Find the sum of $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+4}\right)$

$$S_n=\left(\frac{1}{1}-\cancel{\frac{1}{5}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{6}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{7}}\right)+\left(\frac{1}{4}-\cancel{\frac{1}{8}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{9}}\right)+\cdots+\left(\frac{1}{\infty}-\frac{1}{\infty}\right)$$

$$=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$

$$=\frac{25}{12}$$

Determine if the series is convergent, if so find the Sum:

$$4+3+\frac{9}{4}+\frac{27}{16}+\ldots$$

$$a_n=\frac{3^n}{4^{n-1}}$$

$$=\frac{3^n}{4^n\frac{1}{4}}$$

$$=4\,\frac{3^n}{4^n}$$

$$=4\left(\frac{3}{4}\right)^n$$

Geometric Series

$$S=\frac{a}{1-r}$$

$$=\frac{4}{1-\frac{3}{4}}$$

$$=16$$

Determine if the series is convergent, if so find the sum:

$4 + 3 + \frac{9}{4} + \frac{27}{16} + \ldots$

$a_n = (-1)^n \left(\frac{5^n}{3^{n-1}}\right)$

$= (-1)^n \, 3 \left(\frac{5}{3}\right)^n$

Diverges

Determine if the series is convergent, if so find the sum:

$\sum_{n=1}^{\infty} \frac{n}{n+3}$

Divergence Test

$\lim_{n \to \infty} \frac{n}{n+3} = \frac{\infty}{\infty}$ by LH $\rightarrow$

$\lim_{n \to \infty} \frac{1}{1} = 1$

Divergent

Determine if the series is convergent, if so find the sum:

$\sum_{n=1}^{\infty} \frac{1}{(n+6)(n+7)}$

$$\rightarrow \frac{1}{(n+6)(n+7)}$$

$$= \frac{A}{n+6} + \frac{B}{n+7}$$

$$= \frac{A(n+7) + B(n+6)}{(n+6)(n+7)}$$

$$An + 7A + Bn + 6B = 1$$

$$\rightarrow An + Bn = 0$$
$$7A + 6B = 1$$

$$A + B = 0$$
$$7A + 6B = 1$$

$$-6A - 6B = 0$$
$$7A + 6B = 1$$

$$A = 1$$

$$B = -1$$

$$= \frac{1}{n+6} - \frac{1}{n+7}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n+6} - \frac{1}{n+7}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n+6} - \frac{1}{(n+1)+6}$$

Telescoping Series

$S = a_1$

$= \frac{1}{1+6}$

$= \frac{1}{7}$

Detemine if the series is convergent, if so find the Sum:

$\sum_{n=1}^{\infty} 3\operatorname{arcsec}(n)$

Divergence Test

$\lim_{n\to\infty} 3\operatorname{arcsec}(n)$

$= 3\lim_{n\to\infty} \operatorname{arcsec}(n)$

$= 3\frac{\pi}{2}$

$= \frac{3\pi}{2}$

$\frac{3\pi}{2} \neq 0$

Diverges

Determine if the series is convergent, if so find the Sum:

$$\sum_{n=1}^{\infty} \frac{3^n+7^n}{21^n}$$

$$=\sum_{n=1}^{\infty}\left(\frac{3^n}{21^n}+\frac{7^n}{21^n}\right)$$

$$=\sum_{n=1}^{\infty}\left[\left(\frac{3}{21}\right)^n+\left(\frac{7}{21}\right)^n\right]$$

$$=\sum_{n=1}^{\infty}\left[\left(\frac{1}{7}\right)^n+\left(\frac{1}{3}\right)^n\right]$$

$$=\sum_{n=1}^{\infty}\left(\frac{1}{7}\right)^n+\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n$$

Two Geometric Series

$$=\sum_{n=0}^{\infty}\left(\frac{1}{7}\right)^{n+1}+\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^{n+1}$$

$$=\sum_{n=0}^{\infty}\frac{1}{7}\left(\frac{1}{7}\right)^n+\sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{1}{3}\right)^n$$

$$S=\frac{\frac{1}{7}}{1-\frac{1}{7}}+\frac{\frac{1}{3}}{1-\frac{1}{3}}$$

$$=\frac{1}{7}\cdot\frac{7}{6}+\frac{1}{3}\cdot\frac{3}{2}$$

$$=\frac{1}{6}+\frac{1}{2}$$

$$=\frac{4}{6}$$

$$=\frac{2}{3}$$

Express $3.2\overline{18}$ as a rational number

$$a_n = 3.2 + \frac{18}{1000}\left(\frac{1}{100}\right)^n$$

$$S = 3.2 + \sum_{n=0}^{\infty} \frac{18}{1000}\left(\frac{1}{100}\right)^n$$

Geometric Series

$$= 3.2 + \frac{\frac{18}{1000}}{1 - \frac{1}{100}}$$