Find a vector parametric equation of the line **segment** from $P_1(2, -6, 7)$ to $P_2(0, 4, 1)$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{r}(t) =$ [ ] with $0 \le t \le 1$

$\vec{V} = P_2 - P_1$

$= \langle -2, 10, -6 \rangle$

$\vec{L} = \langle -2t + 2, 10t - 6, -6t + 7 \rangle$

Reparametrize the curve $\vec{r}(t) = \langle e^t, \cos(t), e^{-t} \rangle$ using $t = 3\tau + 2$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{r_1}(\tau) =$ [ ]

$\vec{r}_1(\tau) = \langle e^{3\tau+2}, \cos(3\tau+2), e^{-3\tau-2} \rangle$

Find the point where the helix $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ intersects the plane $z = \frac{\pi}{2}$.

$(x, y, z) = ($ [ ] $)$

Selve for t

$t = \frac{\pi}{2}$

$P = \left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right), \frac{\pi}{2}\right)$

Find the family of antiderivatives of $\frac{d}{dt}\vec{r}(t) = \langle t^3, t - 9, t^2 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

[ ] $+ \langle C_1, C_2, C_3 \rangle$

Find the unique parametrized curve that satisfies the initial condition $\vec{r}(0) = \langle 9, 3, -12 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{r}(t) =$ [ ]

Part A

$\int \frac{d\vec{r}}{dt}\, dt = \int \langle t^3, t-9, t^2 \rangle\, dt$

$$= \langle \tfrac{1}{4}t^4, \tfrac{1}{2}t^2 - 9t, \tfrac{1}{3}t^3 \rangle + \langle C_1, C_2, C_3 \rangle$$

Part B

Initial Condition $\vec{r}(0) = \langle 9, 3, -12 \rangle$, solve for Cs

$9 = \tfrac{1}{4}(0)^4 + C_1$ $\quad 3 = 0 - 0 + C_2$ $\quad -12 = 0 + C_3$

$C_1 = 9$ $\quad C_2 = 3$ $\quad C_3 = -12$

$$\vec{r}(t) = \langle \tfrac{1}{4}t^4 + 3, \tfrac{1}{2}t^2 - 9t + 3, \tfrac{1}{3}t^3 - 12 \rangle$$

Find the family of vector-valued functions whose second derivatives are given by $\frac{d^2}{dt^2}\vec{r}(t) = \langle -2\cos(t), -2\sin(t), 9 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

[ ] $+ \vec{C_1}t + \vec{C_2}$

Find the unique parametrized curve that satisfies the initial conditions $\vec{r}(0) = \langle 1, -1, 5 \rangle$ and $\frac{d}{dt}\vec{r}(0) = \langle 2, -10, 7 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{r}(t) =$ [ ]

Part A

$$\iint \frac{d^2r}{dt^2}\,dt = \iint \langle -2\cos(t), -2\sin(t), 9 \rangle\,dt$$

$$= \int \langle -2\sin(t), 2\cos(t), 9t \rangle\,dt + \vec{C_1}$$

$$= \langle 2\cos(t), 2\sin(t), \tfrac{9}{2}t^2 \rangle + \vec{C_1}t + \vec{C_2}$$

Part B

Initial condition $\frac{d}{dt} r(0) = \langle 2, -10, 7 \rangle$, Solve for $\vec{C}_1$

$$\langle 2, -10, 7 \rangle = \langle -2\sin(0), 2\cos(0), 0 \rangle + \vec{C}_1$$

$$\langle 2, -10, 7 \rangle = \langle 0, 2, 0 \rangle \vec{C}_1$$

$$\vec{C}_1 = \langle 2, -12, 7 \rangle$$

Initial condition $r(0) = \langle 1, -1, 5 \rangle$, Solve for $\vec{C}_2$

$$\langle 1, -1, 5 \rangle = \langle 2\cos(0), 2\sin(0), 0 \rangle + \vec{C}_1 \cdot 0 + \vec{C}_2$$

$$\langle 1, -1, 5 \rangle = \langle 2, 0, 0 \rangle + \vec{C}_2$$

$$\vec{C}_2 = \langle -1, -1, 5 \rangle$$

$$\vec{r}(t) = \langle 2\cos(t) + 2t - 1, 2\sin(t) - 12t - 1, \tfrac{g}{2}t^2 + 7t + 5 \rangle$$

An object at the top of a building with height 122 feet is thrown upward with an initial speed of 29 ft/s. Find its position $z(t)$ above the ground t seconds after being thrown. (Use g = 32 ft/sec^2.)

$z(t)$ =

Find the time, in seconds, it takes for the object to hit the ground.

s

Part A

$$v(t) = -32t + 29$$

$$r(t) = -16t^2 + 29t + 122$$

Part B

$$0 = -16t^2 + 29t + 122$$

$$t = \frac{-29 \pm \sqrt{29^2 - 4 \cdot -16 \cdot 122}}{2 \cdot -16}$$

$$= \frac{-29 \pm \sqrt{8649}}{-32}$$

$$= \frac{29 \pm 93}{32}$$

time cant be negative

$$t = \frac{29+93}{32} \quad \text{or} \quad \cancel{t = \frac{29-93}{32}}$$

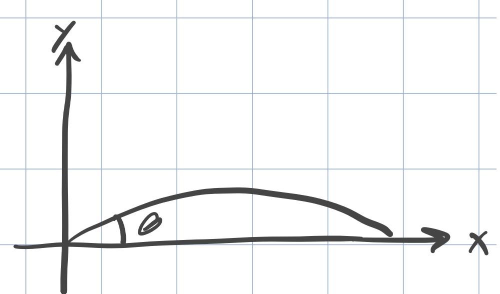
$$t = \frac{122}{32}$$

A soccer ball kicked by a Real Madrid midfielder leaves the player's foot at a speed of 58 mph at an angle θ above the horizontal. Find the angle θ, in radians, that will maximize the distance traveled downrange. (Use g = 32 ft/sec^2.)

Find the maximum height and distance traveled downrange, in ft, at the optimal angle. (Round your answers to two decimal places.)

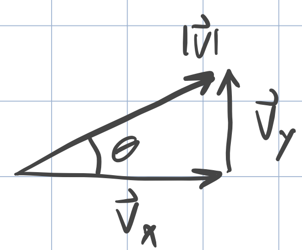
maximum height		ft
distance traveled downrange		ft

Part A



$$\vec{v}(t) = \langle |\vec{v}| \cos(\theta), -32t + |\vec{v}| \sin(\theta) \rangle$$

$$\vec{r}(t) = \int \vec{v}(t)\, dt$$



$$= \langle |\vec{v}| \cos(\theta) t, -16t^2 + |\vec{v}| \sin(\theta) t \rangle$$

Solve $\vec{r}_y(t) = 0$ for t in terms of θ

$$0 = -16t^2 + |\vec{v}|\sin(\theta)t$$

$$0 = -16t + |\vec{v}|\sin(\theta)$$

$$16t = |\vec{v}|\sin(\theta)$$

$$t = \frac{|\vec{v}|\sin(\theta)}{16}$$

Solve for distance as a function of θ

$$x = |\vec{v}|\cos(\theta)\frac{|\vec{v}|\sin(\theta)}{16}$$

$$x = \frac{1}{16}|\vec{v}|^2\cos(\theta)\sin(\theta)$$

Max x is at $\theta = \frac{\pi}{4}$

Part B

$58_{mph} = 85.0\overline{6}$ fps

Find max x

$$x = \frac{1}{16}85.0\overline{6}\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= 266.135 \text{ ft}$$

Part C

Time when the ball hits the ground

$$t = \frac{85.0\overline{6}\,\sin(\theta)}{16}$$

Time when the ball is at its max height

$$t = \frac{85.0\overline{6}\,\sin(\theta)}{16} \cdot \frac{1}{2}$$

$$= 1.880 \text{ s}$$

Max height $\quad -16t^2 + |\vec{v}|\sin(\theta)t$

$$y = -16(1.88)^2 + 85.0\overline{6}\,\sin\left(\frac{\pi}{4}\right)(1.88)$$

$$= 56.533 \text{ s}$$

Consider a trajectory in space with acceleration vector $\vec{a}(t) = \langle e^{-t}, -1, 2t\rangle$. Solve the initial value problem to find the trajectory $\vec{r}(t)$ for the problem with initial velocity $\vec{v}(0) = \langle 7, 0, -3\rangle$ and initial position $\vec{r}(0) = \langle 5, 4, -9\rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{r}(t) =$

$$\vec{V}(t) = \int \vec{a}(t)\,dt$$

$$= \langle -e^{-t}, -t, t^2\rangle + C_1$$

$$\vec{V}(0) = \langle 7, 0, -3\rangle$$

$$\langle 7, 0, -3\rangle = \langle -1, 0, 0\rangle + \vec{C}_1$$

$$\vec{C}_1 = \langle 8, 0, -3\rangle$$

$$\vec{V}(t) = \langle -e^{-t} + 8, -t, t^2 - 3\rangle$$

$$\vec{r}(t) = \int v(t)\,dt$$

$$= \langle e^{-t} + 8t, -\tfrac{1}{2}t^2, \tfrac{1}{3}t^3 - 3t\rangle$$

$$\vec{r}(0) = \langle 5, 4, -9\rangle$$

$$\langle 5, 4, -9\rangle = \langle 1, 0, 0\rangle + \vec{C}_2$$

$$\vec{C}_2 = \langle 4, 4, -9\rangle$$

$$\vec{r}(t) = \langle e^{-t} + 8t + 4, -\tfrac{1}{2}t^2 + 4, \tfrac{1}{3}t^3 - 3t - 9\rangle$$