Optimizing Flight Patterns

17 February 2022

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Introduction

A freight airline based in Des Moines, Iowa handles the cargo of a Des Moines manufacturer that has additional warehouses in Minneapolis, Duluth, Green Bay, and Chicago. There are eleven possible flights among these five cities, all of which need to be made every day. Additionally, there are four pilots available to navigate these flights. It must be discovered which of the four pilots should be used and the most efficient flight schedules, such that costs are limited while covering all eleven flights.

There are several important stipulations to be considered. Two pilots fly out of Des Moines and two Green Bay. Additionally, all pilots must return to the warehouse at which they began their day. Each pilot is paid $35 per takeoff and $40 per hour of flight. Lastly, each pilot can fly no more than 10 hours per day.

With these considerations, flight optimization was deemed a minimum covering problem. In other words, it is a matter of how to cover all eleven flights most efficiently. Efficiency will be gauged by cost. A process of integer programming was used with the employment of Microsoft Excel's Solver tool. This was done to minimize the cost of the flight schedules while ensuring every flight is made daily. Additionally, this program yielded the number of pilots necessary for these schedules.

Methods

Integer programming was utilized to minimize the costs of these eleven freight flights. Since there were only so many feasible flight patterns for each pilot while all flights must be made, this was deemed to be a minimum covering problem. Since covering is a matter of whether a flight is made, binary variables were used by way of integer programming. The criterion used to decide the flights was cost. The program was thus designed to minimize the costs of the flight patterns navigated by some number of necessary pilots to be determined by Microsoft's Solver tool.

Assumptions

The most important assumption made by this model are the flight patterns. There are eleven possible flights, all of which need to be made daily. These flights among the five cities are graphically represented in Figure 1.

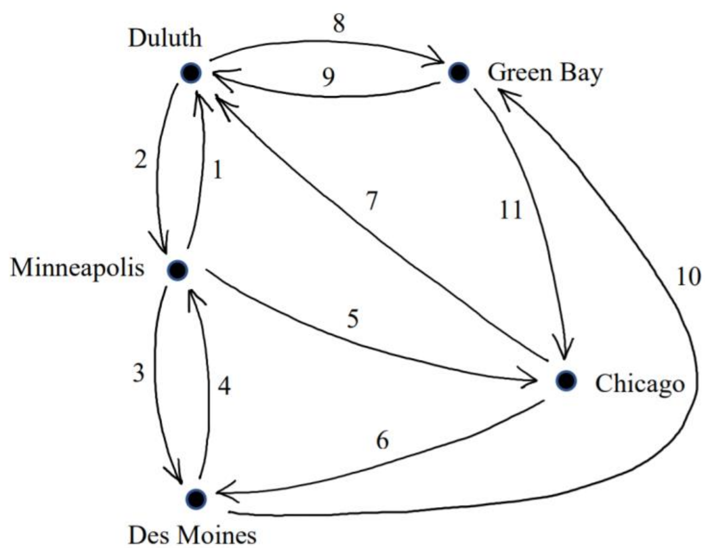

Figure 1

The duration of each flight – including preparation, runway traffic, delays, etc. – are shown in Table 1.

Table 1

Flight Number	1	2	3	4	5	6	7	8	9	10	11
Duration (hours)	1	1	1	1	4	2	3	4	2	5	2

The researchers discovered all possible flight patterns that would not put a pilot above 10 hours of flying while ensuring the pilot finished at his or her starting point. There are 16 total patterns that fit these criteria (labeled A through P in the workbook). Nine are available to Des Moines pilots, seven to Green Bay.

For the Des Moines pilots, there is a range of flight patterns from two to six flights. The shortest pattern was two hours of flying. This would involve two takeoffs and two hours in the air, costing $150. However, this pattern covers only two flights. Similarly, the least number of flights that could be made from Green Bay were two, but this would total six hours of flight, costing $310. On the other end of the extreme, one pattern out of Des Moines makes six flights, totaling 10 hours in the air at $610. This same pattern can be flown by a pilot out of Green Bay. There are a total of three patterns that cover the same flights regardless of the starting point.

In conclusion, there is no way to cover all eleven flights with a single pilot out of each starting point. Moreover, at least three pilots must fly, and the model has 16 flight patterns from which to schedule daily routes.

Parameters

This model has an objective function tasked with minimizing the flight patterns chosen. It is parameterized by the patterns covering specific flights, hours per flight, and the costs of these patterns. Whether the flight pattern is chosen is binary; one if the pattern is chosen, zero if not. The cost associated with each flight pattern is factored into the objective function as a coefficient scaling the binary pattern variable. The detail of each parameter follows.

Patterns and Flights

Whether a pattern covers a flight is a binary variable. Thus, the default for every cell is 0, but if the flight is covered by the pattern, the cell value will be 1. For example, pattern A covers flights 3 and 4 and has two values of 1 corresponding to these two flights. The flights within the coefficient matrix are valued as

$$f_{ij} = \begin{cases} 1, & if\ the\ pattern\ j\ includes\ flight\ i \\ 0, & otherwise \end{cases}$$

with $i = 1, 2, \ldots, 11$ for the flights and $j = A, B, \ldots, P$ for the patterns.

Additionally, the total number of flights and hours flown in every pattern is summated. That is, the sum of all f_i for each respective j. This total is calculated as the sum of its binary values. As such, all sums are integers between 2 and 6, inclusive.

Similarly, the total hours flown is the sum of products. That is, each flight's duration is multiplied by the binary value in each pattern, and these products are summated. Thus, the duration of a given flight is counted if and only if the pattern includes the flight. Therefore, the total hours of each flight pattern, t_j, is calculated with the product of hours h_i and f_{ij} in a pattern. The formulation is

$$t_j = \sum_{i=1}^{11} h_i f_{ij}$$

for each $j = A, B, \ldots, P$ pattern.

A pilot is paid $35 per takeoff and $40 per hour, rendering each flight pattern a discrete cost. This is calculated as a cost C_j with the total flights f_j running some total hours t_j. The formula is

$$C_j = f_j * 35 + t_j * 40$$

for $j = 1,2, \ldots, 16$. All the parameters in the workbook are shown in Figure 2.

Figure 2: Coeffiecient Matrix

Flights	Duration (hours)	Patterns															
		A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	1	0	1	1	0	0	0	0	1	1	0	0	1	1	0	1	0
2	1	0	1	1	0	1	0	1	0	0	0	0	1	1	0	0	0
3	1	1	1	1	0	1	0	1	0	0	0	0	0	1	0	0	0
4	1	1	1	1	1	1	0	0	1	1	0	1	0	1	0	1	0
5	4	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
6	2	0	0	0	1	0	1	0	1	1	0	1	0	0	0	1	1
7	3	0	0	0	0	1	0	0	1	0	0	0	0	0	1	0	0
8	4	0	0	1	0	0	0	0	0	1	1	1	1	1	1	1	0
9	2	0	0	1	0	0	0	1	0	0	1	0	1	1	0	0	0
10	5	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1
11	2	0	0	0	0	0	1	0	0	1	0	1	0	0	1	1	1
	Total Flights	2	4	6	3	5	3	4	4	5	2	4	4	6	3	5	3
	Total Hours	2	4	10	7	10	9	9	7	10	6	9	8	10	9	10	9
	Total Cost	$150	$300	$610	$385	$575	$465	$500	$420	$575	$310	$500	$460	$610	$465	$575	$465
		Des Moines-based Sequences									Green Bay-based Sequences						

Pattern A consists of flights 3 and 4. Thus, the cells f_{31} and f_{41} contain the binary 1 with all other f_{i1} equal to 0. The "Duration" column is how many hours it takes to make each respective flight, and it is conditionally formatted such that shorter flights are green, long flights red, with a spectrum between. Similarly, the "Total Hours" and "Total Cost" rows for each respective flight pattern f_j are conditionally formatted with the same color scheme. Lastly, as the bottom of Figure 2 states, the Des Moines patterns are salmon colored, Green Bay are pale blue.

Decision Variables and Objective Function

Microsoft Excel's Solver tool was employed to choose the optimal patterns for the pilots. The list of patterns $A - P$ is set with binary decision variables, as can be seen in Figure 3. If the program chooses pattern D, for example, the corresponding cell will be valued 1. If a pattern is not chosen, it will remain 0. Coefficients are utilized to scale the selection of patterns. The coefficients are exactly the cost of the flight pattern from Figure 2. Thus, Solver's choices for the objective function will be among values of 1 multiplied by the respective costs of the flight patterns. Therefore, the function will optimize for the lowest cost while fulfilling all eleven flights.

Figure 3

Coefficients	150	300	610	385	575	465	500	420	575	310	500	460	610	465	575	465
Decision Variables	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Des Moines Pilots Needed:								0	Green Bay Pilots Needed:						0

The objective function is

$$z = MIN \sum_{k=A}^{P} d_k$$

for d_k as the corresponding decision variable with $k = A, B, \dots, P$ patterns. For example, if A is selected by Solver, 1 will be entered into the corresponding cell below A in Figure 3. Then, this cell will be multiplied by the corresponding cost of flight pattern A, which will be the input for the objective function. The objective function is simply the sum of all selected products. Thus, the Solver will be tasked to minimize an objective function with the sum of cost inputs, yielding the minimal cost for some patterns that cover all eleven flights.

Constraints

Some of the assumptions above are constraints for this model. However, four constraints were formally entered into the program. The first is that the decision variables must be binary, as was discussed above. Another is in the issue of covering. There are eleven total flights and all of them must be made every day. Thus, Solver must ensure there is at least one coefficient of 1 in each row. In other words, if Solver selects four patterns, they must combine to cover every flight at least once. This part of the unsolved program is visualized in Table 2.

Table 2

Covered	>=	Needed
0	>=	1
0	>=	1
0	>=	1
0	>=	1
0	>=	1

0	>=	1
0	>=	1
0	>=	1
0	>=	1
0	>=	1
0	>=	1

Formally, each cell of Table 2 is the sum of the products of the decision variables and their respective flights. That is, each cell in Table 2 is to add the number of times each flight is flown, and each flight flown is the product of decision variables $A - P$ with the flights $1 - 11$. The formula is

$$F_i = \sum_{k=A}^{P} d_k \, f_{ik}$$

for $i = 1, 2, \ldots, 11$ flights.

The last constraint is on the number of pilots that can be designated to the patterns. There are only four available, two of which begin their flights in Des Moines, and two in Green Bay. Thus, the model must be programmed such that no more than two patterns between A and I may be chosen, and no more than two for J through P. Solver is constrained to make the Des Moines cell in Figure 3 fewer than or equal to 2, and likewise for the Green Bay cell.

Results

The Solver tool found the ideal sequencing to be flight patterns E, L, and P. Respectively, these patterns cost \$575, \$460, and \$465. Thus, given all the constraints noted above, the objective function yields a total of \$1,500 in costs per day using two pilots from Green Bay and one from Des Moines (Figure 4).

Objective Function	z = $1,500															
								Figure 4								
Coefficients	150	300	610	385	575	465	500	420	575	310	500	460	610	465	575	465
Decision Variables	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0
	Des Moines Pilots Needed:								2	Green Bay Pilots Needed:						1

Validation

There are five criteria by which this model can be validated:

1. Eleven flights covered
2. At least three pilots utilized
3. No more than two pilots used in Des Moines or Green Bay, respectively
4. The cost is a reasonable dollar value
5. Pilots land where they began their day.

First, all eleven flights are covered (Table 3).

Table 3

Covered	>=	Needed
1	>=	1
2	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1
1	>=	1

Every flight will be made at least once. Secondly, Figure 4 above shows that three pilots will be necessary to schedule these flights. Because there were limits of takeoff locations and daily flying hours, it was decided prior to the creation of the model that two pilots could not make all eleven flights per day. Figure 4 also shows that no more than two pilots fly out of Des Moines and Green Bay, satisfying the third criterion.

Fourth, the minimal cost of all eleven flights, all things considered, is $\$1500$. This can be validated by manually calculating the cost of each pattern chosen, with $\$35$ per takeoff and $\$40$ per hour. Pattern E is the order of flights $4-5-7-2-3$. That is,

- three 1-hour flights
- one 3-hour flight
- one 4-hour flight

Thus, the cost should be five takeoffs plus ten hours of flight, which is $\$575$. Similarly, pattern L and P are flights $9-2-1-8$ and $11-6-10$, respectively. That is,

- two 1-hour flights
- one 2-hour flight
- one 4-hour flight

for L and,

- two 2-hour flights
- one 5-hour flight

for P. Thus, L and P require four takeoffs plus eight hours of flight and three takeoffs plus nine hours of flight, respectively. The costs of L and P are $\$460$ and $\$465$, respectively. The sum of these three flight patterns is

$$\$575+\$460+\$465=\$1500$$

which corresponds to the value obtained by the programs objective function. Lastly, each pilot must land in the same location at which he or she began the day.

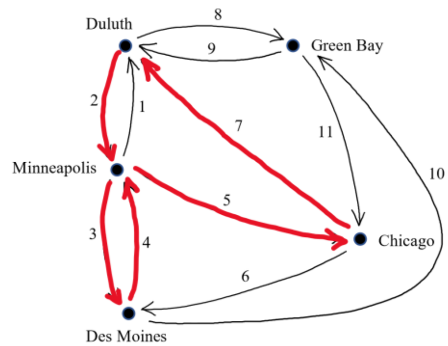

Figure 5

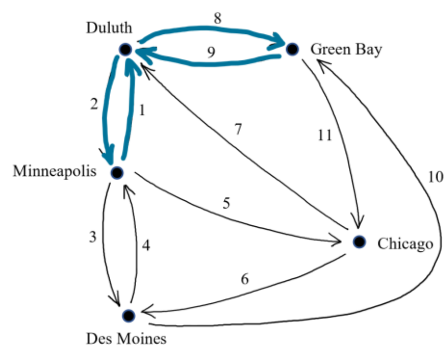

Figure 6

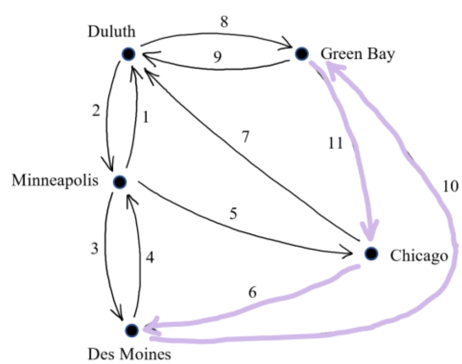

Figure 7

Figure 5 shows the pilot flying pattern E returning beginning and returning to Des Moines. Figure 6 shows the pilot flying pattern L beginning and returning to Green Bay; figure 7 shows likewise for the other Green Bay pilot. Thus, with all five criteria satisfied, it is reasonable to conclude that this model is valid.

Conclusion

The objective of this project was to discover the most cost-effective flight patterns given several important stipulations. Namely, there are eleven flights that must be made daily, four pilots starting in specific locations, airtime limitations, landing requirements, and certain costs per takeoff and hour of flight. This is a minimum covering problem and integer programming was utilized to solve it.

There are three patterns found to optimize time and resources. They are E, L, and P. Which flight scheduling this will require will be discussed in Recommendations below. These flight patterns will total $\$1500$ and only one of the flights will be made twice; namely, flight 2 from Duluth to Minneapolis. These three patterns most efficiently cover all eleven flights that must be made daily. Moreover, these patterns minimize the airline's costs.

Recommendations

The model suggests two Green Bay pilots should fly each day. The two ideal patterns for these pilots are $9-2-1-8$ and $11-6-10$. Scheduling the flights will follow these patterns:

1. Green Bay to Duluth
2. Duluth to Minneapolis
3. Minneapolis to Duluth
4. Duluth to Green Bay

and

1. Green Bay to Chicago
2. Chicago to Des Moines
3. Des Moines to Green Bay.

These two flight patterns make for the most efficient and cost-effective schedules for the Green Bay pilots. The first requires eight hours of airtime, the second, nine. As for Des Moines, the recommended flight pattern is $4-5-7-2-3$. That would correspond to the following route:

1. Des Moines to Minneapolis
2. Minneapolis to Chicago
3. Chicago to Duluth
4. Duluth to Minneapolis
5. Minneapolis to Des Moines.

This uses all ten available hours of flight for this pilot. It also utilizes only one of two available pilots. Perhaps the Des Moines pilots could alternate days flying this route. At most, one Des Moines pilot would work three days in a five-day period; the other would work two days. Between the two pilots, this would leave five days per week to fulfill other contracts.

In summary, this program found three ideal flight patterns, two of which must be flown out of Green Bay, the other out of Des Moines. This schedule would cost $\$1500$ daily, which is the minimal cost of all possible flight patterns making the eleven necessary flights.