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The DCS Theorem

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Abstract. We present a probabilistic proof of the *DCS Triangle* [1][2]. We use the triangle to show decentralized consensus systems cannot cannot scale to support the transactional demands of centralized consensus systems, and therefore any *single system* can have *Decentralization*, *Consensus*, or *Scale*, but not all three properties simultaneously.

Definitions

A *system* is defined as any set of components (see Scope) following *precise rules* in order to provide service(s) to the users of the system. These services constitute the system's *intended behavior*.

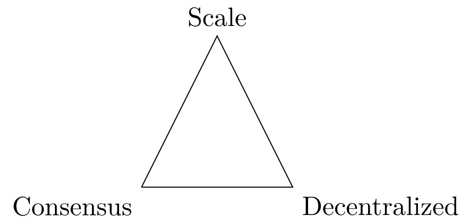
In other words, a system S consists of a set of components, called its *scope* $\{S\}$, and a program ("state transition function", f_S), that together define the system's *intended behavior*, which means: upon receipt of message m, S uses f_S to update the internal state from s to s' and send back reply y within a time interval S_τ.

$$S(t) = \begin{cases} \{S\} & = \{component_1, component_2, \cdots\} \\ f_S(m, s) & = \{s', y\} \end{cases}$$

We note, additionally:

- The *scope* $\{S\}$ may change over time, but there are always several components of a consistent *type* (i.e. "all systems always have at least one *CPU*, one *developer*, and one *user*")
- The system's state s includes all data necessary for the system to compute f_S given a message m
- S is considered *compromised* if it fails to perform its intended behavior within the interval S_τ

We will proceed to prove that any single such system may possess, at most, two of three properties:



- **Consensus** means the system's state, s is a *shared state* that is updated by nodes running a *consensus algorithm* over a network, and that furthermore, the output of *consensus algorithm* determines the network's accepted output of f_S, and whether or not f_S completes within S_τ.
- **Scale** means the system is capable of handling the transactional demands of any competing system providing the same service to the same arbitrary set of users across the globe (*"at scale"*).[1]
- **Decentralized** means the system has no *single point of failure or control* (SPoF). Another way to state this is: the system continues to perform its intended behavior if any single element is removed from $\{S\}$, and no single component in $\{S\}$ has the power to redefine f_S on its own.

Systems whose intended behavior can be modified without the consent of their users are considered *centralized* due to the presence of a central point of control over the definition of the system.

Decentralization Scope & Relativity of Decentralization

Implicit in our definition of a *decentralized system* is the idea that the system is not compromised. A non-functioning system does not fulfill its intended behavior, and therefore, by our definition, is not decentralized.

Imagine a decentralized system S, whose intended behavior (its purpose) is to maintain the integrity of a database while being responsive to queries. It does so by attempting to eliminate all single points of failure within a given *scope.*

Definition. The *scope* of a system refers to all subcomponents and all entities "reasonably relevant" to a system's functioning.

If we consider the scope of our "decentralized" database to be a computer with two CPUs and two hard disks (one primary, another backup), then we can say S is "decentralized" at $t = 0$ (has no single point of failure). However, if at $t = 1$ one of the hard disk fails, it is no longer decentralized since now there does exist a single component capable of compromising the entire system.

[1]Examples of "services" include: streaming video, sending messages, maintaining balances on a ledger, etc.

This means:

- Whether or not a system is decentralized can change over time.
- Any system can be called "decentralized" if we define the scope narrowly enough.
- All decentralized systems can be called "centralized" if we define their scope broadly enough.[2]

The narrowing and enlarging of the scope is called the *relativity of decentralization*, and it is why first agreeing on a reasonable definition for a system's scope is vital before deciding whether or not it is "decentralized".

Computational throughput of consensus systems

The *computational throughput* $T(S)$ of any consensus system depends on three factors:

1. The computational power of each *consensus participant* (those able to select transactions which are *written* to the shared state).[3]
2. The amount of time the consensus algorithm considers messages to be lost (the *timeout* period).
3. The consensus *threshold* the consensus algorithm uses to decide whether consensus has been reached (e.g. "how big of a quorum is required").

Note that if the computational power of a consensus participant is significantly less than that of the other participants, they are more likely to be excluded from the deciding quorum for several reasons:

- If there are no network partitions to determine otherwise, fast consensus participants will process messages more quickly and therefore will be first to create a quorum.
- If there are enough fast consensus participants to create a large enough quorum to exceed the system's consensus threshold, then there is no need to wait for the slow participants to move the system forward.
- Slow consensus participants are more likely than fast consensus participants to hit the system's timeout period for processing and responding to messages, and therefore are more at risk of being excluded from the consensus process entirely.

Therefore, $T(S)$ is a function that is *limited by the slowest consensus participants not excluded in the deciding quorum.*

[2] The entire Internet could be considered centralized if we include the entire solar system as part of the scope. The "single point of failure" could be the Earth itself, its atmosphere, the Sun, etc. Or, perhaps in the not distant future, a single ISP.

[3] Note that participants who are only allowed to *verify* state, but not participate in *creating* it, are not considered consensus *participants*. Instead, they are called state *validators*.

Coordination costs

Relevant for our proof is the notion of *coordination costs*, or the difficulty for one entity to engage another and work toward a common goal.

For example, when Bitcoin was first launched, it would be difficult for any miner to find enough collaborating miners to create a cartel with 51%+ of the hash power, simply because there were many "relevant miners" (consensus participants) distributed all over the world.

Today, however, there are significantly fewer consensus participants in Bitcoin, and it is much easier to (1) identify them, and (2) bring them together in a single room to coordinate around some goal. Therefore, we say the coordination costs are lower today than before.

We can approximate the coordination costs $C(S)$ of any consensus system as simply the number of consensus participants:

$$C(S) = \|\texttt{consensus\_participants}(\{S\})\|$$

Proof

We seek to prove the following restatement of the DCS Triangle:

Theorem 1. *Decentralized consensus systems that scale to meet the demands of competing (and functionally equivalent) centralized consensus systems, become centralized.*

Given these axioms:

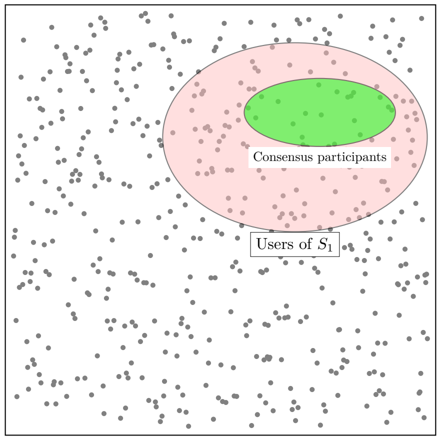
Axiom 1. *In any sufficiently large population (at scale), individual access to computational power is not distributed uniformly. Most individuals have access to average computational power, and a few have access to large amounts.*

Axiom 2. *In any two systems offering the same service to the same large population, the transactional demands of the average user converge at scale.*

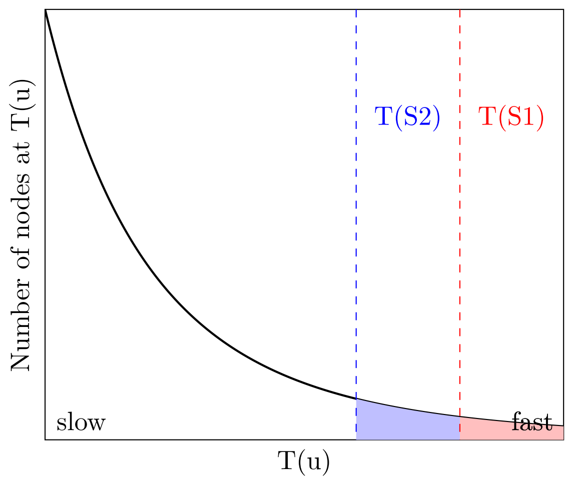
Lemma 1. *Let S_1 and S_2 be functioning consensus systems offering the same service to the same population of users at scale. Let N be a function returning the number of active users on the system. If $N(S_1) > N(S_2)$, then $T(S_1) > T(S_2)$ converges to a true statement as the value $N(S_1) - N(S_2)$ increases.*

Proof. This follows directly from (Axiom 2). □

Lemma 2. *The number of consensus participants decreases at scale, and therefore coordination costs decrease for systems at scale.*



Universe of users



Proof. This follows from (Axiom 1), (Lemma 2), and our definition of $C(S)$. *[TBD. details.]* □

Lemma 3. *"Increasing decentralization" of a consensus system at scale means making it slower and less capable of scale, therefore increase scale makes a system less capable of decentralization.*

Follows from slow participants concept.

Lemma 4. *The probability that $\{S\}$ contains a cartel capable of colluding to censor transactions approaches 1 at scale.*

Proof of the Main Theorem. foobar. □

[This is where the paper currently ends. What follows below are "brain dumps" of random thoughts about how to go about proving the theorem. I expect the entire paper to be no more than 5 pages long.]

References

[1] T. McConaghy, "The DCS Triangle," 10-Jul-2016. [Online]. Available: https://medium.com/the-bigchaindb-blog/the-dcs-triangle-5ce0e9e0f1dc.

[2] G. Slepak, "Slepak's Triangle," *Rebooting Web-of-Trust*, 17-Oct-2016. [Online]. Available: https://github.com/WebOfTrustInfo/rebooting-the-web-of-trust-fall2016/blob/master/topics-and-advance-readings/Slepaks-Triangle.pdf.