

# Template

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1A

1.1.vimrc

```
set nu ai ci si mouse=a ts=2 sts=2 sw=2
nmap<F2> : vs %<.in <CR>
nmap<F3> : !gedit % <CR>
nmap<F8> : !time ./%< %<.in <CR>
nmap<F9> : :w <CR> :!g++ % -o %< -O2 -g -std=c++11 -Wall <CR>

nmap<F5> : !.%< <CR>
nmap<F10> : :w <CR> :make %< <CR>
```

1.2 FastIO

```
// read untill EOF (xint)
struct FastIO {
    static const int S = 1310720;
    int wpos;
    char wbuf[S];
    bool ed;
    FastIO() : wpos(0), ed(0) { }
    inline int xchar() {
        static char buf[S];
        static int len = 0, pos = 0;
        if (pos == len) pos = 0, len = fread(buf, 1, S, stdin);
        if (pos == len) return -1;
        return buf[pos++];
    }
    inline int xint() {
        int c = xchar(), x = 0, s = 1;
        while (c <= 32) {
            if (!c) return ed = 1;
            c = xchar();
        }
        if (c == '-') s = -1, c = xchar();
        for (; '0' <= c && c <= '9'; c = xchar()) x = x * 10 + c - '0';
        return x * s;
    }
    inline int xuint() {
        int c = xchar(), x = 0;
        while (c <= 32) c = xchar();
        for (; '0' <= c && c <= '9'; c = xchar()) x = x * 10 + c - '0';
        return x;
    }
    inline void xstring(char *s) {
        int c = xchar();
        while (c <= 32) c = xchar();
        for (; c > 32; c = xchar()) *s++ = c;
        *s = 0;
    }
    inline void wchar(int x) {
        if (wpos == S) fwrite(wbuf, 1, S, stdout), wpos = 0;
        wbuf[wpos++] = x;
    }
}
```

```
}
inline void wint(int x) {
    if (x < 0) wchar('-', x = -x);
    char s[24];
    int n = 0;
    while (x || !n) s[n++] = '0' + x % 10, x /= 10;
    while (n--) wchar(s[n]);
}
inline void wstring(const char *s) {
    while (*s) wchar(*s++);
}
~FastIO() {
    if (wpos) fwrite(wbuf, 1, wpos, stdout), wpos = 0;
}
} io;
```

1.3 Head

```
#include<bits/stdc++.h>
using namespace std;
#define fi first
#define se second
#define mp make_pair
#define pb push_back
#define rep(i, a, b) for(int i=(a); i<(b); i++)
#define per(i, a, b) for(int i=(b)-1; i>=(a); i--)
#define sz(a) (int)a.size()
#define de(a) cout << #a << " = " << a << endl
#define dd(a) cout << #a << " = " << a << " "
#define all(a) a.begin(), a.end()
#define pw(x) (1ll<<(x))
#define endl "\n"
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
typedef double db;

int main() {
    std::ios::sync_with_stdio(false);
    std::cin.tie(0);
    // cout << setiosflags(ios::fixed);
    // cout << setprecision(3);
    return 0;
}
```

1.4 duipai

```
#!/bin/bash
while true; do
    ./gen > gen.in
    ./sol <gen.in >sol.out
    ./dp <gen.in >dp.out
    if diff sol.out dp.out; then
        printf "AC\n"
```

```
else
    printf "wa\n"
    exit 0
fi
done
// sh duipai.sh
```

## 2 DP

### 2.1 DigDP

```
// fill f -1
ll f[]; // 自顶向下限制
ll dfs(int pos, ..., bool lim){
    if(pos == -1) return ?; // ...
    if(!lim && ~f[...]) return f[...];
    ll res = 0;
    int up = lim ? dig[pos] : 9; // ...
    rep(i, 0, up+1) {
        if(...) res += dfs(pos-1, ..., lim & (i == up));
    }
    if(!lim) f[] = res;
    return res;
}

ll solve(ll x){
    int pos = 0;
    while(x) dig[pos++] = x % 10, x /= 10;
    return dfs(pos-1, ..., 1);
}
```

## 3 DataStructure

### 3.1 2DST

```
namespace ST_2D{
    const int N = 1030;
    int LOG[N], P[20], dep1, dep2;
    short st[11][11][N][N];
    void build(int n, int m, short a[][N]){
        rep(i, 0, 11) P[i] = 1<<i;
        rep(i, 2, 1025) LOG[i] = LOG[i>>1]+1;
        for(dep1 = 0; (1<<dep1) < n; dep1++);
        for(dep2 = 0; (1<<dep2) < m; dep2++);
        rep(i, 1, n+1)
            rep(j, 1, m+1)
                st[0][0][i][j] = a[i][j]; // modi
        rep(i, 1, n+1)
            rep(j, 1, dep2+1)
                rep(k, P[j], m+1)
                    st[0][j][i][k] = max(st[0][j-1][i][k], st[0][j-1][i][k-P[j-1]]);
        rep(i, 1, dep1+1)
```

```
rep(j, P[i], n+1)
    rep(k, 0, dep2+1) //attention to range of k
        rep(l, P[k], m+1)
            st[i][k][j][l] = max(st[i-1][k][j-P[i-1]][l], st[i-1][k][j][l]);
    }
    int qry(int x1, int y1, int x2, int y2){
        int l1 = LOG[x2-x1+1], l2 = LOG[y2-y1+1];
        int res1 = max(st[l1][l2][x1+P[l1]-1][y1+P[l2]-1], st[l1][l2][x2][y2]);
        int res2 = max(st[l1][l2][x1+P[l1]-1][y2], st[l1][l2][x2][y1+P[l2]-1]);
        return max(res1, res2);
    }
}
```

### 3.2 2DSegTree

```
// 区域覆盖、标记永久化、标记单调
const int N=1010;
int n,m,q;
struct seg {
    int ma[N<<2], la[N<<2];
    void upd(int L,int R,int c,int l=0,int r=m,int rt=1) {
        ma[rt]=max(ma[rt], c);
        if(L<=l&&r<=R) {
            la[rt]=max(la[rt], c);
            return ;
        }
        int mid=l+r>>1;
        if(L<=mid) upd(L, R, c, l, mid, rt<<1);
        if(R>=mid+1) upd(L, R, c, mid+1, r, rt<<1|1);
    }
    int qry(int L,int R,int l=0,int r=m,int rt=1) {
        int ans=0;
        ans=max(ans, la[rt]);
        if(L<=l&&r<=R) {
            ans=max(ans, ma[rt]);
            return ans;
        }
        int mid=l+r>>1;
        if(L<=mid) ans=max(ans, qry(L, R, l, mid, rt<<1));
        if(R>=mid+1) ans=max(ans, qry(L, R, mid+1, r, rt<<1|1));
        return ans;
    }
};

struct Seg {
    seg ma[N<<2], la[N<<2];
    void upd(int x1,int x2,int y1,int y2,int c,int l=0,int r=n,int rt=1) {
        ma[rt].upd(y1, y2, c);
        if(x1<=l&&r<=x2) {
            la[rt].upd(y1, y2, c);
            return ;
        }
        int mid=l+r>>1;
        if(x1<=mid) upd(x1, x2, y1, y2, c, l, mid, rt<<1);
        if(x2>=mid+1) upd(x1, x2, y1, y2, c, mid+1, r, rt<<1|1);
    }
}
```

```

int qry(int x1,int x2,int y1,int y2,int l=0,int r=n,int rt=1) {
    int ans=0;
    ans=max(ans, la[rt].qry(y1, y2));
    if(x1<=l&&r<=x2) {
        ans=max(ans, ma[rt].qry(y1, y2));
        return ans;
    }
    int mid=l+r>>1;
    if(x1<=mid) ans=max(ans, qry(x1, x2, y1, y2, l, mid, rt<<1));
    if(x2>=mid+1) ans=max(ans, qry(x1, x2, y1, y2, mid+1, r, rt<<1|1));
    return ans;
}

}T;
int main() {
    scanf("%d%d%d", &n, &m, &q);
    while(q--) {
        int d, s, h, x, y; scanf("%d%d%d%d", &d, &s, &h, &x, &y);
        int t=T.qry(x, x+d-1, y, y+s-1);
        T.upd(x, x+d-1, y, y+s-1, h+t);
    }
    printf("%d\n", T.qry(0, n, 0, m));
    return 0;
}

```

### 3.3 CDQ

```

const int N = 200005;
int p1, p2, pos, n, k, nn, ans[N];
struct node {
    int x, y, z, num, ans;
    bool operator == (const node & b) const {
        return x == b.x && y == b.y && z == b.z;
    }
} a[N], tmp[N];
bool cmp(node a, node b) {
    if (a.x != b.x) return a.x < b.x;
    if (a.y != b.y) return a.y < b.y;
    return a.z < b.z;
}
bool cmp2(node a, node b) {
    //if (a.y != b.y) return a.y < b.y;
    //return a.z < b.z;
    return a.y < b.y;
}
void CDQ(int l, int r) {
    if (l == r) {
        a[l].ans = a[l].num - 1;
        return;
    }
    int mid = l + r >> 1;
    CDQ(l, mid); CDQ(mid+1, r);
    pos = 1;
    rep(i, mid+1, r+1) {
        while (pos <= mid && a[pos].y <= a[i].y) {
            fen.add(fen.a1, a[pos].z, a[pos].num);

```

```

        pos++;
    }
    a[i].ans += fen.sum(fen.a1, a[i].z);
}
rep(i, 1, pos) fen.add(fen.a1, a[i].z, -a[i].num);
p1 = 1; p2 = mid+1;
rep(i, 1, r+1) {
    if (p1 > mid) {tmp[i] = a[p2]; p2++;}
    else if (p2 > r) {tmp[i] = a[p1]; p1++;}
    else if (a[p1].y <= a[p2].y) {tmp[i] = a[p1]; p1++;}
    else {tmp[i] = a[p2]; p2++;}
}
rep(i, 1, r+1) a[i] = tmp[i];
}

int main() {
    cin >> n >> k;
    rep(i, 1, n+1) cin >> a[i].x >> a[i].y >> a[i].z;
    sort(a+1, a+n+1, cmp);
    nn = 0;
    rep(i, 1, n+1) {
        if (i > 1 && a[i] == a[i-1]) { a[nn].num++;
        } else {
            a[++nn] = a[i];
            a[nn].num = 1;
        }
    }
    fen.ini(N);
    CDQ(1, nn);
    rep(i, 1, nn+1) ans[a[i].ans] += a[i].num;
    rep(i, 0, n) cout << ans[i] << endl;
    return 0;
}

```

### 3.4 CartesianTree

```

// desc : bud a cartesian tree from a[0] .. a[n-1]
// time : O(N)
// !!! : return rt, a[n] will be rewrite
int ls[N], rs[N];
int cartesianTree(int a[], int n) {
    a[n] = INT_MAX;
    vi v(1, n);
    fill_n(ls, n, -1), fill_n(rs, n, -1);
    rep(i, 0, n) {
        while (a[v.back()] < a[i])
            ls[i] = v.back(), v.pop_back();
        v.pb(rs[v.back()] = i);
    }
    return v[1];
}

```

### 3.5 Fenwick

```

typedef int T; // modify
namespace KDT {
    const int N = 1e6 + 7, D = 2;
    const T INF = 1e9 + 7;
    const db a1 = 0.75;

    int rt, L, top, w, sta[N];
    struct P { T x[D]; bool operator < (const P &c) const { return x[w] < c.x[w]; } } p[N];
    struct Node { T mi[D], ma[D]; int son[2], sz; P val; } nd[N];

    void init() { rt = L = top = 0; }
    int newnode() { return top ? sta[top--] : ++L; }
    void up(int k) {
        rep(i, 0, D) {
            nd[k].mi[i] = nd[k].ma[i] = nd[k].val.x[i];
            rep(o, 0, 2) if(nd[k].son[o]) {
                int s = nd[k].son[o];
                nd[k].mi[i] = min(nd[k].mi[i], nd[s].mi[i]);
                nd[k].ma[i] = max(nd[k].ma[i], nd[s].ma[i]);
            }
        }
        nd[k].sz = 1; rep(i, 0, 2) nd[k].sz += nd[nd[k].son[i]].sz;
    }
    int build(int l, int r, int w) {
        if(l > r) return 0;
        int mid = l + r >> 1, k = newnode();
        w = w, nth_element(p+l, p+mid, p+r+1, nd[k].val = p[mid]);
        nd[k].son[0] = build(l, mid-1, (w+1)%D);
        nd[k].son[1] = build(mid+1, r, (w+1)%D);
        up(k); return k;
    }
    void pia(int k, int &cnt) {
        if(nd[k].son[0]) pia(nd[k].son[0], cnt);
        p[++cnt] = nd[k].val, sta[++top] = k;
        if(nd[k].son[1]) pia(nd[k].son[1], cnt);
    }
    void check(int &k, int w) {
        bool o = 0;
        rep(i, 0, 2) if(a1 * nd[k].sz < nd[nd[k].son[i]].sz) o = 1;
        if(o) { int cnt = 0; pia(k, cnt), k = build(1, cnt, w); }
    }
    void ins(P p, int &k, int w) {
        if(!k) { k = newnode(), nd[k].val = p, nd[k].son[0] = nd[k].son[1] = 0, up(k); return; }
        ins(p, nd[k].son[nd[k].val.x[w] < p.x[w]], (w+1)%D);
        up(k), check(k, w);
    }
    // 抄上面这部分就好了，下面部分是视具体题目目的
    // 最近点
    T dis(P p, int k) {
        T ans = 0; rep(i, 0, D) ans += max(0, p.x[i] - nd[k].ma[i]) + max(0, nd[k].mi[i] - p.x[i]); // modify
        return ans;
    }
    T dis(P a, P b) {
        T ans = 0; rep(i, 0, D) ans += abs(a.x[i] - b.x[i]);
    }
}

```

```

// index : [1, n]
// time : nlogn
// support : segment add, sum
// !!!! : use before init()!
template<class T>
struct Fenwick {
    static const int N = 2e5+7;
    int n; T a1[N], a2[N];
    void ini(int n) {
        fill_n(a1+1, n, 0); fill_n(a2+1, n, 0);
    }
    void add(T *a, int p, T d) { for(; p<=n; p+=p&-p) a[p]+=d; }
    void add(int l, int r, T d) {
        add(a1, l, d), add(a1, r+1, -d);
        add(a2, l, d * (1-1)), add(a2, r+1, -d * r);
    }
    T sum(T *a, int p) { T r=0; for(; p>=1; p-=p&-p) r+=a[p]; return r; }
    T pre(int p) { return ip ? 0 : sum(a1, p) * p - sum(a2, p); }
    T qry(int l, int r) { return pre(r)-pre(l-1); }
};

```

### 3.6 IntervalMaximumChangeTimes

```

inline int qry(int L, int R, ll &v, int o, bool spe, int l, int r, int rt) {
    if(L > R) return 0;
    if(!spe) {
        if(ma[rt] < v) return 0;
        if(L <= 1 && r <= R) {
            if(l == r) return v = ma[rt], 1;
            int mid = l + r >> 1;
            down(l, r, mid, rt);
            if(ma[l] | o | < v) return o ? qry(L, R, v, o, 0, l, mid, ls) : qry(L, R, v, o, 0, 1, mid, ls);
            int ans = cnt[rt][o] - cnt[ls | o][o] + (o == 0 ? qry(L, R, v, o, 0, l, mid, ls) :
                qry(L, R, v, o, 0, mid+1, r, rs));
            return v = ma[rt], ans;
        }
    }
    int mid = l + r >> 1, ans = 0;
    down(l, r, mid, rt);
    if(o == 0 && L <= mid) ans += qry(L, R, v, o, 0, 0, l, mid, ls);
    if(R > mid) ans += qry(L, R, v, o, 0, mid+1, r, rs);
    if(o == 1 && L <= mid) ans += qry(L, R, v, o, 0, 0, l, mid, ls);
    return ans;
}
void up(int l, int r, int rt) {
    ma[rt] = max(ma[ls], ma[rs]);
    rep(o, 0, 2) { ll v = 0; cnt[rt][o] = qry(l, r, v, o, 1, l, r, rt); }
}

```

### 3.7 KDT

```

// init

```

```

int ls = nd[x].son[0], rs = nd[x].son[1];
nd[x].sum = nd[ls].sum + nd[rs].sum + nd[x].val;
}
void gao(int x) {
    if(!x) return;
    nd[x].rev ^= 1, swap(nd[x].son[0], nd[x].son[1]);
}
void down(int x) { if(nd[x].rev) gao(nd[x].son[0]), gao(nd[x].son[1]), nd[x].rev = 0; }
int id(int u) { return nd[nd[u].fa].son[1] == u; }
void rot(int x) {
    int y = nd[x].fa, z = nd[y].fa;
    int l = id(x), r = (l ^ 1), s = nd[x].son[r];
    if(nrt(y)) nd[z].son[id(y)] = x; nd[x].son[r] = y; nd[y].son[1] = s;
    if(s) nd[s].fa = y; nd[y].fa = x; nd[x].fa = z;
    up(y), up(x);
}
void splay(int x) {
    int top = 0;
    for(int i = x; i = nd[i].fa) {
        sta[++top] = i;
        if(!nrt(i)) break;
    }
    while(top) down(sta[top--]);
    while(nrt(x)) {
        int y = nd[x].fa;
        if(nrt(y)) (id(x) ^ id(y)) ? rot(x) : rot(y);
        rot(x);
    }
}
void access(int x) { for(int y = 0; x = nd[x].fa) splay(x), nd[x].son[1] = y, up(x); }
// 换根
void makeroot(int x) { access(x); splay(x); gao(x); }
// 找根
int findroot(int x) {
    access(x); splay(x);
    while(nd[x].son[0]) down(x), x = nd[x].son[0];
    splay(x);
    return x;
}
// 加边
void link(int x, int y) {
    makeroot(x);
    if(findroot(y) != x) nd[x].fa = y;
}
// 删边
void cut(int x, int y) {
    makeroot(x);
    if(findroot(y) == x && nd[y].fa == x && !nd[y].son[0]) nd[x].son[1] = 0, up(x);
}
// nd[y]: 路径信息
void path(int x, int y) { makeroot(x); access(y); splay(y); }
// 单点修改

```

```

return ans;
}
void qry(P p, int k, T &ans) {
    ans = min(ans, dis(p, nd[k].val));
    int ls = nd[k].son[0], rs = nd[k].son[1];
    T dl = ls ? dis(p, ls) : INF;
    T dr = rs ? dis(p, rs) : INF;
    if(dl > dr) swap(dl, dr), swap(ls, rs);
    if(dl < ans) qry(p, ls, ans);
    if(dr < ans) qry(p, rs, ans);
}
// 矩形区域的最大值 (伪代码)
void qry(int u, int &ans) {
    if(!u || ma < ans) return;
    if(!u.in) { ans = max(ans, ma); return; }
    if(u.in) ans = max(ans, u.val);
    rep(i, 0, 2) if(nd[u].son[i]) qry(nd[u].son[i], ans);
}
// 距离点 u 第 k 远
priority_queue<ll> ans;
void init() {
    while(!ans.empty()) ans.pop();
    rep(i, 0, k) ans.push(1);
}
ll sqr(int x) { return 1ll * x * x; }
ll Dis(P p, int u) {
    ll ans = 0; rep(d, 0, D) ans += max(sqr(nd[u].mi[d] - p.x[d]), sqr(nd[u].ma[d] - p.x[d]));
    return ans;
}
void qry(P p, int u) {
    ll dis = 0; rep(d, 0, D) dis += sqr(nd[u].val.x[d] - p.x[d]);
    ans.push(-dis), ans.pop();
    int ls = nd[u].son[0], rs = nd[u].son[1];
    ll dl = ls ? Dis(p, ls) : -1;
    ll dr = rs ? Dis(p, rs) : -1;
    if(dl > dr) swap(dl, dr), swap(ls, rs);
    if(dr > -ans.top()) qry(p, rs);
    if(dl > -ans.top()) qry(p, ls);
}
}
}

```

### 3.8 LCT

```

struct Node { int val, sum, fa, son[2]; bool rev; };
struct LCT {
    static const int N = ::N;
    Node nd[N]; int sta[N];
    // if(no root) return 1
    bool nrt(int x) {
        int fa = nd[x].fa;
        return nd[fa].son[0] == x || nd[fa].son[1] == x;
    }
    void up(int x) {
        if(!x) return;
    }
}

```

```

    void upd(int x, int c) { splay(x); nd[x].val = c; up(x); }
};

```

### 3.9 Rope

```

#include <ext/rope>
using namespace __gnu_cxx;

//index : [0..sz(rp))
rope<char> rp;

// 在任意处插入字符串
rp.push_back(ch);
// 在末尾插入
rp.insert(cur, 字符串);
// 在 cur 处插入字符串

// 删除任意片段
rp.erase(cur, len);
// 删除 cur 开始的 len 个字符

rp.replace(cur, 字符串);
// 删除 cur 处的字符, 换成字符串

// 取出任意片段
rp.copy(cur, len, 字符串);
// 复制 cur 处开始的 len 个字符到字符串
rp.at(cur);
// 取第 cur 个字符
rp[cur];
// 同上
rp.substr(cur, len);
// 提取从 cur 处开始的 len 个字符

// 可持久化
rp[i] = rp[i - 1];
// 可持久化, O(1), 直接拷贝根节点

/*
* 一) 翻转操作
* 1. 维护一正一反两个 rope
* 2. 翻转等价于交换两个子串

* 二) 区间循环位移
* 1. 拆成多个子串连在一起

* 三) 区间 a -> b, b -> c, c -> d ... z -> a
* 1. 维护 26 个 rope

*/

```

### 3.10 ST

```

// [0, n)
// 实现不同功能请谨慎复用
// 求下标最好用 pair 存
struct ST {
    static const int N = 101010;
    int a[20][N], lg[N];
    void build(int *v, int n) {
        rep(i, 2, n + 1) lg[i] = lg[i >> 1] + 1;
        rep(i, 0, n) a[0][i] = v[i];
    }
};

```

```

rep(i, 1, lg[n] + 1) rep(j, 0, n - (1 << i) + 1) {
    a[i][j] = max(a[i - 1][j], a[i - 1][j + (1 << i >> 1)]);
}

int qry(int l, int r) {
    if(1 > r) swap(l, r);
    int i = lg[r - l + 1];
    return max(a[i][l], a[i][r + 1 - (1 << i)]);
}
};

```

### 3.11 SegIntervalMax

```

// O(n log n)
// 区间取 max, 区间求和
struct Seg {
    #define ls rt << 1
    #define rs ls | 1

    static const int N = 1e5;
    int sum[N];
    int mi[N][2], cnt[N];

    void up(int rt) {
        sum[rt] = sum[ls] + sum[rs];
        rep(i, 0, 2) mi[rt][i] = min(mi[ls][i], mi[rs][i]);
        cnt[rt] = 0;
        rep(i, 0, 2) {
            if(mi[rt][0] == mi[ls | i][0]) cnt[rt] += cnt[ls | i];
            else mi[rt][1] = min(mi[rt][1], mi[ls | i][0]);
        }
    }

    void build(int l, int r, int rt) {
        if(l == r) {
            sum[rt] = mi[rt][0] = 1; // modify
            mi[rt][1] = inf;
            cnt[rt] = 1;
            return;
        }
        int mid = l + r >> 1;
        build(l, mid, ls);
        build(mid + 1, r, rs);
        up(rt);
    }

    void gao(int rt, int c) {
        if(c <= mi[rt][0]) return;
        sum[rt] += 1ll * cnt[rt] * (c - mi[rt][0]);
        mi[rt][0] = c;
    }

    void down(int rt) {
        gao(ls, mi[rt][0]);
        gao(rs, mi[rt][0]);
    }

    void upd(int l, int R, int c, int l1, int r1, int rt) {
        if(l > R) return;
        if(l <= l1 && r1 <= R && c < mi[rt][1]) {
            gao(rt, c);
        }
    }
};

```



```

    }
}T;

```

### 3.13 fhqTreap

```

// init
// id starts from 1
// 不要修改 0 节点的值
struct Node { int val, cnt, sz, ls, rs; ll r; bool rev; };
struct fhqTreap {
    static const int N = 1e5;
    int rt, L; Node nd[N];
    void init() { rt = L = 0; srand(time(0)); }
    ll Rand() { return ((rand() * 1111 << 32) ^ (rand() * 1111 << 16) ^ rand()); }
    int newnode(int c) {
        nd[++L].r = Rand();
        nd[L].val = c;
        nd[L].cnt = nd[L].sz = 1;
        nd[L].ls = nd[L].rs = nd[L].rev = 0;
        return L;
    }
    void up(int x) {
        if(!x) return;
        int ls = nd[x].ls, rs = nd[x].rs;
        nd[x].sz = nd[ls].sz + nd[rs].sz + nd[x].cnt;
    }
    void gao(int x) {
        if(!x) return;
        nd[x].rev ^= 1, swap(nd[x].ls, nd[x].rs);
    }
    void down(int x) {
        if(nd[x].rev) gao(nd[x].ls), gao(nd[x].rs), nd[x].rev = 0;
    }
    // u -> (<= c) (> c)
    void split(int u, int c, int &x, int &y) {
        if(u) {
            down(u);
            if(nd[u].val <= c) x = u, split(nd[u].rs, c, nd[u].rs, y);
            else y = u, split(nd[u].ls, c, x, nd[u].ls);
            up(u);
        } else {
            x = y = 0;
        }
    }
    // u -> (1 ~ k) (k+1 ~ L)
    // !!!: nd[0].cnt == 1
    void split(int u, int k, int &x, int &y) {
        if(u) {
            down(u);
            int sz = nd[nd[u].ls].sz;
            if(sz < k) x = u, split(nd[u].rs, k - sz - 1, nd[u].rs, y);
            else y = u, split(nd[u].ls, k, x, nd[u].ls);
            up(u);
        } else {
            x = y = 0;
        }
    }
}T;

```

```

return;
}
int mid = 1 + r >> 1;
down(rt);
if(L <= mid) upd(L, R, c, 1, mid, ls);
if(R > mid) upd(L, R, c, mid + 1, r, rs);
up(rt);
}
}seg;

```

### 3.12 Splay

```

// init
// id starts from 1
// if go to vertex p, must splay(p)
struct Node { int val, fa, son[2], cnt, sz; bool rev; };
struct Splay {
    static const int N = 1e5;
    int rt, L; Node nd[N];
    int newnode(int c, int fa = 0, int o = 0) {
        nd[++L].fa = fa;
        nd[L].son[o] = L;
        nd[L].val = c;
        nd[L].son[0] = nd[L].son[1] = nd[L].rev = 0;
        nd[L].cnt = nd[L].sz = 1;
        return L;
    }
    void init(int n) { rt = L = 0; }
    void gao(int u) {
        if(!u) return;
        nd[u].rev ^= 1, swap(nd[u].son[0], nd[u].son[1]);
    }
    void down(int u) {
        if(nd[u].rev) gao(nd[u].son[0]), gao(nd[u].son[1]), nd[u].rev = 0;
    }
    void up(int u) {
        if(!u) return;
        int ls = nd[u].son[0], rs = nd[u].son[1];
        nd[u].sz = nd[ls].sz + nd[rs].sz + nd[u].cnt;
    }
    int id(int u) { return nd[nd[u].fa].son[1] == u; }
    void rot(int x) {
        int y = nd[x].fa, z = nd[y].fa;
        int l = id(x), r = (1 ^ l), s = nd[x].son[r];
        if(z) nd[z].son[id(y)] = x; nd[x].son[r] = y; nd[y].son[1] = s;
        if(s) nd[s].fa = y; nd[y].fa = x; nd[x].fa = z;
        up(y), up(x);
    }
    void splay(int x, int g = 0) {
        while(nd[x].fa != g) {
            int y = nd[x].fa, z = nd[y].fa;
            if(z != g) (id(x) ^ id(y)) ? rot(x) : rot(y);
            rot(x);
        }
        if(!g) rt = x;
    }
}T;

```

```

    }
}
int merge(int x, int y) {
    if(x && y) {
        if(nd[x].r < nd[y].r) { down(x), nd[x].rs = merge(nd[x].rs, y), up(x); return x
        ; }
        else { down(y), nd[y].ls = merge(x, nd[y].ls), up(y); return y; }
    }
}T;
}

```

### 3.14 lcSegTree

```

// need init
// 1. use id
// 2. init mi/nd as max/min val
struct Node {
    ll k, b;
    Node() : k(0), b(0) {}
    Node(ll k, ll b) : k(k), b(b) {}
    ll getf(int x) const {
        return k * x + b;
    }
};
struct Seg {
    #define ls rt << 1
    #define rs ls | 1
    static const int N = :N << 2;
    Node nd[N], mi[N];
    void _upd(Node k, int l, int r, int rt) {
        int mid = l + r >> 1;
        if(k.getf(v[mid]) > nd[rt].getf(v[mid])) swap(k, nd[rt]);
        if(l == r) return ;
        if(min(nd[rt].getf(v[l]), nd[rt].getf(v[r])) >= max(k.getf(v[l]), k.getf(v[r])))
            return ;
        if(nd[rt].k > k.k) _upd(k, l, mid, ls);
        else _upd(k, mid + 1, r, rs);
    }
    void _min(Node k, int l, int r, int rt) {
        int mid = l + r >> 1;
        if(k.getf(v[mid]) < mi[rt].getf(v[mid])) swap(k, mi[rt]);
        if(l == r) return ;
        if(max(mi[rt].getf(v[l]), mi[rt].getf(v[r])) <= min(k.getf(v[l]), k.getf(v[r])))
            return ;
        if(mi[rt].k <= k.k) _min(k, l, mid, ls);
        else _min(k, mid + 1, r, rs);
    }
    void upd(int l, int R, Node c, int l, int r, int rt) {
        if(l > R) return ;
        if(l <= 1 && r <= R) {
            _upd(c, l, r, rt);
            _min(c, l, r, rt);
            return ;
        }
        int mid = l + r >> 1;

```

```

        if(l <= mid) upd(L, R, c, l, mid, ls);
        if(R > mid) upd(L, R, c, mid + 1, r, rs);
    }
    ll qry(int p, int l, int r, int rt) {
        ll ans = max(abs(nd[rt].getf(v[p])), abs(mi[rt].getf(v[p])));
        if(l == r) return ans;
        int mid = l + r >> 1;
        if(p <= mid) ans = max(ans, qry(p, l, mid, ls));
        else ans = max(ans, qry(p, mid + 1, r, rs));
        return ans;
    }
}seg;

```

### 3.15 perTreap

```

// init
// id starts from 1
// 不要修改 0 节点的值
struct Node { int val, cnt, sz, ls, rs; ll r, sum; bool rev; };
static const int N = 3e7;
int rt[::N], L; Node nd[N];
void init() { L = 0; srand(time(0)); }
ll Rand() { return ((rand() * 111 << 32) ^ (rand() * 111 << 16) ^ rand()); }
int newnode(int c) {
    nd[++L].r = Rand();
    nd[L].val = nd[L].sum = c;
    nd[L].cnt = nd[L].sz = 1;
    nd[L].ls = nd[L].rs = nd[L].rev = 0;
    return L;
}
int newcopy(int x) {
    nd[++L] = nd[x];
    return L;
}
void up(int x) {
    if(!x) return ;
    int ls = nd[x].ls, rs = nd[x].rs;
    nd[x].sz = nd[ls].sz + nd[rs].sz + nd[x].cnt;
    nd[x].sum = nd[ls].sum + nd[rs].sum + nd[x].val;
}
void gao(int &x) {
    if(!x) return ;
    x = newcopy(x);
    nd[x].rev ^= 1, swap(nd[x].ls, nd[x].rs);
}
void down(int x) {
    if(nd[x].rev) gao(nd[x].ls), gao(nd[x].rs), nd[x].rev = 0;
}
// u -> (<= c) (> c)
void split(int u, int c, int &x, int &y) {
    if(u) {
        u = newcopy(u), down(u);
        if(nd[u].val <= c) x = u, splitc(nd[u].rs, c, nd[u].rs, y);
        else y = u, splitc(nd[u].ls, c, x, nd[u].ls);
    }
}

```

```

if(l == r) return l;
int mid = l+r>>1;
int lc = 0;
for(auto i : add) lc += cnt[ls[i]];
for(auto i : sub) lc -= cnt[ls[i]];
if(lc>=k) {
    rep(i, 0, sz(add)) add[i] = ls[add[i]];
    rep(i, 0, sz(sub)) sub[i] = ls[sub[i]];
    return qry(L, R, k, l, mid);
} else {
    rep(i, 0, sz(add)) add[i] = rs[add[i]];
    rep(i, 0, sz(sub)) sub[i] = rs[sub[i]];
    return qry(L, R, k-lc, mid+1, r);
}
}
}seg;
struct Fenwick {
#define lb(x) ((x)&(-x))
void init() { fill_n(rt+1+n, n, 0); }
void upd(int x, int p, int c) {
    for(; x<=n; x+=lb(x)) seg.upd(rt[x+n], rt[x+n], p, c, 0, sz(V)-1);
}
int qry(int l, int r, int k) {
    add.clear();sub.clear();
    add.pb(rt[r]);sub.pb(rt[l-1]);
    int x = r;
    for(; x; x^=lb(x)) add.pb(rt[n+x]);
    x = l-1;
    for(; x; x^=lb(x)) sub.pb(rt[n+x]);
    return seg.qry(l, r, k, 0, sz(V)-1);
}
}fw;
int main() {
    int T; cin >> T;
    while(T--) {
        cin >> n >> m;
        V.clear(); seg.init(); fw.init();
        ///read
        rep(i, 1, n+1) cin >> a[i], V.pb(a[i]);
        rep(i, 1, m+1) {
            string s;
            cin >> s >> q[i].a >> q[i].b;
            q[i].op = (s[0]=='Q');
            if(s[0]=='Q') cin >> q[i].k;
            else V.pb(q[i].b);
        }
        ///solve
        sort(all(V));
        V.erase(unique(all(V)), V.end());
        rep(i, 1, n+1) seg.upd(rt[i-1], rt[i], rk(a[i]), 1, 0, sz(V)-1);
        rep(i, 1, m+1) {
            if(q[i].op) { cout << V[fw.qry(q[i].a, q[i].b, q[i].k)] << endl;
            } else {
                int p = q[i].a, c = q[i].b;
                fw.upd(p, rk(a[p]), -1);
            }
        }
    }
}

```

```

up(u);
} else {
    x = y = 0;
}
}
// u -> (1 ~ k) (k+1 ~ L)
// !!!: nd[].cnt == 1
void splstk(int u, int k, int &x, int &y) {
    if(u) {
        u = newcopy(u), down(u);
        int sz = nd[nd[u].ls].sz;
        if(sz < k) x = u, splstk(nd[u].rs, k - sz - 1, nd[u].rs, y);
        else y = u, splstk(nd[u].ls, k, x, nd[u].ls);
        up(u);
    } else {
        x = y = 0;
    }
}
// sometimes do not need to newcopy
int merge(int x, int y) {
    if(x && y) {
        if(nd[x].r < nd[y].r) { x = newcopy(x), down(x), nd[x].rs = merge(nd[x].rs, y),
            up(x); return x; }
        else { y = newcopy(y), down(y), nd[y].ls = merge(x, nd[y].ls), up(y); return y;
        }
    } else return x + y;
}
}T;

```

### 3.16 动态 k 大

```

// zoj 2112 动态区间 k 大
const int N = 50505, M = 10101;
int n, m, a[N], rt[N<<1];
vi V, add, sub;
inline int rk(int x) { return lower_bound(all(V), x) - V.begin(); }
struct Q {
    bool op;
    int a, b, k;
}q[M];
struct Seg {
    static const int N = 2500005; // (1:N + 32 * :M) * 16;
    int cntn, cnt[N], ls[N], rs[N];
    void init() { fill_n(rt+1, n, cntn = 0); }
    void upd(int pre, int &now, int p, int c, int l, int r) {
        now = ++cntn;
        cnt[now] = cnt[pre] + c;
        ls[now] = ls[pre];
        rs[now] = rs[pre];
        if(l == r) return;
        int mid = l+r>>1;
        if(p<=mid) upd(ls[pre], ls[now], p, c, l, mid);
        else upd(rs[pre], rs[now], p, c, mid+1, r);
    }
    int qry(int l, int R, int k, int l, int r) {

```

<pre>fw.upd(p, rk(a[p] = c), 1);     } } return 0; }</pre>	<h3>3.17 动态树上路径 k 大</h3> <pre>int n, m, L, dfn, val[N], rt[N], cnt[M], ls[M], rs[M], st[N], ed[N], fw[N], par[N]; pair&lt;int, pii&gt; Q[N]; vi v, res[2], g[N]; LCARMQ R; int F(int x) { return lower_bound(all(v), x) - v.begin() + 1; } // seg void upd(int &amp;now, int pre, int p, int c, int l, int r) {     now = ++L;     cnt[now] = cnt[pre] + c;     ls[now] = ls[pre];     rs[now] = rs[pre];     if(l == r) return ;     int mid = l + r &gt;&gt; 1;     if(p &lt;= mid) upd(ls[now], ls[pre], p, c, l, mid);     else upd(rs[now], rs[pre], p, c, mid + 1, r); } int qry(int k, int l, int r) {     if(l == r) return l;     int mid = l + r &gt;&gt; 1;     int cntr = 0;     rep(i, 0, sz(res[0])) cntr += cnt[rs[res[0][i]]];     rep(i, 0, sz(res[1])) cntr -= cnt[rs[res[1][i]]];     if(cntr &gt;= k) {         rep(i, 0, sz(res[0])) res[0][i] = rs[res[0][i]];         rep(i, 0, sz(res[1])) res[1][i] = rs[res[1][i]];         return qry(k, mid + 1, r);     } else {         rep(i, 0, sz(res[0])) res[0][i] = ls[res[0][i]];         rep(i, 0, sz(res[1])) res[1][i] = ls[res[1][i]];         return qry(k - cntr, l, mid);     } } // build 主席树 void dfs(int u, int fa) {     st[u] = ++dfn;     par[u] = fa;     upd(rt[u], rt[fa], F(val[u]), 1, 1, sz(V));     rep(i, 0, sz(g[u])) if(g[u][i] != fa) dfs(g[u][i], u);     ed[u] = dfn; } // Fenwick void upd(int p, int o, int c) { for( ; p &lt;= n; p += lb(p)) upd(fw[p], c, o, 1, sz(V)); } void upd(int u, int o) {     res[o].pb(rt[u]);     int p = st[u]; for( ; p; p ^= lb(p)) res[o].pb(fw[p]); } void solve() {     R.Build(g);     dfs(1, 0);     rep(i, 1, m + 1) {</pre>
<pre>if(!Q[i].fi) {     int p = Q[i].se.fi, c = Q[i].se.se;     upd(st[p], -1, F(val[p]));     upd(ed[p] + 1, 1, F(val[p]));     val[p] = c;     upd(st[p], 1, F(val[p]));     upd(ed[p] + 1, -1, F(val[p])); } else {     rep(o, 0, 2) res[o].clear();     int a = Q[i].se.fi, b = Q[i].se.se, k = Q[i].fi;     int c = R.lca(a, b), d = par[c];     upd(a, 0); upd(b, 0);     upd(c, 1); upd(d, 1);     cout &lt;&lt; V[qry(k, 1, sz(V)) - 1] &lt;&lt; endl; } } }</pre>	<h3>3.18 常见转化</h3> <pre>/* * 单点修改，区间查询 -&gt; 单点修改，前缀查询 -&gt; 后缀修改，单点查询 */</pre>
	<h3>3.19 覆盖大于 k 次的矩形面积</h3> <pre>/* * 这里是覆盖次数大于 1 次的 */ struct Seg { #define ls rt &lt;&lt; 1 #define rs ls   1 static const int N = :N &lt;&lt; 2; int la[N], len[2][N]; void up(int rt, int l, int r) {     if(la[rt] &gt;= 2) {         len[0][rt] = r - l + 1;         len[1][rt] = r - l + 1;     } else if(la[rt] &gt;= 1) {         len[0][rt] = r - l + 1;         len[1][rt] = (l == r) ? 0 : len[0][ls] + len[0][rs];     } else {         len[0][rt] = (l == r) ? 0 : len[0][ls] + len[0][rs];         len[1][rt] = (l == r) ? 0 : len[1][ls] + len[1][rs];     } } } void upd(int L, int R, int c, int l, int r, int rt) {     if(L &lt;= l &amp;&amp; r &lt;= R) {         la[rt] += c;         upd(rt, l, r);         return ;     }     int mid = l + r &gt;&gt; 1;     if(L &lt;= mid) upd(L, R, c, l, mid, ls);     if(R &gt; mid) upd(L, R, c, mid + 1, r, rs);     upd(rt, l, r); }</pre>

```
// * 两堆物品，个数 (n, m)(n <= m)，两人轮流从某一堆拿任意数量的物品或同时从两堆中取同样
// 多的物品，每次至少一个，不能操作的人败。
// * 必败态: (m - n) * (1 + sqrt(5)) / 2 == n
// // 威佐夫博弈扩展
// * 两堆物品，个数 (n, m)(n <= m)，两人轮流从某一堆拿任意数量的物品或同时从两堆中取绝对
// 值 <= k 的物品，每次至少一个，不能操作的人败。
// * 必败态:
// * d = k + 1, t^2 + (d - 2) * t - d = 0 -> 解出 t
// * 必败: (m - n) / d * t == n
// // 博弈fib
// * 一堆石子，两人轮流取。先手不能在第一次取光，之后可以取的石子数介于 1 到对手刚取的石子数
// 的两倍之间（左闭右团），不能操作的人败。
// * 必败态: 石子个数是 fib 数
```

## 5 Geometry

### 5.1 GeoAdd

```
/*
 * 平面图欧拉定理: V + F - E = 2
 */
bool cmp(const pii &a, const pii &b) { // 极角排序
    int o = a > pii(0, 0), t = b > pii(0, 0);
    if(o != t) return o < t;
    return det(a, b) > 0;
}
bool isSSr(const L &a, const L &b) { // 线段规范相交
    db c1 = det(a.t - a.s, b.s - a.s), c2 = det(a.t - a.s, b.t - a.s),
    c3 = det(b.t - b.s, a.s - b.s), c4 = det(b.t - b.s, a.t - b.s);
    return sign(c1) * sign(c2) < 0 && sign(c3) * sign(c4) < 0;
}
bool isSS(L a, L b) { // 线段不规范相交
    db c1 = det(a.t - a.s, b.s - a.s), c2 = det(a.t - a.s, b.t - a.s),
    c3 = det(b.t - b.s, a.s - b.s), c4 = det(b.t - b.s, a.t - b.s);
    return sign(c1) * sign(c2) <= 0 && sign(c3) * sign(c4) <= 0 &&
    sign(max(a.s.x, a.t.x) - min(b.s.x, b.t.x)) >= 0 &&
    sign(max(b.s.x, b.t.x) - min(a.s.x, a.t.x)) >= 0 &&
    sign(max(a.s.y, a.t.y) - min(b.s.y, b.t.y)) >= 0 &&
    sign(max(b.s.y, b.t.y) - min(a.s.y, a.t.y)) >= 0;
}
db disSS(L a, L b) { // 线段到线段距离
    if(!isSS(a, b)) return 0;
    return min(min(disToSeg(b, a.s), disToSeg(b, a.t)), min(disToSeg(a, b.s), disToSeg(a,
    b.t)));
}
// 判断直线线段是否相交（端点也算）
bool isLS(P a1, P a2, P b1, P b2) {
    db c1 = det(a2 - a1, b1 - a1), c2 = det(a2 - a1, b2 - a1);
    return sign(c1) * sign(c2) <= 0;
}
P outC(P A, P B, P C) { // 外心
    P b = B - A, c = C - A;
    db dB = b.len2(), dC = c.len2(), d = 2 * det(b, c);
    return A - P(b.y * dC - c.y * dB, c.x * dB - b.x * dC) / d;
}
```

```
}
}seg;
```

### 3.20 高维偏序

```
// 如果有比较快O2，不然可能比较慢要手写bitset
namespace PX{
    const int N = :: N, M = sqrt(N) + 5;
    const int K = 7;
    int n, k, B, pos[K][N];
    vector<N> s[K][M];
    vector<pii> V[K];

    struct node{
        int d[K];
    } a[N];

    void init(int _n, int _k) {
        n = _n; k = _k;
        rep(i, 1, n+1) rep(j, 0, k) cin >> a[i].d[j];
        rep(i, 1, n+1) rep(j, 0, k) V[j].pb(mp(a[i].d[j], i));
        rep(j, 0, k) sort(all(V[j]));
        rep(i, 1, n+1)
            rep(j, 0, k)
                a[i].d[j] = lower_bound(all(V[j]), mp(a[i].d[j], i)) - V[j].begin(), pos[j])[
                a[i].d[j]] = i;
        B = sqrt(n);
        rep(j, 0, k) {
            bitset<N> tmp; int id = 1;
            rep(i, 0, n) {
                tmp.set(pos[j][i]);
                if (i == id * B - 1) s[j][id++] = tmp;
            }
        }
        int qry(node a) {
            bitset<N> ans; ans.set();
            rep(j, 0, k) {
                int ed = lower_bound(all(V[j]), mp(V[j][a.d[j]].fi, n+1)) - V[j].begin() - 1;
                bitset<N> tmp; int id = ed / B, st = id * B;
                if (id) tmp = s[j][id - 1];
                rep(i, st, ed+1) tmp[pos[j][i]] = 1;
                ans &= tmp;
            }
            return ans.count();
        }
    }
}
```

## 4 Game

### 4.1 Game

```
// 威佐夫博弈
```

```

}
bool isconvex(vector<P> A) { // 判断是否是凸包逆时针
    bool ok = 1;
    int n = sz(A);
    rep(i, 0, 2) A.pb(A[i]);
    rep(i, 0, n) ok &= det(A[i + 1] - A[i], A[i + 2] - A[i]) >= 0;
    return ok;
}

db diameter(vector<P> A) { // 求凸包最远点对
    int n = sz(A);
    if(n <= 1) return 0;
    int l = 0, r = 0;
    rep(i, 1, n) (A[i] < A[l]) && (l = i), (A[r] < A[i]) && (r = i);
    db res = (A[l] - A[r]).len();
    int i = 1, j = r;
    do (++det(A[i + 1] - A[i], A[j + 1] - A[j]) >= 0 ? j : i) %= n,
        res = max(res, (A[i] - A[j]).len());
    while(i != 1 || j != r);
    return res;
}

// sz(A) <= 100,000
namespace NearestPoints { // 点集中最近点对
    db solve(int l, int r, vector<P> &p) {
        if(l == r) return 1e100;
        int m = l + r >> 1;
        db Xm = p[m].x, lim = min(solve(l, m, p), solve(m + 1, r, p));
        inplace_merge(p.begin() + l, p.begin() + m + 1, p.begin() + r + 1, [&](P a, P b){
            return a.y < b.y;});
        vector<P> V;
        rep(i, l, r + 1) if(fabs(p[i].x - Xm) <= lim) V.pb(p[i]);
        rep(i, 0, sz(V)) rep(j, i + 1, sz(V)) {
            if(fabs(V[j].y - V[i].y) >= lim) break;
            T dis = (V[i] - V[j]).len();
            lim = min(lim, dis);
        }
        return lim;
    }
}

db solve(vector<P> A) {
    sort(all(A), [&](P a, P b){return a.x < b.x;});
    return solve(0, sz(A) - 1, A);
}

// 注意相等关系
// 相离4: 外切3: 相交2: 内切1: 内含0:
int relCC(C A, C B) { // 两圆关系
    db dis = (A.o - B.o).len();
    if(sign(dis - (A.r + B.r)) == 1) return 4;
    if(sign(dis - (A.r + B.r)) == 0) return 3;
    if(sign(dis - fabs(A.r - B.r)) == 1) return 2;
    if(sign(dis - fabs(A.r - B.r)) == 0) return 1;
    return 0;
}

vector<P> tanCC(const C &c1, const C &c2) { // 求圆与圆的切点
    vector<P> res;
    db dis = (c1.o - c2.o).len();
    if(sign(dis - (c1.r + c2.r)) == 0) {
        res.pb((c1.o - c1.o) * c1.r / (c1.r + c2.r));
    }
    if(sign(dis - fabs(c1.r - c2.r)) == 0) {
        res.pb((c1.o + (c2.o - c1.o) * c1.r / (c1.r - c2.r));
    }
    return res;
}

db rad(P p1, P p2) { return atan2l(det(p1, p2), dot(p1, p2)); } // p1 与 p2 的夹角, 有方向
db areaCT(db r, P s, P t) { // 求圆与三角形交面积, 需要除2
    P p1, p2;
    bool f = isCL(C(P(0, 0), r), l(s, t), p1, p2);
    if(!f) return r * r * rad(s, t);
    bool b1 = sign(s.len2() - r * r) == 1, b2 = sign(t.len2() - r * r) == 1;
    if(b1 && b2) {
        if(sign(dot(s - p1, t - p1)) <= 0 && sign(dot(s - p2, t - p2) <= 0))
            return r * r * (rad(s, p1) + rad(p2, t)) + det(p1, p2);
        else return r * r * rad(s, t);
    } else if(b1) return r * r * rad(s, p1) + det(p1, t);
    else if(b2) return r * r * rad(p2, t) + det(s, p2);
    return det(s, t);
}

db areaCPoly(C c, vector<P> p) { // 求圆与多边形交面积
    int n = sz(p);
    db ans = 0;
    rep(i, 0, n) {
        P u = p[i], v = p[(i + 1) % n];
        ans += areaCT(c.r, u - c.o, v - c.o);
    }
    return fabs(ans) / 2;
}

C Mincir(P *p, int n){ // ? 最小圆覆盖
    random_shuffle(p, p + n);
    P o = p[0]; db r = 0;
    rep(i, 1, n) {
        if(sgn(abs(o - p[i]) - r) <= 0) continue;
        o = p[i], r = 0;
        rep(j, 0, i) {
            if(sgn(abs(o - p[j]) - r) <= 0) continue;
            o = (p[i] + p[j]) / 2, r = abs(o - p[j]);
            rep(k, 0, j) {
                if(sgn(abs(o - p[k]) - r) <= 0) continue;
                o = outC(p[i], p[j], p[k]), r = abs(o - p[k]);
            }
        }
        return C(o, r);
    }
}

namespace CircleIntersection { // ?
    struct E {
        P p; T ang; int delta;
        E(P p, T ang, int delta): p(p), ang(ang), delta(delta) {}
    };
    bool operator < (const E &b) const { return ang < b.ang; }
};

bool overlap(C a, C b) { return sgn(a.r - b.r - abs(a.o - b.o)) >= 0; }
void solve(C *c, int n, T *ans) {

```

```

db s2 = (r[t][g+1] - r[t][g]) / (r[i][j+1] - r[t][g]);
if(du >= 0 && dv < 0) res[sz++] = pdi(s1 / (s1 + s2), 1);
else if(du < 0 && dv >= 0) res[sz++] = pdi(s1 / (s1 + s2), -1);
}
sort(res, res + sz);
int cnt = 0; --sz;
rep(t, 0, sz) {
    cnt += res[t].se;
    if(cnt == 0 && sgn(res[t].fi - res[t+1].fi)) {
        db a = res[t].fi;
        if(a < 0) a = 0; if(a > 1) break;
        db b = res[t+1].fi;
        if(b < 0) continue; if(b > 1) b = 1;
        rt += ((r[i][j+1] - r[i][j]) * a + r[i][j]) / ((r[i][j+1] - r[i][j]) * b +
            r[i][j]);
    }
}
return rt / 2;
}

```

## 5.2 MaxAreaPoly

```

ld solve_poly(vi &s) {
    assert(sz(S) > 0);
    int sum = 0, hi = s[0];
    vi vals;
    rep(i, 1, sz(S)) {
        int cur = S[i];
        if (cur > hi) swap(cur, hi);
        sum += cur;
        vals.pb(cur);
    }
    if (sum <= hi) return 0;
    auto getAngle = [&](ld D) -> ld {
        ld tot = 0;
        for (int l : vals) tot += 2 * asin(ld(l) / ld(D));
        return tot;
    };
    bool isReflex = (getAngle(hi) < PI);
    auto tooSmall = [&](ld D) {
        ld ang = getAngle(D);
        ld hiAng = 2 * asin(ld(hi) / ld(D));
        if (isReflex) return ang < hiAng;
        else return ang + hiAng >= 2 * PI;
    };
    ld mi = hi, ma = hi + 1;
    int numExpand = 0;
    while (tooSmall(ma)) numExpand++, ma += (ma - mi);
    rep(tim, 0, 50 + numExpand) {
        ld md = mi + (ma - mi) / 2;
        if (tooSmall(md)) mi = md;
        else ma = md;
    }
    ld D = mi, area = 0;
    for (int l : vals) area += ld(l) * sqrt(ld(D) * ld(D) - ld(l) * ld(l)) / 4;
    ld hiArea = ld(hi) * sqrt(ld(D) * ld(D) - ld(hi) * ld(hi)) / 4;
    if (isReflex) area -= hiArea;
}

```

```

memset(ans, 0, sizeof(T) * (n + 1));
rep(i, 0, n) {
    int cnt=1;
    vector<E> evt;
    rep(j, 0, i) if(c[i]==c[j]) cnt++;
    rep(j, 0, n) if(j!=i && (c[i]==c[j]) && overlap(c[j], c[i])) cnt++;
    rep(j, 0, n) if(j!=i) {
        vector<P> pts=inscc(c[i], c[j]);
        if(sz(pts)) {
            T a[2];
            rep(j, 0, 2) a[j]=(pts[j]-c[i].o).arg();
            evt.pb(E(pts[0], a[0], 1));
            evt.pb(E(pts[1], a[1], -1));
            cnt += a[0] > a[1];
        }
    }
    if(!sz(evt)) ans[cnt] += pi*c[i].r*c[i].r;
    else {
        sort(all(evt));
        evt.pb(evt.front());
        rep(j, 0, sz(evt)-1) {
            cnt+=evt[j].delta;
            ans[cnt] += evt[j].p / evt[j+1].p / 2;
            db ang = evt[j + 1].ang - evt[j].ang;
            if(ang < 0) ang += pi * 2;
            ans[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
        }
    }
}

namespace ConvecIntersection { // ?
const int N = 1005;
struct Rec {
    P d[10]; int dn; // d[dn] = d[0]
    P operator [] (const int&n) {return d[n];}
} r[N];
typedef pair<db, int> pdi;
int n; pdi res[1000005];
db getLoc(P a, P b, P p) {
    if(sgn(b.x - a.x)) return (p.x - a.x) / (b.x - a.x);
    return (p.y - a.y) / (b.y - a.y);
}
} db work() {
    db rt=0;
    rep(i, 0, n) rep(j, 0, r[i].dn) {
        int sz=0;
        res[sz++] = pdi(0, 0); res[sz++] = pdi(1, 0);
        rep(t, 0, n) {
            if(t == i) continue;
            rep(g, 0, r[t].dn) {
                int du = sgn((r[i][j+1] - r[i][j]) / (r[t][g] - r[i][j]));
                int dv = sgn((r[i][j+1] - r[i][j]) / (r[t][g+1] - r[i][j]));
                if(!du && !dv) {
                    if(sgn((r[i][j+1] - r[i][j]) * (r[t][g+1] - r[t][g])) < 0 || i < t) {
                        res[sz++] = pdi(getLoc(r[i][j], r[i][j+1], r[t][g]), 1);
                        res[sz++] = pdi(getLoc(r[i][j], r[i][j+1], r[t][g+1]), -1);
                    } else {
                        db s1 = (r[i][j] - r[t][g]) / (r[t][g+1] - r[t][g]);

```

```

else area += hiArea;
return area;
}

```

### 5.3 MaxAreaTri

```

// O(n ^ 2)
void maxAreaTri(P *p, int n, P &a, P &b, P &c) {
    int i = 0, j = 1, k = 2;
    a = p[i], b = p[j], c = p[k];
    T res = area(a, b, c), cur = res, tmp;
    do {
        while(1) {
            while(cur <= (tmp = area(p[i], p[j], p[(k + 1) % n]))) (++k) %= n, cur = tmp;
            if(cur <= (tmp = area(p[i], p[(j + 1) % n], p[k]))) (++j) %= n, cur = tmp;
            else break;
        }
        if(cur > res) a = p[i], b = p[j], c = p[k], res = cur;
        (++i) %= n;
        if(i == j) (++j) %= n;
        if(j == k) (++k) %= n;
        cur = area(p[i], p[j], p[k]);
    } while(1);
}

```

## 6 Graph

### 6.1 BCC

```

// key contains the id of edges
// _ starts from 0
namespace BCC{
    const int N = 202020;
    vi key, bcc[N];
    int dfn[N], low[N], id[N], st[N], _st, _;
    void dfs(int c, int dep, vector<pii> g[]){
        int cc=0; st[_st++]=c;
        dfn[c]=low[c]=dep;
        for(auto e:g[c]){
            int t=e.fi;
            if(!dfn[t]){
                dfs(t, dep+1, g);
                low[c]=min(low[c], low[t]);
                if(low[t]>dfn[c]) key.pb(e.se);
            } else if(dfn[t] != dfn[c] - 1 || cc++)
                low[c] = min(low[c], dfn[t]);
        }
        if(low[c]==dfn[c]){
            do{id[st[_st]]=_;}while(st[_st]!:=c);
            _++;
        }
    }
    int solve(int n, vector<pii> g[]){

```

```

fill_n(dfn, n, _=0);
fill_n(low, n, _st=0);
fill_n(bcc, n, key=vi());
rep(i, 0, n) if(!dfn[i]) dfs(i, 1, g);
rep(i, 0, n) for(auto j:g[i]) if(id[i]!=id[j].fi)
    bcc[id[i]].pb(id[j].fi);
return _;
}
}

```

### 6.2 DCC

```

// cactus: n multi by 2
// key is cuts
// dcc i->j, i(points), j(bcc_block)
// st is stack
// _st is top of stack
// _ is number of dcc
// can handle isolate point and not connected graph and multi edge
// can handle self circle?
namespace DCC{
    const int N = 202020;
    vi key, dcc[N];
    int dfn[N], low[N], st[N], _st, _;
    void dfs(int c, int dep, const vi g[]){
        int cc=0, out=1<dep; st[_st++]=c;
        dfn[c]=low[c]=dep;
        for(auto t:g[c])
            if(!dfn[t]){
                dfs(t, dep+1, g);
                low[c]=min(low[c], low[t]);
                if(low[t]>=dfn[c]){
                    if(++out==2) key.pb(c);
                    while(st[_st]!:=t) dcc[st[_st]].pb(_);
                    dcc[c].pb(_); dcc[t].pb(_++);
                }
            } else if(dfn[t] != dfn[c] - 1 || cc++)
                low[c] = min(low[c], dfn[t]);
    }
    int solve(int n, const vi g[]){ // n is size of points
        fill_n(dfn, n, _=0);
        fill_n(low, n, _st=0);
        fill_n(dcc, n, key=vi());
        rep(i, 0, n) if(!dfn[i]) dfs(i, 1, g);
        rep(i, 0, n) if(sz(dcc[i]) == 0) dcc[i].pb(_++);
        return _;
    }
}

```

### 6.3 DMST

```

// id starts from 0
// can handle multi edge, self ring
struct edge {int u, v, d, u, v; bitset<1005> b;};

```



```

struct DMST{
    static const int N = ::N , M = N * N , inf = 2e9;
    edge e[M];int n, m, vis[N], pre[N], id[N], index[N], Pre[N];
    bitset<1005> fang;
    int in[N];
    void ini(int n) {this->n = n, m = 0;}
    void addedge(int u, int v, int d) {e[m] = edge({u,v,d,u,v}); e[m].reset();e[m].b[m] = 1;m++;}
    int run(int root){
        int ans = 0;
        while(1){
            rep(i, 0, n) in[i] = inf;
            rep(i, 0, m){
                int u = e[i].u , v = e[i].v;
                if(e[i].d < in[v] && u != v) in[v] = e[i].d, pre[v] = u, index[v] = i;
            }
            rep(i, 0, n) {
                if(i == root) continue;
                if(in[i] == inf) return -1;
                fang ^= e[index[i]].b;
            }
            int cnt = 0;in[root] = 0;
            memset(id, -1, sizeof(*id)*n);
            memset(vis, -1, sizeof(*vis)*n);
            rep(i, 0, n){
                ans += in[i]; int v = i;
                int t = index[i];
                while(vis[v] != i && id[v] == -1 && v!=root) vis[v] = i, v = pre[v];
                if(v != root && id[v] == -1) {
                    for(int u=pre[v];u != v;u = pre[u]) id[u] = cnt;
                    id[v] = cnt++;
                }
            }
            if(cnt == 0) break;
            rep(i, 0, n) if(id[i] == -1) id[i] = cnt++;
            rep(i, 0, m) {
                int v=e[i].v;
                e[i].u = id[e[i].u]; e[i].v = id[e[i].v];
                if(e[i].u != e[i].v) {e[i].d -= in[v];e[i].b ^= e[index[v]].b;}
            }
            n = cnt; root = id[root];
        }
        return ans;
    }
} dmst;

```

## 6.4 Dijkstra

```

int n, dis[N];
void Dijkstra(int st) {
    priority_queue<pii> q;
    fill_n(dis + 1, n, inf);
    dis[st] = 0;
    q.push(mp(0, st));
    while(!q.empty()) {

```

```

        pii u = q.top();q.pop();
        if(dis[u.se] != -u.fi) continue;
        for(auto v : g[u.se]) {
            if(dis[v.fi] > dis[u.se] + v.se) {
                dis[v.fi] = dis[u.se] + v.se;
                q.push(mp(-dis[v.fi], v.fi));
            }
        }
    }
}

```

## 6.5 Dinic

```

// [0,n) init!!
template<class T>
struct Dinic{
    const static int N = 10101 , M = N * 10;
    int s , t , n , h[N] , cur[N] , level[N] , q[N] , e , ne[M] , to[M];
    T cap[M] , flow;
    void liu(int u,int v,T w){ to[e] = v;ne[e] = v;ne[e] = h[u];cap[e] = w;h[u] = e++;}
    void link(int u,int v,T w){ liu(u , v , w);liu(v , u , 0);}
    void ini(int _n = N) { fill(h , h + (n=_n) , -1);e = 0;}
    bool bfs(){
        int L = 0 , R = 0;
        fill(level , level + n , -1);
        level[q[R++]] = s = 0;
        while(L < R && level[t] == -1){
            int c = q[L++];
            for(int k=h[c];-k;k=ne[k])
                if(cap[k] > 0 && level[to[k]] == -1)
                    level[q[R++]] = to[k] = level[c] + 1;
        }
        return ~level[t];
    }
    T dfs(int c,T mx){
        if(c == t) return mx;
        T ret = 0;
        for(int &k = cur[c];-k;k = ne[k]){
            if(level[to[k]] == level[c] + 1 && cap[k] > 0){
                T flow = dfs(to[k] , min(mx , cap[k]));
                ret += flow;cap[k] -= flow , cap[k^1] += flow;mx -= flow;
                if(!mx) return ret;
            }
        }
        level[c] = -1;
        return ret;
    }
    T run(int _s,int _t){
        s = _s , t = _t;
        flow = 0;
        while(bfs()){
            copy(h , h + n , cur);
            flow += dfs(s,-0x>>1);
        }
        return flow;
    }
}

```

```
}  
};
```

## 6.6 DualMST

对偶图最小生成树，等于平面图所有边边权和减去平面图最大生成树。

## 6.7 EulerianPath

```
vi ans; bool vis[N]; int p[N];  
vector<pii> g[N];  
  
void dfs(int u) {  
    for( ; p[u] < sz(g[u]); ++p[u]) {  
        auto v = g[u][p[u]];  
        if(!vis[abs(v.se)]) {  
            vis[abs(v.se)] = 1;  
            dfs(v.fi);  
            ans.pb(-v.se);  
        }  
    }  
}
```

## 6.8 FindCircle

```
// 支持基环树森林和自环重边  
const int N = 1e5 + 7;  
vector<pair<pii>, int> g[N]; //点编号 边权编号  
int tim, dfn[N], fa[N], d[N], k;  
vi cir[N];  
int ne[N]; // 有向图的出度  
int id[N]; //点属于的环编号  
  
void dfs(int u, int pre) { // 为边编号pre  
    dfn[u] = ++tim;  
    rep(1, 0, sz(g[u])) {  
        if (g[u][i].se == pre) continue;  
        int v = g[u][i].fi.fi, w = g[u][i].fi.se;  
        if (dfn[v] && dfn[v] <= dfn[u]) {  
            k++;  
            int p = u; cir[k].pb(p); id[p] = k;  
            if (p != v) {do { p = fa[p]; cir[k].pb(p); id[p] = k;  
                } while (p != v);}  
            if (sz(cir[k]) > 1 && ne[cir[k][0]] != cir[k][1]) reverse(all(cir[k]));  
            continue;  
        }  
        if (!dfn[v]) {fa[v] = u; d[v] = d[u] + w; dfs(v, g[u][i].se);}  
    }  
}
```

## 6.9 Lindstrom\_Gessel\_Viennot\_Lemma

```
/*  
* 对于一张无边权的 DAG 图，给定 n 个起点和对应的 n 个终点，这 n 条不相交路径的方案数为矩阵  
* e(a1,b1),e(a1,b2)...e(a1,bn)  
* e(a2,b1),e(a2,b2)...e(a2,bn)  
* .....  
* .....  
* e(an,b1),e(an,b2)...e(an,bn)  
* 的行列式。  
* 即 M[i][j]=e(ai,bj)  
* e(a,b) 为 a 到 b 的路径方案数  
*/
```

## 6.10 MMST

```
// 曼哈顿最小距离生成树  
// 这份代码处理的区域是 Y 轴右转 45 度  
namespace MMST {  
    #define lb(x) ((x) & -(x))  
    const int N = 101010, Inf = 1e9 + 7;  
  
    vector<pair<int, pii> > E;  
    vi V;  
  
    pii mi[N];  
    void init() { rep(i, 1, sz(V) + 1) mi[i] = mp(Inf, Inf); }  
    void upd(int p, pii c) {  
        p = sz(V) + 1 - p;  
        for( ; p <= sz(V); p += lb(p)) mi[p] = min(mi[p], c);  
    }  
    pii qry(int p) {  
        p = sz(V) + 1 - p;  
        pii ans = mp(Inf, Inf);  
        for( ; p >= 1; p ^= lb(p)) ans = min(ans, mi[p]);  
        return ans;  
    }  
  
    int F(int x) { return lower_bound(all(V), x) - V.begin() + 1; }  
    void _solve(vector<pair<pii, int> > v) {  
        V.clear();  
        rep(i, 0, sz(v)) v[i].fi.se -= v[i].fi.fi, V.pb(v[i].fi.se);  
        sort(all(V));  
        V.erase(unique(all(V)), V.end());  
        sort(all(v));  
        reverse(all(v));  
        init();  
        for(auto u : v) {  
            pii t = qry(F(u.fi.se));  
            int s = u.fi.fi * 2 + u.fi.se;  
            if(t.se != Inf) E.pb(mp(t.fi - s, mp(t.se, u.se)));  
            upd(F(u.fi.se), mp(s, u.se));  
        }  
    }  
    void solve(vector<pair<pii, int> > v) {  
        _solve(v);  
    }  
}
```

```

rep(i, 0, sz(v)) swap(v[i].fi.fi, v[i].fi.se);
_solve(v);
rep(i, 0, sz(v)) v[i].fi.fi *= -1;
_solve(v);
rep(i, 0, sz(v)) swap(v[i].fi.fi, v[i].fi.se);
_solve(v);
}
}

```

## 6.11 ManhattanDistance

```

(x, y) -> (x + y, x - y)      Manhattan distance -> Chebyshev distance
(x, y) -> (x + y >> 1, x - y >> 1) Chebyshev distance -> Manhattan distance

```

## 6.12 MaxMatch

```

namespace MaxMatch {
const int N = 1050;
int link[N], vis[N], use[N], in[N];
queue<int> Q;
int dfs(int u, vi g[]) {
for(auto v : g[u]) {
if(!vis[v]) {
vis[v] = 1;
if(!link[v] || dfs(link[v], g)) { return link[v] = u, 1; }
}
}
return 0;
}
int solve(int n, int m, vi g[]) {
fill_n(link, m+1, 0);
int ret = 0;
rep(i, 1, n+1) {
fill_n(vis, m+1, 0);
ret += dfs(i, g);
}
return ret;
}
}

```

```

}
void MVC(int n, vi g[]) {
fill_n(vis, n+1, 0);
per(i, 1, n+1) link[link[i]] = i;
rep(i, 1, n+1) if (link[i]) vis[i] = use[i] = 1, Q.push(i);
while (!Q.empty()) {
int u = Q.front(); Q.pop();
if (use[u] == 1) {
for (auto v : g[u]) {
use[v] = 2;
if (!vis[v]) vis[v] = 1, Q.push(v);
}
}
}
}
int v = link[u];
use[v] = 1;
if (!vis[v]) vis[v] = 1, Q.push(v);
}
}

```

```

}
rep(i, 1, n+1) if (link[i] && !use[link[i]]) use[i] = 2;
return;
}
}

```

## 6.13 Max\_clique\_BK

```

//g[i][i] should be 0
//g[i] is i's edge
//index [0..N)
//O(n ^ 3)
typedef unsigned long long T;
struct BK {
static const int N = 100; T g[N];
inline int ctz(T s){ return s ? __builtin_ctzll(s) : 64;}
int n, ans;
void ini(int _n) {
//per(i, 0, n = _n) g[i] = (1ull << n) - 1 - (1ull << i); }
n = _n; rep(i, 0, n) g[i] = 0;
rep(i, 0, n) rep(j, 0, n) if (a[i][j]) g[i] |= 1ull << j;
}
void gao(T cur, T can, T ban) {
if (!can && !ban) { ans = max(ans, __builtin_popcountll(cur)); return; }
if (!can) return;
int piv = ctz(can | ban), ret = 0;
T z = can & ~g[piv];
for(int u = ctz(z); u < n; u += ctz(z >> (u + 1)) + 1) {
gao(cur | (1ull << u), can & g[u], ban & g[u]);
can ^= 1ull << u, ban |= 1ull << u;
}
}
int run() { gao(ans = 0, (1ull << n) - 1, 0); return ans; }
} bk;

```

## 6.14 Max\_clique\_fastest

```

const int N = 130;
typedef bool BB[N];
struct Maxclique {
const BB *e; int pk, lv; db Tlimit;
struct ve {int i, d; ve(int i): i(i), d(0) {}}; //ve : Vertex
struct sc {int a, b; sc() : a(0), b(0) {}}; //sc : StepCount
typedef vector<ve> ves; ves V; //ves: Vertices
typedef vector<int> cc; cc Q, QMAX; //cc : ColorClass
vector<cc> C;
vector<sc> S;
Maxclique(BB *conn, int sz, const db tt = 0.025): pk(0), lv(1), Tlimit(tt) {
rep(i, 0, sz) V.pb(ve(i)); e = conn;
C.resize(sz + 1);
S.resize(sz + 1);
}
static bool desc_deg(const ve &a, const ve &b) { return a.d > b.d; }
void ini_col(ves &v) { per(i, 0, sz(v)) v[i].d = min(i, v[0].d) + 1; }
}

```

```

bool spfa(){
    queue<int> Q;
    fill(dis,dis+n,inf);
    ing[s] = true , dis[s] = 0;
    Q.push(s);
    while(!Q.empty()){
        int c = Q.front();Q.pop();ing[c] = false;
        for(int k=h[c];~k;k=ne[k]){
            int v = to[k];
            if(cap[k] <= 0) continue;
            if(dis[c] + cost[k] < dis[v]){
                dis[v] = dis[c] + cost[k];
                pre[v] = k;
                if(ing[v]) Q.push(v) , ing[v] = true;
            }
        }
        return dis[t] != inf;
    }
    U flow;V mincost;
    pair<U,V> run(int _s,int _t){
        s = _s , t = _t;
        flow = mincost = 0;
        while(spfa()){
            U pl = inf;int p , k;
            for(p=t;p!=s;p=to[k^1]) pl = min(pl , cap[k=pre[p]]);
            for(p=t;p!=s;p=to[k^1]){
                k = pre[p];
                cap[k] -= pl;
                cap[k^1] += pl;
            }
            mincost += pl * dis[t];
            flow += pl;
        }
        return make_pair(flow , mincost);
    }
};

```

## 6.16 SCC

```

// _ starts from 0
namespace SCC{
    const int N = 100050;
    int dfn[N],low[N],id[N],st[N],_st,_cc;
    void dfs(int c,vi g[]){
        dfn[c]=low[c]=++cc;
        st[_st++]=c;
        for(auto t:g[c])
            if(!dfn[t])
                dfs(t,g),low[c]=min(low[c],low[t]);
            else if(!id[t])
                low[c] =min(low[c],dfn[t]);
        if(low[c]==dfn[c]){
            ++_i;
            do{id[st[_st]]=_i;}while(st[_st]!=c);
        }
    }
}

```

```

void set_deg(ves &v) { rep(i, 0, sz(v)){v[i].d = 0; rep(j, 0, sz(v)) v[i].d += e[v[i]
].i[v[j].i]; } }
void deg_sort(ves &R) { set_deg(R); sort(all(R), desc_deg); }
bool cut1(int pi, cc &va) { rep(i, 0, sz(va)) if (e[pi][va[i]]) return true;
return false; }
void cut2(ves &va, ves &vb) { rep(i, 0, sz(va) - 1) if (e[va.back().i][va[i].i]) vb.
pb(va[i].i); }
void co_sort(ves &R) {
    int j = 0, maxno = 1, min_k = max(sz(QMAX) - sz(Q) + 1, 1);
    rep(i, 1, 3) C[i].clear();
    rep(i, 0, sz(R)) {
        int pi = R[i].i, k = 1;
        while (cut1(pi, C[k])) k++;
        if (k > maxno) C[(maxno = k) + 1].clear(); C[k].pb(pi);
        if (k < min_k) R[j++] .i = pi;
    }
    if (j > 0) R[j - 1].d = 0;
    rep(k, min_k, maxno + 1)
        rep(i, 0, sz(C[k]))
            R[j].i = C[k][i], R[j++].d = k;
}
void exp_dyn(ves &R) { // expand_dyn
    S[lv].a += S[lv - 1].a - S[lv].b;
    S[lv].b = S[lv - 1].a;
    for (; sz(R); Q.pop_back(), R.pop_back()) {
        if (sz(Q) + R.back().d <= sz(QMAX)) return;
        Q.pb(R.back().i);
        ves Rp; cut2(R, Rp);
        if (sz(Rp)) {
            if ((db) S[lv].a / ++pk < Tlimit) deg_sort(Rp);
            co_sort(Rp); S[lv++] .a++;
            exp_dyn(Rp); --lv;
        } else if (sz(Q) > sz(QMAX)) QMAX = Q;
    }
}
void mcdyn(int *mxc, int &sz) { // mcdyn(int maxclique, int &sz)
    set_deg(V); sort(all(V), desc_deg);
    ini_col(V); rep(i, 0, sz(V) + 1) S[i].a = S[i].b = 0;
    exp_dyn(V); per(i, 0, sz(QMAX)) mxc[i] = QMAX[i];
    sz = sz(QMAX);
}
};

```

## 6.15 MinCostMaxFlow

```

// [0,n) , init!! , inf modify
template<class U,class V>
struct MCMF{
    static const int N = 204 , M = 101010;
    int h[N] , ing[N] , pre[N] , to[M] , ne[M] , e , s , t , n;
    U cap[M];V dis[N] , cost[M];
    void ini(int _n = N){ fill(h , h + (n=_n) , -1);e = 0;}
    void liu(int u,int v,U c,V w){ to[e] = v;ne[e] = h[u];cap[e] = c;cost[e] = w;h[u] =
e++;}
    void link(int u,int v,U c,V w){ liu(u,v,c,w);liu(v,u,0,-w); }
}

```

```

    }
    vi ng[N];
    int solve(int n, vi g[]){
        fill_n(dfn, n, cc=0);
        fill_n(low, n, st=0);
        fill_n(id, n, _=0);
        rep(i, 0, n) if(idfn[i]) dfs(i, g);
        rep(i, 0, n) _id[i];
        fill_n(ng, _vi());
        rep(i, 0, n) for(auto j: g[i]) if(id[i]!=id[j]) ng[id[i]].pb(id[j]);
        return _;
    }
}

```

## 6.17 SteinerTree

// 要视图的情况使用 *spfa*, *dijkstra*, 多源 *bfs*

```

const int N = 11, M = 10;
const int inf = 0x3f3f3f3f;
int n, m, k, a[N][N], st[N][N], dp[1 << M][N][N], S, ans;
bool use[N][N], vis[1 << M][N][N];
int dx[] = {1, -1, 0, 0};
int dy[] = {0, 0, 1, -1};
queue<pii> q;

struct node {
    int x, y, msk;
    node(int x = 0, int y = 0, int msk = 0): x(x), y(y), msk(msk) {}
} now;

pair<node, node> pre[1 << M][N][N];

```

```

void spfa(int msk) {
    while (!q.empty()) {
        pii u = q.front(); q.pop();
        int x = u.fi, y = u.se;
        vis[msk][x][y] = 0;
        rep(i, 0, 4) {
            int nx = x + dx[i], ny = y + dy[i], t = msk | st[nx][ny];
            if (nx > n || nx < 1 || ny > m || ny < 1) continue;
            int &z = dp[t][nx][ny], w = dp[msk][x][y] + a[nx][ny];
            if (z > w) {
                z = w, pre[t][nx][ny] = mp(node(x, y, msk), node(x, y, 0));
                if (t == msk && !vis[msk][nx][ny]) {
                    vis[msk][nx][ny] = 1;
                    q.push(mp(nx, ny));
                }
            }
        }
    }

    void dfs(node now) {
        pair<node, node> t = pre[now.msk][now.x][now.y];
    }
}

```

```

node t1 = t.fi, t2 = t.se;
use[now.x][now.y] = 1;
if (!t1.x) return;
dfs(t1);
if (t2.msk) dfs(t2);
}

int SteinerTree(int n, int m) {
    memset(dp, 0x3f, sizeof(dp));
    rep(i, 1, n+1) rep(j, 1, m+1) {
        cin >> a[i][j];
        if (!a[i][j]) {
            st[i][j] = pw(k);
            dp[pw(k)][i][j] = 0;
            pre[pw(k+1)][i][j] = mp(node(0, 0, 0), node(0, 0, 0));
        }
        S = pw(k) - 1;
        rep(msk, 1, S+1) {
            rep(i, 1, n+1)
                rep(j, 1, m+1) {
                    if (st[i][j] && !(st[i][j] & msk)) continue;
                    int &z = dp[msk][i][j];
                    for (int t = msk & (msk - 1); t > 0; t = (t - 1) & msk) {
                        int t1 = t | st[i][j], t2 = msk ^ t | st[i][j];
                        int w = dp[t1][i][j] + dp[t2][i][j] - a[i][j];
                        if (z > w) z = w, pre[msk][i][j] = mp(node(i, j, t1), node(i, j, t2));
                    }
                    if (z < inf) q.push(mp(i, j)), vis[msk][i][j] = 1;
                }
            spfa(msk);
        }
        ans = inf;
        rep(i, 1, n+1) rep(j, 1, m+1) if (ans > dp[S][i][j])
            ans = dp[S][i][j], now = node(i, j, S);
        dfs(now);
        return ans == inf ? -1 : ans;
    }
}

```

## 6.18 StoerWagner\_O(n3)

```

struct StoerWagner{
    static const int N = 305;
    static const int INF = 0x3f3f3f3f;;
    int n;
    int g[N][N], val[N];
    bool vis[N], use[N];
    void init(int _n) {
        n = _n;
        fill_n(use + 1, n, 0);
        rep(i, 1, n+1) fill_n(g[i] + 1, n, 0);
    }
    void add_edge(int u, int v, int w) {
        g[u][v] += w;
        g[v][u] += w;
    }
}

```

```

}
void merge(int u, int v) {
    rep(i, 1, n+1) {
        g[v][i] += g[u][i];
        g[i][v] += g[i][u];
    }
    use[u] = 1;
}

int MinimumCutPhase(int cnt, int &s, int &t) {
    fill_n(val + 1, n, 0);
    fill_n(vis + 1, n, 0);
    t = 1;
    while (--cnt) {
        vis[s = t] = 1;
        rep(i, 1, n+1) if (!vis[i] && !use[i]) val[i] += g[t][i];
        int ma = 0;
        rep(i, 1, n+1) if (!vis[i] && !use[i] && val[i] >= ma) ma = val[i], t = i;
        if (!ma) return 0;
    }
    return val[t];
}

int solve() {
    int res = INF;
    for (int i = n, s, t; i > 1; --i) {
        res = min(res, MinimumCutPhase(i, s, t));
        if (res == 0) break;
        merge(s, t);
    }
    return res;
}

} Sw;

```

### 6.19 StoerWagner\_O(nmlog(m))

```

struct StoerWagner {
    static const int N = 3005, M = 100005 * 2, INF = 0x3f3f3f3f;
    int head[N], val[N], e, n;
    int to[M], ne[M], data[M];
    bool vis[N];
    int fa[N], link[N];
    void init(int _n) {
        n = _n;
        fill_n(head + 1, n, -1);
        fill_n(link + 1, n, -1);
        rep(i, 1, n+1) fa[i] = i;
        e = 0;
    }

    void add_edge(int u, int v, int w) {
        to[e] = v; data[e] = w; ne[e] = head[u]; head[u] = e++;
        to[e] = u; data[e] = w; ne[e] = head[v]; head[v] = e++;
    }

    int findset(int u) {
        return u == fa[u] ? u : fa[u] = findset(fa[u]);
    }

    void merge(int u, int v) {

```

```

int p = u;
while (~link[p]) p = link[p];
link[p] = v;
fa[v] = u;
}

int MinimumCutPhase(int cnt, int &s, int &t) {
    fill_n(val + 1, n, 0);
    fill_n(vis + 1, n, 0);
    priority_queue<pii> q;
    t = 1;
    while (--cnt) {
        vis[s = t] = 1;
        for (int u = s; ~u; u = link[u]) {
            for (int p = head[u]; ~p; p = ne[p]) {
                int v = findset(to[p]);
                if (!vis[v]) q.push(mp(val[v] += data[p, v]));
            }
        }
        while (!q.empty() && (vis[q.top().se] || val[q.top().se] != q.top().fi)) {
            q.pop();
        }
        if (q.empty()) return 0;
        t = q.top().se; q.pop();
    }
    return val[t];
}

int solve() {
    int res = INF;
    for (int i = n, s, t; i > 1; --i) {
        res = min(res, MinimumCutPhase(i, s, t));
        if (res == 0) break;
        merge(s, t);
    }
    return res;
}

} Sw;

```

### 6.20 生成树计数与欧拉回路方案数

```

// d[][];
// i!=j d[i][j]=0
// i==j d[i][j]=in_deg(i)
// b[][];
// from i to j has b[i][j] directed edges
// a[][] = d[][] - b[][]

// 无向图生成树个数: a[][] 任何一个 n-1 阶主子式的绝对值
// 有向图以 i 为根的生成树个数: a[][] 去掉第 i 行第 i 列的行列式的绝对值

int det(int n) { // det(a[1..n-1][1..n-1])
    int ans=1;
    rep(i, 1, n) {
        rep(j, i+1, n) while(a[j][i]) {
            int t = a[j][i] / a[j][i];
            rep(k, i, n) a[i][k] = sub(a[i][k], mul(a[j][k], t)), swap(a[i][k], a[j][k]);

```

```
B[0] = 1;
rep(i, 1, N) {
    B[i] = 0;
    rep(j, 0, i) B[i] = add(B[i], MOD - mul(C[i + 1][j], B[j]));
    B[i] = mul(B[i], qpow(C[i + 1][i], MOD - 2)) % MOD;
}
}
int cal(int n, int k) {
    int sum = 0;
    rep(i, 0, k + 1) sum = add(sum, mul(C[k + 1][i], mul(B[i], qpow(n, k + 1 - i))));
    return mul(sum, qpow(k + 1, MOD - 2));
}
};
```

7.3 CRT

```
const int N = 1e5+7;
ll a[N], mod[N];

struct CRT{
    int M, R;
    void exgcd(ll a, ll b, ll &x, ll &y){
        if(b == 0) { x = 1; y = 0; return; }
        exgcd(b, a % b, y, x);
        y -= a / b * x;
    }
    ll Inv(ll a, ll mod){
        ll x = 0, y = 0;
        exgcd(a, mod, x, y);
        x %= mod;
        return x < 0 ? x + mod : x;
    }
    ll CRT(int n, ll *a, ll *mod){
        M = mod[1], R = a[1];
        rep(i, 2, n+1) {
            ll g = __gcd(M, mod[i]);
            ll inv = Inv(M / g, mod[i] / g);
            if ((a[i] - R) % g) return -1; // 无解
            R += inv * ((a[i] - R) / g) % (mod[i] / g) * M;
            M = M / g * mod[i];
            R = (R % M + M) % M; // 可能为 0 看是否需要是正整数
        }
        return R;
    }
} crt;
```

7.4 EulerPower

```
// a[1] ^ a[1+1] ^ a[1+2] ... ^ a[r] % mod 注意结果要再模mod
map<int, int> M;

int phi(int n) {
    if (M.count(n)) return M[n];
    int r = n, nn = n;
```

```
ans = P - ans;
}
if(a[i][i] == 0) return 0;
ans = mul(ans, a[i][i]);
}
return ans;
}

// 有向图要判断每个点的出度入度是否相等
// 无向图要转换成有向图
// tw(G): 以 w 为根的生成树个数
// ec(G) = tw(G) * pi(deg[v] - 1)!
// ans = ec(G) * deg[w]; 如果求的不是本质不同的, 就还需要这个
// 本质相同: 1231341 1341231
// 本质不同: 1231341 1312341
```

7 Math

7.1 BerlekampMassey

```
// O(1en^2)
vi BM(vi s) {
    vi C(1, 1), B(1, 1);
    int L = 0, m = 1, b = 1;
    rep(n, 0, sz(s)) {
        ll d = 0;
        rep(i, 0, L+1) (d += 1ll * C[i] * s[n-i]) %= P;
        if(d == 0) ++m;
        else {
            vi T = C;
            ll c = P - d * kpow(b, P - 2) % P;
            while(sz(C) < sz(B) + m) C.pb(0);
            rep(i, 0, sz(B)) C[i + m] = add(C[i + m], mul(c, B[i]));
            if(2 * L <= n) L = n + 1 - L, B = T, b = d, m = 1;
            else ++m;
        }
    }
    reverse(all(C));
    rep(i, 0, sz(C)) C[i] = P - C[i];
    return vi(C.begin(), C.end() - 1);
}
```

7.2 Bernoulli

```
// desc : 0^k + 1^k + 2^k + .. + (n-1)^k
// time-init : O(n^2)
// time-cal : k + log
namespace Bernoulli {
    const int N = 1000;
    int C[N][N], B[N];
    void ini() {
        rep(i, 0, N) C[i][0] = 1;
        rep(i, 0, N) rep(j, 1, i + 1) C[i][j] = add(C[i - 1][j - 1], C[i - 1][j]);
    }
```

## 7.5 FFT

```

const int M = 1 << 17 << 1;
const db pi = acos(-1);

struct vir{
    db r, i;
    vir(db r = 0.0, db i = 0.0) : r(r), i(i){}
    void print() {printf("%f %f\n", r, i);}
    vir operator +(const vir &c) {return vir(r + c.r, i + c.i);}
    vir operator -(const vir &c) {return vir(r - c.r, i - c.i);}
    vir operator *(const vir &c) {return vir(r * c.r - i * c.i, r * c.i + i * c.r);}
} a[M], b[M], w[2][M];

struct FFT{
    int N, na, nb, rev[M];
    void fft(vir *a, int f){
        vir x, y;
        rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int i = 1; i < N; i <= 1)
            for (int j = 0, t = N/(i<1); j < N; j += i<1)
                x = w[f][1] * a[j+k+i], y = a[j+k], a[j+k] = y+x, a[j+k+i] = y-x;
        if (f) rep(i, 0, N) a[i].r /= N;
    }
};

```



```

void fft(vir x[], int k, int v){
    for(int i=0, j=0; i<k; i++){
        if(i>j)swap(x[i], x[j]);
        for(int l=k>>1; (j^=1)<l; l>>=1);
    }
    w[0] = vir(1, 0);
    for(int i=2; i<=k; i<=1){
        vir g = vir(cos(2*pi/i), (v ? -1 : 1) * sin(2*pi/i));
        for(int j=(i>>1); j>=0; j--=2) w[j] = w[j>>1];
        for(int j=1; j<i>>1; j+=2) w[j] = w[j-1] * g;
        for(int j=0; j<k; j+=i){
            vir *a = x+j, *b = a+(i>>1);
            for(int l=0; l<i>>1; l++){
                vir o = b[l] * w[l];
                b[l] = a[l] - o;
                a[l] = a[l] + o;
            }
        }
    }
    if (v) for(int i=0; i<k; i++) x[i] = vir(x[i].a/k, x[i].b/k);
}

void doit(int *a, int *b, int na, int nb) {
    for(K=1; K<= na*nb>>1; K<= 1);
    rep(1, 0, K) x[1] = y[1] = vir(0, 0);
    for(int i=0; i<=na; i++) (i&1 ? x[i>>1].b : x[i>>1].a) = a[i];
    for(int i=0; i<=nb; i++) (i&1 ? y[i>>1].b : y[i>>1].a) = b[i];
    fft(x, K, 0);
    fft(y, K, 0);
    rep(1, 0, K){
        int j = K-1 & K-i;
        vir tmp = (i&k>>1) ? vir(1, 0) - w[i&k>>1] : w[1] + vir(1, 0);
        z[i] = (x[i]*y[i])*4 - (x[i] - !x[j])*(y[i] - !y[j])*tmp*0.25;
    }
    fft(z, K, 1);
    rep(i, 0, na*nb+1) a[i] = i&1 ? z[i>>1].b + 0.1 : z[i>>1].a + 0.1;
}

```

## 7.8 Fib

```

// sum(fib[1..n]) + 1=fib[n+2]
// gcd(fib[n], fib[m]) = fib[gcd(n, m)]

```

## 7.9 GaussDB

```

namespace GaussDB{
    static const int N = 505;
    db a[N][N], x[N]; //增广矩阵和解集
    bool ok[N]; // 标记变元是否确定
    int free[N], free_num; // 一组合法自由变元
    const db eps = 1e-14;
    int Gauss(int equ, int var){
        int k, col, p;
        fill_n(ok, var, 0); free_num = 0;
    }
}

```

```

}
}
}
vir A[MAXN], B[MAXN], C[MAXN], D[MAXN];

void doit(int *a, int *b, int na, int nb){
    for (N=1; N<na+nb-1; N<=1);
    L=15;
    MASK = (1<<L) - 1;
    rep(i, 0, N) w[i] = vir(cos(2 * i * PI / N), sin(2 * i * PI / N));
    rep(1, 0, N) {
        A[i] = vir(a[i] >> L, a[i] & MASK);
        B[i] = vir(b[i] >> L, b[i] & MASK);
    }
}

void mul() {
    FFT(A, N), FFT(B, N);
    rep(1, 0, N) {
        int j = (N - i) % N;
        vir da = (A[i] - conj(A[j])) * vir(0, -0.5),
            db = (A[i] + conj(A[j])) * vir(0.5, 0),
            dc = (B[i] - conj(B[j])) * vir(0, -0.5),
            dd = (B[i] + conj(B[j])) * vir(0.5, 0);
        C[i] = da * dd + da * dc * vir(0, 1);
        D[j] = db * dd + db * dc * vir(0, 1);
    }
    FFT(C, N), FFT(D, N);
    for (int i = 0; i < N; ++i) {
        ll da = (ll)(C[i].i / N + 0.5) % P,
            db = (ll)(C[i].r / N + 0.5) % P,
            dc = (ll)(D[i].i / N + 0.5) % P,
            dd = (ll)(D[i].r / N + 0.5) % P;
        a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % P;
    }
}

```

## 7.7 FFT\_fast

```

const int N = 1 << 21;
const double pi=acos(-1.0);
struct vir{
    double a,b;
    vir(double r=0.0,double i=0.0) {a=r,b=i;}
    vir operator +(const vir &o) const{return vir(a+o.a,b+o.b);}
    vir operator -(const vir &o) const{return vir(a-o.a,b-o.b);}
    vir operator *(const vir &o) const{return vir(a*o.a-b*o.b,b*o.a+a*o.b);}
    vir operator /(const double &o) const{return vir(a*o,b*o);}
    vir operator !() const{return vir(a,-b);}
} x[N+1], y[N+1], z[N+1], w[N+1];

int K;

```

```

fill_n(x, var, 0);
for(k = col = 0; k < equ && col < var; ++k, ++col){
    p = k; rep(i, k+1, equ) if (fabs(a[i][col]) > fabs(a[p][col])) p = i;
    if (p != k) rep(j, k, var+1) swap(a[p][j], a[k][j]);
    if (fabs(a[k][col]) < eps) {k--; continue;}
    int inv = kpow(a[k][col], P - 2);
    rep(i, col, var+1) a[k][i] = mul(a[k][i], inv);
    rep(i, k+1, equ){
        if (!a[i][col]) continue;
        int t = a[i][col];
        rep(j, col, var+1) a[i][j] = add(a[i][j], -mul(a[k][j], t));
    }
    rep(i, k, equ) if (a[i][var]) return -1;//无解
    if (k < var){
        /*
        int pre = var;
        per(i, 0, k) {
            int num = 0;
            rep(j, 0, var) if (a[i][j]) {
                if (!num) p = j; num++;
            }
            rep(j, 0, i) if (a[j][p]) {
                int t = a[j][p];
                rep(l, p, var+1) a[j][l] = add(a[j][l], -mul(a[i][l], t));
            }
            rep(j, p+1, pre) free[free_num++] = j; pre = p;
            if (num > 1) continue;
            x[p] = a[i][var];
            ok[p] = 1;
        }*/
        return var - k;//自由变元个数
    }
    per(i, 0, var) {
        int t = a[i][var];
        rep(j, i+1, var) if (a[i][j]) t = add(t, -mul(a[i][j], x[j]));
        x[i] = t;
    }
    return 0;
}
}

```

## 7.11 GaussXor

//对 2 取模的 01 方程组

```

namespace Gauss{
    static const int N = 2e3 + 10;
    //有 equ 个方程, var 个变元. 增广矩阵行数为 equ 列数为, [0..var]
    bitset<N> a[N]; //增广矩阵
    bool ok[N]; // 标记变元是否确定
    int x[N]; //解集
    int free[N], free_num; //一组合法自由变元 (多解枚举自由变元可以使用)
    //返回值为 -1 表示无解, 为 0 是唯一解, 否则返回自由变元个数
    int Gauss(int equ, int var){
        int p, col, k; // k 为增广矩阵的秩

```

```

fill_n(x, var, 0);
for(k = col = 0; k < equ && col < var; ++k, ++col){
    p = k;
    rep(i, k+1, equ) if (fabs(a[i][col]) > fabs(a[p][col])) p = i;
    if (p != k) rep(j, k, var+1) swap(a[p][j], a[k][j]);
    if (fabs(a[k][col]) < eps) {k--; continue;}
    rep(i, k+1, equ){
        if (fabs(a[i][col]) < eps) continue;
        db t = a[i][col] / a[k][col];
        rep(j, col, var+1) a[i][j] -= a[k][j] * t;
    }
    rep(i, k, equ) if (fabs(a[i][var]) > eps) return -1;//无解
    if (k < var){
        /*
        int pre = var;
        per(i, 0, k) {
            int num = 0;
            rep(j, 0, var) if (fabs(a[i][j]) > eps) {
                if (!num) p = j; num++;
            }
            rep(j, 0, i) if (fabs(a[j][p]) > eps) {
                db t = a[j][p] / a[i][p];
                rep(l, p, var+1) a[j][l] -= a[i][l] * t;
            }
            rep(j, p+1, pre) free[free_num++] = j; pre = p;
            if (num > 1) continue;
            ok[p] = 1;
            x[p] = a[i][var] / a[i][p];
        }*/
        return var - k;//自由变元个数
    }
    per(i, 0, var) {
        db t = a[i][var];
        rep(j, i+1, var) if (fabs(a[i][j]) > eps) t -= x[j] * a[i][j];
        x[i] = t / a[i][i];
    }
    return 0;
}
}

```

## 7.10 GaussInt

```

namespace GaussInt{
    static const int N = :N, P = 1e9 + 7;
    int a[N][N], x[N]; //增广矩阵和解集
    bool ok[N]; // 标记变元是否确定
    int free[N], free_num; // 一组合法自由变元
    int add(int a, int b) {if ((a += b) >= P) a -= P; return a < 0 ? a + P : a;}
    int mul(int a, int b) {return 1ll * a * b % P;}
    int kpow(int a, int b) {int r=1; for(; b; b>>=1, a=mul(a, a)) {if(b&1)r=mul(r, a);} return r;}
    int Gauss(int equ, int var){
        int k, col, p;
        fill_n(free, var, 0); free_num = 0;

```

```

fill_n(ok, var, 0); free_num = 0;
fill_n(x, var, 0);
for(k = 0, col = 0; k < equ && col < var; k++, col++){
    p = k; rep(i, k, equ) if (a[i][col]) {p = i; break;}
    if (p != k) swap(a[k], a[p]);
    if (!a[k][col]){
        k--; free[free_num++] = col; //这个是自由变元
        continue;
    }
    rep(i, 0, equ) if (i != k && a[i][col]) a[i] ^= a[k];
}
rep(i, col, var) free[free_num++] = i;
rep(i, k, equ) if (a[i][var]) return -1;
if(k < var) {
    /*
    per(i, 0, k) {
        int num = 0;
        rep(j, 0, var) if(a[i][j]) {
            if (!num) p = j; num++;
        }
        if(num > 1) continue;
        ok[p] = 1; x[p] = a[i][var];
    }*/
    return var - k; //自由变元个数
}
//唯一解，回代
per(i, 0, var){
    bool t = a[i][var];
    rep(j, i+1, var) t ^= (a[i][j] && x[j]);
    x[i] = t;
}
return 0;
}
}

```

## 7.12 LinearBasis

```

struct Base{
    ll a[63];
    Base() {memset(a, 0, sizeof(a));}
    void ins(ll x){
        for(int i=62; ~i; ~i) {
            if(x>>i&1) {
                if(a[i]) x^=a[i];
                else{ a[i]=x; break; }
            }
        }
    }
};

```

## 7.13 LinearRecursion

```

// a_{m} = \sum_{j=0}^{m-1} a_{-mj} * c_{-j}  O(m^2lgm)
int linear_recurrence(ll n, int m, vi a, vi c) {

```

```

if (n<m) return (a[n]+P)%P;
vector<ll> v(m, 0), u(m<=1, 0);
v[0] = 1;
for(ll x = 0, W = n ? 1ll<=(63 - __builtin_clzll(n)) : 0; W>= 1, x <= 1) {
    fill(all(u), 0);
    int b = !(n & W); if(b) x++;
    if(x < m) u[x] = 1;
    else {
        rep(i, 0, m) rep(j, 0, m) (u[i + b + j] += v[i] * v[j]) %= P;
        per(i, m, 2*m) rep(j, 0, m) (u[i - m + j] += c[j] * u[i]) %= P;
    }
    copy(u.begin(), u.begin() + m, v.begin());
}
ll ans = 0;
rep(i, 0, m) (ans += v[i] * a[i]) %= P;
return (ans+P)%P;
}

```

## 7.14 MathFunction

```

const int N = 1e6 + 7;
int n, M, f[N], g[N], h[N], phi[N], u[N], p[N]; // f[n] 为 n 的最小质因子, g[n]=f[n]^k,
phi[n] 为欧拉函数, u[n] 为莫比乌斯函数, h[n] 为一般积性函数

void prime(int n) {
    u[1]=phi[1]=1, h[1]=(0); // 1 的时候特判
    rep(i, 2, n+1) {
        if (!f[i]) {
            p[++M]=i;
            f[i] = g[i] = i;
            phi[i] = i - 1;
            u[i] = -1;
            h[i] = (0);
        } // 质数的时候特判
        for (int j = 1, k; j <= M && p[j] <= f[i] && i * p[j] <= n; j++){
            f[k = i * p[j]] = p[j];
            if (p[j] < f[i]) {
                g[k] = p[j];
                phi[k] = phi[i] * phi[p[j]];
                u[k] = u[i] * u[p[j]];
                h[k] = h[i] * h[p[j]];
            } else {
                g[k] = g[i] * p[j];
                phi[k] = phi[i] * p[j];
                u[k] = 0;
                h[k] = h[i] / g[i] * (0);
            } // 质数幂特判
            // phi[i * p[j]] = phi[i] * phi[p[j]] < f[i] ? phi[p[j]] : p[j];
            // u[i * p[j]] = u[i] * (p[j] < f[i] ? u[p[j]] : 0);
        }
        // phi[i * j] = phi[i] * phi[j] (gcd(i, j)=1)
        // phi[i * j] = phi[i] * phi[j] (gcd(i, j)=1)
        // phi[i * j] = phi[i] * phi[j] (gcd(i, j)=1)
        // phi[i * j] = phi[i] * phi[j] (gcd(i, j)=1)
    }
}

```

```
}
}
```

## 7.15 NTT

```
const int M = 1 << 17 << 1;
int a[M], b[M];

struct NTT{
    static const int G = 3, P = 1004535809; //P = C*2^k + 1
    int N, na, nb, w[2][M], rev[M];
    ll kpow(ll a, int b){
        ll c = 1;
        for (; b; b >>= 1, a = a * a % P) if (b & 1) c = c * a % P;
        return c;
    }
    void FFT(int *a, int f){
        rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
        for (int i = 1; i < N; i <= 1)
            for (int j = 0, t = N / (i <= 1); j < N; j += i <= 1)
                for (int k = 0, l = 0, x, y; k < i; k++, l += t)
                    x = (ll) w[f][l] * a[j+k+i] % P, y = a[j+k], a[j+k] = (y+x) % P, a[j+k+i]
                        = (y-x+P) % P;
        if (f) for (int i = 0, x = kpow(N, P-2); i < N; i++) a[i] = (ll)a[i] * x % P;
    }
    void work(){
        int d = __builtin_ctz(N);
        w[0][0] = w[1][0] = 1;
        for (int i = 1, x = kpow(G, (P-1) / N), y = kpow(x, P-2); i < N; i++) {
            rev[i] = (rev[i>>1] >> 1) | ((i&1) << (d-1));
            w[0][i] = (ll)x * w[0][i-1] % P, w[1][i] = (ll)y * w[1][i-1] % P;
        }
    }
    void doit(int *a, int *b, int na, int nb){ // [0, na)
        for (N = 1; N < na + nb - 1; N <= 1);
        rep(i, na, N) a[i] = 0;
        rep(i, nb, N) b[i] = 0;
        work(), FFT(a, 0), FFT(b, 0);
        rep(i, 0, N) a[i] = (ll)a[i] * b[i] % P;
        FFT(a, 1);
        //rep(i, 0, N) cout << a[i] << endl;
    }
} ntt;
```

## 7.16 Polya

```
/*
 * Burnside's Lemma
 * 首先列出所有可能的染色方案，然后找出每个置换下保持不变的方案（不动点）数。
 * 等价类数目：所有置换的不动点数的平均值。
 * Polya enumeration theorem
 */
```

```
* 一个循环的颜色需相同
*/
```

## 7.17 SternBrocotTree

```
namespace SBT {
    typedef long double db;
    typedef int U;
    typedef pair<U, U> pii;
    const U INF = 1e9 + 7;
    typedef __int128 T;
    typedef pair<T, T> V; // V = [double|long double|fraction]
    inline int cmp(const V &a, const V &b) {
        T x = a.fi * b.se - a.se * b.fi;
        return (x > 0) - (x < 0);
    }
    inline bool in(const V &a, const V &b, const V &c) {
        return 0 <= cmp(c, a) && cmp(c, b) < 0;
    }
    pii operator+(const pii &a, const pii &b) {
        return mp(a.fi + b.fi, a.se + b.se);
    }
    pii operator*(const pii &a, U x) {
        return mp(a.fi * x, a.se * x);
    }
    bool search(V v, U MAXB, pii &lo, pii &hi, int f) {
        V x;
        U l = 0, r = f > 0 ? (hi.se ? (MAXB - lo.se) / hi.se : INF) :
            (lo.se ? (MAXB - hi.se) / lo.se : INF);
        while (l + 1 < r) {
            U z = (l + r) >> 1;
            x = f > 0 ? lo + hi * z : lo * z + hi;
            f * cmp(x, v) <= 0 ? l = z : r = z;
        }
        x = f > 0 ? lo + hi * r : lo * r + hi;
        r = f * cmp(x, v) <= 0 ? r : l;
        f > 0 ? lo = lo + hi * r : hi = lo * r + hi;
        return r > 0;
    }
    pii solve(V v, U MAXB) { // find ROUND_HALF_UP(a / b) = v, b <= MAXB
        V L = mp(v.fi * 10 - 5, v.se * 10);
        V R = mp(v.fi * 10 + 5, v.se * 10);
        pii lo(0, 1), hi(1, 0);
        while (true) {
            bool ok = 0;
            //V m = mp(lo.fi + hi.fi, lo.se + hi.se);
            //if (in(L, R, m)) return mp(m.fi, m.se);
            ok |= search(v, MAXB, lo, hi, 1);
            ok |= search(v, MAXB, lo, hi, -1);
            if (!ok) break;
        }
        db t1 = (db) lo.fi / lo.se;
        db t2 = (db) hi.fi / hi.se;
    }
```

```

db t3 = (db) v.fi / v.se;
if (t2 - t3 <= t3 - t1) return hi; else return lo;
//if (in(L, R, lo)) return lo;
//if (in(L, R, hi)) return hi;
return mp(-1, -1);
}
};

```

## 7.18 min\_25

```

struct Min_25 {
    // F(i) 要拆成多个完全积性函数的和
    static const int N = 1e6 + 7;
    int Sqr, m, p[N], id1[N], id2[N], tot, cntp;
    ll g[N], sp[N], h[N], n, w[N];
    bool isp[N];
    // f(p) = p ^ k
    ll f(int p) { return 1; }

    // 要求的积性函数 F(p ^ e)
    ll F(int p, int e) { return e == 1 ? -1 : 0; }

    // 假设都是质数的完全积性函数前缀和去掉 f(1)
    ll calc(ll n) { return n - 1; }

    void prime(int n) {
        cntp = 0; isp[1] = 1;
        rep(i, 2, n+1) {
            if(!isp[i]) p[++cntp] = i;
            for(int j = 1; j <= cntp && i * p[j] <= n; j++) {
                isp[i * p[j]] = 1;
                if(i % p[j] == 0) break;
            }
        }

        rep(i, 1, cntp+1) sp[i] = sp[i-1] + f(p[i]);
        p[++cntp] = INT_MAX;
    }

    ll S(ll x, int y) {
        if(x <= 1 || p[y] > x) return 0;
        int k = (x <= Sqr ? id1[x] : id2[n/x]);
        int ret = -(g[k] - sp[y-1]); // 质数的答案
        for(int i = y; i <= tot && 1ll * p[i] * p[i] <= x; i++) {
            ll t1 = p[i], t2 = 1ll * p[i] * p[i];
            for(int e = 1; t2 <= x; e++, t1 = t2, t2 *= p[i]) {
                if (F(p[i], e)) ret += S(x / t1, i + 1) * F(p[i], e);
                ret += F(p[i], e + 1); // 合数的答案
            }
        }
        return ret;
    }

    ll solve(ll _n) {
        n = _n; if (n == 0) return 0;
        m = 0; Sqr = sqrt(n);
    }
}

```

```

tot = upper_bound(p + 1, p + cntp + 1, Sqr) - (p + 1);
for(ll i = 1, j; i <= n; i = j + 1) {
    j = n / (n / i);
    w[++m] = n / i;
    g[m] = calc(w[m]);
    w[m] <= Sqr ? id1[w[m]] = m : id2[j] = m;
}
rep(j, 1, tot + 1)
    for(int i = 1; i <= m && 1ll * p[j] * p[j] <= w[i]; i++) {
        ll t = w[i] / p[j];
        int k = t <= Sqr ? id1[t] : id2[n / t];
        g[i] -= f(p[j]) * (g[k] - sp[j - 1]);
    }
return S(n, 1) + 1;
}
} _U;

```

## 7.19 polynomial

```

template<class T>
struct polynomial {
    static const int N = 101010;
    static const int P = 998244353;
    T a1[N], b1[N], c[N], a[N], pre[N], suf[N], ifac[N], fac[N];
    T add(T a, T b) { a = (a + b) % P; return a < 0 ? a + P : a; }
    T mul(T a, T b) { a = 1ll * a * b % P; return a < 0 ? a + P : a; }
    T kpow(T a, T b) { T r = 1; for(; b; b >>= 1, a = mul(a, a)) { if(b & 1) r = mul(r, a); } return r; }
    void calc(int n, T *a, T *b) {
        fill_n(c, n+1, 0);
        rep(i, 0, n+1) rep(j, 0, 2) c[i+j] = add(c[i+j], mul(a[i], b[j]));
        memcpy(a, c, sizeof(a[0]) * (n+1));
    }
    void solve(int n, T *x, T *y) { // a[0]*x^n ... a[n]*x^n
        fill_n(a, n+1, 0);
        rep(i, 0, n+1) {
            fill_n(a1, n+1, 0); a1[0] = 1;
            rep(j, 0, n+1) if (j != i) a1[0] = mul(a1[0], x[i] - x[j]);
            a1[0] = mul(y[i], kpow(a1[0], P - 2));
            rep(j, 0, n+1) if (j != i) {
                b1[0] = -x[j]; b1[1] = 1;
                calc(n, a1, b1);
            }
            rep(j, 0, n+1) a[j] = add(a[j], a1[j]);
        }
    }
    T get(int n, int k, T *x, T *y) { // f(k)
        T res = 0;
        rep(i, 0, n+1) {
            T s1 = y[i], s2 = 1;
            rep(j, 0, n+1) if (j != i) s1 = mul(s1, k - x[j]);
            rep(j, 0, n+1) if (j != i) s2 = mul(s2, x[i] - x[j]);
            res = add(res, mul(s1, kpow(s2, P - 2)));
        }
        return res;
    }
}

```

```

T get(int n, int k, T *y) { // x is [1..n]
    fac[0] = 1; rep(i, 1, n+1) fac[i] = mul(fac[i-1], i);
    ifac[n] = kpow(fac[n], P - 2);
    per(i, 0, n) ifac[i] = mul(ifac[i+1], i+1);
    pre[0] = suf[n+1] = 1;
    rep(i, 1, n+1) pre[i] = mul(pre[i-1], k - i);
    per(i, 1, n+1) suf[i] = mul(suf[i+1], k - i);
    T ans=0;
    rep(i, 1, n+1){
        T s1 = mul(pre[i-1], suf[i+1]);
        T s2 = mul(ifac[i-1], ifac[n-i]);
        T fg = (n-i)&1 ? -1 : 1;
        ans = add(ans, mul(fg*s1, mul(s2, y[i])));
    }
    return ans;
}
};

```

## 7.20 polysum

```

struct polysum {
    static const int D = 101000;
    static const int P = 998244353;
    ll a[D], fac[D], ifac[D], p1[D], p2[D], h[D][2], C[D];
    ll add(ll a, ll b) {a = (a + b) % P; return a < 0 ? a + P : a;}
    ll mul(ll a, ll b) {a = 1ll * a * b % P; return a < 0 ? a + P : a;}
    ll kpow(ll a, ll b) {ll r=1; for(;b;>=1; a=mul(a,a)) {if(b&1)r=mul(r,a);} return r;}
    void init(int M) {
        fac[0] = 1; rep(i, 1, M+5) fac[i] = mul(fac[i-1], i);
        ifac[M+4] = kpow(fac[M+4], P - 2);
        per(i, 0, M+4) ifac[i] = mul(ifac[i+1], i+1);
    }
    ll calcn(int d, ll *a, ll n) { // a[0].. a[d] a[n]
        if (n <= d) return a[n];
        p1[0] = p2[0] = 1;
        rep(i, 0, d+1) p1[i+1] = mul(p1[i], (n - i) % P);
        rep(i, 0, d+1) p2[i+1] = mul(p2[i], (n - d + i) % P);
        ll ans=0;
        rep(i, 0, d+1) {
            ll s1 = mul(p1[i], p2[d - i]);
            ll s2 = mul(ifac[i], ifac[d - i]);
            ll t = mul(mul(s1, s2), a[i]);
            ans = (d-i)&1 ? add(ans, -t) : add(ans, t);
        }
        return ans;
    }
    ll Polysum(ll n, ll *a, ll m) { // a[0].. a[m] \sum_{i=0}^{n-1} a[i]
        a[m+1] = calcn(m, a, m+1);
        rep(i, 1, m+2) a[i] = add(a[i-1], a[i]);
        return calcn(m+1, a, n-1);
    }
    ll qpolysum(ll R, ll n, ll *a, ll m) { // a[0].. a[m] \sum_{i=0}^{n-1} a[i]*R^i
        if (R == 1) return Polysum(n, a, m);
        a[m+1] = calcn(m, a, m+1);
        ll r = kpow(R, P - 2), p3 = 0, p4 = 0, c, ans;

```

```

h[0][0] = 0; h[0][1] = 1;
rep(i, 1, m+2) {
    h[i][0] = mul(h[i-1][0] + a[i-1], r);
    h[i][1] = mul(h[i-1][1], r);
}
rep(i, 0, m+2) {
    ll t = mul(ifac[i], ifac[m+1-i]);
    p3 = i & 1 ? add(p3, -mul(h[i][0], t)) : add(p3, mul(h[i][0], t));
    p4 = i & 1 ? add(p4, -mul(h[i][1], t)) : add(p4, mul(h[i][1], t));
}
c = mul(kpow(p4, P - 2), -p3);
rep(i, 0, m+2) h[i][0] = add(h[i][0], h[i][1] * c);
rep(i, 0, m+2) c[i] = h[i][0];
ans = add(mul(calcn(m, c, n), kpow(R, n)), -c);
return ans;
}
};

```

## 7.21 prime

```

// time : O(n)
// low[] : optional
const int N = 1e6 + 6;
int low[N], cntp, p[N];
bool isp[N];

void getprime() {
    fill_n(isp + 2, N - 2, 1);
    rep(i, 2, N) {
        if (isp[i]) p[cntp++] = i;
        for (int j=0; j<cntp&&p[j]*i<N; j++){
            //low[p[j] * i] = p[j];
            isp[p[j] * i] = 0;
            if (i % p[j] == 0) break;
        }
    }

    // 优化版欧拉筛法 bitset 需要 O2
    const int N = 3e7 + 6;
    const int M = 2e6 + 6;
    //int low[N],
    bitset<N / 3 + 1> isp;
    int cntp, p[M];

    void getprime(int N) {
        cntp = 2; p[0] = 2; p[1] = 3;
        for (int i = 5, k = 1; i <= N; (k & 1) ? i+=2 : i+=4, k++){
            if (!isp[k]) {
                p[cntp++] = i;
                // low[i] = i;
            }
            for (int j = 2; j < cntp && p[j] * i <= N; j++) {
                //low[p[j] * i] = p[j];
                isp[p[j] * i / 3] = 1;
            }
        }
    }
}

```

```
        if (i % p[j] == 0) break;
    }
}

// 优化埃氏筛法空间最小可以不存质数
const int N = 3e8 + 6;
const int M = 2e7 + 6;
int cntp, p[M];
bitset<N / 3 + 1> bit;

void getprime(int n){
    int i, j;
    cntp = 2; p[0] = 2; p[1] = 3;
    for(i = 5, j = 1; i * i <= n; (j & 1) ? i += 2 : i += 4, j++){
        if(bit[j] == 0){
            p[cntp++] = i;
            for(int j = i * i; j <= n; j += i)
                if(j % 2 != 0 && j % 3 != 0) bit[j / 3] = 1;
        }
    }
    for(; i <= n; (j & 1) ? i += 2 : i += 4, j++) if(bit[j] == 0) p[cntp++] = i;
}
```

8 Others

8.1 BitOperation

```
// 枚举子集
for(int i = x; i; (i -= 1) & x) {
    //
}
// 统计子集的答案
rep(i, 0, n) {
    rep(j, 0, 1 << n) if(j >> i & 1) {
        upd(s[j], s[j ^ (1 << i)]);
    }
}
// 统计超集的答案
rep(i, 0, n) {
    for(int j = (1 << n) - 1; ~j; --j) if(! (j >> i & 1)) {
        upd(s[j], s[j | (1 << i)]);
    }
}

//
int __builtin_ffs (unsigned int x)
int __builtin_ffsl (unsigned long)
int __builtin_ffsll (unsigned long long)
Returns one plus the index of the least significant 1-bit of x, or if x is zero, returns zero.

//
int __builtin_clz (unsigned int x)
```

```
Returns the number of leading 0-bits in x, starting at the most significant bit position
. If x is 0, the result is undefined.

//
int __builtin_ctz (unsigned int x)
Returns the number of trailing 0-bits in x, starting at the least significant bit
position. If x is 0, the result is undefined.

//
int __builtin_popcount (unsigned int x)
Returns the number of 1-bits in x.

//
int __builtin_parity (unsigned int x)
Returns the parity of x, i.e. the number of 1-bits in x modulo 2.
```

8.2 Bitset

```
// Base
b.any(); // has 1 ?
b.none(); // all 0 ?
b.count(); // cnt of 1

b.set(); // all to 1
b.reset(); // all to 0
b.flip(); // all = 0 <-> 1

b.set(p); // b[p] = 1
b.test(p); // b[p] is 1
b.reset(p); // b[p] = 0
b.flip(p); // b[p] = 0 <-> 1

// Black tech
// __builtin_ctz in bitst
b._Find_first();
// travel all 1
for (int i = b._Find_first(); i < sz(b); i = b._Find_next(i));
```

8.3 RomanNumerals

```
const int rom[30] = {
    3000, 2000, 1000, 900, 800, 700, 600, 500, 400, 300, 200, 100,
    90, 80, 70, 60, 50, 40, 30, 20, 10,
    9, 8, 7, 6, 5, 4, 3, 2, 1
};
string smb[30] = {
    "MMM", "MM", "M",
    "CM", "DCCC", "DCC", "DC", "CD", "CCC", "CC", "C",
    "XC", "LXXX", "LXX", "LX", "L", "XL", "XXX", "XX", "X",
    "IX", "VIII", "VII", "VI", "V", "IV", "III", "II", "I"
};
string toRoman(ll d) {
    string r;
```

```
rep(i, 0, 30) if (d >= rom[i]) d -= rom[i], r += smb[i];
return r;
}
```

8.4 Strtok

```
char s[111];
gets(s);
vector<string> a;
for(char* p=strtok(s, ".,()");p; p=strtok(NULL, ".,()")) a.pb(p);
```

9 String

9.1 ACAutomaton

```
/*
 * [0, L) , N-1 is virtual , 0 is rt
 * init!!
 * addtion: end[] end[c]=end[fail[c]]
 */
struct Trie{
    static const int N = 101010 , M = 26;
    int ne[N][M] , fail[N] , fa[N] , rt , L;
    void ini(){ fill_n(ne, fail, 0) = N-1, M, 0; L = 0; rt = newnode(); }
    int newnode(){ fill_n(ne[L], M, 0); return L++; }
    void add(char *s){
        int p = rt;
        for(int i=0; s[i]; ++i){
            int c = s[i] - 'a'; // modify
            if(!ne[p][c]) ne[p][c] = newnode() , fa[L-1] = p;
            p = ne[p][c];
        }
    }
    void Build(){
        vi v; v.pb(rt);
        rep(i, 0, sz(v)){
            int c = v[i];
            rep(i, 0, M) ne[c][i] ?
                v.pb(ne[c][i]) , fail[ne[c][i]] = ne[fail[c]][i] :
                ne[c][i] = ne[fail[c]][i];
        }
    }
};
```

9.2 DoublingArray

```
namespace Doubling{
    static const int N = 101010;
    // sa[0~n]: 排名第i的后缀是以i sa[i] 开头
    // h[1~n]: S[sa[i-1]] 与 S[sa[i]] 的最长公共前缀长度为 h[i]
    int t[N] , wa[N] , wb[N] , sa[N] , h[N];
    void sort(int *x, int *y, int n, int m){
        rep(i, 0, m) t[i] = 0;
```

```
rep(i, 0, n) t[x[y[i]]]++;
rep(i, 1, m) t[i] += t[i-1];
per(i, 0, n) sa[-t[x[y[i]]]] = y[i];
}
bool cmp(int *x, int a, int b, int d){
    return x[a] == x[b] && x[a+d] == x[b+d];
}
void da(int *s, int n, int m){
    int *x=wa, *y=wb;
    rep(i, 0, n) x[i] = s[i] , y[i] = i;
    sort(x , y , n , m);
    for(int j=1, p=1; p<n; m=p, j<=<=1){
        p = 0; rep(i, n-j, n) y[p++] = i;
        rep(i, 0, n) if(sa[i] >= j) y[p++] = sa[i] - j;
        sort(x , y , n , m);
        swap(x , y); p = 1; x[sa[0]] = 0;
        rep(i, 1, n) x[sa[i]] = cmp(y, sa[i], sa[i-1], j)?p-1:p++;
    }
}
void cal_h(int *s, int n, int *rk){
    int j, k=0;
    for(int i=1; i<=n; ++i) rk[sa[i]] = i;
    for(int i=0; i<n; ++i) rk[rk[i++]] = k;
    for(k&&--k, j=sa[rk[i]-1]; s[i+k]==s[j+k]; ++k);
}
}
// rank[0~n-1]: 以 i 开头的后缀排名 rank[i]
struct DA{ // [0, n] , in[n] = 0 , n load
    static const int N = 101010;
    int p[18][N] , rk[N] , in[N] , Log[N] , n;
    void Build(){
        Doubling: da(in, n+1, 300);
        Doubling: cal_h(in, n, rk);
        Log[0] = -1; for(int i=1; i<=n; ++i) Log[i] = Log[i-1] + (i==(i&(-i)));
        for(int i=1; i<=n; ++i) p[0][i] = Doubling::h[i];
        for(int j=1; 1<j<=n; ++j){
            int lim = n+1-(1<j);
            for(int i=1; i<=lim; ++i)
                p[j][i] = min(p[j-1][i] , p[j-1][i+(1<j)>>1]);
        }
        // 某两个后缀的最长公共前缀
    }
    int lcp(int a, int b){
        a = rk[a] , b = rk[b];
        if(a > b) swap(a , b); ++a;
        int t = Log[b-a+1];
        return min(p[t][a] , p[t][b-(1<t)+1]);
    }
};
```

9.3 Exkmp

```
/*
 * s 串的每个后缀与 t 串的最长公共前缀
 * t: a b a
```



```
void Manacher(char *s,int n,int *pa){
    pa[0] = 1;
    for(int i=1,j=0;i<(n<<1)-1;++i){
        int p = i >> 1, q = i - p, r = ((j + 1)>>1) + pa[j] - 1;
        pa[i] = r < q ? 0 : min(r - q + 1, pa[(j<<1) - i]);
        while(0 <= p - pa[i] && q + pa[i] < n && s[p - pa[i]] == s[q + pa[i]])
            pa[i]++;
        if(q + pa[i] - 1 > r) j = i;
    }
}
```

9.6 PalindromicTree

```
// [0,p) , 0(even) and 1(odd) is virtual , init!!
struct Palindromic_Tree {
    static const int N = ::N, M = 26;
    int ne[N][M], fail[N], len[N], S[N], last, n, p, cnt[N], las[N];
    int newnode(int l){
        fill(ne[p], ne[p] + M, 0);
        len[p] = l;
        las[p] = n;
        cnt[p] = 0;
        return p++;
    }
    void ini(){
        p = 0;newnode(0);newnode(-1);
        S[n] = last = 0] = -1;
        fail[0] = 1;
    }
    int get_fail(int x){
        while(S[n - len[x] - 1] != S[n]) x = fail[x];
        return x;
    }
    void add(int c){
        S[++n] = c;
        int cur = get_fail(last);
        if(!ne[cur][c]){
            int now = newnode(len[cur] + 2);
            fail[now] = ne[get_fail(fail[cur])][c];
            ne[cur][c] = now;
        }
        last = ne[cur][c];
        cnt[last]++;
    }
    void build() {
        for(int i = p - 1; ~i; --i) cnt[fail[i]] += cnt[i];
    }
}pam;
```

9.7 SAIS

```
/**
 * Ensure that str[n] is the unique lexicographically smallest character in str.
 * time complexity: O(n)
 */
```

```
* nt: 0 0 1
* s: a b a c a b a
* ns: 3 0 1 0 3 0 1
*/
void exkmp(char *s,int *z,char *t,int *p){
    int lens = strlen(s);
    int lent = strlen(t);
    p[0]=0;
    for(int i=0,x=0,y=0;i<lens;++i){
        z[i] = i <= y ? min(y-i,p[i-x]) : 0;
        while(i + z[i] < lens && z[i] < lent && s[i + z[i]] == t[z[i]]) ++z[i];
        if(y <= i + z[i]) x = i, y = i + z[i];
    }
}
```

```
void Exkmp(){
    scanf("%s%s",s,t);
    exkmp(t+1,nt+1,t,nt);
    exkmp(s,ns,t,nt);
}
```

9.4 Kmp

```
/*
t: a b a
nt:-1 -1 0
s: a b a c a b a
ns: 0 1 2 -1 0 1 2
*/
void kmp(char *s,int *ns,char *t,int *nt){
    int lens = strlen(s);
    int lent = strlen(t);
    nt[0] = -1;
    for(int i=0,j=-1;i<lens;++i){
        while(j >= 0 && s[i] != t[j + 1]) j = nt[j];
        if(s[i] == t[j + 1]) ++j;
        ns[i] = j;
        if(j + 1 == lent) j = nt[j];
    }
}
void KMP(){
    scanf("%s%s",s,t);
    kmp(t+1,nt+1,t,nt);
    kmp(s,ns,t,nt);
}
```

9.5 Manacher

```
/*
 * length of pa is two size of str
 * i: [0, n) pa[i<<1] : odd string 整个回文长度为 2*pa[i<<1]-1
 * i: [0, n - 1) pa[i<<1+1] : even string 整个回文长度为 2*pa[i<<1]
 * N>2*n
 */
```

```

int sta[N<<1], cnt[N<<1];
ll c[N];
inline int gao(int k) {
    int cc = 0;
    while(top && sta[top] > k) {
        cc += cnt[top];
        cnt[top] = 0;
        c[cc] += sta[top] - max(k, sta[top-1]);
        --top;
    }
    return cc;
}
inline void push(int x, int y) {
    if(!top || sta[top] != x) sta[++top] = x;
    cnt[top] += y;
}
inline void build(int n) {
    top = 0;
    fill_n(c+1, n, 0);
    rep(i, 1, n+1) {
        int lcp = SA::ht[i], cc = gao(lcp);
        push(lcp, cc);
        push(n - SA::sa[i], 1);
    }
    gao(0);
}
};

```

## 9.8 StrHash

```

inline int upd(int a, int b, int P) {
    if((a += b) >= P) a -= P;
    return a < 0 ? a + P : a;
}
inline int mul(int a, int b, int P) {return 1ll * a * b % P; }

struct Int{
    static const int P = 1e9 + 7, Q = 1e9 + 9;
    int a, b;
    Int(int a = 0, int b = 0) : a(a), b(b) {}
    inline Int operator + (const Int &c) const { return Int(upd(a, c.a, P), upd(b, c.b, Q)); }
    inline Int operator - (const Int &c) const { return Int(upd(a, -c.a, P), upd(b, -c.b, Q)); }
    inline Int operator * (const Int &c) const { return Int(mul(a, c.a, P), mul(b, c.b, Q)); }
    inline bool operator == (const Int &c) const {return a == c.a && b == c.b;}
} _0 = Int(), _1 = Int(1, 1), B[N];

void init(int n){
    B[0] = _1; B[1] = Int(233, 233);
    rep(i, 2, n+1) B[i] = B[i-1] * B[1];
}

struct Str{

```

```

*/
namespace SA {
    const static int N = 100000 + 10;
    int sa[N], rk[N], ht[N], s[N<<1], t[N<<1], p[N], cnt[N], cur[N];
    #define pushS(x) sa[cur[s[x]]++] = x
    #define pushL(x) sa[cur[s[x]]++] = x
    #define inducedSort(v) std::fill_n(sa, n, -1); std::fill_n(cnt, m, 0);
    for (int i = 0; i < n; i++) cnt[s[i]]++;
    for (int i = 1; i < m; i++) cnt[s[i]] += cnt[i-1];
    for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;
    for (int i = n1-1; ~i; i--) pushS(v[i]);
    for (int i = 1; i < m; i++) cur[i] = cnt[i-1];
    for (int i = 0; i < m; i++) if (sa[i] > 0 && t[sa[i]-1]) pushL(sa[i]-1);
    for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;
    for (int i = n-1; ~i; i--) if (sa[i] > 0 && t[sa[i]-1]) pushS(sa[i]-1)
    void sais(int n, int m, int *s, int *t, int *p) {
        int n1 = t[n-1] = 0, ch = rk[0] = -1, *s1 = s+n;
        for (int i = n-2; ~i; i--) t[i] = s[i] == s[i+1] ? t[i+1] : s[i] > s[i+1];
        for (int i = 1; i < n; i++) rk[i] = t[i-1] && t[i] ? (p[n1] = i, n1++) : -1;
        inducedSort(p);
        for (int i = 0, x, y; i < n; i++) if (-(x = rk[sa[i]]) > 0) {
            if (ch < 1 || p[x+1] - p[x] != p[y+1] - p[y]) ch++;
            else for (int j = p[x], k = p[y]; j <= p[x+1]; j++, k++)
                if ((s[j]<<1|t[j]) != (s[k]<<1|t[k])) {ch++; break;}
            s1[y = x] = ch;
        }
        if (ch+1 < n1) sais(n1, ch+1, s1, t+n, p+n1);
        else for (int i = 0; i < n1; i++) sa[s1[i]] = i;
        for (int i = 0; i < n1; i++) s1[i] = p[sa[i]];
        inducedSort(s1);
    }
}

template<typename T>
int mapCharToInt(int n, const T *str) {
    int m = *max_element(str, str+n);
    std::fill_n(rk, m+1, 0);
    for (int i = 0; i < n; i++) rk[str[i]] = 1;
    for (int i = 0; i < m; i++) rk[i+1] += rk[i];
    for (int i = 0; i < n; i++) s[i] = rk[str[i]] - 1;
    return rk[m];
}

template<typename T>
void suffixArray(int n, const T *str) {
    int m = mapCharToInt(n, str);
    sais(n, m, s, t, p);
    for (int i = 0; i < n; i++) rk[sa[i]] = i;
    for (int i = 0, h = ht[0] = 0; i < n-1; i++) {
        int j = sa[rk[i]-1];
        while (i+h < n && j+h < n && s[i+h] == s[j+h]) h++;
        if (ht[rk[i]] = h) h--;
    }
}

};
// 出现 i 次的子串有 c[i] 个
namespace S {
    int top;

```

```
Int a; int len;
Str(Int a = _0, int len = 0) : a(a), len(len) {}
Str(int x) {a = Int(x, x); len = 1;}
inline Str operator + (const Str &c) const { return Str(a * B[c.len] + c.a, len + c.len); }
inline Str operator - (const Str &c) const { return Str(a - c.a * B[len - c.len], len - c.len); } // 减去一个前缀
inline bool operator == (const Str &c) const { return a == c.a && len == c.len; }
} ha[N], hb[N];

void init(vi &s, Str *ha) {
    rep(i, 0, sz(s)) ha[i] = i > 0 ? ha[i-1] + Str(s[i] + 1) : Str(s[0] + 1);
}

Str sub(Str *ha, int l, int r) {
    if (l > r) return Str();
    return l > 0 ? ha[r] - ha[l-1] : ha[r];
}
```

```
    }
}
void ini() {
    rt = last = L = 1;
    fill(ne[rt], ne[rt] + M, 0);
    l[0] = -1;
}
}
// BucketSort
rep(i, 1, L + 1) ++cnt[l[i]];
rep(i, 1, L + 1) cnt[i] += cnt[i - 1];
rep(i, 1, L + 1) cur[cnt[l[i]] - 1] = i;
```

10 Tree

10.1 Centroid

```
// id starts from 1
namespace Centroid {
    const int N = ::N;
    bool vis[N];
    int sz[N];
    void dfsz(int c, int fa, int Sz, int &rt) {
        sz[c] = 1;
        for(auto t : g[c]) if(!vis[t] && t != fa) dfsz(t, c, Sz, rt) , sz[c] += sz[t];
        if(!rt && sz[c] * 2 > Sz) rt = c;
    }
    void dfs(int c) {
        int rt = 0; dfsz(c, 0, 0, rt); dfsz(c, 0, sz[c], rt = 0);
        vis[rt] = true;
        /*
         * 注意计算以 rt 为起点的路径、只包含 rt 的路径
         * 注意 v != vis[rt]
         */
        for(auto t : g[rt]) if(!vis[t]) dfs(t);
    }
};
```

10.2 DsuOnTree

```
// id starts with 1
namespace QuerySubtree {
    static const int N = ::N;
    int sz[N], wson[N], par[N];
    void dfs(int c, int fa, vi g[]) {
        sz[c] = 1; par[c] = fa; int &swson[c] = 0;
        for(auto t : g[c]) if(t != fa)
            dfs(t, c, g), sz[c] += sz[t], (sz[t] >= sz[s]) && (s = t);
    }
    vi nd;
    void solve(int c, int fa, bool iswson, vi g[]) {
        for(auto t : g[c]) if(t != wson[c] && t != fa) solve(t, c, false, g);
    }
};
```

9.9 SuffixAutomaton

```
/* [0,L], 0 is virtual, 1 is rt, init!!
 * l[par[s]] + 1, l[s]
 * 好像暴力向上跳的复杂度是对的。
 */
struct SAM {
    static const int N = ::N << 1, M = 26;
    int par[N], l[N], ne[N][M];
    int rt, last, L;
    void add(int c) {
        int p = last;
        /* ex
         if(ne[p][c] && l[ne[p][c]] == l[p] + 1) {
             last = ne[p][c];
             return ;
         }
         */
        int np = ++L;
        fill(ne[np], ne[np] + M, 0);
        l[np] = l[p] + 1;
        last = np;
        while(p && !ne[p][c]) ne[p][c] = np, p = par[p];
        if(!p) par[np] = rt;
        else {
            int q = ne[p][c];
            if(l[q] == l[p] + 1) par[np] = q;
            else {
                int nq = ++L;
                l[nq] = l[p] + 1;
                copy(ne[q], ne[q] + M, ne[nq]);
                par[nq] = par[q];
                par[q] = par[np];
                while(p && ne[p][c] == q) ne[p][c] = nq, p = par[p];
            }
        }
    }
};
```

<pre>if(wson[c]) solve(wson[c] , c , true , g); for(auto t : g[c]) if(t != wson[c] &amp;&amp; t != fa) {     // 将孩子树的信息加入 } // 将当前节点的信息加入 // 查询 if(!iswson) {     // 删除整棵子树的信息 nd.clear() } } void solve(vi g[]){     dfs(1,0,g);     solve(1,0,false,g); // 如果输入是单组数据, 改成 true 可以优化 } }</pre>	<pre>dfs2(1, 0, g); } }hc;</pre>
<h3>10.3 HeavyChain</h3> <pre>// id starts with 1 struct HeavyChain{     static const int N = ::N;     int sz[N], wson[N], top[N], dep[N], id[N], _r, par[N], who[N];     void dfs(int c, int fa, vi g[]){         sz[c] = 1;         par[c] = fa;         dep[c] = dep[fa] + 1;         int &amp;s = wson[c] = top[c] = 0;         for(auto t : g[c]) if(t != fa) {             dfs(t, c, g);             sz[c] += sz[t];             if(sz[t] &gt;= sz[s]) s = t;         }     }     void dfs2(int c, int fa, vi g[]){         id[c] = ++_i;         who[_i] = c;         int s = wson[c];         if(!top[c]) top[c] = c;         if(s) top[s] = top[c], dfs2(s, c, g);         for(auto t : g[c]) if(t != fa &amp;&amp; t != s) dfs2(t, c, g);     }     void Query(int a, int b){         int fa = top[a], fb = top[b];         while(fa != fb){             if(dep[fa] &lt; dep[fb]) swap(a, b), swap(fa, fb);             // Cal id[fa] .. id[a]             a = par[fa]; fa = top[a];         }         if(dep[a] &lt; dep[b]) swap(a, b);         // Cal id[b] .. id[a]         // b is lca     }     void Build(vi g[]){         dfs(1, 0, g);         _=0;     } }</pre>	<h3>10.4 LCARMQ</h3> <pre>// N is 2 size of tree , id of nodes start from 1 struct LCARMQ{     static const int N = 101010 &lt;&lt; 1;     int a[20][N] , lft[N] , dep[N] , lg[N] , L;     int rmin(int x,int y){return dep[x] &lt; dep[y] ? x : y;}     void add(int x){ a[0][L++] = x;}     void dfs(int c,int fa,const vi g[]){         lft[c]=L;add(c);         for(auto t : g[c]) if(t!=fa) dep[t]=dep[c]+1,dfs(t,c,g),add(c);     }     void Build(const vi g[]){         L = 0;dfs(1,0,g);dep[0] = -1;         rep(i,2,L) lg[i]=lg[i&gt;&gt;1]+1;         rep(i,1,20){             int lim = L+1-(1&lt;&lt;i);             rep(j,0,lim) a[i][j] = rmin(a[i-1][j] , a[i-1][j+(1&lt;&lt;i&gt;&gt;1)]);         }         int lca(int x,int y){             x = lft[x] , y = lft[y];             if(x &gt; y) swap(x , y);             int i = lg[y-x+1];             return rmin(a[i][x] , a[i][y+1-(1&lt;&lt;i)]);         }     }; }</pre>
	<h3>10.5 LongChain</h3> <pre>struct LongChain{     static const int N = ::N;     int wson[N] , top[N] , dep[N] , lg[N];     int jump[N][20] , id[N] , who[N] , rwho[N] , _;     void dfs(int c,int fa,vi g[]){         dep[c]=1;int &amp;swson[c]=top[c]=0;         jump[c][0]=fa;rep(i,1,20) jump[c][i]=jump[jump[c][i-1]][i-1];         for(auto t:g[c]) if(t!=fa)             dfs(t,c,g),dep[c]=max(dep[t]+1,dep[c]),(dep[t]&gt;=dep[s])&amp;&amp;(s=t);     }     void dfs2(int c,int fa,int rc,vi g[]){         if(!top[c]) top[c]=c,rc=c;         who[id[c]=++_]=c;rwho[_]=rc;         int swson[c];         if(s) top[s]=top[c],dfs2(s,c,jump[rc][0],g);         for(auto t:g[c]) if(t!=fa&amp;&amp;t!=s) dfs2(t,c,t,g);     }     void Build(vi g[]){         dfs(1,0,g);_=0;dfs2(1,0,1,g);         rep(i,2,N) lg[i]=lg[i&gt;&gt;1]+1;     } }</pre>

```

if(dep[x] > dep[y]) swap(x, y);
per(i, 0, M) if(dep[pre[y][i]] >= dep[x]) y = pre[y][i];
per(i, 0, M) if(pre[x][i] != pre[y][i]) x = pre[x][i], y = pre[y][i];
if(x == y) return x;
return pre[x][0];
}
void adde(int u, int v, int id) {
    if(st[u] > st[v]) swap(u, v);
    int f = lca(u, v);
    if(f == u) {
        nds.pb(Node(id, st[u], st[v]));
    } else {
        int l = ed[u], r = st[v];
        if(l > r) swap(l, r);
        nds.pb(Node(id, l, r, f));
    }
}
// p is index in tree
void add(int p) {
}
void sub(int p) {
}
void upd(int p, int c) {
    p = dfn[p];
    cnt[p] += c;
    (cnt[p] == 1) ? add(p) : sub(p);
}
void solve(vi g[]) {
    rep(i, 0, N << 1) B[i] = i / SZ;
    dfs(1, cd = 0, g);
    // adde(u, v)
    sort(all(nds));
    int l = 1, r = 0;
    for(auto &nd : nds) {
        while(r < nd.r) upd(++r, 1);
        while(l > nd.l) upd(--l, 1);
        while(r > nd.r) upd(r--, -1);
        while(l < nd.l) upd(l++, -1);
        if(nd.lca) upd(st[nd.lca], 1);
        // save ans
        if(nd.lca) upd(st[nd.lca], -1);
    }
}
}
}

```

10.7 VTree

```

// nodes should sorted in dfs order
namespace Vtree {
    const int N = 101010;
    int tp[N], _;
    vi g[N];
    void solve(vi&v, LCARMQ&R) {
        _ = 0;
        vi del, del.pb(tp[_++] = v[0]);
    }
}

```

```

}
void solve(int c, int fa, vi g[]) {
    for(auto t : g[c]) if(t != fa) solve(t, c, g);
    if(wson[c]) {
        // upd c by wson[c], O(1) or O(log(n))
    } else {
        // c is leaf
    }
    for(auto t : g[c]) if(t != fa && t != wson[c]) {
        // brute force upd c by t
    }
    // 注意统计以 c 为起点的链的答案，注意深度的限制（两棵子树都要注意）
}
// kth_par should exist
int kth_par(int x, int k) {
    if(k == 0) return x;
    int j0 = 1 << lg[k];
    int p0 = jump[X][lg[k]];
    int j1 = k - j0;
    int del = id[p0] - id[top[p0]];
    if(del >= j1) return who[id[p0] - j1];
    else return rwho[id[top[p0]] + j1 - del];
}
}hc;

```

10.6 MoOnTree\_Path

```

/*
 * 带修改莫队：块大小  $N^{1/2/3}$  按照 l 所在块，r 所在块，time 排序
 */
namespace MoOnTree {
    const int N = 1e5, SZ = sqrt(N), M = 17;

    int cd; // starts from 1
    int dep[N], pre[N][M], st[N], ed[N], dfn[N << 1], B[N << 1], cnt[N];
    struct Node {
        int l, r, id, lca;
        Node(int id, int l, int r, int lca = 0) : id(id), l(l), r(r), lca(lca) {}
    }
    bool operator < (const Node &c) const {
        if(B[l] != B[c.l]) return B[l] < B[c.l];
        return (r < c.r) ^ (B[l] & 1);
    }
};
vector<Node> nds;

void dfs(int u, int fa, vi g[]) {
    dep[u] = dep[fa] + 1;
    pre[u][0] = fa;
    for(int i = 1; i < M && pre[u][i - 1]; ++i) pre[u][i] = pre[pre[u][i - 1]][i - 1];
    dfn[++cd] = u, st[u] = cd;
    for(auto v : g[u]) if(v != fa) dfs(v, u, g);
    dfn[++cd] = u, ed[u] = cd;
}

int lca(int x, int y) {
}

```

```
rep(i,1,sz(v)){
    int lca = R.lca(tp[_-1] , v[i]);
    vi l;while(_ > 0 && R.dep[lca] < R.dep[tp[_-1]]) l.pb(tp[_-]);
    if(_ == 0 || lca != tp[_-1]) del.pb(tp[_++] = lca);
    l.pb(tp[_-1]);del.pb(tp[_++] = v[i]);
    rep(i,1,sz(l)) g[l[i]].pb(l[i-1]);
}
rep(i,0,_-1) g[tp[i]].pb(tp[i+1]);
// root = tp[0]
// dfs()
for(auto t : del){
    // Cal()
    g[t].clear();
}
}
```