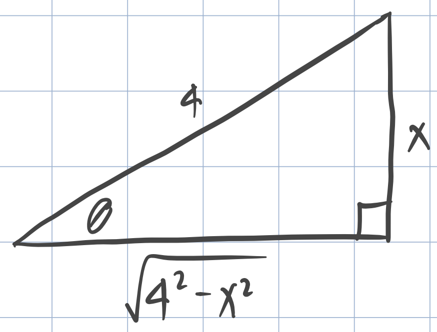
Evaluate $\int \frac{\sqrt{16-x^2}}{x^2}\,dx$



$= \int \frac{\sqrt{4^2-x^2}}{x^2}\,dx$

$= \int \frac{4\cos(\theta)}{16\sin^2(\theta)} \cdot 4\cos\theta \, d\theta$

$= \int \frac{16\cos^2(\theta)}{16\sin^2(\theta)}\,d\theta$

$= \int \cot^2(\theta)\,d\theta$

$= \int \csc^2(\theta) - 1 \, d\theta$

$= -\cot(\theta) - \theta + C$

$= -\cot(\arcsin(\frac{x}{4})) - \arcsin(\frac{x}{4}) + C$

$\cos(\theta) = \frac{\sqrt{4^2-x^2}}{4}$

$\underline{\sqrt{4^2-x^2} = 4\cos(\theta)}$

$x = 4\sin(\theta)$

$\underline{\theta = \arcsin(\frac{x}{4})}$

$\sin(\theta) = \frac{x}{4}$

$x = 4\sin(\theta)$

$x = 4\sin(\theta)$

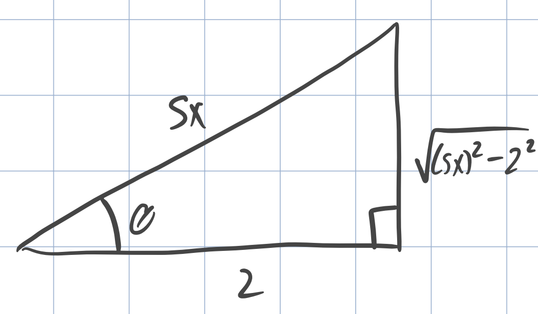
$\underline{x^2 = 16\sin^2(\theta)}$

$x = 4\sin(\theta)$

$\frac{dx}{d\theta} = 4\cos(\theta)$

$\underline{dx = 4\cos(\theta)\,d\theta}$

Evaluate $\int \frac{1}{\sqrt{25x^2-4}}\,dx$



$= \int \frac{1}{\sqrt{(5x)^2-2^2}}\,dx$

$= \int \frac{1}{2\tan(\theta)} \cdot \frac{2}{5}\frac{\sin(\theta)}{\cos^2(\theta)}\,d\theta$

$$= \frac{1}{5}\int \frac{1}{\frac{\sin(\theta)}{\cos(\theta)}} \cdot \frac{\sin(\theta)}{\cos^2(\theta)}\, d\theta$$

$$= \frac{1}{5}\int \frac{\cancel{\cos(\theta)}}{\cancel{\sin(\theta)}} \cdot \frac{\cancel{\sin(\theta)}}{\cos^2(\theta)}\, d\theta$$

$$= \frac{1}{5}\int \frac{1}{\cos(\theta)}\, d\theta$$

$$= \frac{1}{5}\int \sec(\theta)\, d\theta$$

$$= \frac{1}{5}(\ln|\tan(\theta) + \sec(\theta)|) + C$$

$$= \frac{1}{5}\ln(|\tan(\theta) + \sec(\theta)|) + C$$

$$= \frac{1}{5}\ln\left(\left|\frac{\sin(\theta)}{\cos(\theta)} + \frac{1}{\cos(\theta)}\right|\right) + C$$

$$= \frac{1}{5}\ln\left(\left|\frac{\frac{\sqrt{(5x)^2 - 2^2}}{5x}}{\frac{2}{5x}} + \frac{1}{\frac{2}{5x}}\right|\right) + C$$

$$= \frac{1}{5}\ln\left(\left|\frac{\sqrt{(5x)^2 - 2^2}}{\cancel{5x}} \cdot \frac{\cancel{5x}}{2} + \frac{5x}{2}\right|\right) + C$$

$$= \frac{1}{5}\ln\left(\left|\frac{\sqrt{(5x)^2 - 2^2}}{2} + \frac{5x}{2}\right|\right) + C$$

$$= \frac{1}{5}\ln\left(\left|\frac{\sqrt{25x^2 - 4} + 5x}{2}\right|\right) + C$$

$$\tan(\theta) = \frac{\sqrt{(5x)^2 - 2^2}}{2} \qquad \cos(\theta) = \frac{2}{5x}$$

$$\underline{\sqrt{(5x)^2 - 2^2} = 2\tan(\theta)} \qquad \underline{x = \frac{2}{5}\cos^{-1}(\theta)}$$

$$x = \frac{2}{5}(\cos(\theta))^{-1}$$

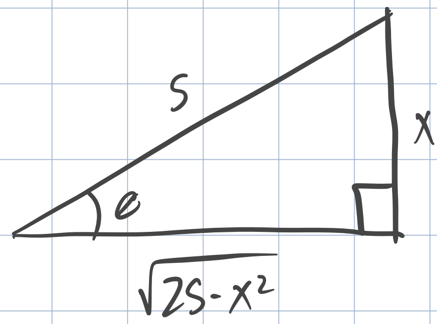
$$\frac{dx}{d\theta} = \frac{2}{5} \cdot -1(\cos(\theta))^{-2} \cdot -\sin(\theta)$$

$$\underline{dx = \frac{2}{5}\frac{\sin(\theta)}{\cos^2(\theta)}\, d\theta}$$

Evaluate $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{25 - x^2}}\, dx$

$$= \int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{5^2 - x^2}}\, dx$$

$$= \int_0^{2\sqrt{3}} \frac{125\sin^3(\theta)}{\cancel{5}\cancel{\cos(\theta)}} \cdot \cancel{5}\cancel{\cos(\theta)}\, d\theta$$



$$\cos(\theta) = \frac{\sqrt{25-x^2}}{5} \qquad \sin(\theta) = \frac{x}{5}$$

$$\underline{\sqrt{25-x^2} = 5\cos(\theta)} \qquad \underline{x = 5\sin(\theta)}$$

$$x = 5\sin(\theta) \qquad \underline{x^3 = 125\sin^3(\theta)}$$

$$\frac{dx}{d\theta} = 5\cos(\theta)$$

$$\underline{dx = 5\cos(\theta)\, d\theta}$$

$$= 125\int_0^{2\sqrt{3}} \sin^3(\theta)\, d\theta$$

$$= 125\int_0^{2\sqrt{3}} \sin^2(\theta)\sin(\theta)\, d\theta$$

$$= 125\int_0^{2\sqrt{3}} (1-\cos^2(\theta))\sin(\theta)\, d\theta$$

$$u = \cos(\theta)$$

$$\frac{du}{d\theta} = -\sin(\theta)$$

$$-du = \sin(\theta)\, d\theta$$

$$= 125\int_0^{2\sqrt{3}} 1-u^2 \cdot -du$$

$$= -125\int_0^{2\sqrt{3}} 1-u^2\, du$$

$$= -125\left(u - \frac{1}{3}u^3 \Big|_0^{2\sqrt{3}}\right)$$

$$= -125\left(\cos(\theta) - \frac{1}{3}(\cos(\theta))^3 \Big|_0^{2\sqrt{3}}\right)$$

$$= -125\left(\frac{\sqrt{25-x^2}}{5} - \frac{1}{3}\left(\frac{\sqrt{25-x^2}}{5}\right)^3 \Big|_0^{2\sqrt{3}}\right)$$

$$= -125\left[\left(\frac{\sqrt{25-(2\sqrt{3})^2}}{5} - \frac{1}{3}\left(\frac{\sqrt{25-(2\sqrt{3})^2}}{5}\right)^3\right) - \left(\frac{\sqrt{25-(0)^2}}{5} - \frac{1}{3}\left(\frac{\sqrt{25-(0)^2}}{5}\right)^3\right)\right]$$

$$= -125\left[\left(\frac{\sqrt{25-12}}{5} - \frac{1}{3}\left(\frac{\sqrt{25-12}}{5}\right)^3\right) - \left(1 - \frac{1}{3}\right)\right]$$

Evaluate $\int \frac{x^2}{(1-16x^2)^{3/2}}\,dx$

$= \int \frac{x^2}{(\sqrt{1-(4x)^2})^3}\,dx$

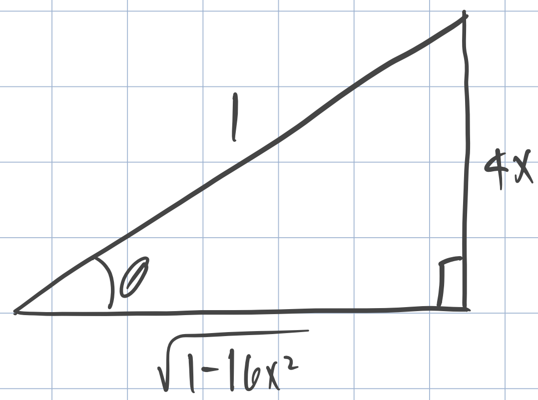
$= \int \frac{\frac{1}{16}\sin^2(\theta)}{\cos^{\cancel{3}2}(\theta)} \cdot \frac{1}{4}\cancel{\cos(\theta)}\,d\theta$

$= \int \frac{1}{16}\tan^2(\theta)\cdot\frac{1}{4}\,d\theta$

$= \frac{1}{64}\int \tan^2(\theta)\,d\theta$

$= \frac{1}{64}(-\theta + \tan(\theta))\,d\theta$

$= \frac{1}{64}\left(-\text{arc}^{-1}(\sqrt{1-16x^2}) + \frac{4x}{\sqrt{1-16x^2}}\right) + C$



$\sin(\theta) = \frac{4x}{1}$ $\qquad$ $\cos(\theta) = \frac{\sqrt{1-16x^2}}{1}$

$x = \frac{1}{4}\sin(\theta)$ $\qquad$ $\sqrt{1-16x^2} = \cos(\theta)$

$\underline{x^2 = \frac{1}{16}\sin^2(\theta)}$ $\qquad$ $\underline{(\sqrt{1-16x^2})^3 = \cos^3(\theta)}$

$x = \frac{1}{4}\sin(\theta)$ $\qquad$ $\cos(\theta) = \sqrt{1-16x^2}$

$\frac{dx}{d\theta} = \frac{1}{4}\cos(\theta)$ $\qquad$ $\underline{\theta = \arccos(\sqrt{1-16x^2})}$

$\underline{dx = \frac{1}{4}\cos(\theta)\,d\theta}$

Evaluate $\int \frac{x}{\sqrt{x^2-8x}}\,dx$

$= \int \frac{x}{\sqrt{(x^2-8x+16)-16}}\,dx$

$= \int \frac{x}{\sqrt{(x-4)^2-16}}\,dx$

$u = x-4$ $\qquad$ $u+4 = x$

$\frac{du}{dx} = 1$

$du = dx$

$$= \int \frac{u+4}{\sqrt{u^2-16}}\, du$$

$$= \underline{\int \frac{u}{\sqrt{u^2-16}}\, dx} + \int \frac{4}{\sqrt{u^2-16}}\, du$$

$$\rightarrow \int \frac{u}{\sqrt{u^2-16}}\, du$$

$v = u^2 - 16$

$\frac{dv}{du} = 2u$

$\frac{1}{2} dv = u\, du$

$$= \int \frac{1}{\sqrt{v}} \cdot \frac{1}{2}\, dv$$

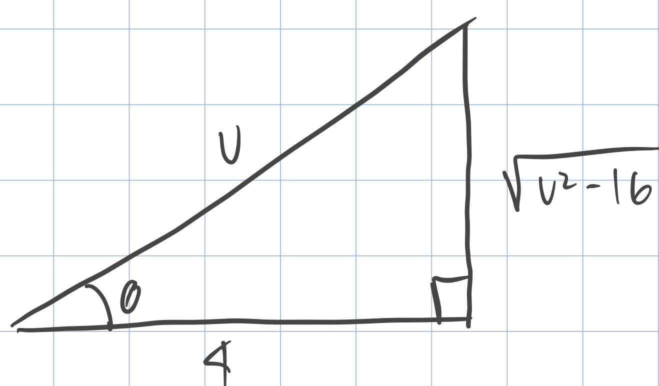
$$= \frac{1}{2} \int v^{-\frac{1}{2}}\, dv$$

$$= \frac{1}{2}\left(2v^{\frac{1}{2}}\right) + C$$

$$= \sqrt{u^2-16} + C$$

$$= \sqrt{u^2-16} + \underline{\int \frac{4}{\sqrt{u^2-16}}\, du}$$

$$\rightarrow \int \frac{4}{\sqrt{u^2-16}}\, du$$



$$= \int \frac{\cancel{4}}{\cancel{4\tan(\theta)}} \cdot 4\sec(\theta)\cancel{\tan(\theta)}\,d\theta \qquad \tan(\theta) = \frac{\sqrt{u^2-16}}{4} \qquad \cos(\theta) = \frac{4}{u}$$

$$= 4\int \sec(\theta)\,d\theta \qquad \sqrt{u^2-16} = 4\tan(\theta) \qquad u = 4\sec(\theta)$$

$$= 4\ln(|\tan(\theta) + \sec(\theta)|) + C \qquad \frac{du}{d\theta} = 4\sec(\theta)\tan(\theta)$$

$$= 4\ln\left(\left|\tan(\theta) + \frac{1}{\cos(\theta)}\right|\right) + C \qquad du = 4\sec(\theta)\tan(\theta)\,d\theta$$

$$= 4\ln\left(\left|\frac{\sqrt{u^2-16}}{4} + \frac{1}{\frac{4}{u}}\right|\right) + C$$

$$= 4\ln\left(\left|\frac{\sqrt{u^2-16}}{\cancel{4}} + \frac{u}{4}\right|\right)$$

$$= \sqrt{u^2-16} + 4\ln\left(\left|\frac{\sqrt{u^2-16}}{\cancel{4}} + \frac{u}{4}\right|\right) + C$$

$$u^2-16 = u^2-16$$

$$= (x-4)^2 - 16$$

$$= (x^2 - 8x + 16) - 16$$

$$= x^2 - 8x$$

$$= \sqrt{x^2-8x} + 4\ln\left(\left|\frac{\sqrt{x^2-8x}}{4} + \frac{x-4}{4}\right|\right) + C$$

Evaluate $\int \frac{x^2}{\sqrt{13-4x+x^2}}\,dx$

$\rightarrow 13 - 4x + x^2 = x^2 - 4x + 13$

$$= x^2 - 4x + 4 + 13 - 4$$

$$= (x-2)^2 + 9$$

$$= \int \frac{x^2}{\sqrt{(x-2)^2+9}}\,dx$$

$u = x-2 \qquad x = u+2$

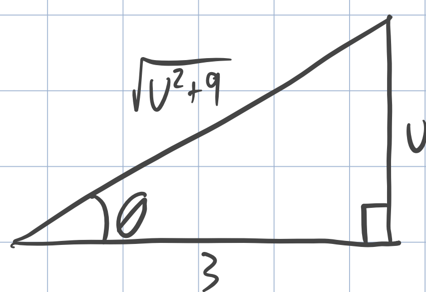
$du = dx \qquad x^2 = (u-2)^2$

$$= \int \frac{(u-2)^2}{\sqrt{u^2+9}}\,du$$

$$= \int \frac{u^2-4u+4}{\sqrt{u^2+9}}\,du$$

$$= \underline{\int \frac{u^2}{\sqrt{u^2+9}}\,du} - 4\int \frac{u}{\sqrt{u^2+9}}\,du + 4\int \frac{1}{\sqrt{u^2+9}}\,du$$

$$\rightarrow \int \frac{u^2}{\sqrt{u^2+9}}\,du$$



$$= \int \frac{9\tan^2(\theta)}{\cancel{3\sec(\theta)}} \cdot \cancel{3}\sec^2(\theta)\,d\theta$$

$$= 9\int \tan^2(\theta)\sec(\theta)\,d\theta$$

$$= 9\int (\sec^2(\theta)-1)\sec(\theta)\,d\theta$$

$$= 9\int \sec^3(\theta) - \sec(\theta)\,d\theta$$

$$= 9\int \sec^3(\theta)\,d\theta - \int \sec(\theta)\,d\theta$$

$\cos(\theta) = \frac{3}{\sqrt{u^2+9}} \qquad \tan(\theta) = \frac{u}{3}$

$\underline{\sqrt{u^2+9} = 3\sec(\theta)} \qquad u = 3\tan(\theta)$

$u = 3\tan(\theta) \qquad u = 3\tan(\theta)$

$\frac{du}{d\theta} = 3\sec^2(\theta)$

$u^2 = 9\tan^2(\theta)$

$= 9\int \sec^3(\theta)\,d\theta - \ln(|\tan(\theta) - \sec(\theta)|)$

$du = 3\sec^2(\theta)\,d\theta$

Rest of this was not worth the effort

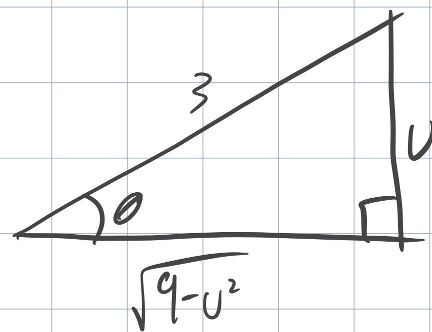
Evaluate $\int \cos(x)\sqrt{9-\sin^2(x)}\,dx$

$u = \sin(x)$

$\frac{du}{dx} = \cos(\theta)$

$du = \cos(\theta)\,dx$

$= \int \sqrt{9-u^2}\,du$



$= 3\int \cos(\theta) \cdot 3\cos(\theta)\,d\theta$

$\cos(\theta) = \frac{\sqrt{9-u^2}}{3}$

$= 9\int \cos^2(\theta)\,d\theta$

$\sqrt{9-u^2} = 3\cos(\theta)$

$= 9\int \frac{1}{2}(1+\cos(2\theta))\,d\theta$

$\sin(\theta) = \frac{u}{3}$

$= \frac{9}{2}\int 1 + \cos(2\theta)\,d\theta$

$u = 3\sin(\theta)$

$= \frac{9}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C$

$\frac{du}{d\theta} = 3\cos(\theta)$

$= \frac{9}{2}\left(\arcsin\left(\frac{u}{3}\right) + \frac{1}{2}\sin\left(2\arcsin\left(\frac{u}{3}\right)\right)\right) + C$

$$= \frac{9}{2}\left(\arcsin\left(\frac{\sin(x)}{3}\right) + \frac{1}{2}\sin\left(2\arcsin\left(\frac{\sin(x)}{3}\right)\right)\right) + C$$

$dx = 3\cos(\theta)\,d\theta$