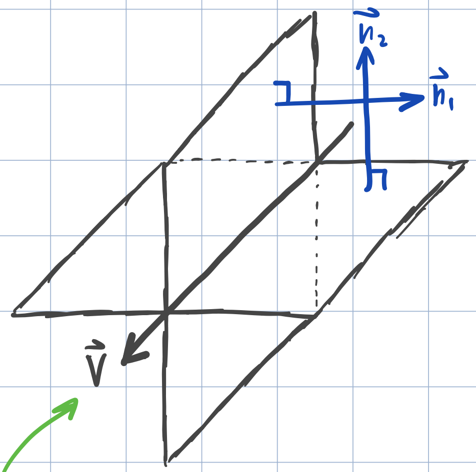
Determine parametric equations for the line in which the planes $2x - y + z = 9$ and $x + y - z = 3$ intersect. (Enter your answers as a comma-separated list of equations. Let t be the parameter.)

$\vec{n}_1 = \langle 2, -1, 1 \rangle$

$\vec{n}_2 = \langle 1, 1, -1 \rangle$

$\vec{v} = \langle 1-1, -(-2-1), 2+1 \rangle$

$= \langle 0, 3, 3 \rangle$



Goal

$$\begin{array}{c|c|c} 2 & -1 & 1 \\ 1 & 1 & -1 \end{array}$$

Find a point on $\vec{v}$

$2x - y + z = 9$
$x + y - z = 3$

$y = 2x + z - 9$
$y = 3 + z - x$

$2x \cancel{+z} - 9 = 3 \cancel{+z} - x$

$3x = 12$

$x = 4$

Solve for y and z

$4 + y - z = 3$

$y = z - 1$

Let $z = 1$

$y = 0$

Point = $(4, 0, 1)$

Combine $\vec{V}$ and Point

$x = 4$
$y = 3t$
$z = 3t + 1$

Determine parametric equations of the line segment from point $P(2, 4, -7)$ to $Q(-6, 3, 1)$. (Enter your answers as a comma-separated list of equations. Let $0 \le t \le 1$ be the parameter.)

$\vec{V} = Q - P$

$= \langle -8, -1, 8 \rangle$

Combine $\vec{V}$ and P

$x = -8t + 2$
$y = -1t + 4$
$z = 8t - 7$

Are the following two lines L_1 and L_2 parallel, intersecting, or neither parallel nor intersecting?

L_1: $x = 2 + t$, $y = 1 - 3t$, $z = 8 + 2t$
L_2: $x = 5 - 4s$, $y = 12s$, $z = -2 - 8s$

- parallel
- intersecting
- neither

$\vec{V}_1 = \langle 1, -3, 2 \rangle$

$\vec{V}_2 = \langle -4, 12, -8 \rangle$

$\vec{V}_2 = -4\vec{V}_1$

Vectors are parallel

Determine an equation for the plane passing through the three points $P(5, 0, 1)$, $Q(0, 4, -1)$, and $R(-1, 3, 0)$.

$\vec{V}_1 = P - Q$

$= \langle 5, -4, 2 \rangle$

$\vec{V}_3 = P - R$

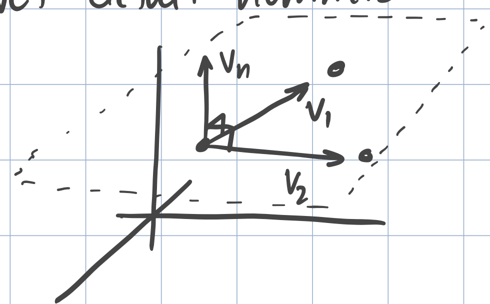
$= \langle 6, -3, 1 \rangle$

$\vec{V}_n = \vec{V}_1 \times \vec{V}_3$

$= \langle -4 + 6, -(5 - 12), -15 + 24 \rangle$

$= \langle 2, 7, 9 \rangle$

Not actual numbers



$$\begin{array}{c|c|c} 5 & -4 & 2 \\ 6 & -3 & 1 \end{array}$$

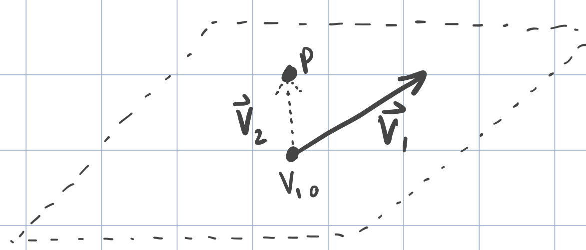
Combine $\vec{V}_n$ and P

$$2(x-5)+7(y-0)+9(z-1)=0$$

Determine an equation for the plane passing through the point $P(-4, 2, 0)$ and containing the line with parametric equations $x = 3 + 7t$, $y = 1 - 3t$, and $z = -2 + 5t$.

$$\vec{V}_1 = \begin{pmatrix} x = 3+7t \\ y = 1-3t \\ z = -2+5t \end{pmatrix}$$

Not actual numbers



$$\vec{V}_1 = \langle 7, -3, 5 \rangle$$

Point where vector starts

$$V_{10} = (3, 1, -2)$$

$$\vec{V}_2 = P - V_{10}$$

$$= (-4, 2, 0) - (3, 1, -2)$$

$$= \langle -7, 1, 2 \rangle$$

$$\begin{array}{c|c|c} 7 & -3 & 5 \\ -7 & 1 & 2 \end{array}$$

$$\vec{V}_n = \vec{V}_1 \times \vec{V}_2$$

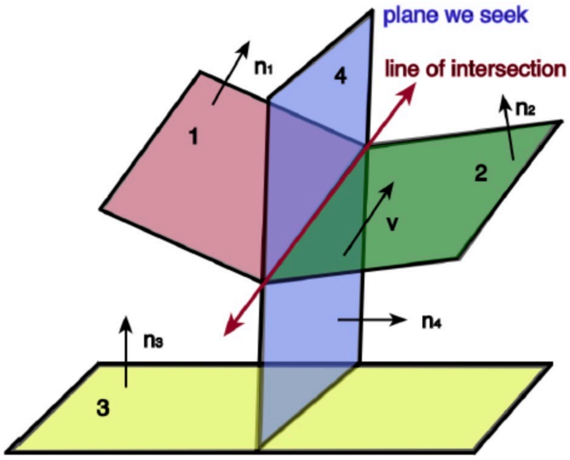
$$= \langle -6-5, -(14+35), 7-21 \rangle$$

$$= \langle -11, -49, -14 \rangle$$

Combine $\vec{V}_n$ and point V_{10}

$$\text{Plane} = -11(x-3) - 49(y-1) - 14(z+2) = 0$$

Determine an equation for the plane passing through the line of intersection of the two planes, Plane #1, $x - 2y = 3$, and Plane #2, $y + 3z = 9$, and perpendicular to Plane #3, $7x + 2y - z = -8$. Consult the figure below for a visualization of how Plane #4 relates to the other three.



$$\vec{n}_1 = \langle 1, -2, 0 \rangle$$

$$\vec{n}_2 = \langle 0, 1, 3 \rangle$$

$$\begin{array}{c|c|c} 1 & -2 & 0 \\ 0 & 1 & 3 \end{array}$$

$$\vec{V} = \vec{n}_1 \times \vec{n}_2$$

$$= \langle -6, -(3), 1 \rangle$$

$$= \langle -6, -3, 1 \rangle$$

$$\vec{n}_3 = \langle 7, 2, -1 \rangle$$

$$\begin{array}{c|c|c} -6 & -3 & 1 \\ 7 & 2 & -1 \end{array}$$

$$\vec{n}_4 = \vec{V} \times \vec{n}_3$$

$$= \langle 3-2, -(6-7), -12+21 \rangle$$

$$= \langle 1, 1, 9 \rangle$$

Find a point on the line of intersection of plane 1 and 2

$x - 2y = 3$

$y + 3z = 9$

Let $y = 0$

$x = 3$

$3z = 9$

$z = 3$

$P = (3, 0, 3)$

Combine $\vec{n}_4$ and p

Plane $= 1(x-3) + 1(y-0) + 9(z-3) = 0$

Determine the point at which the line passing through the points $P(1, 0, 6)$ and $Q(9, -1, 9)$ intersects the plane given by the equation $x + y - z = 7$.

$(x, y, z) = (\quad)$

$\vec{V}_L = P - Q$

$= \langle -8, 1, -3 \rangle$

Parametric Equations of L

$x = -8t + 1$
$y = t$
$z = -3t + 6$

Plug into plane equation and solve for t

$(-8t+1) + (t) - (-3t + 6) = 7$

$-8t + 1 + t + 3t - 6 = 7$

$-4t = 12$

$t = -3$

Point of intersection $= (24+1, -3, 9+6)$

$= (25, -3, 15)$

Find the angle in degrees between the two planes $x - 10y - 7z = 1$ and $2x + 5y + 7z = 11$. (Round your answer to two decimal places.)

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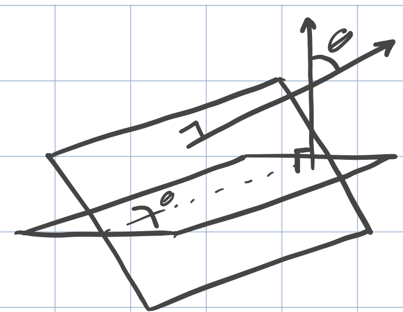
$\vec{n}_1 = \langle 1, -10, -7 \rangle$

$\vec{n}_2 = \langle 2, 5, 7 \rangle$



Find the angle between the normal vectors

$$|\vec{n}_1| = \sqrt{1 + 100 + 49}$$

$$|\vec{n}_2| = \sqrt{4 + 25 + 49}$$

$$\vec{n}_1 \cdot \vec{n}_2 = 2 - 50 - 49$$

$$= -97$$

$$\vec{n}_1 \cdot \vec{n}_2 = |\vec{n}_1||\vec{n}_2| \cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} \right)$$

$$= 153.736^\circ$$

Smallest angle $= 180 - 153.736$

$$= 26.264^\circ$$

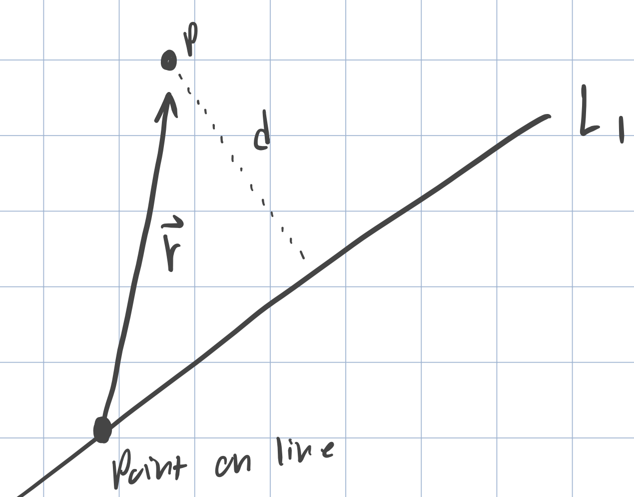
Find the distance from the point P to the given line L.

$P(0, -1, 2)$ and L: $x = 2 + 3t$, $y = -1 - 2t$, $z = -1 + 2t$

$$L = \langle 3, -2, 2 \rangle t + \langle 2, -1, -1 \rangle$$

$$V_L = \langle 3, -2, 2 \rangle$$

$$L_0 = \langle 2, -1, -1 \rangle$$



$\vec{r} = P - L_0$

$= \langle -2, 0, 3 \rangle$

$$\begin{vmatrix} -2 & 0 & 3 \\ 3 & -2 & 2 \end{vmatrix}$$

$d = \dfrac{|\vec{r} \times \vec{v}|}{|\vec{v}|}$

$= \dfrac{|\langle 6, -(-4-9), 4 \rangle|}{\sqrt{9+4+4}}$

$= \dfrac{|\langle 6, 13, 4 \rangle|}{\sqrt{17}}$

$= \dfrac{\sqrt{221}}{\sqrt{17}}$

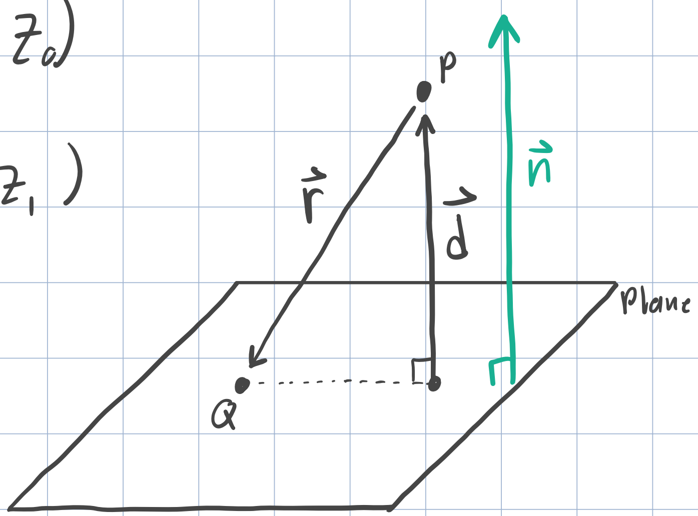
Find the distance from the point P to the given plane.

$P(-3, -1, 0)$ and the plane is $4x - 2y - 6z = 5$

Plane: $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$

$Q=(x_0, y_0, z_0)$

$P=(x_1, y_1, z_1)$



$|\vec{d}|$ is the shortest distance between the point and plane and so is what we seek.

We will use $\vec{n}$ for projection later so we dont care abut its length, only direction. We can use the normal vector of the plane as $\vec{n}$ because it requires the least work.

$$\vec{n}=\langle A, B, C\rangle$$

$\vec{d}$ is $\vec{r}$ projected onto $\vec{n}$

$$\vec{d}=\text{proj}_{\vec{n}}(\vec{r})$$

$$=\frac{|\vec{r}\cdot\vec{n}|}{|\vec{n}|}\hat{n}$$

We dont care abut the direction of $\vec{d}$ so eliminate it.

$$|\vec{d}| = \frac{|\vec{r} \cdot \vec{n}|}{|\vec{n}|}$$

Find $\vec{r}$

$$\vec{r} = Q - P$$

$$= \langle x_0 - x_1, y_0 - y_1, z_0 - z_1 \rangle$$

Plug $\vec{r}$ into the equation for $|\vec{d}|$

$$|\vec{d}| = \frac{|A(x_0 - x_1) + B(y_0 - y_1) + C(z_0 - z_1)|}{|\vec{n}|}$$

$$= \frac{|A(x_0 - x_1) + B(y_0 - y_1) + C(z_0 - z_1)|}{\sqrt{A^2 + B^2 + C^2}}$$