A False Theorem in Mathematics

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Abstract

In this paper, we present a false theorem that appears to hold true at first glance. We provide a brief proof of the theorem, demonstrating its apparent validity. However, upon closer examination, we reveal the subtle mistake in the proof that invalidates the theorem. This exercise serves to remind us of the importance of rigorous proof techniques and the potential pitfalls in mathematical reasoning.

1 Introduction

Mathematics is a field that relies on precise definitions and rigorous proofs. Occasionally, however, seemingly plausible proofs may be flawed due to subtle errors in reasoning. In this paper, we present a false theorem and provide a brief proof that appears to support the theorem's validity. The goal of this exercise is to encourage readers to critically evaluate mathematical proofs and recognize potential mistakes.

2 A False Theorem

We now present the false theorem:

Theorem 1. *For all real numbers a and b, if $a < b$, then $a^2 < b^2$.*

Proof. Assume $a < b$. Multiplying both sides of the inequality by a, we have $a^2 < ab$. Similarly, multiplying both sides of the original inequality by b, we obtain $ab < b^2$. Combining these inequalities, we arrive at the conclusion that $a^2 < ab < b^2$, which implies that $a^2 < b^2$. □

3 Revealing the Error

While the above proof appears to be valid, there is a subtle mistake that invalidates the theorem. The error occurs in the first step of the proof, where we assume that multiplying both sides of the inequality $a < b$ by a maintains the inequality. This assumption holds true when a is positive, but when a is negative, the inequality reverses.

As an example, consider $a = -2$ and $b = -1$. We have $a < b$, but $a^2 = 4$ and $b^2 = 1$. Thus, $a^2 > b^2$, contradicting the theorem.

4 Conclusion

The false theorem presented in this paper serves as a reminder of the importance of careful and rigorous proof techniques in mathematics. It demonstrates how subtle errors in reasoning can lead to incorrect conclusions. By examining and critically evaluating proofs, we can better understand the underlying mathematics and avoid potential pitfalls in our own work.