Compute the work done by the force $\vec{F} = \langle 2x^2y, -xz, 3z \rangle$ in moving an object along the parametrized curve $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ with $0 \le t \le 1$ when force is measured in Newtons and distance in meters.

[ ] J

$$W = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \nabla \vec{r}\, dt$$

$$= \int_0^1 \langle 2(t)^2(t^2), -(t)(t^3), 3(t^3) \rangle \cdot \langle 1, 2t, 3t^2 \rangle\, dt$$

$$= \int_0^1 \langle 2t^4, -t^4, 3t^3 \rangle \cdot \langle 1, 2t, 3t^2 \rangle\, dt$$

$$= \int_0^1 2t^4 - 2t^5 + 9t^5\, dt$$

$$= \frac{2}{5} - \frac{2}{6} + \frac{9}{6}$$

$$= \frac{2}{5} - \frac{1}{3} + \frac{3}{2}$$

Compute the work done by the force $\vec{F} = \langle \sin(x + y), xy, x^2z \rangle$ in moving an object along the trajectory that is the line segment from (1, 1, 1) to (2, 2, 2) followed by the line segment from (2, 2, 2) to (−3, 6, 6) when force is measured in Newtons and distance in meters.

$W =$ [ ] J

$\vec{r}_1 = \langle t, t, t \rangle$ from $1 \le t \le 2$

$\vec{r}_2 = \langle x(g), y(g), z(g) \rangle$ from $0 \le g \le 1$

$x(g) = -5g + 2$

$y(g) = 4g + 2$

$z(g) = 4g + 2$

$$\vec{r}_2 = \langle -5g+2, 4g+2, 4g+2 \rangle$$

$$W = \int_{C_1} \vec{F} \cdot \hat{T} \left|\frac{d\vec{r}}{dt}\right| dt + \int_{C_2} \vec{F} \cdot \hat{T} \left|\frac{d\vec{r}}{dt}\right| dt$$

$$\rightarrow \int_{C_1} \vec{F} \cdot \hat{T} \left|\frac{d\vec{r}}{dt}\right| dt$$

$$= \int_1^2 \langle \sin(2t), t^2, t^3 \rangle \cdot \langle 1,1,1 \rangle \, dt$$

$$= \int_1^2 \sin(2t) + t^2 + t^3 \, dt$$

Evaluated With Calculator

$$\rightarrow \int_{C_2} \vec{F} \cdot \hat{T} \left|\frac{d\vec{r}}{dt}\right| dt$$

$$= \int_0^1 \langle \sin(-5g+2+4g+2), (-5g+2)(4g+2), (-5g+2)^2(4g+2) \rangle \cdot \langle -5,4,4 \rangle \, dt$$

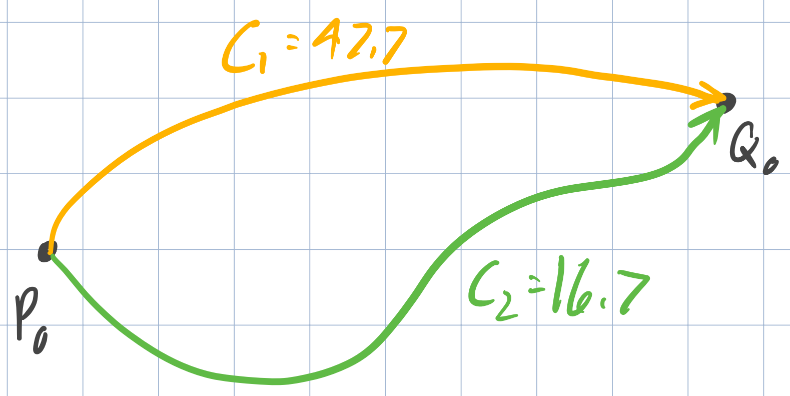
$$= \int_0^1 \langle \sin(-g+4), (-5g+2)(4g+2), (-5g+2)^2(4g+2) \cdot \langle -5,4,4 \rangle \, dg$$

$$= \int_0^1 -5\sin(-g+4) + 4(-5g+2)(4g+2) + 4(-5g+2)^2(4g+2) \, dg$$

Evaluated with Calculator

$$= -5\cos(3) + \frac{425}{12} + \frac{9\cos(4) + \cos(2)}{2}$$

Let C_1 and C_2 be two smooth parameterized curves that start at P_0 and end at $Q_0 \neq P_0$, but do not otherwise intersect. If the line integral of the function $f(x, y, z)$ along C_1 is equal to 47.7 and the line integral of $f(x, y, z)$ along C_2 is 16.7, what is the line integral around the closed loop formed by first following C_1 from P_0 to Q_0, followed by the curve from Q_0 to P_0 along C_2, but moving in the opposite direction?



$$47.7 - 16.7 = 31$$

Determine if the given vector field $\vec{F}$ is conservative or not.

$$\vec{F} = \langle -8y,\ 12y^2 - 8z^2 - 8x - 8z,\ -16yz - 8y \rangle$$

- conservative
- not conservative

If $\vec{F}$ is conservative, find all potential functions f for $\vec{F}$ so that $\vec{F} = \nabla f$. (If $\vec{F}$ is not conservative, enter NOT CONSERVATIVE. Use C as an arbitrary constant.)

$f(x, y, z) =$

$$\vec{F} = \langle P, Q, R \rangle$$

$$\frac{\partial P}{\partial x} = 0 \qquad \frac{\partial P}{\partial y} = -8 \qquad \frac{\partial P}{\partial z} = 0$$

$$\frac{\partial Q}{\partial x} = -8 \qquad \frac{\partial Q}{\partial y} = 24y \qquad \frac{\partial Q}{\partial z} = -16z - 8$$

$$\frac{\partial R}{\partial x} = 0 \qquad \frac{\partial R}{\partial y} = -16z - 8 \qquad \frac{\partial R}{\partial z} = -16y$$

Conservative

$$\nabla f = \vec{F} \quad \text{or} \quad \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$f = \int \frac{\partial f}{\partial x}\, dx$$

$$= \int -8y \, dx$$

$$= -8xy + k_1(y,z)$$

$$f = \int \frac{\partial f}{\partial y}\, dy$$

$$= \int 12y^2 - 8z^2 - 8x - 8z \, dy$$

$$= \frac{12}{3}y^3 - 8yz^2 - 8xy - 8yz + k_2(x,z)$$

$$= 4y^3 - 8yz^2 - 8xy - 8yz + k_2(x,z)$$

$$f = \int \frac{\partial f}{\partial y}\, dz$$

$$= \int -16yz - 8y \, dz$$

$$= -\frac{16}{2}yz^2 - 8yz + k_3(x,y)$$

$$= -8yz^2 - 8yz + k_3(x,y)$$

Combine Equations without repeating terms

$$f = -8xy + 4y^3 - 8yz^2 - 8yz + C$$

Determine if the given vector field $\vec{F}$ is conservative or not.

$\vec{F} = \langle (y + 6z + 4)\sin(x), -\cos(x), -6\cos(x) \rangle$

- ○ conservative
- ○ not conservative

If $\vec{F}$ is conservative, find all potential functions f for $\vec{F}$ so that $\vec{F} = \nabla f$. (If $\vec{F}$ is not conservative, enter NOT CONSERVATIVE. Use C as an arbitrary constant.)

$f(x, y, z) =$ ______

$$\vec{F} = \langle P, Q, R \rangle$$

$$\frac{\partial P}{\partial x} = (y+6z+4)\cos(x) \qquad \frac{\partial P}{\partial y} = \sin(x) \checkmark \qquad \frac{\partial P}{\partial z} = 6\sin(x) \checkmark$$

$$\frac{\partial Q}{\partial x} = \sin(x) \checkmark \qquad \frac{\partial Q}{\partial y} = 0 \qquad \frac{\partial Q}{\partial z} = 0 \checkmark$$

$$\frac{\partial R}{\partial x} = 6\sin(x) \checkmark \qquad \frac{\partial R}{\partial y} = 0 \checkmark \qquad \frac{\partial R}{\partial z} = 0$$

Conservative

$$\nabla f = \vec{F} \quad \text{or} \quad \nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$f = \int \frac{\partial f}{\partial x}\,dx$$

$$= \int (y+6z+4)\sin(x)\,dx$$

$$= -(y+6z+4)\cos(x) + k_1(y,z)$$

$$f = \int \frac{\partial f}{\partial y}\,dy$$

$$= \int -\cos(x)\,dy$$

$$= -\cos(x)y + k_2(x,z)$$

$$f = \int \frac{\partial f}{\partial z}\, dz$$

$$= \int -6\cos(x)\, dz$$

$$= -6\cos(x) z + k_3(x,y)$$

Add up all terms without repeating

$$f = -(y + 6z + 4)\cos(x) + C$$

Let $f(x, y, z) = xy^3z^2$ and let C be the curve $\vec{r}(t) = \left\langle e^{t\cos(t^2+1)}, \ln(t^2+1), \frac{1}{\sqrt{t^2+1}} \right\rangle$ with $0 \le t \le 1$. Compute the line integral of ∇f along C.

$$\int_C \nabla f(x,y,z) \cdot d\vec{r}$$

$$= f(\vec{r}(1)) - f(\vec{r}(0))$$

$$= f(\langle e^{\cos(2)}, \ln(2), \tfrac{1}{\sqrt{2}} \rangle) - f(\langle 1, 0, 1 \rangle)$$

$$= \left(e^{\cos(2)} \ln(2)^3 \tfrac{1}{2}\right) - (0)$$

$$= \frac{1}{2} e^{\cos(2)} \ln(2)^3$$