Find the critical points of the given function. Then use the second derivative test to determine if the critical points correspond to local maxima, local minima, or saddle points of the graph of the function, or if the test is inconclusive.

$f(x, y) = -x^3 - y^3 + 3xy$

$(x, y) = (\quad)$ (smaller x-value) ---Select---

$(x, y) = (\quad)$ (larger x-value) ---Select---

Find critical points

$f_x = -3x^2 + 3y$
$f_y = -3y^2 + 3x$

$0 = -3x^2 + 3y$
$0 = -3y^2 + 3x$

$0 = -x^2 + y$
$0 = -y^2 + x$

$0 = -x^2 + y$
$y^2 = x$

$0 = -x^2 + y$
$y^4 = x^2$

$y^4 = y$

$y^4 - y = 0$

$y(y^3 - 1) = 0$

$y = 0 \qquad y = 1$

$(0,0)$

$(1,1)$

Second Derivative Test

$f_x = -3x^2 + 3y$

$f_{xx} = -6x$

$f_y = -3y^2 + 3x$

$f_{yy} = -6y$

$f_{xy} = 3$

$D(x,y) = (-6x)(-6y) - (3)^2$

$= 36xy - 9$

Test Point $(0,0)$

$D(0,0) = -9$

Saddle Point

Test Point $(1,1)$

$D(1,1) = 36 - 9 = 27$

Max or min

$f_{xx}(1,1) = -6$

Local max

Find the critical points of the given function. Then use the second derivative test to determine if the critical points correspond to local maxima, local minima, or saddle points of the graph of the function, or if the test is inconclusive. (Order your answers from smallest to largest x, then from smallest to largest y.)

$f(x, y) = (x + y)(2x + xy)$

$(x, y) = (\quad)$ ---Select---

$(x, y) = (\quad)$ ---Select---

$(x, y) = (\quad)$ ---Select---

$(x, y) = (\quad)$ ---Select---

Find critical points

$f = (x+y)(2x+xy)$

$= 2x^2 + 2xy + x^2y + xy^2$

	x	y
$2x$	$2x^2$	$2xy$
xy	x^2y	xy^2

$f_x = 4x + 2y + 2xy + y^2$

$f_y = 2x + x^2 + 2xy$

$0 = 4x + 2y + 2xy + y^2$

$0 = 2x + x^2 + 2xy$

$0 = x(4 + 2y) + 2y + y^2$

$0 = x(2 + x + 2y)$

Solve for $0 = x$ and $0 = 2 + x + 2y$

$0 = x(4 + 2y) + 2y + y^2$
$0 = x$

$0 = 2y + y^2$

$0 = y(2 + y)$

$y = 0$ $\qquad\qquad$ $y = -2$

$0 = x(4)$ $\qquad\qquad$ $0 = x(4 - 4) - 4 + 4$

$x = 0$ $\qquad\qquad$ $0 = 0$

Use other equation

$0 = x(2 + x - 4)$

Solve for $x = 0$ and $0 = 2 + x - 4$

$x = 0$

$x = 2$

$(0,0)$, $(0,-2)$, and $(2,-2)$

$0 = x(4+2y) + 2y + y^2$
$0 = 2 + x + 2y$

$0 = x(4+2y) + 2y + y^2$
$x = -2 - 2y$

$0 = (-2-2y)(4+2y) + 2y + y^2$

	4	$2y$
-2	-8	$-4y$
$-2y$	$-8y$	$-4y^2$

$0 = -8 - 4y - 8y - 4y^2 + 2y + y^2$

$0 = -3y^2 - 10y - 8$

$-3y \qquad y = -4y$
$-4 \qquad 2 = -6y$

$0 = (-3y+4)(y-2)$

$0 = (-3y-4)(y+2)$

$y = -\frac{4}{3}$ $\qquad$ $y = -2$

$x = -2 - 2(-\frac{4}{3})$ $\qquad$ $x = -2 - 2(-2)$

$= -\frac{6}{3} + \frac{8}{3}$ $\qquad$ $x = -2 + 4$

$= \frac{2}{3}$ $\qquad$ $x = 2$

$(\frac{2}{3}, -\frac{4}{3})$ and $(2, -2)$

Critical Points

$(0,0)$, $(0,-2)$, $(2,-2)$, $(\frac{2}{3}, -\frac{4}{3})$

Second Derivativate Test

$f_{xx} = 4 + 2y$

$f_{yy} = 2x$

$f_{xy} = 2 + 2x + 2y$

$D(x,y) = (4+2y)(2x) - (2+2x+2y)^2$

Test Point $(0,0)$

$D(0,0) = (4+0)(0) - (2+0+0)^2$

$= -4$

Saddle Point

Test Point $(0,-2)$

$D(0,-2) = (4+2(-2))(0) - (2+0+2(-2))^2$

$= -(-4)^2$

$= -16$

Saddle Point

Test Point $(2,-2)$

$$D(2,-2) = (4+2(-2))(2(2)) - (2+2(2)+2(-2))^2$$

$$= (0)(4) - (6-4)^2$$

$$= -4$$

Saddle point

Test Point $\left(\frac{2}{3}, -\frac{4}{3}\right)$

$$D\left(\frac{2}{3}, -\frac{4}{3}\right) = \left(4+2\left(\frac{2}{3}\right)\right)\left(2\left(\frac{2}{3}\right)\right) - \left(2+2\left(\frac{2}{3}\right)+2\left(-\frac{4}{3}\right)\right)^2$$

$$= \left(\frac{12}{3}+\frac{4}{3}\right)\left(\frac{4}{3}\right) - \left(\frac{6}{3}+\frac{8}{3}-\frac{8}{3}\right)^2$$

$$= \left(\frac{48}{3}\right)\left(\frac{4}{3}\right) - \left(\frac{6}{3}\right)^2$$

$$= \frac{192}{9} - \frac{36}{9}$$

$$= \frac{156}{9}$$

$$f_{xx}\left(\frac{2}{3}, -\frac{4}{3}\right) = 4 + 2\left(-\frac{4}{3}\right)$$

$$= \frac{12}{3} - \frac{8}{3}$$

$= \frac{4}{3}$

Max

Find the global extreme values of $f(x, y) = x^2 - xy + y^2 + 3$ on the closed triangular region in the first quadrant bounded by the lines $x = 6$, $y = 0$, and $y = x$.

global maximum

global minimum

Critical Points of f

$f_x = 2x - y$
$f_y = -x + 2y$

$0 = 2x - y$
$0 = -x + 2y$

$0 = 2x - y$
$0 = -2x + 4y$

$0 = 3y$

$y = 0$

$x = 0$

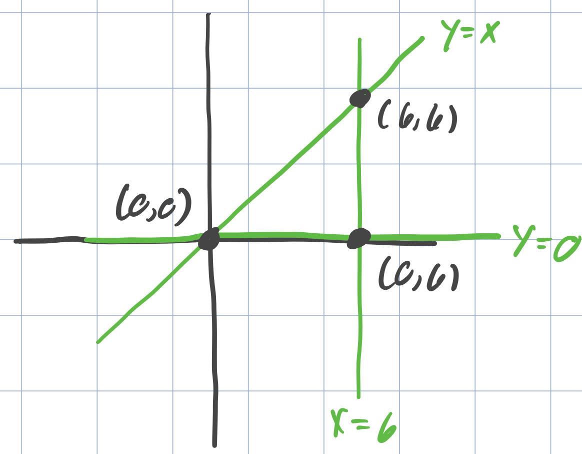
$(0, 0)$

Check bound $x = 6$ in $0 \leq y \leq 6$

No critical points

End points

(0, 6) and (6, 6)



Check band $y=0$ in $0 \le x \le 6$

No critical points

End Points

(0,0) and (0, 6)

Check band $y=x$ in $0 \le x \le 6$

No critical points

End points

(0,0) and (6, 6)

Points of Intrest

(x,y)	(0,0)	(0,6)	(6,6)
$f(x,y)$	3	39	39

Global Max is $z = 39$

Global Min is $z = 3$

Find the critical points of the given function. Then use the second derivative test to determine if the critical points correspond to local maxima, local minima, or saddle points of the graph of the function, or if the test is inconclusive. (Order your answers from smallest to largest x, then from smallest to largest y.)

$f(x, y) = 8xye^{-x^2 - y^2}$

$(x, y) =$ ( ) ---Select---

$(x, y) =$ ( ) ---Select---

$(x, y) =$ ( ) ---Select---

$(x, y) =$ ( ) ---Select---

$(x, y) =$ ( ) ---Select---

$f = 8y(xe^{-x^2-y^2})$

$f = 8x(ye^{-x^2-y^2})$

Critical Points

$f_x = 8y[(x)(-2xe^{-x^2-y^2}) + (e^{-x^2-y^2})(1)]$

$= 8ye^{-x^2-y^2}(-2x^2+1)$

$f_y = 8x[(y)(-2ye^{-x^2-y^2}) + (e^{-x^2-y^2})(1)]$

$= 8xe^{-x^2-y^2}(-2y^2+1)$

$0 = 8ye^{-x^2-y^2}(-2x^2+1)$

$0 = 8xe^{-x^2-y^2}(-2y^2+1)$

$0=(y)(e^{-x^2-y^2})(-2x^2+1)$ (1)

$0=(x)(e^{-x^2-y^2})(-2y^2+1)$ (2)

Cases

e to the power of anything will never be 0

1a.) $y=0$ 1b.) $0=e^{-x^2-y^2}$ 1c.) $0=-2x^2+1$

2a.) $x=0$ 2b.) $0=e^{-x^2-y}$ 2c.) $0=-2y^2+1$

Case 1a2a

$y=0$

$x=0$

$(0,0)$

Case 1a2b

$y=0$

$0=e^{-x^2+0}$

$y=0$

$\ln(0)=-x^2$

$y=0$

DNE

Eliminate case

Case 1a2c

$y=0$

$0 \neq 1$

Eliminate Case

Case 1b2a

$0=e^{-x^2-y^2}$

$x=0$

$\ln(0) = -y^2$

$x=0$

DNE

$x=0$

Eliminate Case

Case 1b2b

$0=e^{-x^2-y^2}$

$0=e^{-x^2-y^2}$

True but not useful

Case 1b 2c

$0 = e^{-x^2-y^2}$
$0 = -2y^2 + 1$

$0 = e^{-x^2-y^2}$
$2y^2 = 1$

$0 = e^{-x^2-y^2}$
$y = \sqrt{1/2}$

$0 = e^{-x^2-1/2}$
$y = \sqrt{1/2}$

$\ln(0) = -x^2 - 1/2$
$y = \sqrt{1/2}$

DNE
$y = \sqrt{1/2}$

Eliminate Case

Case 1c2a

$0 = -2x^2 + 1$

$x = 0$

$0 \neq 1$

Eliminate Case

Case 1c2b

$0 = -2x^2 + 1$
$0 = e^{-x^2 - y^2}$

$2x^2 = 1$
$0 = e^{-x^2 - y^2}$

$x = \sqrt{\frac{1}{2}}$
$\ln(0) = -\frac{1}{2} - y^2$

$x = \sqrt{1/2}$
DNE

Eliminate Case

Case 1c2c

$0 = -2x^2 + 1$
$0 = -2y^2 + 1$

$2x^2 = 1$

$2y^2 = 1$

$x = \pm\sqrt{1/2}$
$y = \pm\sqrt{1/2}$

$(\sqrt{1/2}, \sqrt{1/2}), (-\sqrt{1/2}, \sqrt{1/2}), (-\sqrt{1/2}, -\sqrt{1/2})$ and $(\sqrt{1/2}, -\sqrt{1/2})$

Points of Intrest

$(\sqrt{1/2}, \sqrt{1/2}), (-\sqrt{1/2}, \sqrt{1/2}), (-\sqrt{1/2}, -\sqrt{1/2}), (\sqrt{1/2}, -\sqrt{1/2})$, and $(0,0)$

Used GeoGebra to identify max and mins

Find three non-negative numbers a, b, and c such that their product is a maximum when the numbers a, b, and c are constrained by the relation $7a + b + c = 10$.

a =

b =

c =

Optimize abc subject to $7a + b + c = 10$

$f(a,b,c) = abc$

$g(a,b,c) = 7a + b + c - 10$

$\nabla f = \lambda \nabla g$

$bc = \lambda(7)$
$ac = \lambda(1)$
$ab = \lambda(1)$

$bc = 7\lambda$ (1)

$ac = \lambda$ (2)

$ab = \lambda$ (3)

$7a + b + c = 10$ (4)

a, b, c, and λ cannot be 0 because it would mean $0 = 10$

(1) and (2)

$bc = 7(ac)$

$b = 7a$

(2) and (3)

$ac = ab$

$c = b$

Sub into (4)

$7a + 7a + 7a = 10$

$21a = 10$

$a = \frac{10}{21}$

$$b = 7\left(\frac{10}{21}\right)$$

$$= \frac{10}{3}$$

$$c = \frac{10}{3}$$