A car's cooling system has a capacity of 20 quarts. Initially, the system contains a mixture of 3 quarts of antifreeze and 17 quarts of water. Water runs into the system at the rate of 1 $\frac{\text{gal}}{\text{min}}$, then the homogeneous mixture runs out at the same rate. In quarts, how much antifreeze is in the system at the end of 5 minutes? (Round your answer to two decimal places.)

$$A'(t) = \left(0 \cdot 4 \frac{qt}{min}\right) - \left(\frac{A(t)}{20} \cdot 4 \frac{qt}{min}\right)$$

$$= -\frac{A(t)}{5}$$

$$= -\frac{1}{5}A(t)$$

$$\frac{dA}{dt} = -\frac{1}{5}A$$

$$\frac{1}{A}\,dA = -\frac{1}{5}\,dt$$

$$\int \frac{1}{A}\,dA = -\frac{1}{5}\int 1\,dt$$

$$\ln(|A|) = -\frac{1}{5}t + C$$

$$A(t) = e^{-\frac{1}{5}t + C}$$

$$= Ce^{-\frac{1}{5}t}$$

Initial Condition $A(0) = 3$

$$3 = Ce^{0}$$

$$C = 3$$

Unique Solution

$A(t) = 3e^{-\frac{1}{5}t}$

After 5 min

$A(5) = 3e^{-\frac{1}{5}(5)}$

$= 3e^{-1}$

$= \frac{3}{e}$ qt

400 gallons of pesticide is accidentally spilled into a lake and uniformly mixes with the water. The volume of the lake including the pesticide is 10^8 gallons. A river flows into the lake bringing 5,000 gallons of fresh water per minute, and the uniform mixture spills over the dam at the same rate. How long, in minutes, will it take to reduce the pesticide in the lake to a safe level of 1 part per million gallons? (Round your answer to the nearest minute.)

$A'(t) = (0 \cdot 5000) - \left(\frac{A(t)}{10^8} \cdot 5000\right)$

$= -5e\text{-}5\,A(t)$

$\frac{dA}{dt} = -5e\text{-}5\,A$

$\cancel{\frac{1}{A}}\, dA = -5e\text{-}5\, dt$

$\ln(|A|) = -5e\text{-}5 \int 1\, dt$

$A(t) = Ce^{-5e\text{-}5\,t}$

Initial Condition $A(0) = 400$

$400 = Ce^{-5e\text{-}5(0)}$

$400 = C$

Unique Solution

$A(t) = 400e^{-5e-5t}$

Find t when A(t) is safe at 1 gal per 1e6 gal

$$\frac{1\text{gal}}{1e6\text{ gal}} = \frac{x}{1e8\text{ gal}}$$

$1e6x = 1e8$

$x = 100$

$100 = 400e^{-5e-5t}$

$\frac{1}{4} = e^{-5e-5t}$

$\frac{1}{4} = \frac{1}{e^{5e-5t}}$

$\frac{1}{4}e^{5e-5t} = 1$

$e^{5e-5t} = 4$

$5e-5t = \ln(4)$

$$t = \frac{\ln(4)}{5e-5}$$

A tank contains 500 gallons of salt-free water. A brine containing 0.25 lb of salt per gallon runs into the tank at the rate of 5 $\frac{\text{gal}}{\text{min}}$, and the well-stirred mixture runs out at 5 $\frac{\text{gal}}{\text{min}}$. In pounds per gallon, what is the concentration of salt in the tank at the end of 10 minutes? (Round your answer to four decimal places.)

$$A'(t) = \left(0.25 \frac{\text{lb}}{\text{gal}} \cdot 5 \frac{\text{gal}}{\text{min}}\right) - \left(\frac{A(t)}{500} \cdot 5\right)$$

$$= \frac{5}{4} - \frac{1}{100} A(t)$$

$$\frac{dA}{dt} = \frac{5}{4} - \frac{1}{100} A(t)$$

$$\frac{dA}{dt} = \frac{125}{100} - \frac{A(t)}{100}$$

$$\frac{dA}{dt} = \frac{125 - A(t)}{100}$$

$$\frac{1}{125 - A(t)} dA = \frac{1}{100} dt$$

$$\frac{1}{A(t) - 125} dA = -\frac{1}{100} dt$$

$$\ln(|A(t) - 125|) = -\frac{1}{100} t + C$$

$$A(t) - 125 = e^{-\frac{1}{100}t + C}$$

$$A(t) = Ce^{-\frac{1}{100}t} + 125$$

Initial Condition $A(0) = 0$

$$0 = Ce^{-\frac{1}{100}(0)} + 125$$

$0 = Ce^0 + 125$

$C = -125$

Unique Solution

$A(t) = -125e^{-\frac{1}{100}t} + 125$

Find the concentration when $t = 10$

$A(10) = -125e^{-\frac{1}{100}(10)} + 125$

$= 11.8951b$

$\text{Concentration} = \frac{11.895}{500}$

$= 0.02397 \frac{lb}{gall}$

Radium decays exponentially; It has a half life of 1600 years. Find a formula for the amount $q(t)$ remaining from 70 mg of pure radium after t years.

$\frac{dq}{dt} = kq$

$\frac{1}{q}\, dq = k\, dt$

$\ln(|q|) = kt + C$

$q = e^{kt+C}$

$q = Ce^{kt}$

Initial Condition $q(0) = 70$

$70 = Ce^{0}$

$C = 70$

$q = 70e^{kt}$

Half life condition $q(1600) = 35$

$35 = 70e^{k(1600)}$

$\frac{1}{2} = e^{1600k}$

$\ln(\frac{1}{2}) = 1600k$

$k = \frac{\ln(\frac{1}{2})}{1600}$

$q(t) = 70e^{\frac{\ln(\frac{1}{2})}{1600}t}$

After how many years will there be 10 mg of Radium left?

$10 = 70e^{\frac{\ln(\frac{1}{2})}{1600}t}$

$$\frac{1}{7}=e^{\frac{\ln(\frac{1}{2})}{1600}t}$$

$$\ln\left(\frac{1}{7}\right)=\frac{\ln(\frac{1}{2})}{1600}t$$

$$t=\frac{1600\ln(\frac{1}{7})}{\ln(\frac{1}{2})}$$

$$=4492 \text{ years}$$

A 3D printer was purchased for \$20,000 and has a value of \$12,000 after 2 years of use. Assuming an exponential decay, find the value of the printer after 8 years of use.

$$y=y_0e^{kt}$$

Initial Condition $y(0)=20{,}000$ and $y(2)=12{,}000$

$$Y_0=20{,}000$$

$$y=20{,}000e^{kt}$$

$$12{,}000=20{,}000e^{k(2)}$$

$$\frac{12}{20}=e^{2k}$$

$$\frac{3}{5}=e^{2k}$$

$$\ln\left(\frac{3}{5}\right)=2k$$

$k = \frac{1}{2}\ln\left(\frac{3}{5}\right)$

$y = 20,000e^{\frac{1}{2}\ln\left(\frac{3}{5}\right)t}$

$y(8) = 20,000e^{\frac{1}{2}\ln\left(\frac{3}{5}\right)(8)}$

$= 2592$ years

Suppose that \$10,000 is invested in an accaunt for which interest is compemding continiaesly at 3.11%. What is the valve after 5 years? 10 years? After how many years will the amant dable?

$A = Pe^{rt}$

$A = 10,000e^{0.0311t}$

$A(5) = 11682$

$A(10) = 13647$

$20,000 = 10,000e^{0.0311t}$

$2 = e^{0.0311t}$

$\ln(2) = 0.0311t$

$t = \frac{\ln(2)}{0.0311}$

The Lincoln wheat penny was designed by Victor D. Brenner in 1909. Currently, the most valued penny is the 1909 S VDB penny. It was minted in San Francisco and only 484,000 were minted with the initials on the back. In 2015, the S VDB penny in uncirculated condition is worth $3,200. When will it be worth $15,000? (Round your answer to the nearest year.)

$y(t) = y_0 a^t$

Initial Condition of \$0.01

$y(t) = 0.01a^t$

Initial Condition of $3200 = y(106)$

$3200 = 0.01a^{106}$

$3.2e5 = a^{106}$

$\ln(3.2e5) = \ln(a^{106})$

$\ln(3.2e5) = 106 \ln(a)$

$\ln(a) = \frac{\ln(3.2e5)}{106}$

$a = e^{\frac{\ln(3.2e5)}{106}}$

$a = 1.127$

General Equation

$y(t) = 0.01 \cdot 1.127^t$

Solve for $1.5e4 = y(t)$

$1.5e4 = 0.01 \cdot 1.127^t$

$1.5e6 = 1.127^t$

$\ln(1.5e6) = t\ln(1.127)$

$t = 118.9$

Year 2028

The temperature of a hot tub is 103° and the room tempature is 75°. The water cools to 90° in 10 min. What is the water tempature after 20 min?

$T(t) = (T_0 - T_m)e^{-kt} + T_m$

Plug in values

$T(t) = (103 - 75)e^{-kt} + 75$

$= 28e^{-kt} + 75$

Initial Condition $90 = T(10)$

$90 = 28e^{-10k} + 75$

$15 = 28e^{-10k}$

$\frac{15}{28} = e^{-10k}$

$\frac{15}{28} = \frac{1}{e^{10k}}$

$e^{10k} = \frac{28}{15}$

$10k = \ln(\frac{28}{15})$

$k = \frac{\ln(\frac{28}{15})}{10}$

$= 6.24e{-2}$

Unique Solution

$T(t) = 28e^{-6.24e-2t} + 75$

Solve for $t = 20$

$T(20) = 83.038^{\circ}$

After how many minutes will the tempature be 80°?

$$80 = 28e^{-6.24e-2t} + 75$$

$$5 = 28e^{-6.24e-2t}$$

$$\frac{5}{28} = e^{-6.24e-2t}$$

$$\frac{28}{5} = e^{6.24e-2t}$$

$$\ln\left(\frac{28}{5}\right) = 6.24e-2\,t$$

$$t = \frac{\ln\left(\frac{28}{5}\right)}{6.24e-2}$$

$$= 27.6 \text{ minuetes}$$

The population of the United States has grown at different rates over ten-year increments as shown by the following table.

Year	Population of U.S.
1930	123.1 million
1940	132.1 million
1950	152.3 million
1960	180.7 million

If the maximum supportable population of the U.S. is 600 million people, use the logistic model to predict the population (in millions of people) of the U.S. in 2005 by using the following years as data points. (Round your answers to one decimal place.)

Using 1930 and 1940 as data

$$P(t) = \frac{kP_0}{P_0 + (k - P_0)e^{-rt}}$$

$$= \frac{600P_0}{P_0 + (600 - P_0)e^{-rt}}$$

Let 1930 be $t=0$

$$P(t) = \frac{7.386e4}{123.1 + (476.9)e^{-rt}}$$

Initial Condition $P(10) = 132.1$

$$132.1 = \frac{7.386e4}{123.1 + (476.9)e^{-10r}}$$

$$1.789e\text{-}3 = \frac{1}{123.1 + 476.9e^{-10r}}$$

$$123.1 + 476.9e^{-10r} = 559.121$$

$$476.9e^{-10r} = 436.022$$

$$e^{-10r} = 0.914$$

$$e^{10r} = 1.094$$

$$10r = \ln(1.094)$$

$$r = 8.96e\text{-}3$$

Unique Solution

$$P(t) = \frac{7.386e4}{123.1 + 476.9e^{-8.96e\text{-}3t}}$$

$$P(75) = 201.4475$$