Marcus Quan **CSC_226_001_SPRG_2024**
Assignment Homework3 due 01/29/2024 at 11:59pm EST

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1. (2 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

There are 21 mathematic majors and 141 computer science majors at a college.

A) How many ways are there to pick 7 representatives, so that 2 is/are mathematics majors and the other 5 is/are computer science majors?

$\binom{21}{2} \cdot \binom{141}{5}$

______ $= \frac{21!}{2!\,19!} \cdot \frac{141!}{5!\,136!}$

B) How many ways are there to pick two representatives who are either a mathematic major or a computer science major?

$\binom{162}{2}$

______ $= \frac{162!}{2!\,160!}$

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2. (1 point)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

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order matters repetition allowed

How many license plates can be made using either 4 letters followed by 2 digits or 3 letters followed by 3 digits?

26 letters (4 letters and 2 digits) or (3 letters and 3 digits)

10 digits ______ $(26^4 \cdot 10^2) + (26^3 \cdot 10^3)$

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3. (3 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Pick 6 | 1 bride | order matters
 | 1 groom | repetition not allowed
 | 8 guests |

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if...

A) The bride must be in the picture?

______ $\left(\text{6 spots for bride}\right) \cdot P(9,5)$

$= 6 \cdot \frac{9!}{4!}$

B) Both the bride and groom must be in the picture?

______ $\left(\text{6 spots for bride}\right) \cdot \left(\text{5 spots for groom}\right) \cdot P(8,4)$

$= 6 \cdot 5 \cdot \frac{8!}{4!}$

C) Exactly one of the bride and the groom is in the picture.

______ $\left(\text{bride or groom}\right) \cdot \left(\text{6 spot for bride/groom}\right) \cdot P(8,5)$

$2 \cdot 6 \cdot \frac{8!}{3!}$

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4. (1 point)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

How many bit strings of length 10 contain five consecutive 0's or five consecutive 1's ? $\left(\text{5 zeros in a row}\right) \cup \left(\text{5 ones in a row}\right)$

______ $= \left|\text{5 zeros in a row}\right| + \left|\text{5 ones in a row}\right| - \left|\text{5 zeros in a row and 5 ones in a row}\right|$

000001 _ _ _ _
_ 000001 _ _ _
_ _ 000001 _ _
_ _ _ 000001 _
_ _ _ _ 000001
_ _ _ _ _ 00000

$\left|\text{5 zeros in a row}\right| = \left|\text{5 ones in a row}\right| = 5 \cdot 2^4 + 2^5 = 112$

$= 112 + 112 - 2$

$= 222$

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5. (5 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Show that in any set of six classes there must be two that meet on the same day, assuming that no classes are held on weekends.

Number of Pigeons = 6

Number of Holes = 5

Celing Function

⌈ 6 / 5 ⌉ = 2, which proves our original statement by the pigeonhole principle.

6. (5 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Let d be a positive integer. Show that among any group of $d+1$ (not necessarily consecutive) integers, there are at least two with the same remainder, when they are divided by d.

Number of Pigeons = d+1

Number of Holes = d

⌈ d+1 / d ⌉ = 2, which proves our original statement by the pigeonhole principle.

7. (1 point)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

List all of the permutations of $\{1,2,3\}$.

IMPORTANT: Be sure to use parentheses to enclose each permutation, and seperate terms and permutations with a , (comma). E.g., $(1,2),(2,1)$

{ (1,2,3),(1,3,2),... (3,2,1) }

8. (1 point)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

In how many different orders can five runners finish a race?

5!

9. (9 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

One hundred tickets, numbered 1, 2, 3, ..., 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). No ticket can win more than one prize.

A) How many ways are there to award the prizes?

$P(100,4) = \frac{100!}{96!}$

B) How many ways are there to award the prizes if the person holding ticket 47 wins the grand prize?

$P(99,3) = \frac{99!}{96!}$

C) How many ways are there to award the prizes if the person holding ticket 47 wins one of the prizes?

$(4)\cdot P(99,3) = 4\cdot\frac{99!}{96!}$

D) How many ways are there to award the prizes if the person holding ticket 47 does not win a prize?

$P(99,4) = \frac{99!}{95!}$

E) How many ways are there to award the prizes if the people holding tickets 19 and 47 both win prizes?

$P(4,2)\cdot P(98,2)$

$= \frac{4!}{2!}\cdot\frac{98!}{96!}$

F) How many ways are there to award the prizes if the people holding tickets 19, 47, 73, and 97 all win prizes?

$P(4,4) = 4!$

G) How many ways are there to award the prizes if none of the people holding tickets 19, 47, 73, and 97 wins a prize?

$P(96,4) = \frac{96!}{92!}$

H) How many ways are there to award the prizes if the grand prize winner is a person holding ticket 18, 47, 73, or 97?

$= (4) \cdot P(99,3)$

$= 4 \cdot \frac{99!}{96!}$

I) How many ways are there to award the prizes if the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?

$= P(4,2) \cdot P(96,2)$

$= \frac{4!}{2!} \cdot \frac{96!}{94!}$

10. (1 point)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Suppose that a department contains **10** men and **15** women. How many ways are there to form a committee with **6 members** if it must have more women than men?

6 members

must have no more than 2 men

Cases:

- 0 men: $\binom{15}{6}$
- 1 man: $\binom{10}{1} \cdot \binom{15}{5}$
- 2 men: $\binom{10}{2} \cdot \binom{15}{4}$

$= \binom{15}{6} + \binom{10}{1}\binom{15}{5} + \binom{10}{2}\binom{15}{4}$

$= \frac{15!}{6!\,9!} + 10\,\frac{15!}{5!\,10!} + \frac{10!}{2!\,8!}\,\frac{15!}{4!\,11!}$

11. (2 points)

Here is **a list of functions and symbols** that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

There are 10 programs queued to run on a quad-core processor (processor with four cores). If each program can only be assigned to at most one core, and each core will have one program running on it, how many different combinations of programs can be run at the same time? $\binom{10}{4}$

If a core is not required to be running a program, how many different combinations of programs can be run at the same time?

Cases:

4 cores: $\binom{10}{4}$

3 cores: $\binom{10}{3}$

2 cores: $\binom{10}{2}$

1 core: $\binom{10}{1} = 10$

0 cores: 1

$= \binom{10}{4} + \binom{10}{3} + \binom{10}{2} + 10 + 1$