Table of Integrals

Basic Forms

(1) $\displaystyle\int u\,dv = uv - \int v\,du$

(2) $\displaystyle\int u^n\,du = \frac{1}{n+1}u^{n+1} + C,\ n \neq -1$

(3) $\displaystyle\int \frac{du}{u} = \ln|u| + C$

(4) $\displaystyle\int e^u\,du = e^u + C$

(5) $\displaystyle\int a^u\,du = \frac{1}{\ln a}a^u + C$

(6) $\displaystyle\int \sin u\,du = -\cos u + C$

(7) $\displaystyle\int \cos u\,du = \sin u + C$

(8) $\displaystyle\int \sec^2 u\,du = \tan u + C$

(9) $\displaystyle\int \csc^2 u\,du = -\cot u + C$

(10) $\displaystyle\int \sec u\tan u\,du = \sec u + C$

(11) $\displaystyle\int \csc u\cot u\,du = -\csc u + C$

(12) $\displaystyle\int \tan u\,du = \ln|\sec u| + C$

(13) $\displaystyle\int \cot u\,du = \ln|\sin u| + C$

(14) $\displaystyle\int \sec u\,du = \ln|\sec u + \tan u| + C$

(15) $\displaystyle\int \csc u\,du = \ln|\csc u - \cot u| + C$

(16) $$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$$

(17) $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

(18) $$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

(19) $$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

(20) $$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

Forms Involving $\sqrt{a^2+u^2}$

(21) $$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

(22) $$\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

(23) $$\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$$

(24) $$\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u^2} + \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

(25) $$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

(26) $$\int \frac{u^2\,du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

(27) $$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$$

(28) $$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$$

(29) $$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

Forms Involving $\sqrt{a^2-u^2}$

(30) $$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

(31) $$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$$

(32) $$\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

(33) $$\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C$$

(34) $$\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$$

(35) $$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

(36) $$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C$$

(37) $$\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$$

(38) $$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

Forms Involving $\sqrt{u^2 - a^2}$

(39) $$\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

(40) $$\int u^2 \sqrt{u^2 - a^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{a^4}{8} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

(41) $$\int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{u} + C$$

(42) $$\int \frac{\sqrt{u^2 - a^2}}{u^2}\, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

(43) $$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

(44) $$\int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2} \ln \left| u + \sqrt{u^2 - a^2} \right| + C$$

(45) $$\int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

(46) $$\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$$

Forms Involving $a + bu$

(47) $$\int \frac{u\, du}{a + bu} = \frac{1}{b^2}\left(a + bu - a \ln |a + bu|\right) + C$$

(48) $$\int \frac{u^2\, du}{a + bu} = \frac{1}{2b^3}\left[(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|\right] + C$$

(49) $$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

(50) $$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

(51) $$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$$

(52) $$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

(53) $$\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$$

(54) $$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

(55) $$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$$

(56) $$\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu} + C$$

(57) $$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C \quad \text{if } a > 0$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C \quad \text{if } a < 0$$

(58) $$\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$$

(59) $$\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$$

(60) $$\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2na}{b(2n+3)}\int \frac{u^{n-1}}{\sqrt{a+bu}}\,du$$

(61) $$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}}{\sqrt{a+bu}}\,du$$

(62) $$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$$

Trigonometric Forms

(63) $$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

(64) $$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

(65) $$\int \tan^2 u\,du = \tan u - u + C$$

(66) $$\int \cot^2 u\,du = -\cot u - u + C$$

(67) $$\int \sin^3 u\,du = -\frac{1}{3}(2+\sin^2 u)\cos u + C$$

(68) $$\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

(69) $$\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$$

(70) $$\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$$

(71) $$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

(72) $$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

(73) $$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

(74) $$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

(75) $$\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$$

(76) $$\int \cot^n u \, du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$$

(77) $$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

(78) $$\int \csc^n u\, du = -\frac{1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\, du$$

(79) $$\int \sin au \sin bu\, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

(80) $$\int \cos au \cos bu\, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

(81) $$\int \sin au \cos bu\, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

(82) $$\int u \sin u\, du = \sin u - u\cos u + C$$

(83) $$\int u \cos u\, du = \cos u + u\sin u + C$$

(84) $$\int u^n \sin u\, du = -u^n \cos u + n\int u^{n-1}\cos u\, du$$

(85) $$\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1}\sin u\, du$$

(86) $$\int \sin^n u \cos^m u\, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2} u \cos^m u\, du$$

$$\int \sin^n u \cos^m u\, du = \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \sin^n u \cos^{m-2} u\, du$$

Inverse Trigonometric Forms

(87) $$\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$$

(88) $$\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

(89) $$\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln (1+u^2) + C$$

(90) $$\int u \sin^{-1} u \, du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

(91) $$\int u \cos^{-1} u \, du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

(92) $$\int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

(93) $$\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], \; n \neq -1$$

(94) $$\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} \, du}{\sqrt{1-u^2}} \right], \; n \neq -1$$

(95) $$\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} \, du}{1+u^2} \right], \; n \neq -1$$

Exponential and Logarithmic Forms

(96) $$\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$$

(97) $$\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$$

(98) $$\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$$

(99) $$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$$

(100) $$\int \ln u\,du = u\ln u - u + C$$

(101) $$\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$$

(102) $$\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$$

Hyperbolic Forms

(103) $$\int \sinh u\,du = \cosh u + C$$

(104) $$\int \cosh u\,du = \sinh u + C$$

(105) $$\int \tanh u\,du = \ln\cosh u + C$$

(106) $$\int \coth u \, du = \ln|\sinh u| + C$$

(107) $$\int \operatorname{sech} u \, du = \tan^{-1}|\sinh u| + C$$

(108) $$\int \operatorname{csch} u \, du = \ln|\tanh \frac{1}{2}u| + C$$

(109) $$\int \operatorname{sech}^2 u \, du = \tanh u + C$$

(110) $$\int \operatorname{csch}^2 u \, du = -\coth u + C$$

(111) $$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

(112) $$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

Forms Involving $\sqrt{2au - u^2}$

(113) $$\int \sqrt{2au - u^2} \, du = \frac{u-a}{2}\sqrt{2au - u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(114) $$\int u\sqrt{2au - u^2} \, du = \frac{2u^2 - au - 3a^2}{6}\sqrt{2au - u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(115) $$\int \frac{\sqrt{2au - u^2}}{u} \, du = \sqrt{2au - u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(116) $$\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(117) $$\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(118) $$\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(119) $$\int \frac{u^n\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$$

(120) $$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$