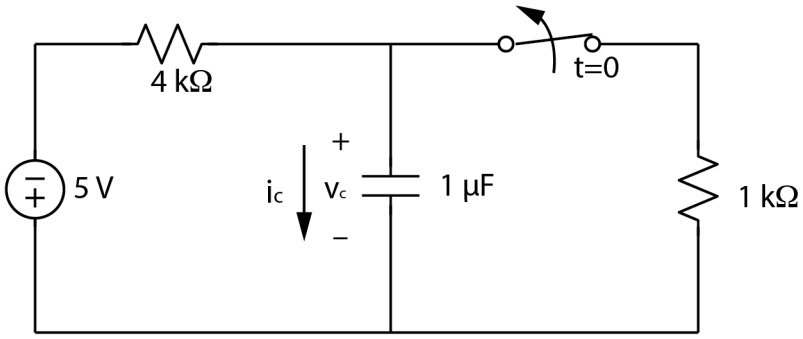
Homework 4

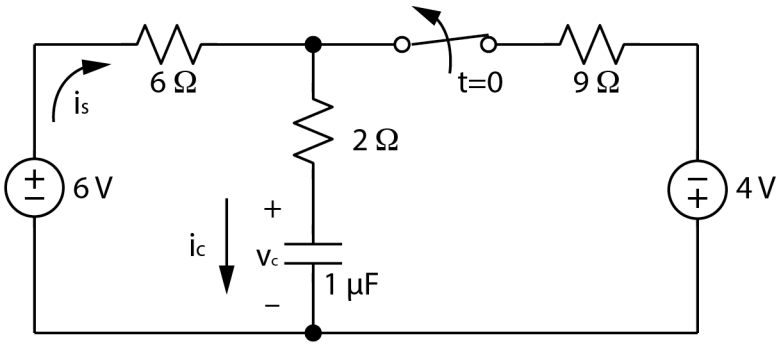
Clearly describe the reasoning behind the work done in each problem.

SOLVE 5 OF ANY OF THE FOLLOWING PROBLEMS.

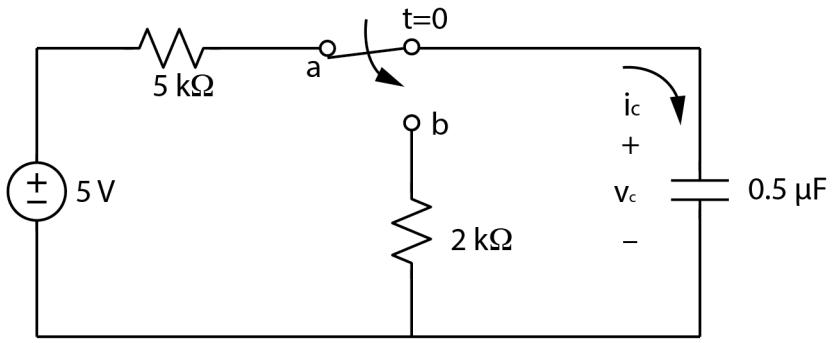
Problem 1: (*20 points*)
Solve for the capacitor's current and voltage, i_c and v_c, at $t = 0^-$, $t = 0^+$ and $t = \infty$.



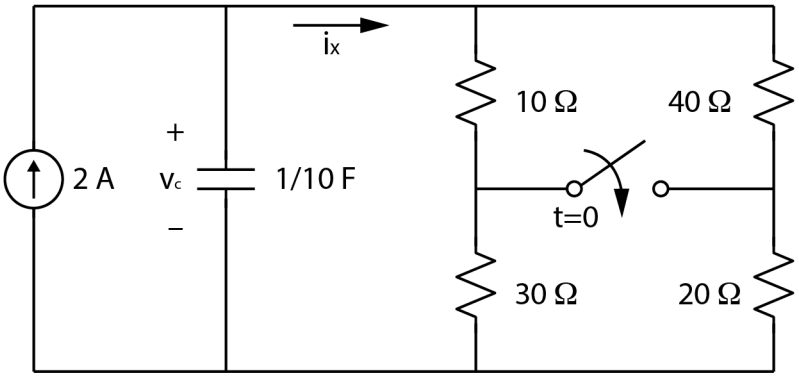
Problem 2: (*20 points*)
Solve for v_c, i_c and i_s at $t = 0^-$, $t = 0^+$ and $t = \infty$. Explain whether the capacitor is charging or discharging.



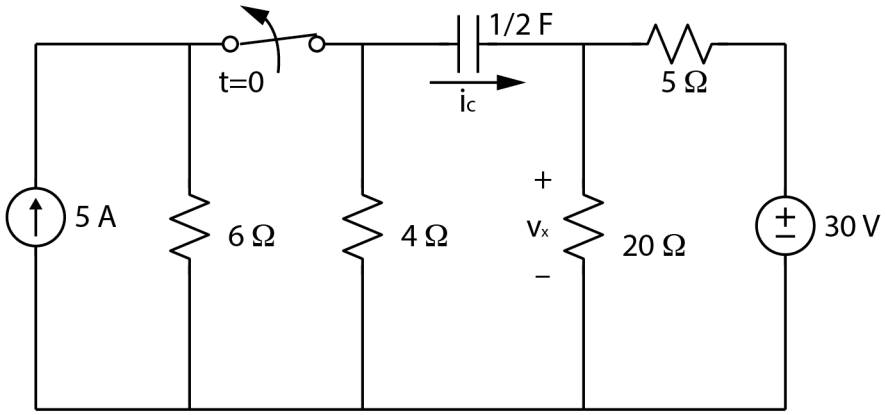
Problem 3: (*20 points*)
Consider the two-position switch in the circuit, which has been in position a for a long time. At time t=0 the switch moves to position b. Determine the time for the capacitor to discharge to 2V and calculate the current at that time.



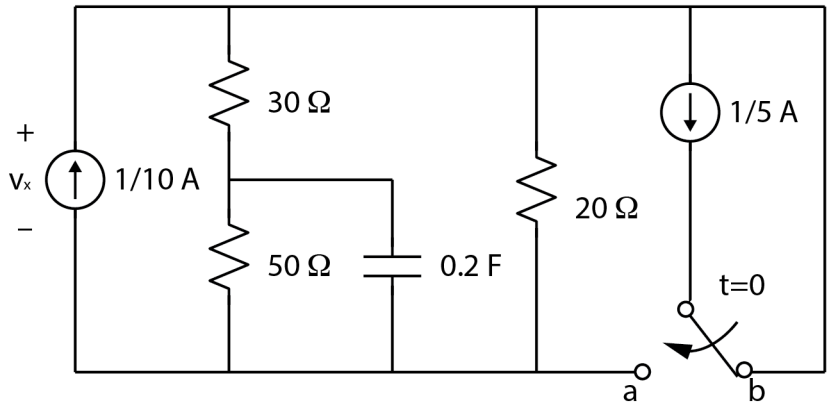
Problem 4: (*20 points*)
Solve for i_x at $t = 0^-$, $t = 0^+$ and $t = \infty$.



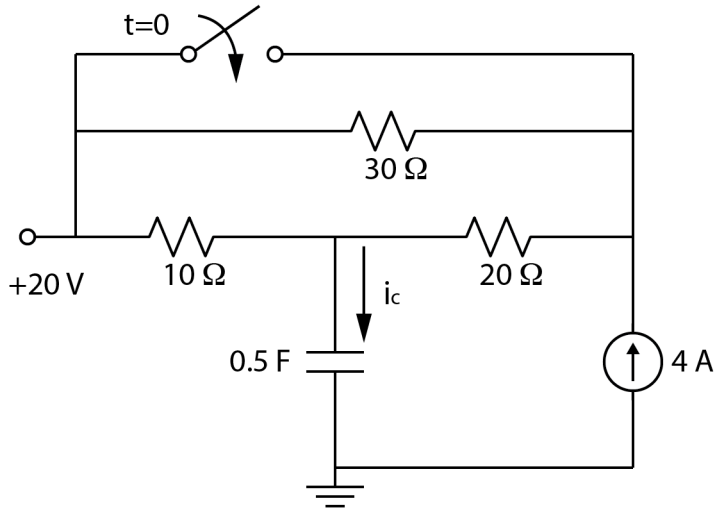
Problem 5: (*20 points*)
Solve for v_x and i_c at $t = 0^-$, $t = 0^+$ and $t = \infty$.



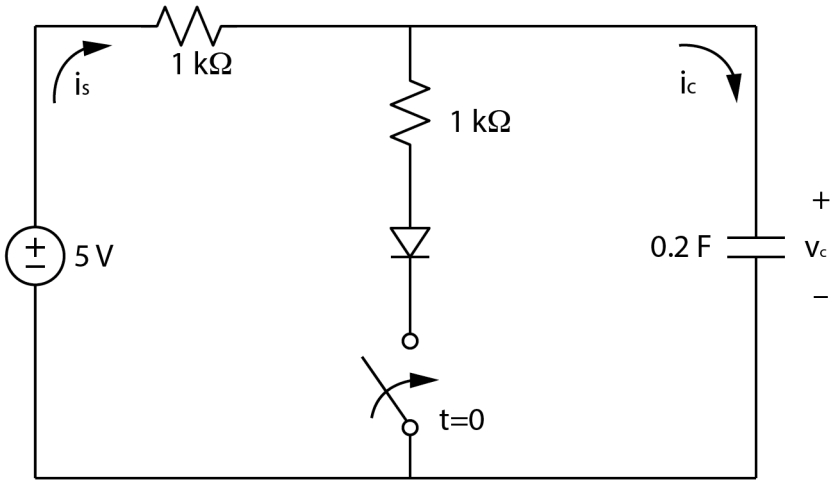
Problem 6: (*30 points*)
Solve for v_x at $t = 0^-$, $t = 0^+$ and $t = \infty$.



Problem 7: (*20 points*)
Solve for i_c at $t = 0^-$, $t = 0^+$ and $t = \infty$.

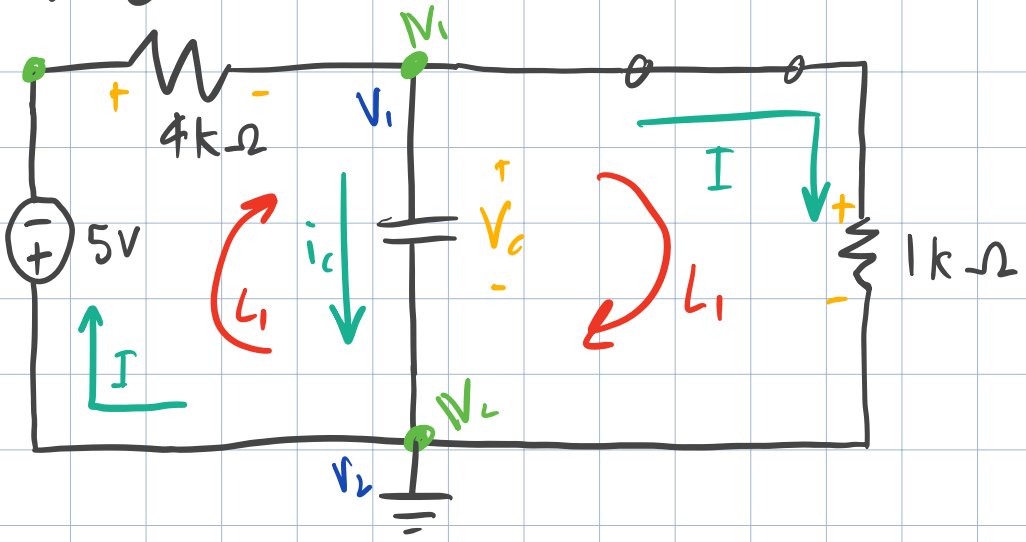


Problem 8: (*30 points*)
Solve for v_c, i_c and i_s at $t = 0^-$, $t = 0^+$ and $t = \infty$. $V\gamma = 1$ V.



Problem 1a

When $t = 0^-$



$i_c = 0\,A$

$V_c = -1\,V$

Solve for V_c

$V_2 = 0$

$V_1 = -5 - 4000\,I$

Solve for I

KVL on L_1:

$5 + 1000\,I + 4000\,I = 0$

$5000\,I = -5$

$$I = \frac{-1}{1000}\ A$$

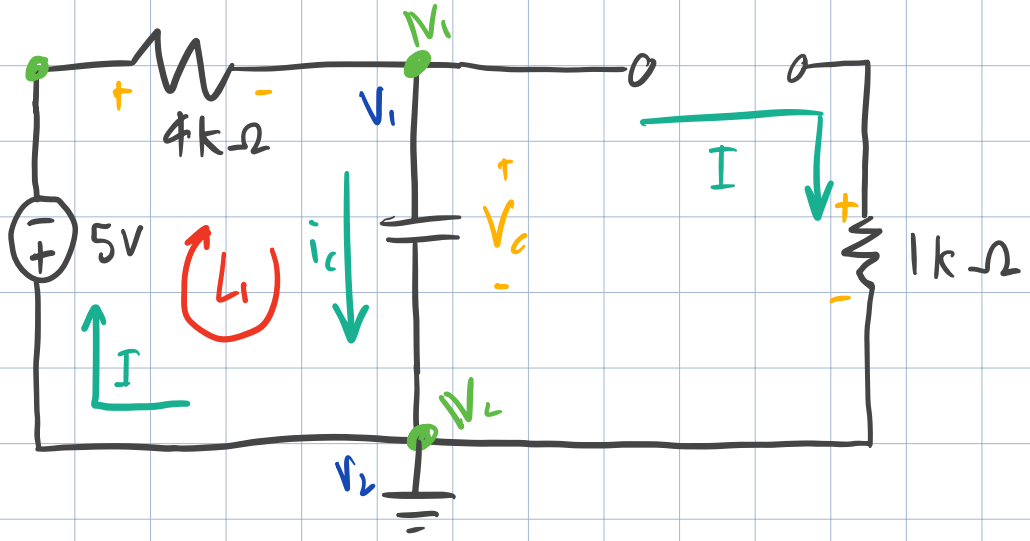
$$V_1 = -5 + 4$$

$$= -1\ V$$

$$V_c = V_1 - V_2$$

$$= -1\ V$$

When $t = 0^+$



$$i_c = -\frac{1}{1000}\ A$$

$$V_c = -1\ V$$

Solve for I

$$I = i_c$$

KVL on L_1

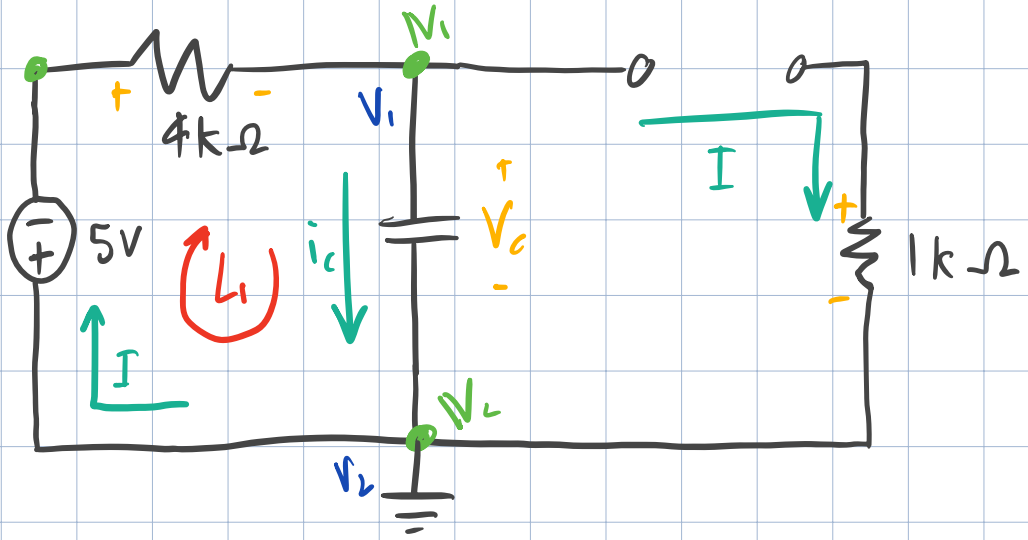
$5 + V_c + 4000I = 0$

$5 - 1 + 4000I = 0$

$I = -\frac{4}{4000}$ A

$= -\frac{1}{1000}$ A

When $t = \infty$



$i_c = 0$A

$V_c = -5$v

Solve for V_c

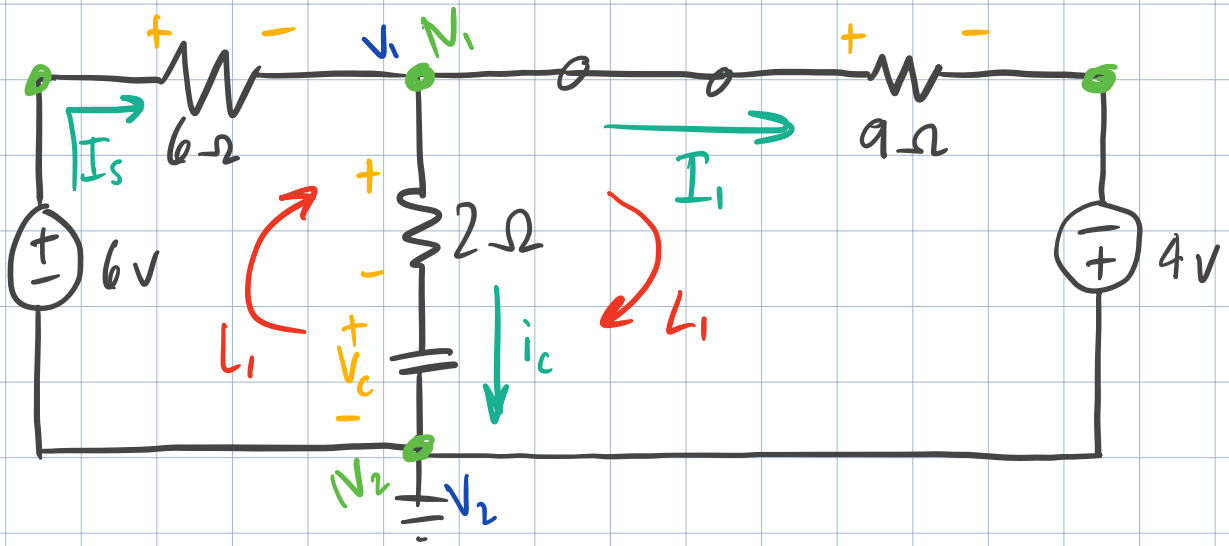
KVL en L_1

$5 + V_c + 4000I = 0$

$V_c = -5V$

Problem Two

When $t = 0^-$



$i_c = 0A$

$V_c = 2v$

$I_s = \frac{2}{3}A$

Solve for I_s

$I_s = I_1$

Kvl en L_1

$6 - 6I - 9I + 4 = 0$

$-15I = -10$

$$I = \frac{10}{15} = \frac{2}{3}\ A$$

Solve for V_c

$$V_c = V_1 - V_2$$

$$V_2 = 0\ v$$

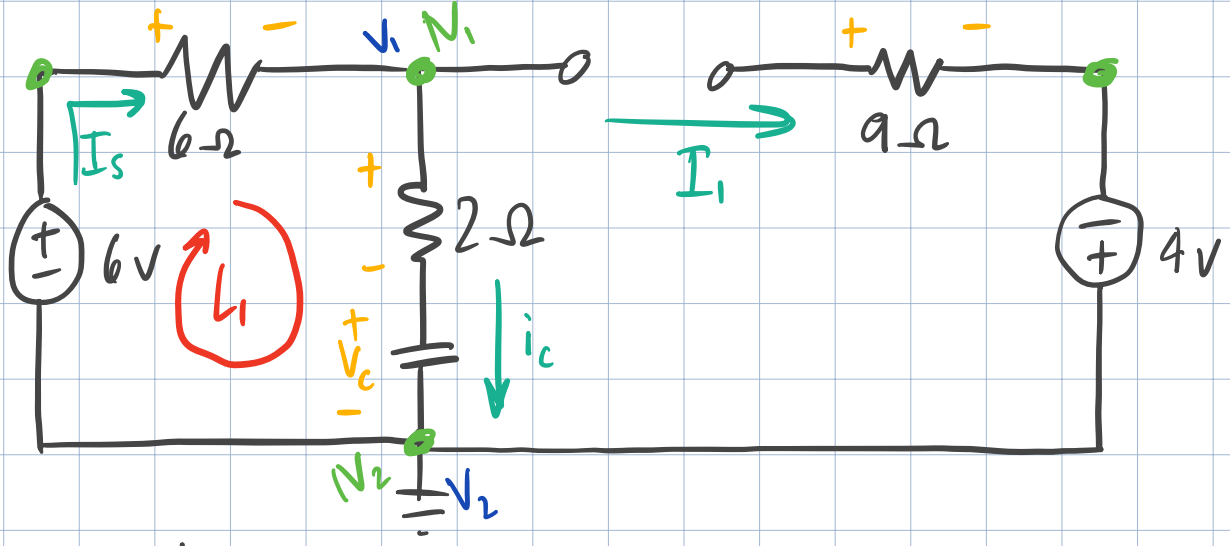
$$V_1 = 6 - 6I$$

$$= 6 - \frac{12}{3}$$

$$= 6 - 4$$

$$= 2\ v$$

When $t = 0^+$



$$i_c = \frac{1}{2}A$$

$V_c = 2V$

$I_s = \frac{1}{2}A$

Solve for i_c

$I_s = i_c$

KCL on L_1

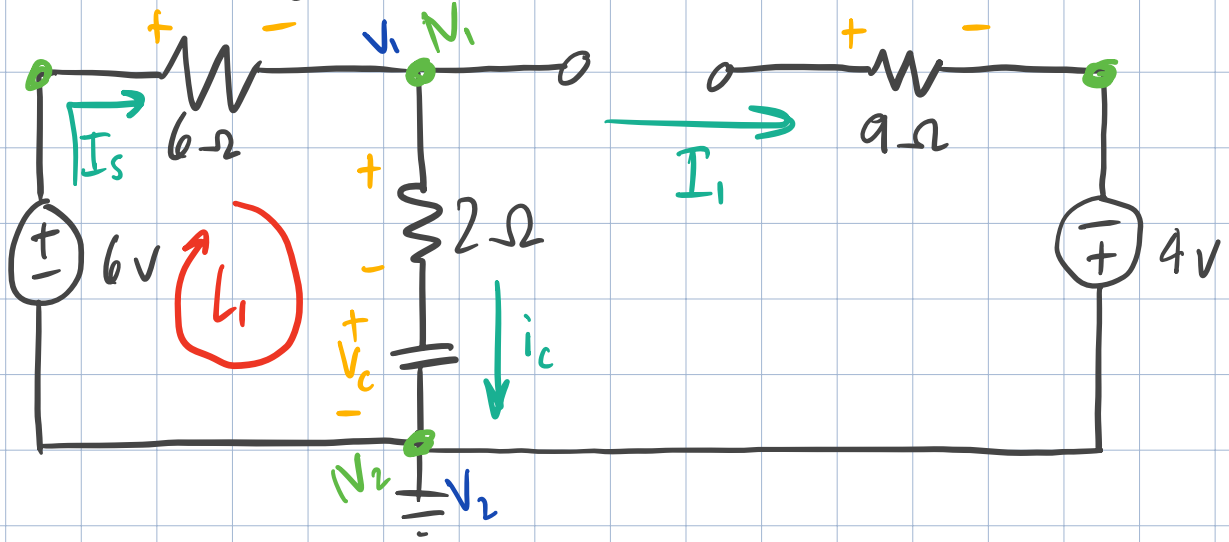
$6 - 6I_s - 2I_s - 2 = 0$

$4 - 8I_s = 0$

$I_s = \frac{4}{8}$

$= \frac{1}{2}A$

When $t = \infty$



$i_c = 0A$

$V_c = 6V$

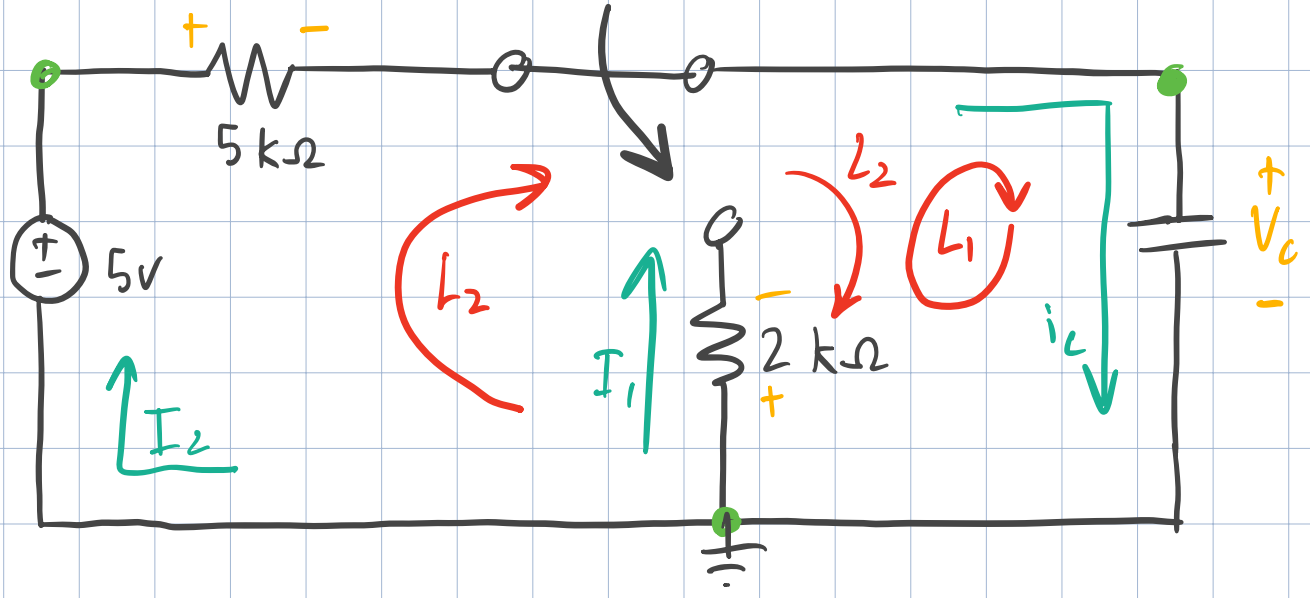
$I_s = 0A$

At $t = 0^-$ the capacitor is charged and not charging or discharging.

At $t = 0^+$ the capacitor is charging because the current flow through the capacitor is positive.

At $t = \infty$ the capacitor is charged and not charging or discharging.

Problem Three



At $t = 0^-$: $V_c = 5V$

When $0^+ < t < \infty$

Kcl on L_1

$$V_c + 2000 i_c = 0$$

$$V_c + 2000 \frac{dq}{dt} = 0$$

$$V_c + 2000 C \frac{dV_c}{dt} = 0$$

Differential Equation Solving

$$V_c = V_c(0^+) e^{-\frac{t}{RC}}$$

$$\frac{V_c}{V_c(0^+)} = e^{-\frac{t}{RC}}$$

$$-\frac{t}{RC} = \ln\left(\frac{V_c}{V_c(0^+)}\right)$$

$$t = -RC \ln\left(\frac{V_c}{V_c(0^+)}\right)$$

$$= -(2000)(0.5e-6) \ln\left(\frac{V_c}{5}\right)$$

Solve for t when $V_c = 2v$

$$t = -(2000)(0.5e-6) \ln\left(\frac{2}{5}\right)$$

$$= 9.1629e-4 s$$

Solve for i_c at $t = 9.1629e-4 s$

KVL on h_1

$$2000 i_c - V_c(t) = 0$$

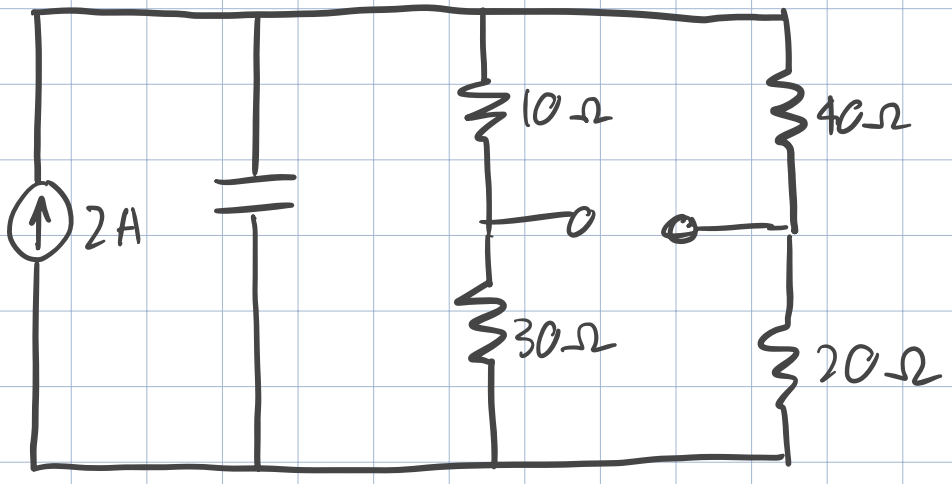
$$i_c = \frac{V_c(t)}{2000}$$

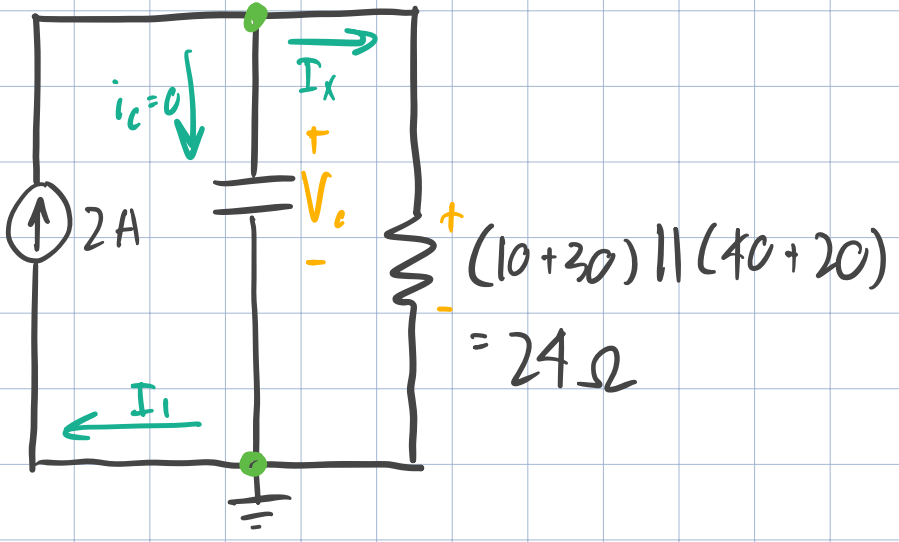
$$= \frac{2}{2000}$$

$$= \frac{1}{1000} A$$

Problem Four

When $t = 0^-$



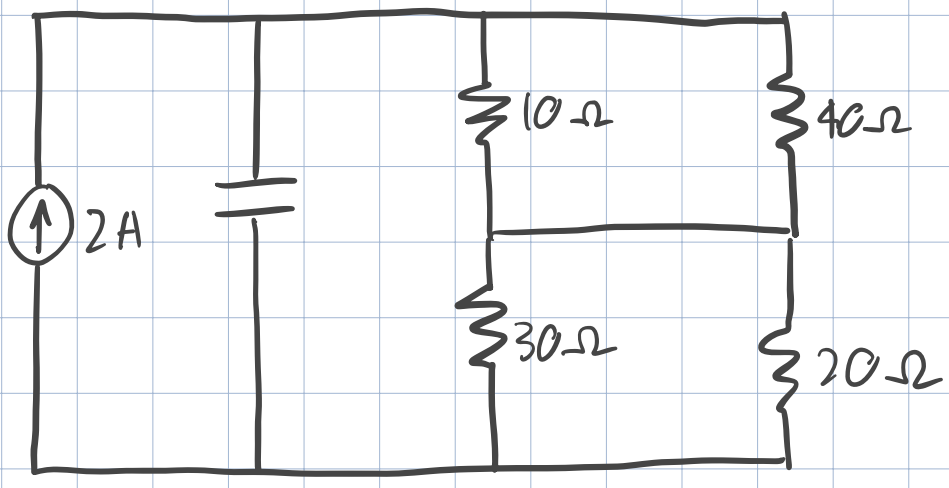


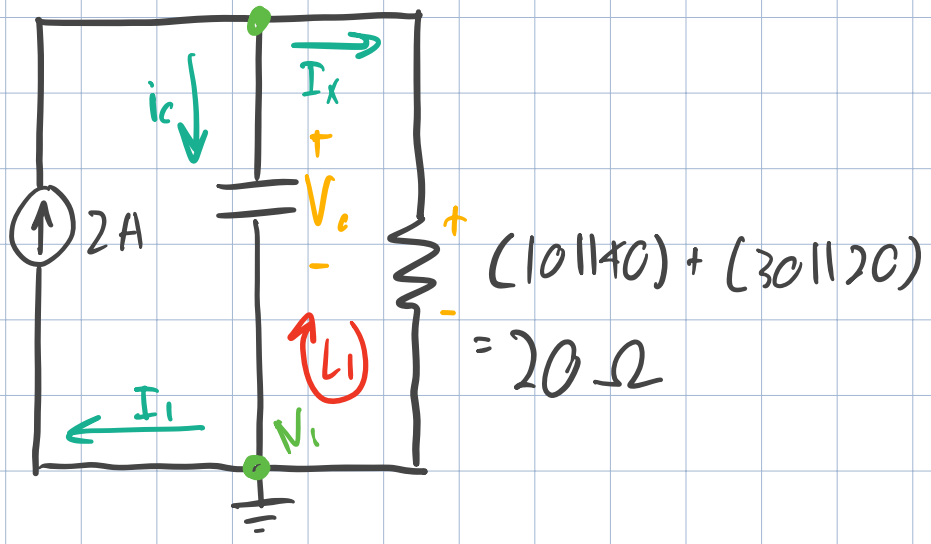
$(10+30) \| (40+20)$

$= 24\Omega$

$I_x = 2A$

When $t = 0^+$





$(10 \| 40) + (30 \| 20)$

$= 20\Omega$

Solve for I_x

KVL on L_1

$V_c(0^+) - 20I_x = 0$

$V_c(0^-) - 20I_x = 0$

$V_c(0^-) = 48V$

$48 - 20I_x = 0$

$$I_x = \frac{48}{20}\,A$$

$$= \frac{12}{5}\,A$$

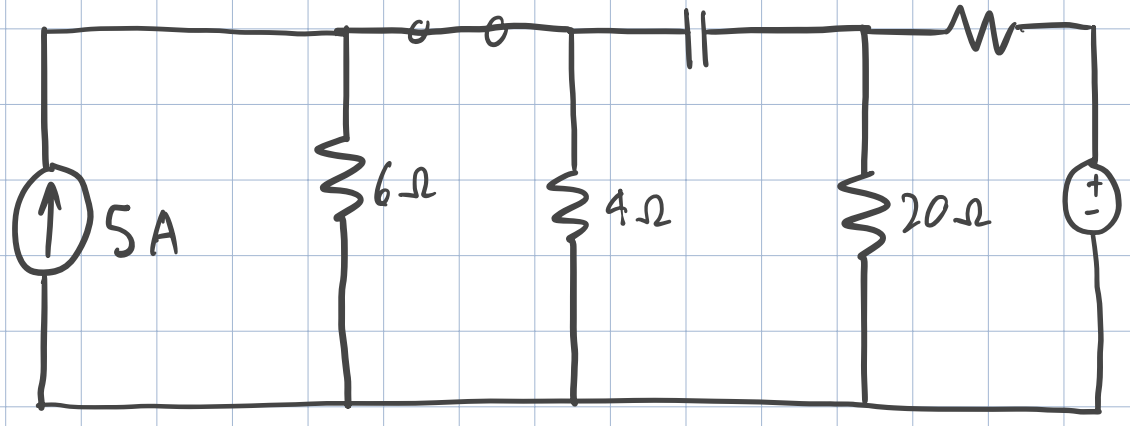
When $t = \infty$

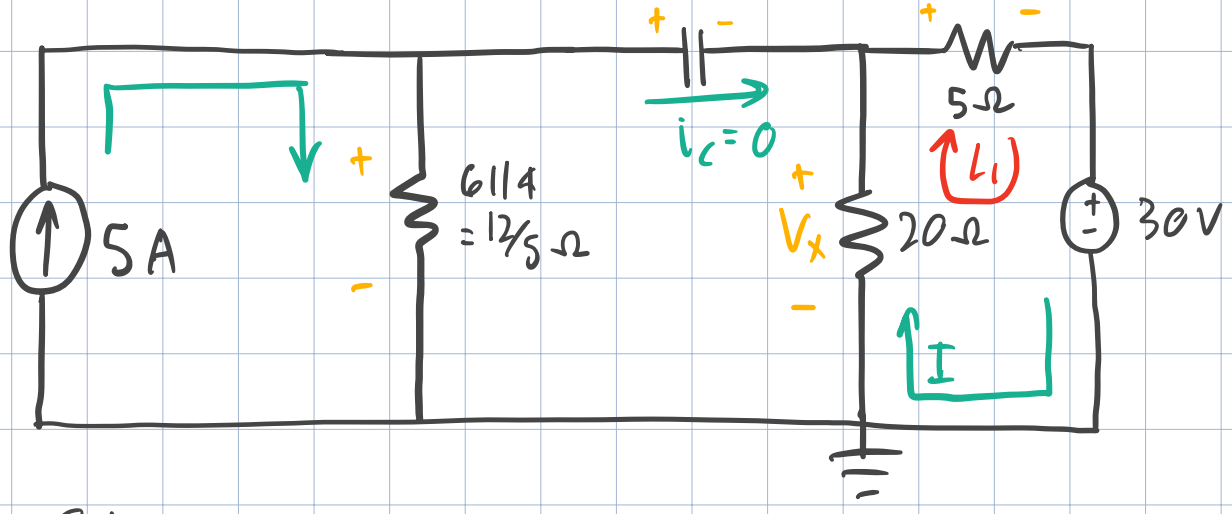
$$I_x = 2 + i_c(\infty)$$

$$= 2A$$

Problem Five

When $t = 0^-$





Solve for V_x

Kvl on L_1

$20I + 5I - 30 = 0$

$25I = 30$

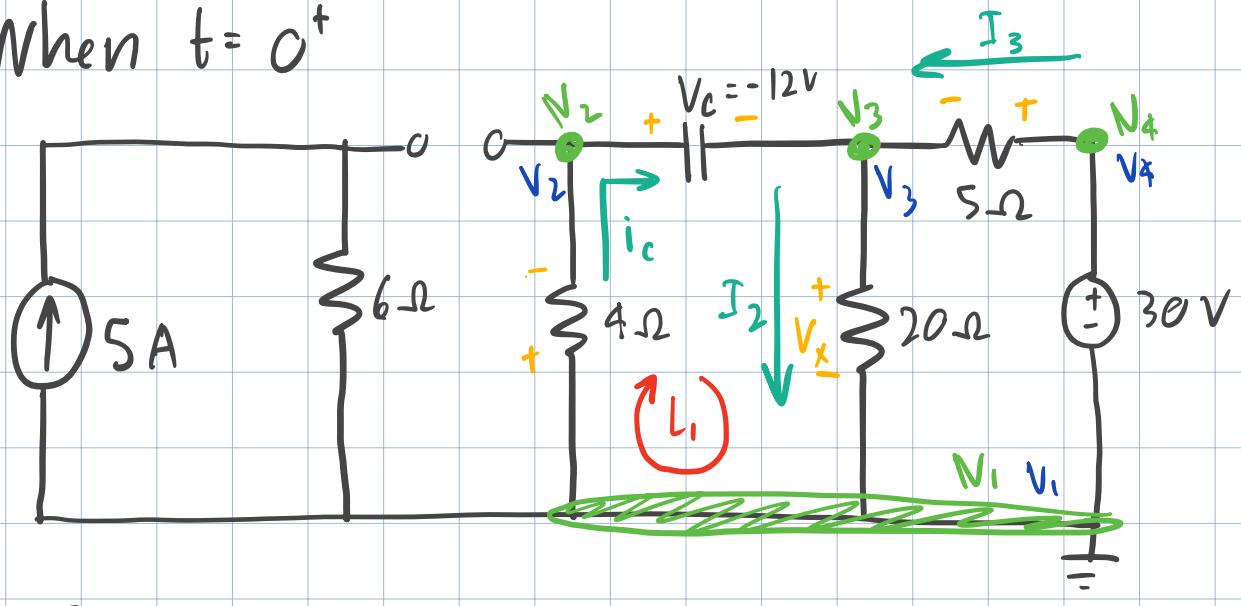
$I = \frac{6}{5}\,A$

$V_x = IR$

$= \frac{6}{5} \cdot 20$

$= 24\,V$

When $t = 0^+$



Solve for $V_c(0^+)$

$V_c(0^+) = V_c(0^-)$

$= \frac{12}{5} \cdot -5$

$= -12V$

Solve for V_x

Use Node Voltage Analysis

4 nodes - 1 ground - 2 voltage sources = 1 unknown

Solve for voltages

Voltages

$V_1 = 0V$

$V_4 = 30V$

KCL at V_3

$I_2 = i_c + I_3$

$$\frac{V_3 - V_1}{20} = \frac{V_1 - V_2}{4} + \frac{V_4 - V_3}{5}$$

$$V_3 - V_1 = 5(V_1 - V_2) + 4(V_4 - V_3)$$

$$V_3 = -5V_2 + 4V_4 - 4V_3$$

$$V_2 = V_3 - 12$$

$$V_3 = -5(V_3 - 12) + 4(30) - 4V_3$$

$$V_3 = -5V_3 + 60 + 120 - 4V_3$$

$$10V_3 = 180$$

$V_3 = 18\ V$

$V_x = V_3$

$= 18 V$

Solve for i_c

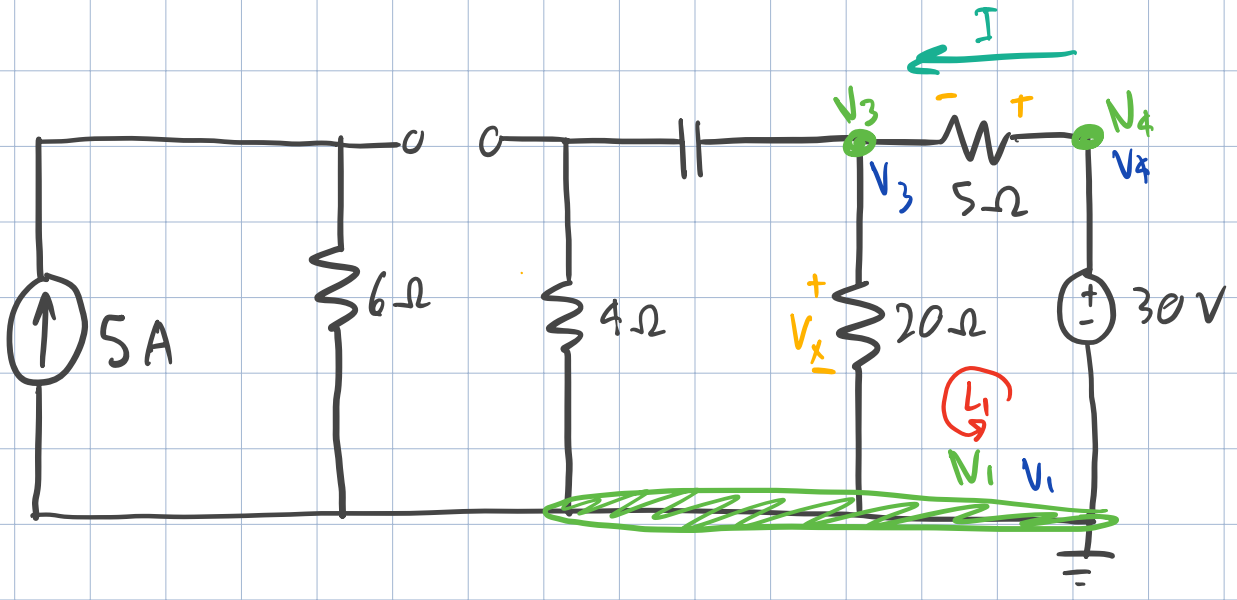
$$i_c = \frac{V_1 - V_2}{4}$$

$$= -\frac{V_1 - (V_3 - 12)}{4}$$

$$= -\frac{-18 + 12}{4}$$

$$= -\frac{3}{2} A$$

When $t = \infty$



$V_x = 24 V$

$i_C = C\,A$

Solve for V_x

KVL on L_1

$30 - 5I - 20I = 0$

$25I = 30$

$I = \frac{30}{25}$

$= \frac{6}{5} A$

$V_x = 20I$

$= 24V$