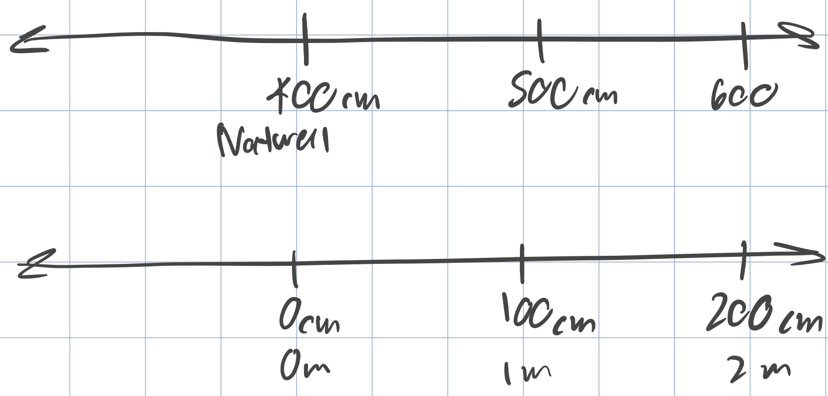
It takes 30J of work to stretch a spring from its natural length of 400 cm to 500 cm. Find the work required to stretch it from 500 cm to 600 cm

$W_i = F_i \cdot d_i$



$F_i = kx$

$W_i = kx\,\Delta x$

$W = k\int_a^b x\,dx$

$30 = k\int_0^1 x\,dx$

$30 = k\left(\frac{1}{2}x^2\right)$

$30 = \frac{1}{2}k$

$k = 60$

$W = 60 \cdot \int_1^2 x\,dx$

$= 60\left(\frac{1}{2}x^2\Big|_1^2\right)$

$= 30\left(x^2\Big|_1^2\right)$

$= 30(4-1)$

$= 30(3)$

$= 90\text{ J}$

Find the center of mass of a region bounded by $y=2x$, the x axis, and $x=3$

$$\bar{x} = \frac{M_x}{m} = \frac{\cancel{\rho} \cdot \int_a^b x f(x)\, dx}{\cancel{\rho} \cdot \int_a^b f(x)\, dx} = \frac{18}{9} = 2$$

$COM = (2, 2)$

$$\bar{y} = \frac{M_y}{m} = \frac{\cancel{\rho} \cdot \int_a^b \frac{1}{2} f(x)^2\, dx}{\cancel{\rho} \cdot \int_a^b f(x)\, dx} = \frac{18}{9} = 2$$

$$\int_0^3 2x\, dx = x^2 \Big|_0^3$$

$$= 9$$

$$\int_0^3 x \cdot 2x\, dx = \int_0^3 2x^2\, dx$$

$$= \frac{2}{3} x^3 \Big|_0^3$$

$$= \frac{2}{3}(27)$$

$$= 18$$

$$\int_0^3 \frac{1}{2}(2x)^2\, dx = \int_0^3 \frac{1}{2}(4x^2)\, dx$$

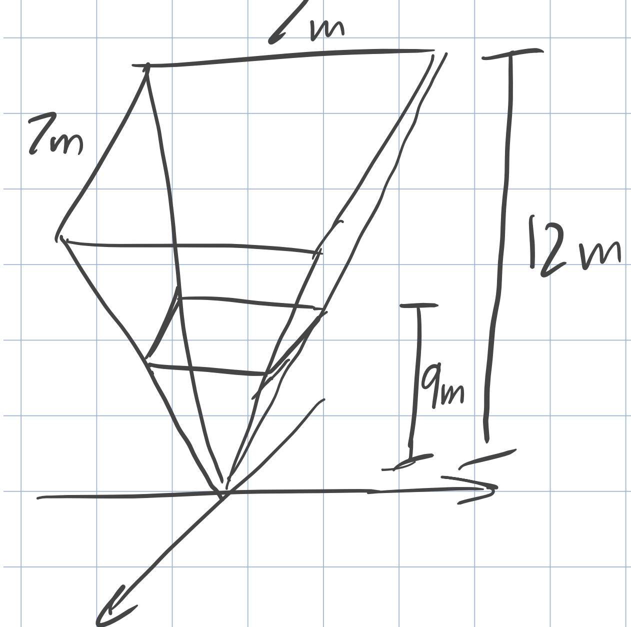
$$= \int_0^3 2x^2\, dx$$

$$= \frac{2}{3} x^3 \Big|_0^3$$

$$= \frac{2}{3}(27)$$

$= 18$

A tank in the shape of an inverted pyramid with a square cross section has a height of 12 m with a 49 m^2 cross section at the top. The tank is filled with water of a density of 1000 kg/m^3 up to the 9 m mark. Set up the integral to pump all the water to the top of the tank.



$W_i = F_i \cdot d_i$

$d_i = 12 - Y_i$

$F_i = \rho \cdot 9.8 \cdot V_i$

$V_i = S^2 \Delta y_i$

$= (2x_i)^2 \Delta y_i$

$= (2 \cdot \frac{7}{24} Y_i)^2 \Delta y_i$

$= (\frac{7}{12} Y_i)^2 \Delta y$

$= \frac{49}{144} Y_i^2 \Delta y$

$F_i = 9.8 \rho (\frac{49}{144} Y_i^2 \Delta y)$

$W_i = (9.8 \rho (\frac{49}{144} y^2 \Delta y))(12 - Y_i)$

$Y = mx + b$

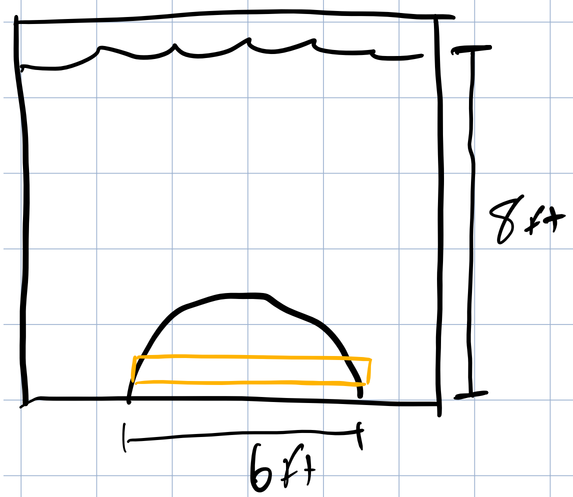
$m = \frac{12}{\frac{7}{2}} = \frac{12}{1} \cdot \frac{2}{7} = \frac{24}{7}$

$Y = \frac{24}{7} x$

$X_i = \frac{7}{24} Y_i$

$$W = 9.8\rho \frac{49}{144} \cdot \int_0^9 (y^2)(12 - y)\, dy$$

A vertical dam has a semicircular gate at the bottom as shown below. Set up the integral to find the hydrostatic force on it.



$$F_i = \rho A_i d_i$$

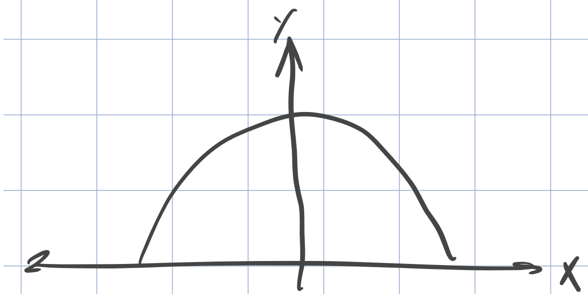
$$d_i = 8 - y_i$$

$$A_i = 2x_i \Delta y$$

$$= 2(\sqrt{9 - y_2})\Delta y$$

$$F_i = 2\rho(\sqrt{9 - y_2})(8 - y_i)\Delta y$$

$$F = 2\rho \cdot \int_0^3 \sqrt{9 - y_2}\,(8 - y)\, dy$$



$$3^2 = x^2 + y^2$$

$$x^2 = 9 - y^2$$

$$x_i = \sqrt{9 - y^2}$$