Evaluate $\int t(7t^2-1)^3 dt$

$u = 7t^2 - 1 \qquad du = 14t\,dt$

$\frac{du}{dx} = 14t \qquad \frac{1}{14}du = t\,dt$

$= \int u^3 \cdot \frac{1}{14} du$

$= \frac{1}{4}u^4 \cdot \frac{1}{14} + C$

$= \frac{1}{56}(7t^2-1)^4 + C$

Evaluate $\int \frac{(\sqrt{x}+1)^3}{\sqrt{x}} dx$

$u = x^{1/2} + 1$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$2du = \frac{1}{\sqrt{x}} dx$

$= \int 2u^3 du$

$= 2 \cdot \frac{1}{4}u^4 + C$

$= \frac{1}{2}(\sqrt{x}+1)^4 + C$

Evaluate $\int \frac{1}{1+16x^2} dx$

$= \int \frac{1}{1+(4x)^2} dx$

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4\,dx$

$\frac{1}{4}du = dx$

$= \int \frac{1}{1+u^2} \cdot \frac{1}{4} \, du$

$= \frac{1}{4} \int \frac{1}{1+u^2} \, du$

$= \frac{1}{4} \arctan(u) + C$

$= \frac{1}{4} \arctan(4x) + C$

Evaluate $\int \sec(4x - \frac{\pi}{2}) \tan(4x - \frac{\pi}{2}) dx$

$y = 4x - \frac{\pi}{2}$
$\frac{dy}{dx} = 4$
$dy = 4dx$
$\frac{1}{4} dy = dx$

$= \int \sec(y) \tan(y) \frac{1}{4} dy$

$= \frac{1}{4} \int \sec(y) \tan(y) dy$

$= \frac{1}{4} \sec(y) + C$

$= \frac{1}{4} \sec(4x - \frac{\pi}{2}) + C$

Evaluate $= \int 8x^3 e^{-x^4}$

$= 8 \int x^3 e^{-x^4} dx$

$u = x^4$
$\frac{du}{dx} = 4x^3$
$du = 4x^3 dx$
$\frac{1}{4} du = x^3 dx$

$= 8\int \frac{1}{4} e^{-v}\,dv$

$= 2\int e^{-v}\,dv$

$= 2 \cdot -e^{-v} + C$

$= -2e^{-x^4} + C$

Evaluate $\int \frac{1}{8x+4}\,dx$

$= \int \frac{1}{4(2x+1)}\,dx$

$= \frac{1}{4}\int \frac{1}{2x+1}\,dx$

$u = 2x+1$
$\frac{du}{dx} = 2$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$= \frac{1}{4}\int \frac{1}{2} \cdot \frac{1}{u}\,du$

$= \frac{1}{8}\int u^{-1}\,du$

$= \frac{1}{8} \cdot \ln(|u|) + C$

$= \frac{1}{8}\ln(|\frac{1}{8x+4}|) + C$

Evaluate $\int \cos^3(8x)\sin(8x)\,dx$

$u = 8x$
$\frac{du}{dx} = 8$
$du = 8dx$

$\frac{1}{8}du = dx$

$= \int \frac{1}{8}\cos(u)^3 \sin(u)\, du$

$= \frac{1}{8}\int \cos(u)^3 \sin(u)\, du$

$v = \cos(u)$
$\frac{dv}{du} = -\sin(u)$
$dv = -\sin(u)du$
$-dv = \sin(u)du$

$= -\frac{1}{8}\int v^3\, dv$

$= -\frac{1}{8} \cdot \frac{1}{4} v^4 + C$

$= -\frac{1}{32} v^4 + C$

$= -\frac{1}{32} \cos(u)^4 + C$

$= -\frac{1}{32} \cos(8x)^4 + C$

Evaluate $\int \tan(5x+1)\, dx$

$u = 5x+1$
$\frac{du}{dx} = 5$
$du = 5dx$
$\frac{1}{5}du = dx$

$= \int \frac{1}{5}\tan(u)\, du$

$= \frac{1}{5}\int \tan(u)\, du$

$= -\frac{1}{5}\ln(|\cos(u)|) + C$

$= -\frac{1}{5}\ln(|\cos(5x+1|) + C$

Evaluate $\int (x+3)\sqrt{x-3}\,dx$

$= \int (x+3)(x-3)^{\frac{1}{2}}dx$

$u = (x+3)$ $\qquad\qquad$ $dv = (x-3)^{\frac{1}{2}}dx$

$\frac{du}{dx} = 1$ $\qquad\qquad$ $\frac{dv}{dx} = (x-3)^{\frac{1}{2}}$

$du = dx$ $\qquad\qquad$ $v = \frac{2}{3}(x-3)^{\frac{3}{2}} + C$

$= \frac{2}{3}(x+3)(x-3)^{\frac{3}{2}} - \int \frac{2}{3}(x-3)^{\frac{3}{2}}dx$

$= \frac{2}{3}(x+3)(x-3)^{\frac{3}{2}} - \frac{2}{3}\int (x-3)^{\frac{3}{2}}dx$

$= \frac{2}{3}(x+3)(x-3)^{3/2} - \frac{2}{3}\left(\frac{2}{5}(x-3)^{5/2}\right) + C$

$= \frac{2}{3}(x+3)(x-3)^{3/2} - \frac{4}{15}(x-3)^{\frac{5}{2}} + C$

Evaluate $\int x^3 \sin(x^2)dx$

$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{1}{2}dy = x\,dx$

$= \int x^2 \sin(x^2)\,x\,dx$

$= \int y\sin(y)\frac{1}{2}dy$

$= \frac{1}{2}\int y\sin(y)dy$

$u = y \qquad dv = \sin(y)\,dy$

$\frac{du}{dy} = 1 \qquad \frac{dv}{dy} = \sin(y)$

$du = dy \qquad v = -\cos(y)$

$= \frac{1}{2}(-y\cos(y) - \int -\cos(y)\,dy)$

$= \frac{1}{2}(-y\cos(y) - (-\sin(y) + C))$

$= \frac{1}{2}(-y\cos(y) + \sin(y) + C)$

$= \frac{1}{2}(-x^2\cos(x^2) + \sin(x^2) + C)$

$= \frac{1}{2}(-x^2\cos(x^2) + \sin(x^2)) + C$

Evaluate $\int x\csc(x)^2\,dx$

$u = x \qquad dv = \csc(x)^2\,dx$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \csc(x)^2\,dx$

$du = dx \qquad v = -\cot(x)$

$= -x\cot(x) - \int -\cot(x)\,dx$

$= -x\cot(x) + \int \cot(x)\,dx$

$= -x\cot(x) + \int \frac{\cos(x)}{\sin(x)}\,dx$

$= -x\cot(x) + \int \cos(x)\sin(x)^{-1}\,dx$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$du = \cos(x)\,dx$

$= -x\cot(x) + \int u^{-1}\,du$

$= -x\cot(x) + \ln(|u|) + C$

$= -x\cot(x) + \ln(|\sin(x)|) + C$

Evaluate $\int 5x^2 e^{6x}\,dx$

$= 5\int x^2 e^{6x}\,dx$

$u = x^2 \qquad dv = e^{6x}\,dx$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = e^{6x}$

$du = 2x\,dx \qquad v = \frac{1}{6}e^{6x}$

$= 5\left(x^2 \cdot \frac{1}{6}e^{6x} - \int \frac{1}{6}e^{6x} \cdot 2x\,dx\right)$

$= 5\left(\frac{1}{6}x^2 e^{6x} - \frac{2}{6}\int x e^{6x}\,dx\right)$

$u = x \qquad dv = e^{6x}\,dx$

$du = dx \qquad \frac{dv}{dx} = e^{6x}$

$\qquad v = \frac{1}{6}e^{6x}$

$= 5\left(\frac{1}{6}x^2 e^{6x} - \frac{1}{3}\left(\frac{1}{6}xe^{6x} - \int \frac{1}{6}e^{6x}\,dx\right)\right)$

$= 5\left(\frac{1}{6}x^2 e^{6x} - \frac{1}{3}\left(\frac{1}{6}xe^{6x} - \frac{1}{6}\int e^{6x}\,dx\right)\right)$

$= 5\left(\frac{1}{6}x^2 e^{6x} - \frac{1}{3}\left(\frac{1}{6}xe^{6x} - \frac{1}{6}\left(\frac{1}{6}e^{6x} + C\right)\right)\right)$

$= 5\left(\frac{1}{6}x^2 e^{6x} - \frac{1}{18}\left(xe^{6x} - \frac{1}{6}e^{6x}\right)\right) + C$

$= \frac{5}{6}\left(x^2 e^{6x} - \frac{1}{3}\left(xe^{6x} - \frac{1}{6}e^{6x}\right)\right) + C$

Evaluate $\int \sin^{-1}(x)\,dx$

$u = \sin^{-1}(x)$ $\qquad dv = dx$

$\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{dv}{dx} = 1$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\qquad v = x$

$= x\sin^{-1}(x) - \int x(1-x^2)^{-\frac{1}{2}}\,dx$

$= x\sin^{-1}(x) - \int (1-x^2)^{-\frac{1}{2}}\,x\,dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$= x\sin^{-1}(x) - \int u^{-\frac{1}{2}} \cdot -\frac{1}{2}\,du$

$= x\sin^{-1}(x) + \frac{1}{2}\int u^{-\frac{1}{2}}\,du$

$= x\sin^{-1}(x) + \frac{1}{2}(2u^{1/2}) + C$

$= x\sin^{-1}(x) + \sqrt{u}$

$= x\sin^{-1}(x) + \sqrt{1-x^2} + C$

Evaluate $\int e^{5x}\cos(3x)\,dx$

$u = e^{5x}$ $\qquad dv = \cos(3x)\,dx$

$\frac{du}{dx} = e^{5x} \cdot 5$ $\qquad \frac{dv}{dx} = \cos(3x)$

$du = 5e^{5x}\,dx$ $\qquad v = \frac{1}{3}\sin(3x)$

$= \frac{1}{3}\sin(3x)e^{5x} - \int \frac{5}{3}\sin(3x)e^{5x}\,dx$

$= \frac{1}{3}\sin(3x)e^{5x} - \frac{5}{3}\int \sin(3x)e^{5x}dx$

$u = e^{5x}$ $\quad dv = \sin(3x)\,dx$

$\frac{du}{dx} = e^{5x}\cdot 5$ $\quad \frac{dv}{dx} = \sin(3x)$

$du = 5e^{5x}dx$ $\quad v = -\frac{1}{3}\cos(3x)$

$= \frac{1}{3}\sin(3x)e^{5x} - \frac{5}{3}\left(-\frac{1}{3}\cos(3x)e^{5x} - \int -\frac{5}{3}\cos(3x)e^{5x}dx\right)$

$= \frac{1}{3}\sin(3x)e^{5x} - \frac{5}{3}\left(-\frac{1}{3}\cos(3x)e^{5x} + \frac{5}{3}\int \cos(3x)e^{5x}dx\right)$

$= \frac{1}{3}\sin(3x)e^{5x} + \frac{5}{9}\cos(3x)e^{5x} - \frac{25}{9}\int \cos(3x)e^{5x}dx$

Include the original expression again

$\int \cos(3x)e^{5x}dx = \frac{1}{3}\sin(3x)e^{5x} + \frac{5}{9}\cos(3x)e^{5x} - \frac{25}{9}\int \cos(3x)e^{5x}dx$

$\int \cos(3x)e^{5x}dx + \frac{25}{9}\int \cos(3x)e^{5x}dx = \frac{1}{3}\sin(3x)e^{5x} + \frac{5}{9}\cos(3x)e^{5x}$

$\frac{9}{9}\int \cos(3x)e^{5x}dx + \frac{25}{9}\int \cos(3x)e^{5x}dx = \frac{1}{3}\sin(3x)e^{5x} + \frac{5}{9}\cos(3x)e^{5x}$

$\frac{34}{9}\int \cos(3x)e^{5x}dx = \frac{1}{3}\sin(3x)e^{5x} + \frac{5}{9}\cos(3x)e^{5x}$

$\int \cos(3x)e^{5x}dx = \frac{\cancel{9}^{3}}{34}\cdot\frac{1}{\cancel{3}}\sin(3x)e^{5x} + \frac{\cancel{9}}{34}\cdot\frac{5}{\cancel{9}}\cos(3x)e^{5x}$

$\int \cos(3x)e^{5x}dx = \frac{3}{34}\sin(3x)e^{5x} + \frac{5}{34}\cos(3x)e^{5x}$

$= \frac{3}{34}\sin(3x)e^{5x} + \frac{5}{34}\cos(3x)e^{5x} + C$

Evaluate $\int_0^2 x^3(x^2+1)^{-\frac{1}{2}}dx$

$u = x^2 + 1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int_0^2 (x^2+1-1)(x^2+1)^{-\frac{1}{2}} x\,dx$

$= \frac{1}{2}\int_0^2 (u-1)(u)^{-\frac{1}{2}}\,du$

$= \frac{1}{2}\int_0^2 u^{1/2} - u^{-1/2}\,du$

$= \cancel{\frac{1}{2}}\left(\frac{\cancel{2}}{3}u^{3/2} - \cancel{2}u^{1/2}\Big|_0^2\right)$

$= \frac{1}{3}u^{\frac{3}{2}} - u^{\frac{1}{2}}\Big|_0^2$

$= \frac{1}{3}(x^2+1)^{\frac{3}{2}} - (x^2+1)^{\frac{1}{2}}\Big|_0^2$

$= \left(\frac{1}{3}(2^2+1)^{3/2} - (2^2+1)^{1/2}\right) - \left(\frac{1}{3}(0^2+1)^{3/2} - (0^2+1)^{1/2}\right)$

$= \left(\frac{1}{3}(5)^{3/2} - (5)^{1/2}\right) - \left(\frac{1}{3}(1)^{3/2} - (1)^{1/2}\right)$

Evaluate $\int_0^{\pi/4} x\cos(2x)\,dx$

$u = x$ $\qquad dv = \cos(2x)\,dx$
$\frac{du}{dx} = 1$ $\qquad \frac{dv}{dx} = \cos(2x)$
$du = dx$ $\qquad v = \frac{1}{2}\sin(2x)$

$= \frac{1}{2}x\sin(2x)\Big|_0^{\pi/4} - \int_0 \frac{1}{2}\sin(2x)\,dx$

$= \frac{1}{2}x\sin(2x)\Big|_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0 \sin(2x)\,dx$

$= \frac{1}{2}x\sin(2x)\Big|_0^{\frac{\pi}{4}} - \frac{1}{2}\left(-\frac{1}{2}\cos(2x)\Big|_0\right)$

$= \frac{1}{2}x\sin(2x) + \frac{1}{2}\left(\frac{1}{2}\cos(2x)\right)\Big|_0^{\frac{\pi}{4}}$

$= \frac{1}{2} x\sin(2x) + \frac{1}{4}\cos(2x)\Big|_0^{\frac{\pi}{4}}$

$= \left(\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos\left(\frac{\pi}{2}\right)\right) - \left(0 + \frac{1}{4}\cos(0)\right)$

$= \left(\frac{\pi}{8}(1) + 0\right) - \left(\frac{1}{4}(1)\right)$

$= \frac{\pi}{8} - \frac{1}{4}$

Evaluate $\int_2^3 \frac{\ln(x)}{4x}\,dx$

$u = \ln(x)$
$\frac{du}{dx} = \frac{1}{x}$
$du = \frac{1}{x}\,dx$

$= \frac{1}{4}\int_2^3 \ln(x)\,\frac{1}{x}\,dx$

$= \frac{1}{4}\int_2^3 u\,du$

$= \frac{1}{4}\left(\frac{1}{2}u^2\Big|_2^3\right)$

$= \frac{1}{4}\left(\frac{1}{2}\ln(x)^2\Big|_2^3\right)$

$= \frac{1}{8}\left(\ln(x)^2\Big|_2^3\right)$

$= \frac{1}{8}\left[\ln(3)^2 - \ln(2)^2\right]$

Evaluate $\int e^{\sqrt{x}}\,dx$

$= \int e^{x^{1/2}}\,dx$

$u = x^{1/2}$
$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$du = \frac{1}{2} x^{-\frac{1}{2}}$

$2du = x^{-\frac{1}{2}} dx$

$2du = u^{-1} dx$

$2du = \frac{1}{u} dx$

$2u\,du = dx$

$= \int e^u \cdot 2u\,du$

$= 2\int u e^u du$

$y = u$

$= 2\int y e^y dy$

$u = y$ $\quad$ $dv = e^y dy$

$\frac{du}{dy} = 1$ $\quad$ $\frac{dv}{dy} = e^y$

$du = dy$ $\quad$ $v = e^y$

$= 2(ye^y - \int e^y dy)$

$= 2(ye^y - (e^y)) + C$

$= 2(ye^y - e^y) + C$

$= 2(\sqrt{x}\, e^{\sqrt{x}} - e^{\sqrt{x}}) + C$

Evaluate $\int \cos^3(3x)dx$

$= \int \cos(3x)^3 dx$

$u = 3x$

$\frac{du}{dx} = 3$

$du = 3dx$
$\frac{1}{3}du = dx$

$= \int \cos(u)^3 \cdot \frac{1}{3} du$

$= \frac{1}{3} \int \cos(u)^3 du$

$u = y$

$= \frac{1}{3} \int \cos(y)^3 dy$

$= \frac{1}{3} \int \cos(y)^2 \cos(y) dy$

$= \frac{1}{3} \int (1 - \sin(y)^2) \cos(y) dy$

$u = \sin(y)$
$\frac{du}{dy} = \cos(y)$
$du = \cos(y) dy$

$= \frac{1}{3} (\int (1 - u^2) du)$

$= \frac{1}{3} (u - \frac{1}{3} u^3) + C$

$= \frac{1}{3} (\sin(y) - \frac{1}{3} \sin(y)^3) + C$

$= \frac{1}{3} (\sin(3x) - \frac{1}{3} \sin(3x)^3) + C$