

periodic pulse.
↓
Express in the fourier series form.



T = period → repeats itself after T.
→ Here width is taken as diff parameters.

$$g(t) = \sum_{n=-\alpha}^{\alpha} C_n e^{jn\omega t} \quad \text{(Exponential Fourier Series).}$$

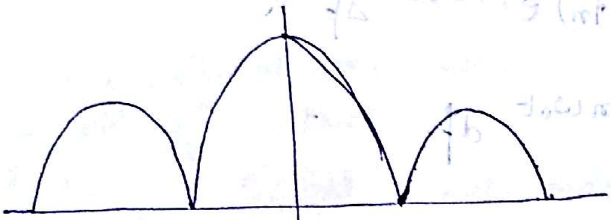
$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t) \cdot e^{-jn\omega t}\, dt$$

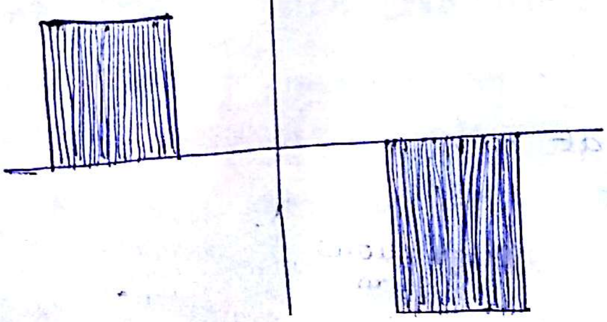
$$= 1/T \cdot \int_{-T_0/4}^{T_0/4} e^{-jn\omega t}\, dt = 1/T \left[\frac{e^{-jn\omega t}}{-jn\omega}\right]_{-T_0/4}^{T_0/4}$$

$$= \frac{\cancel{-j}\, 2\sin \omega n T_0/4}{\cancel{-j} n\, \omega T} = \frac{2 \sin n \times 2\pi/4 \times T_0/T}{n\, 2\pi/\cancel{4}}$$

$$= \frac{\cancel{2} \sin \frac{n\pi}{2} \cdot \frac{T_0}{T}}{\cancel{2}\, n\pi}$$

$T_0/T = 5 =$ freq.
so 5 discrete spectra.
$T_0/T =$ duty cycle.
Reciprocal of duty cycle gives freq.



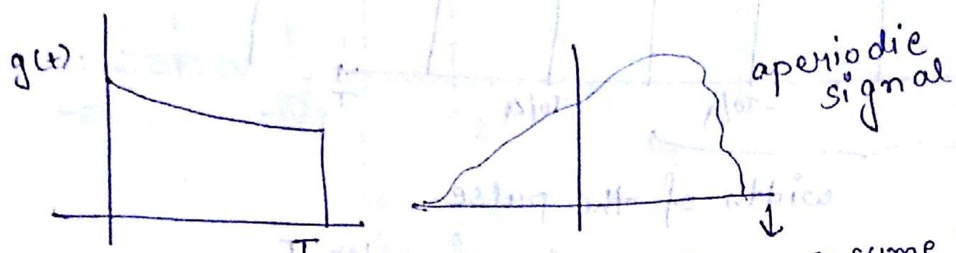


orthogonal as $\phi(t) x_1 = 0$ and $\phi(t) x_2 = 0$.

product of both lastly gives 0.

$$P_g = a_0^2 + \sum_{n=1}^{\alpha} a_n^2 + \sum_{n=1}^{\alpha} b_n^2$$

Inverse Fourier Transform :-



$g_{T_0}(t)$ assume it to be periodic of period ∞.

$$g(t) = \lim_{T_0 \to \infty} g_{T_0}(t)$$

$$= \underset{T_0 \to \infty}{Lt} \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

where $C_n = \frac{1}{T_0} \int_{T_0} g(t) e^{-jn\omega_0 t} dt$.

we define,

$$\Delta f = \frac{1}{T_0}, \; f_n = \frac{n}{T_0} \Rightarrow \frac{a}{T_0} + \frac{b}{T_0} n \Delta f$$

when $T_0 \to \infty$
$f_n = f$.

$$G(f_n) = C_n T_0$$

$$g(t) = \underset{T_0 \to \infty}{Lt} \sum_{n=-\infty}^{\infty} C_n T_0 e^{jn\omega_0 t} \cdot 1/T_0$$

$$= \underset{T_0 \to \infty}{Lt} \sum_{n=-\infty}^{\infty} G(f_n) e^{jn\omega_0 t} \Delta f$$

under this f_n becomes continuous sum $\rightarrow$

$$= \int_{-\infty}^{\infty} g(f) e^{jn\omega_0 t} df.$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t) e^{-jn\omega_0 t} dt$$

$$G(f_n) = \int_{-T_0/2}^{T_0/2} g(t) e^{-jn\omega_0 t} dt.$$

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-jn\omega_0 t} dt$$

Continuous sum.

$$g(t) = \int_{-\infty}^{\infty} G(t) \cdot e^{jn\omega_0 t} dt.$$

- source → It originates a message, such as a television picture, a teletype message or data. If data is non-electrical, it must be converted by an input transducer into an electrical waveform referred to as the baseband signal or message signal
- transmitter → modifies the baseband signal for

- signal → a set of information or data.
- system → is an entity that processes a set of signals (inputs) to yield another set of signals.

cont. time signals → Telephone and video camera outputs.
discrete time signals → monthly sales, stock markets.

- analog signal → A signal whose amplitude can take on any value in a continuous range is an analog signal.
- digital signal → It's amplitude can take on only a finite number of values.

- Every signal generated in lab or observed in real life is energy signal.
- A power signal must necessarily have infinite duration. otherwise its power, which is its avg. energy will not approach a non-zero limit.
- Discretising means restricting variables to specific values only (e.g., integers).
- Continuous and discrete time both can be & analog in nature.
- <u>unit impulse function</u>: The unit impulse can be regarded as a rectangular pulse with a width that has become infitesimally small, a height that has become infitesimally large and an overall area that has been maintained at unity.

* jump discontinuity prop: When there is a jump discontinuity in a periodic signal, its fourier series at the pt. of discontinuity converges to an avg. of the left-hand and right-hand limits of g(t).

18/6/18

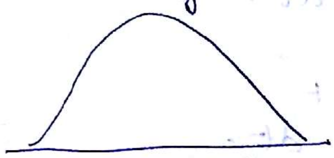
If we have a periodic signal $g_{T_0}(t)$, we get frequency spectrum.

$$g_{T_0}(t) = \sum C_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T_0}$$

where $$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-jn\omega_0 t}$$

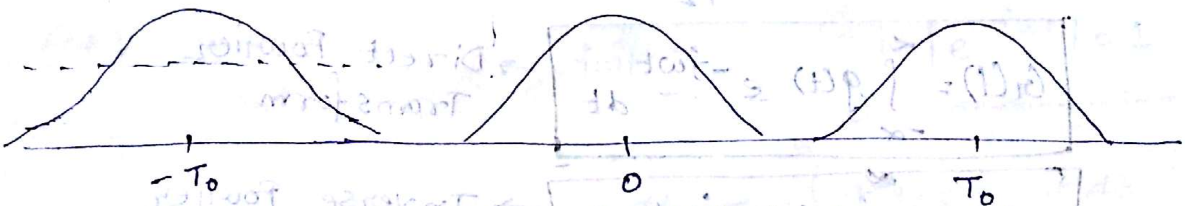
Q. How can we get the frequency information of an aperiodic signal $g(t)$.

$g(t)$ → we need to visualise this as a periodic signal.



↑

How do we get its frequency spectrum?



$$g(t) = \lim_{T_0 \to \infty} g_{T_0}(t)$$

where, $$g(t) = \lim_{T_0 \to \infty} \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-jn\omega_0 t}\, dt.$$

$\Delta f = 1/T_0$

$f_n = \frac{n}{T_0}$

$G(f_n) = C_n T_0$

when $T_0 \to \infty$, $\Delta f \to df$

f_n (discrete freq.) $\to f$ (cont. freq.).

$\Sigma \to \int$

$$g(t) = \lim_{T_0 \to \infty} \sum_{n=-\infty}^{\infty} C_n e^{jn\frac{2\pi}{T_0}t}.$$

$$g(t) = \lim_{T_0 \to \infty} \sum_{n=-\infty}^{\infty} C_n T_0\, e^{jn\frac{2\pi}{T_0}\times t} \times \frac{1}{T_0}$$

$$= \lim_{T_0 \to \infty} \sum_{n=-\infty}^{\infty} G(f_n)\, e^{jn\left(\frac{2\pi}{T_0}\right)t} \times \Delta f$$

under the limiting condit^n, $T_0 \to \infty$

$$g(t) = \int_{-\infty}^{\infty} G(f_n)\, e^{j\times 2\pi \times f_n t} \times \Delta f$$

- Under limiting conditions, f_n (discrete freq.) is equal to f (cont. freq.).

$$\therefore g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{j2\pi ft} \times \Delta f \quad \Delta f = df \text{ (in limiting conditn)}.$$

$$\therefore \boxed{g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{2\pi jft}\, df}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\Rightarrow G(f_n) = C_n T_0 = \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\lim_{T_0 \to \alpha} G(f_n) = \lim_{T_0 \to \alpha} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\boxed{G(f) = \int_{-\alpha}^{\alpha} g(t)\, e^{-j\omega t}\, dt} \rightarrow \text{Direct Fourier Transform}$$

$$\boxed{g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{j\omega t}\, df} \rightarrow \text{Inverse Fourier transform.}$$

- When we take $-\alpha \leq t < \alpha$, we are interested in measuring the global freq.

→ Both of them together is called a <u>Fourier transform pair</u>

$$g(t) \Leftrightarrow G(f).$$

• Under limiting conditions, fn (discrete freq.) is equal to f (cont. freq.).

$$\therefore g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{j2\pi f t}$$

$\Delta f = df$ (in limiting conditn).

$$\therefore \boxed{g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{2\pi j f t}\, df}$$

$$C_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\Rightarrow G(f_n) = C_n T_0 = \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\lim_{T_0 \to \alpha} G(f_n) = \lim_{T_0 \to \alpha} \int_{-T_0/2}^{T_0/2} g_{T_0}(t)\, e^{-j2\pi(n/T_0)t}\, dt.$$

$$\boxed{G(f) = \int_{-\alpha}^{\alpha} g(t)\, e^{-j\omega t}\, dt}$$ → Direct Fourier Transform

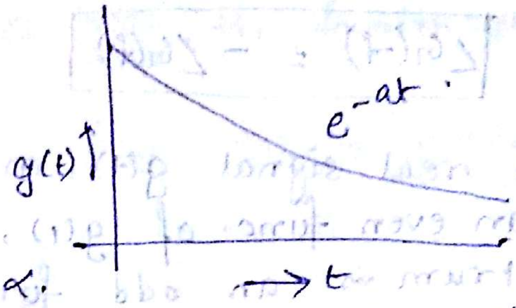
$$\boxed{g(t) = \int_{-\alpha}^{\alpha} G(f)\, e^{j\omega t}\, df}$$ → Inverse Fourier transform.

• When we take $-\alpha \leq t < \alpha$, we are interested in measuring the global freq.

→ Both of them together is called a <u>Fourier transform pair</u>

$$g(t) \Leftrightarrow G(f).$$

Q. $g(t) = e^{-at} u(t)$

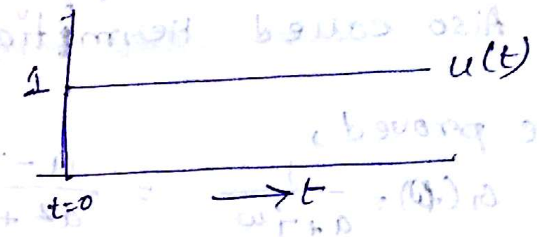


~~$G(t)$~~

- presence of $u(t)$ makes the limit go from 0 to ∞.

$$\therefore G(f) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt.$$

just bcz it is multiplied by $u(t)$, the func. does not start from infinity.



$$= \int_0^{\infty} e^{-at} \cdot e^{-j\omega t} dt.$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt.$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty} = \frac{1}{a+j\omega} \quad \text{provided } a>0.$$

we know $\boxed{|e^{-j\omega t}| = 1}$

(OR)

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} g(t) \cos\omega t \, dt - \int_{-\infty}^{\infty} g(t) \sin\omega t \, dt$$

If $g(t)$ is real value for 't'.

$G(f) = a - jb$

↳ It is complex (It has two real values).

$|G(f)|$ vs f → magnitude spectrum.

$\angle G(f)$ vs f → Phase spectrum.

Now, $G(-f) = a + jb$.

If my $g(t)$ is real, $G(f) = a - jb$ and $G(-f) = a+jb$.

$$\Rightarrow \boxed{|G(-f)| = \sqrt{a^2+b^2} = |G(f)|}$$

$\angle G(f) = \angle \theta_g(f) = \text{phase} = -\tan^{-1}(b/a)$.

$\angle G(-f) = \angle \theta_g(-f) = \tan^{-1}(b/a)$

$$\boxed{\angle G(-f) = -\angle G(f)}$$

- For a real signal $g(t)$, magnitude spectrum is an even func. of $g(t)$, whereas phase spectrum is an odd func.

→ Conjugate Symmetry.

$$G(-f) = G^*(f)$$

Also called Hermitian symmetry.

we proved,

$$G(\omega) = \frac{1}{a+j\omega} = \frac{a-j\omega}{a^2+\omega^2}$$

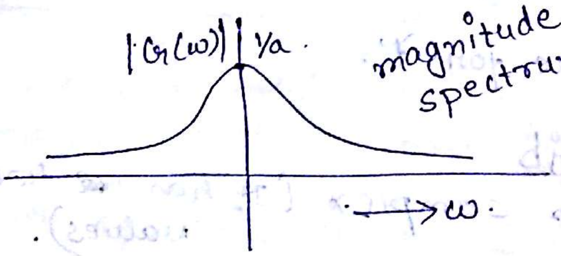
$$= \frac{a}{a^2+\omega^2} - j\frac{\omega}{a^2+\omega^2}$$

$$\therefore |G(\omega)| = \frac{1}{a^2+\omega^2}\sqrt{a^2+\omega^2}$$

$$\Rightarrow |G(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

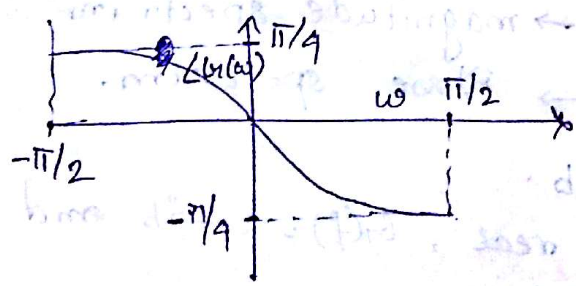
$$\angle G(\omega) = -\tan^{-1}(\omega/a)$$

$$G(f) = \frac{1}{2\pi} G(\omega)$$



magnitude spectrum

odd f^n of freq.



$$\boxed{\int_{-\alpha}^{\alpha} g(t)\,dt < \alpha}$$

For an aperiodic signal g(t), the fourier series expansion of G(t),
depends on whether the func. g(t) is absolutely integrable or not.

$$\int_{-\alpha}^{\alpha} |g(t)|\, dt \leq P < \alpha .$$

↳ some finite value.

- For some func. g(t), if fourier transformation does not exist, Laplace transform may exist.

$L[g(t)]$