A spring is 0.28 m long when it is relaxed. When a force of magnitude of 380N is applied, the spring becomes 0.36 m long.

What is the stiffness of the spring?

$$|\vec{F}_{net}| = k|s|$$

$$|s| = |0.36m - 0.28m|$$

$$= 0.08m$$

$$380 = 0.08k$$

$$k = 4750 \frac{N}{m}$$

Next the spring is compressed so that its length is 0.21 m. What magnitude of force is required to do this?

$$|\vec{F}_{net}| = k|s|$$

$$|s| = |0.21m - 0.28m|$$

$$= 0.07$$

$$|\vec{F}_{net}| = 4750 \cdot 0.07$$

$$= 332.5N$$

You push on a spring whose stiffness is 22 N/m, compressing it until it is 1.8 cm shorter than its relaxed length. What is the magnitude of the force that the spring now exerts on your hand?

$1.8\,cm = 0.018\,m$

$|s| = 0.018\,m$

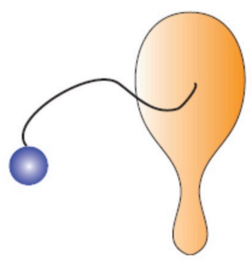
$|\vec{F}_{net}| = k|s|$

$= 22 \cdot 0.018$

$= 0.396\,N$

Use the approximation that $\vec{v}_{avg} = \vec{p}_f/m$ for each time step.

A paddle ball toy consists of a flat wooden paddle and a small rubber ball that are attached to each other by an elastic band (figure). You have a paddle ball toy for which the mass of the ball is 0.011 kg, the stiffness of the elastic band is 0.925 N/m, and the relaxed length of the elastic band is 0.295 m. You are holding the paddle so the ball hangs suspended under it, when your cat comes along and bats the ball around, setting it in motion. At a particular instant the momentum of the ball is $\langle -0.02, -0.01, -0.02 \rangle$ kg · m/s, and the moving ball is at location $\langle -0.2, -0.61, 0 \rangle$ m relative to an origin located at the point where the elastic band is attached to the paddle. Define this instant as $t = 0$.



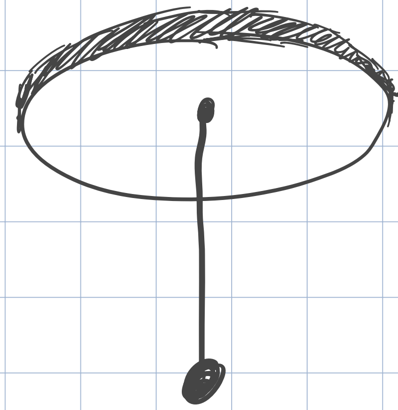
Determine the position of the ball 0.1s later, using a Δt of 0.1s

$m = 0.011\,kg$ $\quad \vec{p}_i = \langle -0.02, -0.01, -0.02 \rangle\,kg\,m/s$

$k = 0.925\,N/m$ $\quad \vec{r}_i = \langle -0.2, -0.61, 0 \rangle\,m$

$L_0 = 0.295\,m$ $\quad \Delta t = 0.1\,s$

Iterative Motion Prediction:



1. Find Net Force

2. Update Momentum

3. Update Position

1. Find Net Force

$$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{gravity}$$

↳ $\vec{F}_{gravity} = \langle 0, -9.8\frac{m}{s^2} \cdot m, 0 \rangle$

$= \langle 0, -0.1078, 0 \rangle N$

↳ $\vec{F}_{spring} = -ks\hat{L}$

↳ $s = |\vec{L}| - L_0$

↳ $|\vec{L}| = |\vec{r}_i|$

$= \sqrt{r_{ix}^2 + r_{iy}^2 + r_{iz}^2}$

$= \sqrt{0.04 + 0.3721 + 0}$

$= \sqrt{0.4121}$

$= 0.6420\,m$

$s = 0.6420\,m - 0.295\,m$

$= 0.347$

$\hookrightarrow \hat{L} = \frac{\vec{L}}{|\vec{L}|}$

$= \frac{\langle -0.2, -0.61, 0 \rangle m}{0.6420\,m}$

$= \langle -0.3116, -0.9502, 0 \rangle m$

$\vec{F}_{spring} = -(0.925)(0.347)(\langle -0.3116, -0.9502, 0 \rangle)$

$= -0.3210 \langle -0.3116, -0.9502, 0 \rangle$

$= \langle 0.1000, 0.3045, 0 \rangle N$

$\vec{F}_{net} = \langle 0.1000, 0.3045, 0 \rangle + \langle 0, -0.1078, 0 \rangle$

$= \langle 0.1000, 0.1967, 0 \rangle N$

2. Update Momentum

$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$

$= \langle -0.02, -0.01, -0.02 \rangle + \langle 0.1000, 0.1967, 0 \rangle \cdot 0.1$

$= \langle -0.02, -0.01, -0.02 \rangle + \langle 0.01, 0.01967, 0 \rangle$

$= \langle -0.01, 0.00967, -0.02 \rangle \, kg \cdot \frac{m}{s}$

3. Update Position

$\vec{V}_f = \frac{\vec{p}_f}{m}$

$= \frac{\langle -0.01, 0.00967, -0.02 \rangle}{0.011}$

$= \langle -0.9091, 0.8791, -1.8182 \rangle \frac{m}{s}$

$\vec{r}_f = \vec{r}_i + \vec{V}_f \Delta t$

$= \langle -0.2, -0.61, 0 \rangle + \langle -0.09091, 0.8791, -1.8182 \rangle \cdot 0.1$

$= \langle -0.2, -0.61, 0 \rangle + \langle -0.09091, 0.08791, -0.18182 \rangle$

$= \langle -0.2909, -0.5221, -0.18182 \rangle \, m$

Starting with the same initial position and momentum, find the position of the ball at 0.05s.

$m = 0.011 \, kg$ $\quad \vec{p}_i = \langle -0.02, -0.01, -0.02 \rangle \, kg \, m/s$

$k = 0.925 \, N/m$ $\quad \vec{r}_i = \langle -0.2, -0.61, 0 \rangle \, m$

$L_0 = 0.295 \, m$ $\quad \Delta t = 0.05 s$

1. Find Net Force

→ Same as the first section

$F_{net} = \langle 0.1000, 0.1967, 0 \rangle N$

2. Update Momentum

$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$

$= \langle -0.02, -0.01, -0.02 \rangle + \langle 0.1000, 0.1967, 0 \rangle \cdot 0.05$

$= \langle -0.02, -0.01, -0.02 \rangle + \langle 0.005, 0.009835, 0 \rangle$

$= \langle -0.015, -0.000165, -0.02 \rangle kg \cdot \frac{m}{s}$

3. Update Position

$\vec{V}_f = \frac{\vec{p}_f}{m}$

$= \frac{\langle -0.015, -0.000165, -0.02 \rangle}{0.011}$

$= \langle -1.3636, -0.015, -1.8182 \rangle$

$\vec{r}_f = \vec{r}_i + \vec{V}_f \Delta t$

$= \langle -0.2, -0.61, 0 \rangle + \langle -1.3636, -0.015, -1.8182 \rangle \cdot 0.05$

$= \langle -0.2, -0.61, 0 \rangle + \langle -0.06818, -0.00075, -0.09091 \rangle$

$= \langle -0.2682, -0.6108, -0.09091 \rangle$

Using the position and momentum you just calculated, find the position of the ball at $t = 0.1$

Values Updated from Python script to account for rounding

$m = 0.011\,kg$ $\quad \vec{p}_i = \langle -0.01500, -0.0001422, -0.02 \rangle$

$k = 0.925\,N/m$ $\quad \vec{r}_i = \langle -0.2682, -0.6106, -0.09091 \rangle$

$L_0 = 0.295\,m$ $\quad \Delta t = 0.05\,s$

1. Find Net Force

$\vec{F}_{net} = \vec{F}_{spring} + \vec{F}_{gravity}$

$\hookrightarrow \vec{F}_{gravity} = \langle 0, -9.8 \cdot 0.011, 0 \rangle$

$= \langle 0, -0.1078, 0 \rangle\,N$

$\hookrightarrow \vec{F}_{spring} = -ks\hat{L}$

$\hookrightarrow s = |\vec{L}| - L_0$

$\hookrightarrow |\vec{L}| = |\vec{r}_i|$

$= \sqrt{0.07193 + 0.3728 + 0.008265}$

$= 0.6731\,m$

$s = 0.3781\ m$

$\hookrightarrow \hat{L} = \frac{\vec{L}}{|\vec{L}|}$

$= \frac{\langle -0.2682, -0.6106, -0.09091 \rangle}{0.6731}$

$= \langle -0.3985, -0.9071, -0.1351 \rangle\ m$

$\vec{F}_{spring} = -(0.925)(0.3781) \cdot \langle -0.3985, -0.9071, -0.1351 \rangle$

$= -0.3497 \cdot \langle -0.3985, -0.9071, -0.1351 \rangle$

$= \langle 0.1394, 0.3173, 0.04724 \rangle\ N$

$\vec{F}_{net} = \langle 0.1394, 0.2095, 0.04724 \rangle\ N$

2. Update Momentum

$\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$

$= \vec{p}_i + \langle 0.00697, 0.01048, 0.002362 \rangle$

$= \langle -0.00803, 0.01033, -0.01764 \rangle$

3. Update Position

$$\vec{r}_f = \vec{r}_i + \frac{p_f}{m} \Delta t$$

$$= \vec{r}_i + \langle -0.73, 0.9393, -1.6035 \rangle \Delta t$$

$$= \vec{r}_i + \langle -0.0365, 0.04697, -0.08017 \rangle$$

$$= \langle -0.3047, -0.5636, -0.1711 \rangle \, m$$