Use Lagrange multiplier techniques to find the local extreme values of the given function subject to the stated constraint. If appropriate, determine if the extrema are global. (If a local or global extreme value does not exist enter DNE.)

$f(x, y) = x^2 + y^2 + 2x + 2$ with constraint $x^2 + y^2 = 64$

local max [ ] ---Select---

local min [ ] ---Select---

Let $g(x,y) = x^2+y^2-64$

$\nabla f = \lambda \nabla g$

$2x+2 = \lambda(2x)$
$2y = \lambda(2y)$
$x^2+y^2 = 64$

$x+1 = \lambda x$ (1)
$y = \lambda y$ (2)
$x^2+y^2 = 64$ (3)

Find Cases

~~1a $x=0$~~

$0 \neq 1$

Eliminate case

1b $x \neq 0$

$x - \lambda x = -1$

$x(1-\lambda) = -1$

$x = -\dfrac{1}{1-\lambda}$

2a $y=0$

$0=0$

2b $y \neq 0$

$\lambda = 1$

Check Case 1b2a

$Y=0$

$x^2+0=64$

$x = \pm 8$

$(8,0)$ and $(-8,0)$

Check Case 1b2b

$\lambda = 1$

$x = -\frac{1}{1-1}$

$x = DNE$

Eliminate Case

Points of Intrest

$(8,0)$ and $(-8,0)$

$f(0,8) = 64 + 0 + 2(8) + 2$

$= 82$

$f(0,-8) = 64 + 0 + 2(-8) + 2$

$= 50$

Global Max at $(0,8)$ of 82

Global Min at $(0,-8)$ of 50

Use Lagrange multiplier techniques to find the local extreme values of the given function subject to the stated constraint. If appropriate, determine if the extrema are global. (If a local or global extreme value does not exist enter DNE.)

$f(x, y) = 5x + y + 7$ with constraint $g(x, y) = xy = 1$

local max [ ] ---Select---

local min [ ] ---Select---

Let $g(x,y) = xy - 1$

$\nabla f = \lambda \nabla g$

$5 = \lambda(y)$
$1 = \lambda(x)$
$xy = 1$

$5 = \lambda y \quad (1)$
$1 = \lambda x \quad (2)$
$xy = 1 \quad (3)$

Find Cases

~~1a $y=0$~~

$5 \neq 0$

Eliminate Case

1b $y \neq 0$

$y = \frac{5}{\lambda}$

~~2a $x=0$~~

$1 \neq 0$

Eliminate Case

2b $x \neq 0$

$x = \frac{1}{\lambda}$

Check Cases

Case 1b 2b

$y = \frac{5}{\lambda}$ and $x = \frac{1}{\lambda}$

$\left(\frac{5}{\lambda}\right)\left(\frac{1}{\lambda}\right) = 1$

$\frac{5}{\lambda^2} = 1$

$\lambda^2 = 5$

$\lambda = \pm\sqrt{5}$

$x = \pm\frac{1}{\sqrt{5}}$ and $y = \pm\frac{5}{\sqrt{5}}$, where either both x and y are positive or both are negative

$\left(\frac{1}{\sqrt{5}}, \frac{5}{\sqrt{5}}\right)$ and $\left(-\frac{1}{\sqrt{5}}, -\frac{5}{\sqrt{5}}\right)$

Points of Intrest

$f\left(\frac{1}{\sqrt{5}}, \frac{5}{\sqrt{5}}\right) = \frac{5}{\sqrt{5}} + \frac{5}{\sqrt{5}} + 7$

$= \frac{10}{\sqrt{5}} + 7$

$f\left(-\frac{1}{5}, -\frac{5}{\sqrt{5}}\right) = -\frac{5}{\sqrt{5}} - \frac{5}{\sqrt{5}} + 7$

$= \frac{-10}{\sqrt{5}} + 7$

Use Lagrange multiplier techniques to find the local extreme values of the given function subject to the stated constraint. (If a local or global extreme value does not exist enter DNE.)

$f(x, y) = e^{3xy}$ with constraint $g(x, y) = x^3 + y^3 = 16$

local max

local min

$g(x, y) = x^3 + y^3 - 16$

$\nabla f = \lambda \nabla g$

$$e^{3xy}(\cancel{3}y) = \lambda(\cancel{3}x^2)$$
$$e^{3xy}(\cancel{3}x) = \lambda(\cancel{3}y^2)$$

$$ye^{3xy} = \lambda x^2 \qquad (1)$$
$$xe^{3xy} = \lambda y^2 \qquad (2)$$
$$x^3 + y^3 = 16 \qquad (3)$$

x, y, and λ cannot equal 0 because it would mean that $0 = 16$

(1) and (2)

$$e^{3xy} = \frac{\lambda x^2}{y}$$

$$e^{3x} = \frac{\lambda y^2}{x}$$

$$\frac{\cancel{\lambda} x^{\cancel{2}}}{\cancel{y}} = \frac{\cancel{\lambda} y^{\cancel{2}}}{\cancel{x}}$$

$$x = y$$

Plus into constraint

$$x^3 + x^3 = 16$$

$$2x^3 = 16$$

$$x^3 = 8$$

$x = 2$

$y = 2$

$(2,2)$

Points of Intrest

$(2,2)$

$f(2,2) = e^{12}$

Second Derivative Test

$f_x = 3y e^{3xy}$

$f_{xx} = 9y^2 e^{3xy}$

$f_y = 3x e^{3xy}$

$f_{yy} = 9x^2 e^{3xy}$

$f_{xy} = 3[(y)(3x e^{3xy}) + (e^{3xy})(1)]$

$= 3(3xy e^{3xy} + e^{3xy})$

$= 3e^{3xy}(3xy + 1)$

$$D(x,y) = (9y^2 e^{3xy})(9x^2 e^{3xy}) - (3e^{3xy}(3xy+1))^2$$

$$= 81x^2y^2e^{6xy} - (3e^{3xy}(3xy+1))^2$$

$$= 81x^2y^2e^{6xy} - 9e^{6xy}(3xy+1)^2$$

Check Point $(2,2)$

$$D(x,y) = -5.96e12$$

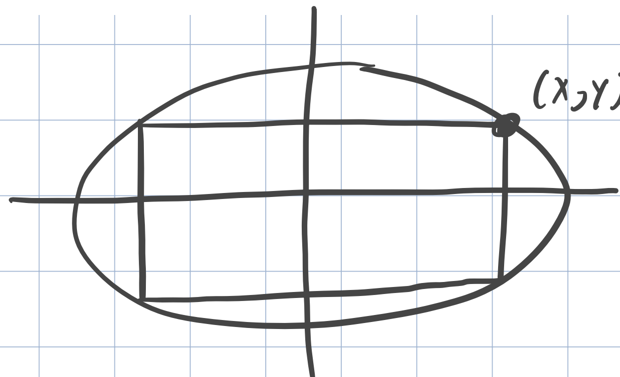
For some reasen the second derivative test is wrong on this problem

Use Lagrange multiplier techniques to find the dimensions of the rectangle with largest perimeter that can be inscribed inside an ellipse $\frac{x^2}{144} + \frac{y^2}{25} = 1$, when the sides of the rectangle are parallel to the coordinate axes.

x-dimension

y-dimension

Optimize $F(x,y) = 2x + 2x + 2y + 2y$
with the constraint $g(x) = \frac{x^2}{144} + \frac{y^2}{25} - 1$



$f(x,y) = 4x + 4y$

$g(x,y) = \frac{1}{144}x^2 + \frac{1}{25}y^2 - 1$

$h = 2y$
$W = 2x$

$\nabla f = \lambda \nabla g$

$4 = \lambda\left(\frac{2}{144}x\right)$
$4 = \lambda\left(\frac{2}{25}y\right)$

$\frac{1}{144} x^2 + \frac{1}{25} y^2 = 1$

$$x = \frac{4 \cdot 144}{2\lambda}$$

$$= \frac{288}{\lambda}$$

$$y = \frac{4 \cdot 25}{2\lambda}$$

$$= \frac{50}{\lambda}$$

Plug into constraint

$$\frac{1}{144}\left(\frac{288}{\lambda}\right)^2 + \frac{1}{25}\left(\frac{50}{\lambda}\right)^2 = 1$$

$$\frac{288^2}{144} \frac{1}{\lambda^2} + \frac{50^2}{25} \frac{1}{\lambda^2} = 1$$

$$576 \frac{1}{\lambda^2} + 12 \frac{1}{\lambda^2} = 1$$

$$676 \frac{1}{\lambda^2} = 1$$

$$\lambda^2 = 676$$

$$\lambda = \pm 261$$

If λ is negative x and y would be negative. This is not possible.

$$\lambda = 261$$

$$X = \frac{288}{26}$$

$$Y = \frac{50}{26}$$

$$W = 2 \cdot \frac{288}{26}$$

$$L = 2 \cdot \frac{50}{26}$$

Use Lagrange multiplier techniques to find shortest and longest distances from the origin to the curve $x^2 + xy + y^2 = 2$.

shortest distance

longest distance

Distance function

$$L = \sqrt{x^2 + y^2}$$

$$= (x^2 + y^2)^{1/2}$$

Optimize $f(x,y) = \sqrt{x^2+y^2}$ subject to
$g(x,y) = x^2 + xy + y^2 - 2$

$$\nabla f = \lambda \nabla g$$

$$\cancel{\tfrac{1}{2}}(x^2+y^2)^{-\frac{1}{2}}(\cancel{2}x) = \lambda(2x + y)$$

$$\cancel{\tfrac{1}{2}}(x^2+y^2)^{-\frac{1}{2}}(\cancel{2}y) = \lambda(x + 2y)$$

$$\frac{x}{\sqrt{x^2+y^2}} = \lambda(2x + y)$$

$$\frac{y}{\sqrt{x^2+y^2}} = \lambda(x+2y)$$

$$\frac{x}{\sqrt{x^2+y^2}} = \lambda(2x+y) \quad (1)$$

$$\frac{y}{\sqrt{x^2+y^2}} = \lambda(x+2y) \quad (2)$$

$$x^2+xy+y^2=2 \quad (3)$$

Find Cases and Points of Intrest

x and y cannot equal 0 because that would mean that $0=2$

$\lambda \neq 0$ because then $x=0$ and $y=0$

~~1a $x=0$, $y\neq 0$~~

$$0=\lambda(2(0)+y)$$

$$y=0$$

$$\lambda=0$$

$\lambda \neq 0$ and both x and y cannot be 0. Eliminate case

Neither x or y can be 0 for the reasoning in 1a.
All variables are not 0

2a $x \neq 0$ and $y \neq 0$

(1) and (2)

$$\sqrt{x^2+y^2} = \frac{x}{\lambda(2x+y)}$$

$$\sqrt{x^2+y^2} = \frac{y}{\lambda(x+2y)}$$

$$\frac{x}{\lambda(2x+y)} = \frac{y}{\lambda(x+2y)}$$

$$\cancel{\lambda}x(x+2y) = \cancel{\lambda}y(2x+y)$$

$$x^2 + \cancel{2xy} = \cancel{2xy} + y^2$$

$$x^2 = y^2$$

$$\pm x = \pm y$$

$x = y$ and $x = -y$

Plug into constraint

$x^2 + x^2 + x^2 = 2$ $\qquad$ $(-y)^2 - y^2 + y^2 = 2$

$$3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$x = \pm\sqrt{\frac{2}{3}}$$

$$y^2 - y^2 + y^2 = 1$$

$$y^2 = 1$$

$$y = \pm\sqrt{1}$$