Evaluate $\int_0^{\frac{\pi}{2}} \sin^3(x)\,dx$

$$= \int_0^{\frac{\pi}{2}} \sin^{2+1}(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(x)\sin(x)\,dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos^2(x))\sin(x)\,dx$$

$\frac{\pi}{2} \to \cos(\frac{\pi}{2}) \to 0$

$0 \to \cos(0) \to 1$

$U = \cos(x)$ $\quad \frac{du}{dx} = -\sin(x)$

$-du = \sin(x)\,dx$

$$= \int_1^0 (1-u^2) \cdot -du$$

$$= \int_0^1 1-u^2\,du$$

$$= u - \tfrac{1}{3}u^3 \Big|_0^1$$

$$= 1 - \tfrac{1}{3}(1)^3$$

$$= \tfrac{2}{3}$$

Evaluate $\int \sin^2(2x)\cos^2(2x)\,dx$

$$= \int \tfrac{1}{2}(1-\cos(4x))\tfrac{1}{2}(1+\cos(4x))\,dx$$

$$= \tfrac{1}{4}\int (1-\cos(4x))(1+\cos(4x))\,dx$$

$= \frac{1}{4}\int 1^2 - \cos^2(4x)\ dx$

$= \frac{1}{4}\int 1 - \cos^2(4x)\ dx$

$= \frac{1}{4}\int \sin^2(4x)\ dx$

$= \frac{1}{4}\int \frac{1}{2}(1 - \cos(8x)\ dx$

$= \frac{1}{8}\int 1 - \cos(8x)\ dx$

$= \frac{1}{8}\left(\int 1\ dx - \int \cos(8x)\ dx\right)$

$= \frac{1}{8}(x - \int \cos(8x))\ dx$

$u = 8x \qquad \frac{du}{dx} = 8$

$\frac{1}{8}du = dx$

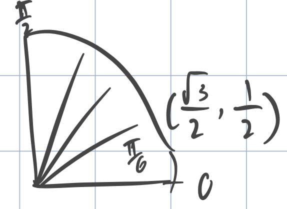
$= \frac{1}{8}(x - \int \cos(u)\ \frac{1}{8}du)$

$= \frac{1}{8}(x - \frac{1}{8}\sin(u)) + C$

$= \frac{1}{8}(x - \frac{1}{8}\sin(8x)) + C$

Evaluate $\int_0^{\pi/6} \tan^2(x)\sec^4(x)\ dx$

$= \int_0^{\pi/6} \tan^2(x)\sec^2(x)\sec^2(x)\ dx$



$$= \int_0^{\pi/6} \tan^2(x)\left(1+\tan^2(x)\right)\sec^2(x)\,dx$$

$u = \tan(x)$ $\quad \frac{du}{dx} = \sec^2(x)$ $\quad \frac{\pi}{6} \rightarrow \tan\left(\frac{\pi}{6}\right) \rightarrow \frac{1}{2}\cdot\frac{2}{\sqrt{3}} \rightarrow \frac{1}{\sqrt{3}}$

$du = \sec^2(x)\,dx$ $\quad 0 \rightarrow \tan(0) \rightarrow 0$

$$= \int_0^{\frac{1}{\sqrt{3}}} u^2(1+u^2)\,du$$

$$= \int_0^{\frac{1}{\sqrt{3}}} u^2 + u^4\,du$$

$$= \frac{1}{3}u^3 + \frac{1}{5}u^5 \Big|_0^{\frac{1}{\sqrt{3}}}$$

$$= \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^5$$

Evaluate $\int \cos^7(4x)\,dx$

$$= \int \cos^6(4x)\cos(4x)\,dx$$

$$= \int \left(\cos^2(4x)\right)^3 \cos(4x)\,dx$$

$$= \int \left(1-\sin^2(4x)\right)^3 \cos(4x)\,dx$$

$u = \sin(4x)$ $\quad \frac{du}{dx} = \cos(4x)4$

$\frac{1}{4}du = \cos(4x)\,dx$

$$= \frac{1}{4}\int \left(1-u^2\right)^3 du$$

$$=\frac{1}{4}\int (1-2u^2+u^4)(1-u^2)\,du$$

$$=\frac{1}{4}\int 1-2u^2+u^4-u^2+2u^4-u^6\,du$$

$$=\frac{1}{4}\int 1-3u^2+3u^4-u^6\,du$$

$$=\frac{1}{4}\left(u-\frac{3}{3}u^3+\frac{3}{5}u^5-\frac{1}{7}u^7\right)+C$$

$$=\frac{1}{4}\left(\sin(4x)-\sin^3(4x)+\frac{3}{5}\sin^5(4x)-\frac{1}{7}\sin^7(4x)\right)+C$$

Evaluate $\int \sin^5(5x)\cos^3(5x)\,dx$

$$=\int \sin^4(5x)\cos^3(5x)\sin(5x)\,dx$$

$$=\int (\sin^2(5x))^2\cos^3(5x)\sin(5x)\,dx$$

$$=\int (1-\cos^2(5x))^2\cos^3(5x)\sin(5x)\,dx$$

$$u=\cos(5x) \qquad \frac{du}{dx}=-\sin(5x)\cdot 5$$

$$-\frac{1}{5}\,du=\sin(5x)\,dx$$

$$=\int (1-u^2)^2 u^3\cdot -\frac{1}{5}\,du$$

$$=-\frac{1}{5}\int (1-u^2)^2 u^3\,du$$

$$=-\frac{1}{5}\int (1-2u^2+u^4)u^3\,du$$

$$= -\frac{1}{5}\int u^3 - 2u^5 + u^7 \, du$$

$$= -\frac{1}{5}\left(\frac{1}{4}u^4 - \frac{2}{6}u^6 + \frac{1}{8}u^8\right) + C$$

$$= -\frac{1}{5}\left(\frac{1}{4}\cos^4(5x) - \frac{1}{3}\cos^6(5x) + \frac{1}{8}\cos^8(5x)\right) + C$$

Evaluate $\int \sin^6(5x)\, dx$

$$= \int (\sin^2(5x))^3 \, dx$$

$$= \int \left(\frac{1}{2}(1-\cos(10x)\right)^3 dx$$

$$= \frac{1}{8}\int (1-\cos(10x))^3 \, dx$$

$$= \frac{1}{8}\int (1-\cos(10x))^2(1-\cos(10x)) \, dx$$

$$= \frac{1}{8}\int (1-2\cos(10x)+\cos^2(10x))(1-\cos(10x)) \, dx$$

$$= \frac{1}{8}\int 1-2\cos(10x)+\cos^2(10x)-\cos(10x)+2\cos^2(10x)-\cos^3(10x) \, dx$$

$$= \frac{1}{8}\int 1-3\cos(10x)+3\cos^2(10x)-\cos^3(10x) \, dx$$

$$= \frac{1}{8}\left(x - \frac{3}{10}\sin(10x) + 3\int \cos^2(10x)\, dx - \int \cos^3(10x)\, dx\right)$$

$$\rightarrow \int \cos^2(10x)\, dx$$

$$= \int \frac{1}{2}(1+\cos(20x))\, dx$$

$$= \frac{1}{2}\int 1 + \cos(20x)\ dx$$

$$= \frac{1}{2}\left(x + \frac{1}{20}\sin(20x)\right)$$

$$= \frac{1}{8}\left(x - \frac{3}{10}\sin(10x) + \frac{3}{2}\left(x + \frac{1}{20}\sin(20x)\right) - \int \cos^3(10x)\ dx\right)$$

$$\int \cos^3(10x)\ dx$$

$$= \int \cos^2(10x)\cos(10x)\ dx$$

$$= \int (1 - \sin^2(10x))\cos(10x)\ dx$$

$$u = \sin(10x) \qquad \frac{du}{dx} = \cos(10x)\cdot 10$$

$$\frac{1}{10}du = \cos(10x)\ dx$$

$$= \frac{1}{10}\int 1 - u^2\ du$$

$$= \frac{1}{10}\left(u - \frac{1}{3}u^3\right)$$

$$= \frac{1}{10}\left(\sin(10x) - \frac{1}{3}\sin^3(10x)\right)$$

$$= \frac{1}{8}\left(x - \frac{3}{10}\sin(10x) + \frac{3}{2}\left(x + \frac{1}{20}\sin(20x)\right) - \frac{1}{10}\left(\sin(10x) - \frac{1}{3}\sin(10x)\right)\right)$$

Evaluate $\int \sec^4(4x)\ dx$

$$= \int \sec^2(4x)\sec^2(4x)\ dx$$

$$= \int (1 + \tan^2(4x)) \sec^2(4x)\, dx$$

$$u = \tan(4x) \qquad \frac{du}{dx} = \sec^2(4x) \cdot 4$$

$$\frac{1}{4} du = \sec^4(4x)\, dx$$

$$= \int (1 + u^2) \cdot \frac{1}{4} du$$

$$= \frac{1}{4} \int 1 + u^2 \, du$$

$$= \frac{1}{4} \left(u + \frac{1}{3} u^3 \right) + C$$

$$= \frac{1}{4} \left(\tan(4x) + \frac{1}{3} \tan^3(4x) \right) + C$$

Evaluate $\int \tan^5(x) \sec(x)\, dx$

$$= \int \tan^4(x) \tan(x) \sec(x)\, dx$$

$$= \int (\tan^2(x))^2 \tan(x) \sec(x)\, dx$$

$$= \int (\sec^2(x) - 1)^2 \tan(x) \sec(x)\, dx$$

$$u = \sec(x) \qquad \frac{du}{dx} = \tan(x) \sec(x)$$

$$du = \tan(x) \sec(x)\, dx$$

$$= \int (u^2 - 1)^2 du$$

$= \int u^4 - 2u^2 + 1 \, du$

$= \frac{1}{5}u^5 - \frac{2}{3}u^3 + u + C$

$= \frac{1}{5}\sec^5(x) - \frac{2}{3}\sec^3(x) + \sec(x) + C$

Evaluate $\int \tan^3(6x)\,dx$

$= \int \tan^2(6x)\tan(x)\,dx$

$= \int (\sec^2(6x) - 1)\tan(6x)\,dx$

$= \int \left(\frac{\sec^2(6x) - 1}{\sec(6x)}\right)\tan(6x)\sec(6x)\,dx$

$u = \sec(\ )$ $\quad \frac{du}{dx} = \sec(6x)\tan(6x) \cdot 6$

$\frac{1}{6}du = \sec(6x)\tan(6x)\,dx$

$= \frac{1}{6}\int \frac{u^2 - 1}{u}\,du$

$= \frac{1}{6}\int u - u^{-1}\,du$

$= \frac{1}{6}\left(\frac{1}{2}u^2 - \ln(|u|)\right) + C$

$= \frac{1}{6}\left(\frac{1}{2}\sec^2(6x) - \ln(|\sec(6x)|)\right) + C$