Let $\vec{a} = \langle 2, 0, 3 \rangle$, $\vec{b} = \langle -2, 6, 3 \rangle$, and $\vec{c} = \langle 3, 2, 4 \rangle$.

(a) Compute $\vec{a} \cdot \vec{b}$.

(b) Compute $\vec{a} \cdot \vec{c}$.

(c) Compute $\vec{b} \cdot \vec{c}$.

(d) Compute $\vec{a} \cdot (\vec{b} - \vec{c})$.

Part A

$$\vec{a} \cdot \vec{b} = -4 + 0 + 9$$
$$= 5$$

Part B

$$\vec{a} \cdot \vec{c} = 6 + 0 + 12$$
$$= 18$$

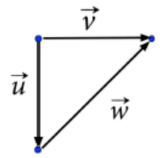
Part C

$$\vec{b} \cdot \vec{c} = -6 + 12 + 12$$
$$= 18$$

Part D

$$\vec{a}\cdot(\vec{b}-\vec{c}) = \vec{a}\cdot\langle -5, 4, -1\rangle$$
$$= -10 + 0 - 3$$
$$= -13$$

The short sides of the right triangle in the figure have length seven.



Find $\vec{u}\cdot\vec{v}$, $\vec{u}\cdot\vec{w}$, and $\vec{w}\cdot\vec{v}$.

$\vec{u}\cdot\vec{v} =$

$\vec{u}\cdot\vec{w} =$

$\vec{w}\cdot\vec{v} =$

Part A

0 because $\vec{v}$ and $\vec{u}$ are perpendicular

Part B

$\vec{u} = \langle 0, -7\rangle$

$\vec{w} = \langle 7, 7\rangle$

$\vec{u}\cdot\vec{w} = -49$

Part C

$\vec{w} = \langle 7, 7 \rangle$

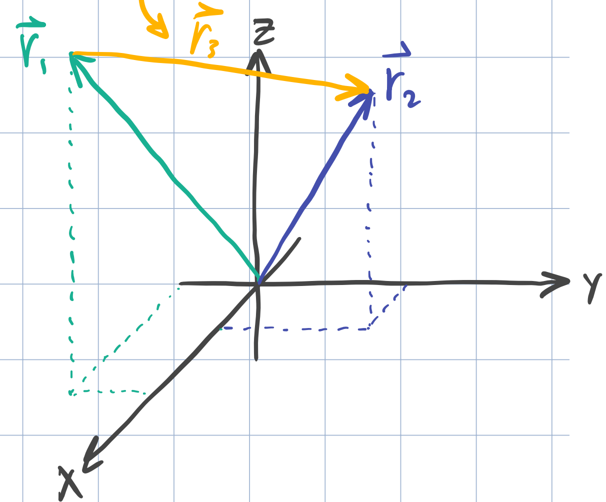
$\vec{v} = \langle 7, 0 \rangle$

$\vec{w} \cdot \vec{v} = 49$

Find the three angles of the triangle formed using the position vectors $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and the line segment connecting their endpoints. Give your answers in degrees to two decimal places. (Enter your answers as a comma-separated list with no degree symbols.)

°

Be careful with this vector and its direction



$\vec{r}_1 = \langle 2, -1, 4 \rangle$

$\vec{r}_2 = \langle 1, 2, 3 \rangle$

$\vec{r}_3 = \langle -1, 3, -1 \rangle$

General Angle Formula

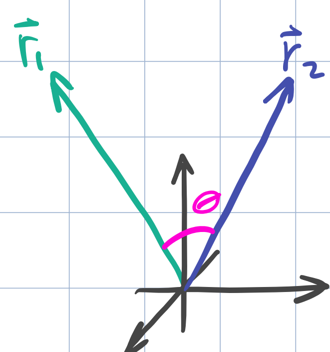
$$A_x B_x + A_y B_y + A_z B_z = |\vec{A}||\vec{B}| \cos(\theta)$$

$$\theta = \arccos\left(\frac{A_x B_x + A_y B_y + A_z B_z}{|\vec{A}||\vec{B}|}\right)$$

Angle between $\vec{r}_1$ and $\vec{r}_2$

Let $\vec{A} = \vec{r}_1$ and $\vec{B} = \vec{r}_2$

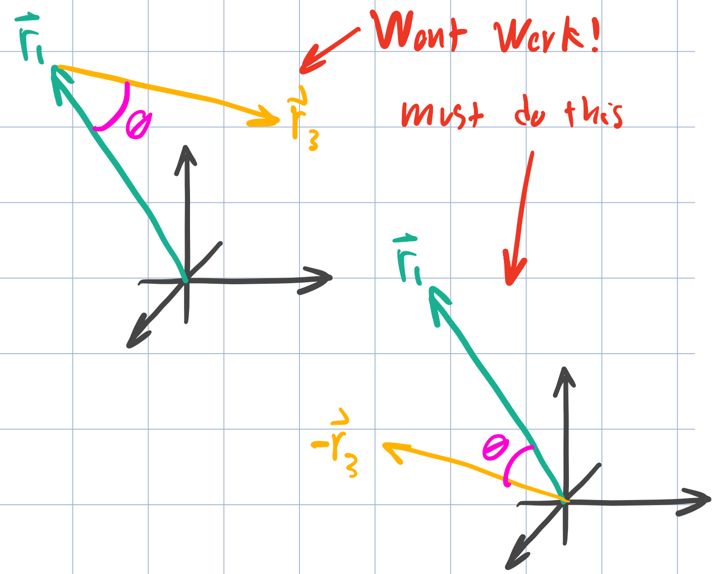
$\theta = 45.585°$



Angle between $\vec{r}_1$ and $\vec{r}_3$

Let $\vec{A} = \vec{r}_1$ and $\vec{B} = -\vec{r}_3$

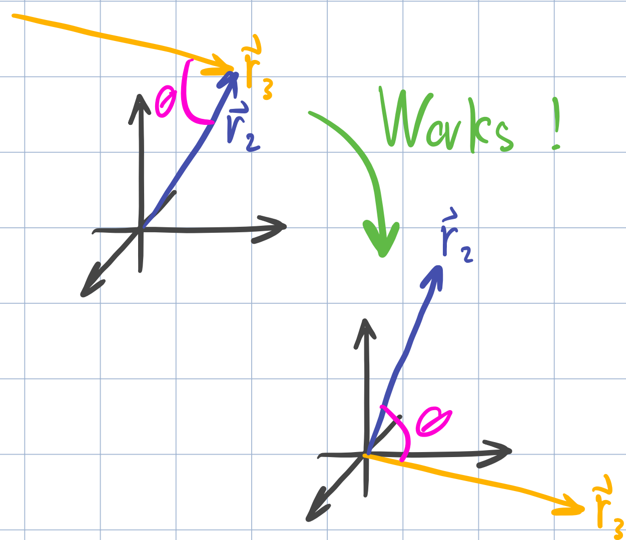
$\theta = 53.690°$



Angle between $\vec{r}_2$ and $\vec{r}_3$

Let $\vec{A} = \vec{r}_1$ and $\vec{B} = \vec{r}_3$

$\theta = 80.726°$



If $\vec{a} \cdot \vec{b} = 30$, $||\vec{a}|| = 5$, and the angle between the vectors $\vec{a}$ and $\vec{b}$ is 30°, what is the magnitude of vector $\vec{b}$?

$$|\vec{a}||\vec{b}|\cos(\theta) = \vec{a} \cdot \vec{b}$$

$$|\vec{b}| = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\cos(\theta)}$$

$$= \frac{30}{5\cos(30)}$$

Find the orthogonal decomposition of vector $\vec{b} = \langle 4, 0, 0\rangle$ with respect to vector $\vec{a} = \langle 6, -5, 0\rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{b}_{\parallel}$ =

$\vec{b}_{\perp}$ =

$$\vec{b}_{\parallel} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$$

$$= \frac{24}{36+25} \langle 6, -5, 0 \rangle$$

$$\vec{b}_\perp = \vec{b} - \vec{b}_\parallel$$

Find the work done in Newton-meters by the constant force $\vec{F} = 20\hat{\imath} + 10\hat{\jmath} + 15\hat{k}$ in moving a particle along the straight line from point $P(7, -2, 6)$ to $Q(9, 3, 3)$, where distance is in meters.

N-m

$$\vec{r} = Q - P$$

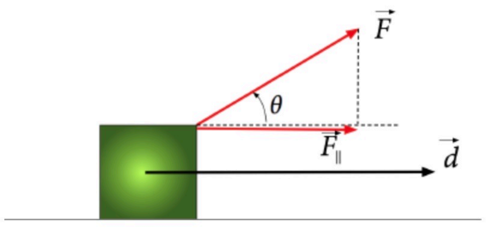
$$= \langle 2, 5, -3 \rangle \text{m}$$

$$\vec{F} = \langle 20, 10, 15 \rangle \text{N}$$

$$\vec{r} \cdot \vec{F} = 40 + 50 - 45$$

$$= 45 \text{ Nm}$$

After a snow storm Bob pulls his sled along a horizontal path with a constant force of $F = 9$ N, with the rope tipped upward $\theta = 30°$ as in the figure.



If Bob pulls the sled 25 m, how much work does he do?

N-m

$$\vec{F}_\parallel = (\vec{F} \cdot \hat{\imath})\hat{\imath}$$

$$= (9\cos(30))\langle 1, 0, 0 \rangle$$

$$= \langle 9\cos(30), 0, 0 \rangle \text{ N}$$

$\vec{W} = d \cdot |\vec{F}_{\parallel}|$

$= 25 \cdot 9 \cos(30)$ N