Find the general solution for $y'' - 9y' + 18y = 0$

Auxiliary Equation

$$r^2 - 9r + 18 = 0$$

$$(r-6)(r-3) = 0$$

$$r_1 = 6 \qquad r_2 = 3$$

General Solution

$$y_c = C_1 e^{6x} + C_2 e^{3x}$$

Find the general solution to $y'' + 6y' + 58y = 0$

Auxiliary Equation

$$r^2 + 6r + 58 = 0$$

$$r = \frac{-6 \pm \sqrt{36 - 232}}{2}$$

$$= \frac{-6 \pm \sqrt{-196}}{2}$$

$$= \frac{-6 \pm 14i}{2}$$

$$= -3 \pm 7i \qquad \alpha = -3 \qquad \beta = 7$$

General Equation

$$y = e^{-3x}(C_1 \cos(7x) + C_2 \sin(7x))$$

Find the general solution to $81 \frac{d^2y}{dx^2} + 72 \frac{dy}{dx} + 16y = 0$

$$81y'' + 72y' + 16y = 0$$

Auxiliary Equation

$$81r^2 + 72r + 16 = 0$$

$a = 81 \quad b = 72 \quad c = 16$

$$r = \frac{-72 \pm \sqrt{72^2 - 4 \cdot 81 \cdot 16}}{2 \cdot 81}$$

$$= \frac{-72 \pm \sqrt{0}}{162}$$

$$= -\frac{72}{162}$$

$$= -\frac{4}{9}$$

General Equation

$$y = C_1 e^{-\frac{4}{9}x} + C_2 x e^{-\frac{4}{9}x}$$

Solve the IVP $3y''+5y'+2y=0$ when $y(0)=5$ and $y'(0)=-4$

Auxiliary Equation

$$3r^2+5r+2=0$$

$$r=\frac{-5\pm\sqrt{25-24}}{6}$$

$$=\frac{-5\pm\sqrt{1}}{6}$$

$r_1=-\frac{2}{3}$ $\qquad$ $r_2=-1$

General Equation

$$y=C_1e^{-\frac{2}{3}x}+C_2e^{-x}$$

$$y'=-\frac{2}{3}C_1e^{-\frac{5}{3}x}-C_2e^{-2x}$$

Initial Condition $y(0)=5$ and $y'(0)=-4$

$$5=C_1e^0+C_2e^0$$
$$-4=-\frac{2}{3}C_1e^0-C_2e^0$$

$$5=C_1+C_2$$
$$-4=-\frac{2}{3}C_1-C_2$$

$$1=\frac{1}{3}C_1$$

$C_1 = 3$

$C_2 = 2$

Unique Solution

$y = 3e^{-\frac{2}{3}x} + 2e^{-x}$

Solve the IVP $y'' - 6y' + 10y = 0$ when $y(0) = 3$ and $y'(0) = 6$

Auxiliary Equation

$r^2 - 6r + 10 = 0$

$r = \frac{6 \pm \sqrt{36-40}}{2}$

$= \frac{6 \pm \sqrt{-4}}{2}$

$= \frac{6 \pm 2i}{2}$

$= 3 \pm i$ $\qquad \alpha = 3$ $\qquad \beta = 1$

General Solution

$y = e^{3x}(C_1 \cos(x) + C_2 \sin(x))$

$= C_1 e^{3x} \cos(x) + C_2 e^{3x} \sin(x)$

$$y' = C_1(e^{3x} \cdot -\sin(x) + \cos(x) \cdot 3e^{3x}) + C_2(e^{3x} \cdot \cos(x) + \sin(x) \cdot 3e^{3x})$$

$$= C_1(-e^{3x}\sin(x) + 3e^{3x}\cos(x)) + C_2(e^{3x}\cos(x) + 3e^{3x}\sin(x))$$

$$= e^{3x}(-C_1\sin(x) + 3C_1\cos(x) + C_2\cos(x) + 3C_2\sin(x))$$

Initial Condition $y(0) = 3$ and $y'(0) = 6$

$$3 = C_1 e^0 \cos(0) + C_2 e^0 \sin(0)$$
$$6 = e^0(-C_1\sin(0) + 3C_1\cos(0) + C_2\cos(0) + 3C_2\sin(0))$$

$$3 = C_1$$
$$6 = 3C_1 + C_2$$

$$C_1 = 3$$
$$C_2 = -3$$

Unique Solution

$$y = e^{3x}(3\cos(x) - 3\sin(x))$$

Solve the IVP $y'' + 25y = 0$ when $y(\frac{\pi}{5}) = 0$ and $y'(\frac{\pi}{5}) = 1$

Auxiliary Equation

$$r^2 + 25 = 0$$

$r^2 = -25$

$r = 5i \qquad \alpha = 0 \qquad \beta = 5$

General Solution

$y = e^0 (C_1 \cos(5x) + C_2 \sin(5x))$

$= C_1 \cos(5x) + C_2 \sin(5x)$

$y' = -5C_1 \sin(5x) + 5C_2 \cos(5x)$

Initial Values $y(\frac{\pi}{5}) = 0$ and $y'(\frac{\pi}{5}) = 1$

$0 = C_1 \cos(\pi) + C_2 \sin(\pi)$
$1 = -5C_1 \sin(\pi) + 5C_2 \cos(\pi)$

$0 = C_1 \cdot -1 + 0$
$1 = 5C_2 \cdot -1$

$C_1 = 0$
$C_2 = -\frac{1}{5}$

Unique Solution

$y = -\frac{1}{5} \sin(5x)$

Solve the IVP $y''+12y'+36y=0$ when $y(0)=0$ and $y(1)=9$

Auxiliary Equation

$$r^2+12r+36=0$$

$$(r+6)(r+6)=0$$

$$r=-6$$

General Equation

$$y=C_1e^{-6x}+C_2xe^{-6x}$$

Initial Values $y(0)=0$ and $y(1)=9$

$$0=C_1$$
$$9=C_1e^{-6}+C_2e^{-6}$$

$$C_1=0$$
$$9=C_2\frac{1}{e^6}$$

$$C_2=9e^6$$

Unique Solution

$$y=9e^6xe^{-6x}$$

Simplify the complex number $(-4-5i)(7+9i)$

$= -28 - 36i - 35i - 45i^2$

$= -28 - 71i + 45$

$= 17 - 71i$

Simplify the complex number $\frac{5+i}{7+2i}$

$= \frac{5+i}{7+2i} \cdot \frac{7-2i}{7-2i}$

$= \frac{35-10i+7i-2i^2}{49-4i^2}$

$= \frac{35-3i+2}{49+4}$

$= \frac{37-3i}{53}$

$= \frac{37}{53} - \frac{3}{53} i$