Evaluate the iterated integral.

$$\int_1^5 \int_0^2 x^2y^3 \, dx \, dy$$

$$\int_1^5 \int_0^2 x^2y^3 \, dx \, dy$$

$$= \int_1^5 \left(\frac{1}{3}x^3y^3\right)\Big|_0^2 \, dy$$

$$= \int_1^5 \frac{1}{3}y^3 \left[(2)^3 - (0)^3\right] dy$$

$$= \int_1^5 \frac{1}{3}y^3 (8) \, dy$$

$$= \frac{8}{3}\int_1^5 y^3 \, dy$$

$$= \frac{8}{3}\left(\frac{1}{4}y^4\right)\Big|_1^5$$

$$= \frac{2}{3}\left[(5)^4 - (1)^4\right]$$

$$= 416$$

Evaluate the iterated integral.

$$\int_0^{\pi} \int_0^{\pi/4} 9 \sin(x + 4y) \, dy \, dx$$

$$\int_0^{\pi} \int_0^{\pi/4} 9\sin(x+4y) \, dy \, dx$$

$$= 9\int_0^{\pi} \int_0^{\pi/4} \sin(x+4y) \, dy \, dx$$

$$= 9\int_0^{\pi} \left(-\cos(x+4y)(4)\right)\Big|_0^{\pi/4} dx$$

$$= -36\int_0^{\pi} \cos\left(x + \cancel{4}\cdot\frac{\pi}{\cancel{4}}\right) dx$$

$$= -36\int_0^{\pi} \cos(x+\pi)\, dx$$

$$= -36\left(\sin(x+\pi)\right)\Big|_0^{\pi}$$

$$= -36\sin(2\pi)$$

$$= 0$$

Evaluate the iterated integral.

$$\int_0^5 \int_1^y e^{2x-3y}\, dx\, dy$$

$$\int_0^5 \int_1^y e^{2x-3y}\, dx\, dy$$

$$= \int_0^5 \left(\frac{1}{2} e^{2x-3y}\right)\Big|_1^y dy$$

$$= \frac{1}{2}\int_0^5 \left[\left(e^{2y-3y}\right) - \left(e^{2-3y}\right)\right] dy$$

$$= \frac{1}{2}\int_0^5 e^{-y} - e^{2-3y}\, dy$$

$$= \frac{1}{2}\left(-e^{-y} - e^{2-3y}\left(-\frac{1}{3}\right)\right)\Big|_0^5)$$

$$= \frac{1}{2}\left(-e^{-y} + \frac{1}{3}e^{2-3y}\right)\Big|_0^5$$

$$= \frac{1}{2}\left[\left(-e^{-5} + \frac{1}{3}e^{2-15}\right) - \left(-e^{0} + \frac{1}{3}e^{2}\right)\right]$$

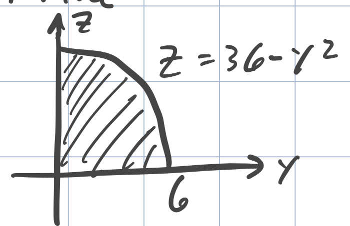
$$=\frac{1}{2}\left(-e^{-5}+\frac{1}{3}e^{-13}+1-\frac{1}{3}e^{2}\right)$$

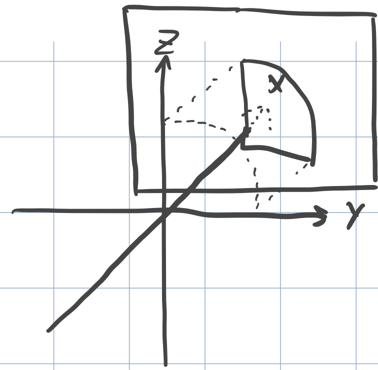
Compute the volume of the region.

the region in the first octant bounded by the parabolic cylinder $z = 36 - y^2$ and the plane $x = 7$

[ ] units^3

Zy Plane



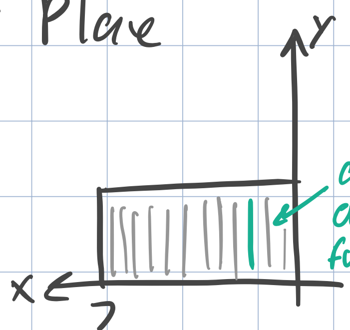


$Z = 36 - y^2$ when $0 \le y \le 6$

Integrate on Zx plane first then on yx

$$A(y) = \int_0^6 36 - y^2 \, dy$$

xy Plane



one stripe has A area so it is the function we care about

$y = A$ when $0 \le x \le 7$

$$V = \int_0^7 A(y) \, dx$$

Solve for Volume

$$= \int_0^7 \int_0^6 36 - y^2 \, dy \, dx$$

$$= \int_0^7 \left(36y - \frac{1}{3}y^3\right)\Big|_0^6 \, dx$$

$$= \int_0^7 216 - \frac{1}{3} \cdot 216 \, dx$$

$= \int_0^7 144 \, dx$

$= 144 \int_0^7 1 \, dx$

$= 144 \cdot 7$

$= 1008$

Evaluate the double integral of the function over the region.

$\iint_D 8x^2 \, dA$ where D is the region in the first quadrant bounded by $y = 3x$ and $y = x^3$

Bounding functions are in terms of x so this is a type 1 region.

Find points of intersection of bounding functions

Let $g(x) = x^3$ and $h(x) = 3x$

$g(x) = h(x)$

$x^3 - 3x = 0$

$x(x^2 - 3) = 0$

$x = 0$ and $x = \pm\sqrt{3}$

$D = \{(x, y) \mid 0 \leq x \leq \sqrt{3}, \; x^3 \leq y \leq 3x\}$

$$V = \iint_D 8x^2\, dA$$

$$= \int_0^{\sqrt{3}} \int_{x^3}^{3x} 8x^2 \, dy \, dx$$

$$= \int_0^{\sqrt{3}} 8x^2 \int_{x^3}^{3x} 1 \, dy \, dx$$

$$= \int_0^{\sqrt{3}} 8x^2 (y) \Big|_{x^3}^{3x} \, dx$$

$$= \int_0^{\sqrt{3}} 8x^2 [(3x) - (x^3)] \, dx$$

$$= 8 \int_0^{\sqrt{3}} x^2 (3x - x^3) \, dx$$

$$= 8 \int_0^{\sqrt{3}} 3x^3 - x^5 \, dx$$

$$= 8 \left(\frac{3}{4} x^4 - \frac{1}{6} x^6 \right) \Big|_0^{\sqrt{3}}$$

$$= 8 \left(\frac{3}{4} (\sqrt{3})^4 - \frac{1}{6} (\sqrt{3})^6 \right)$$

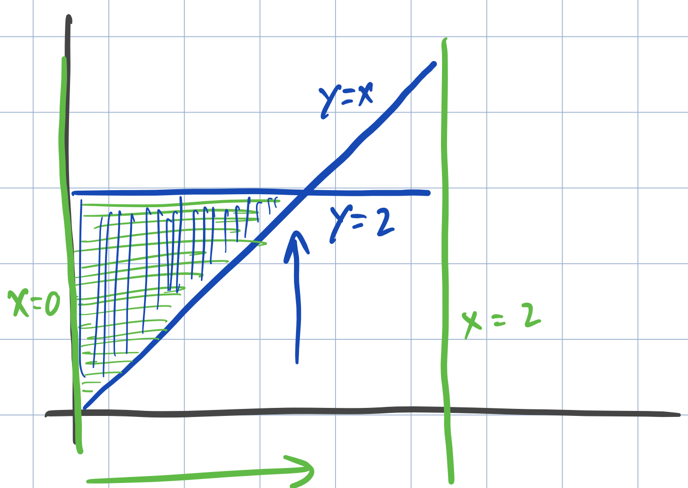
$$= 8 \left(\frac{3}{4} \cdot 3^2 - \frac{1}{6} \cdot 3^3 \right)$$

$$= 8 \left(\frac{27}{4} - \frac{27}{6} \right)$$

$$= 18$$

Interchange the order of integration on $\int_0^2 \int_x^2 f(x, y)\, dy\, dx$.

- $\int_0^2 \int_y^2 f(x, y)\, dx\, dy$
- $\int_0^2 \int_0^y f(x, y)\, dx\, dy$
- $\int_x^2 \int_0^2 f(x, y)\, dx\, dy$
- $\int_2^x \int_0^2 f(x, y)\, dx\, dy$
- $\int_0^2 \int_y^0 f(x, y)\, dx\, dy$

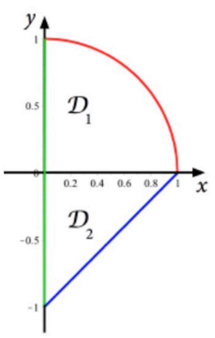


Define as a type 2 region

$D = \{(x,y) \mid 0 \le y \le 2,\ 0 \le x \le y\}$

$V = \int_0^2 \int_0^y f(x,y)\, dx\, dy$

Let f be a continuous function defined on the proper region $D = D_1 \cup D_2 \subset \mathbb{R}^2$ as shown in the figure.



D_1 is the region in the first quadrant inside the circle $x^2 + y^2 = 1$, and D_2 is the region in the fourth quadrant bounded by the coordinate axes and the line $y = x - 1$. Determine the value of $\iint_{D_1} f(x, y)\, dA$ if $\iint_{D_1 \cup D_2} f(x, y)\, dA = 32$ and $\iint_{D_2} f(x, y)\, dA = -10$.

$\iint_{D_1} f(x, y)\, dA =$ ______

$$V_{D_1 \cup D_2} = V_{D_1} + V_{D_2}$$

$$V_{D1} = V_{D_1 \cup D_2} - V_{D_2}$$

$$= 32 + 10$$

$$= 42$$