Paul Burgwardt

#0304700

CS344 – Professor Tamon

Homework 1

Merge Sort Analysis

The plan is to divide the array into two halves, and recursively sort them. Then merge the two halves, to make a sorted whole.

$$T(N) = \begin{cases} 0, & if\ N = 1 \\ 2T\left(\frac{N}{2}\right) + N, & otherwise \end{cases}$$

We can apply the Master Theorem to determine the asymptotic cost:

$a = 2$

$b = 2$

$c = 1$

$Since\ \log_b a = c \ldots \log_2 2 = 1 \ldots \boldsymbol{\Theta}(NlogN)$

Insertion Sort Analysis

The plan is to scan the sequence from left to right, and insert the next element in the correct position, so we maintain a sorted array.

$$T(N)\ is\ the\ arithmetic\ series\ \sum_{k=1}^{n-1} k$$

So the total asymptotic cost will be:

$$T(N) = \frac{N(N-1)}{2} \ldots \boldsymbol{\Theta}(\boldsymbol{N^2})$$

Bubble Sort Analysis

The number of comparisons between elements and the number of swaps can be stated as follows:

$$(N-1) + (N-2) + \cdots + 2 + 1 = \frac{N(N-1)}{2} = \boldsymbol{\Theta}\ (N^2)$$

Selection Sort Analysis

Similar to bubble sort, the number of comparisons between elements and the number of swaps can be stated as follows:

$$(N-1)+(N-2)+\cdots+\ 2+1=\ \frac{N(N-1)}{2}=\boldsymbol{\Theta}\ (N^2)$$

Quick Sort Analysis

MEDIAN:

The best case of quicksort occurs when the pivot point that we choose is such that it divides the array into two exactly equal parts, in every step. So we get the recurrence:

$$2T\left(\frac{N}{2}\right)+N$$

Using the Master Theorem like we did previously, we come to the conclusion:

$$\boldsymbol{\Theta}(NlogN)$$

RANDOMIZED:

In this scenario, the pivot is randomly chosen. When we pick the pivot, we perform (N – 1) comparisons in order to split the array. Now, depending on the pivot, we might split the array into a smaller one of size zero, and a greater of size (N - 1) or into a smaller one of size one, and a greater one of size (N – 2), etc. We can write a recurrence for this idea as:

$$T(N)=(N-1)+\tfrac{1}{N}\textstyle\sum_{i=0}^{N-1}(T(i)+T(N-i-1))$$

So we can use the Substitution Method:

$$T(N)\le(N-1)+\ \frac{2}{N}\sum_{i=1}^{N-1}(ci\times lni)$$

$$T(N)\le(N-1)+\ \frac{2}{N}(\left(\frac{c}{2}\right)N^2lnn-\frac{cN^2}{4}+\ \frac{c}{4}$$

$$T(N)\le cnln(n)$$

$\Theta(N log N)$

Plot

Ranking:

Merge Sort < *Quick Sort (Median)* < *Quick Sort (Random)* < *Selection Sort* < *Insertion Sort* < *Bubble Sort*