Determine the first partial derivatives of $g(x, y, z) = 7\ln\left(\frac{3xy}{7z}\right)$.

$\frac{\partial f}{\partial x}$ =

$\frac{\partial f}{\partial y}$ =

$\frac{\partial f}{\partial z}$ =

$$\frac{\partial f}{\partial x} = 7\left(\frac{1}{\frac{3xy}{7z}} \cdot \frac{3y}{7z}\right)$$

$$= 7\left(\frac{\cancel{7z}}{\cancel{3}x\cancel{y}} \cdot \frac{\cancel{3y}}{\cancel{7z}}\right)$$

$$= \frac{7}{x}$$

$$\frac{\partial f}{\partial y} = 7\left(\frac{1}{\frac{3xy}{7z}} \cdot \frac{3x}{7z}\right)$$

$$= 7\left(\frac{\cancel{7z}}{\cancel{3x}y} \cdot \frac{\cancel{3x}}{\cancel{7z}}\right)$$

$$= \frac{7}{y}$$

$$\frac{\partial f}{\partial y} = 7\left(\frac{1}{\frac{3xy}{7z}} \cdot \frac{3xy}{7}\left(-z^{-2}\right)\right)$$

$$= 7\left(\frac{\cancel{7}z}{\cancel{3xy}} \cdot \frac{\cancel{3xy}}{-\cancel{7}z^{\cancel{2}}}\right)$$

$$= -\frac{7}{z}$$

Determine the first partial derivatives of $h(r, s) = \ln(r^2 + s^2 + 3)$ at the point $P(5, -2)$.

$h_r(5, -2)$ =

$h_s(5, -2)$ =

Part One

$$h_r = \frac{1}{r^2+s^2+3}(2r)$$

$$= \frac{2r}{r^2+s^2+3}$$

$$h_r(5,-2) = \frac{10}{25+4+3}$$

$$= \frac{10}{32}$$

Part Two

$$h_s = \frac{1}{r^2+s^2+3}(2s)$$

$$= \frac{2s}{r^2+s^2+3}$$

$$h_s(5,-2) = \frac{-4}{25+4+3}$$

$$= -\frac{4}{32}$$

Determine the second partial derivatives of $f(x, y, z) = \tan(1 + 2x^2y^4z^2)$.

f_{xx} =

f_{xy} =

f_{xz} =

f_{yy} =

f_{yx} =

f_{yz} =

f_{zz} =

f_{zx} =

f_{zy} =

Part One: First 3 answers

$f_x = \sec^2(1+2x^2y^4z^2)(4xy^4z^2)$

Derivative of $\sec^2$ for later use

$\frac{d}{dx}\sec^2(x) = \frac{d}{dx}(\sec(x))^2$

$= 2\sec(x) \cdot \sec(x)\tan(x)$

$= 2\sec^2(x)\tan(x)$

$f_{xx} = \frac{\partial}{\partial x}(4y^4z^2(x\sec^2(1+2x^2y^4z^2)))$

$= 4y^4z^2(x(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2) \cdot 4xy^4z^2) + \sec^2(1+2x^2y^4z^2))$

$f_{xy} = \frac{\partial}{\partial y}(4xz^2(y^4\sec^2(1+2x^2y^4z^2)))$

$= 4xz^2(y^4(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2) \cdot 8x^2y^3z^2) + \sec^2(1+2x^2y^4z^2)(4y^3))$

$f_{xz} = \frac{\partial}{\partial z}(4xy^4(z^2\sec^2(1+2x^2y^4z^2)))$

$= 4xy^4(z^2(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2) \cdot 4x^2y^4z) + \sec^2(1+2x^2y^4z^2)(2z))$

Part Two: Middle 3 answers

$$f_y = \sec^2(1+2x^2y^4z^2)(8x^2y^3z^2)$$

$$f_{yy} = \frac{\partial}{\partial y}\left(8x^2z^2\left(y^3\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 8x^2z^2\left(y^3\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 8x^2y^3z^2\right) + \sec^2(1+2x^2y^4z^2)(3y^2)\right)$$

$$f_{yx} = \frac{\partial}{\partial x}\left(8y^3z^2\left(x^2\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 8y^3z^2\left(x^2\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 4xy^4z^2\right) + \sec^2(1+2x^2y^4z^2)(2x)\right)$$

$$f_{yz} = \frac{\partial}{\partial z}\left(8x^2y^3\left(z^2\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 8x^2y^3\left(z^2\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 4x^2y^4z\right) + \sec^2(1+2x^2y^4z^2)(2z)\right)$$

Part Three: Last 3 answers

$$f_z = \sec^2(1+2x^2y^4z^2)(4x^2y^4z)$$

$$f_{zz} = \frac{\partial}{\partial z}\left(4x^2y^4\left(z\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 4x^2y^4\left(z\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 4x^2y^4z\right) + \sec^2(1+2x^2y^4z^2)\right)$$

$$f_{zx} = \frac{\partial}{\partial x}\left(4y^4z\left(x^2\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 4y^4 z\left(x^2\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 4xy^4z^2\right) + \sec^2(1+2x^2y^4z^2)(2x)\right)$$

$$f_{zy} = \frac{\partial}{\partial y}\left(4x^2 z\left(y^4\sec^2(1+2x^2y^4z^2)\right)\right)$$

$$= 4x^2 z\left(y^4\left(2\sec^2(1+2x^2y^4z^2)\tan(1+2x^2y^4z^2)\cdot 8x^2y^3z^2\right) + \sec^2(1+2x^2y^4z^2)(4y^3)\right)$$

Find equations of the tangent plane and normal line at the point $P_1(3, 6, -2\sqrt{11})$ to the graph of the hyperboloid of 1-sheet given by $x^2 + y^2 - z^2 = 1$. (Write the normal line as a comma separated list of parametric equations; let t be the parameter.)

tangent plane

normal line

Part One

$$F(x,y,z) = x^2 + y^2 - z^2 - 1$$

$$\vec{n} = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right\rangle$$

$$= \langle 2x, 2y, -2z \rangle$$

$$\vec{n}(3, 6, -2\sqrt{11}) = \langle 6, 12, 4\sqrt{11} \rangle$$

Tangent Plane: $6(x-3) + 12(y-6) + 4\sqrt{11}(z + 2\sqrt{11}) = 0$

Part Two

$$x = 6t + 3$$
$$y = 12t + 6$$

$$z = 4\sqrt{11}\,t - 2\sqrt{11}$$