

A force of 100 lb is required to stretch a spring 4 inches beyond its natural length. How much work is done stretching it from its natural length to 6 inches beyond its natural length

$$F = kx$$

$$W_i = F_i \cdot \Delta x_i$$

$$100 \text{ lb} = k \cdot 4 \text{ in}$$

$$= k x_i \cdot \Delta x_i$$

$$k = 25 \text{ lb/in}$$

$$= 25 x_i \Delta x_i$$

$$W = \int_0^6 25x \, dx$$

$$= 25 \int_0^6 x \, dx$$

$$= 25 \left(\frac{1}{2} x^2 \right) \Big|_0^6$$

$$= 25 \left(\frac{1}{2} \cdot 36 \right)$$

$$= 450$$

Suppose that 2 J of work is required to stretch a spring from its natural length of 30 cm to a length of 42 cm.

How much work is needed to stretch it from 35 cm to 40 cm.

$$W = \int_a^b f(x) \, dx$$

$$2 = \int_0^{0.12} kx \, dx$$

$$2 = k \int_0^{0.12} x \, dx$$

$$2 = k \left(\frac{1}{2} x^2 \Big|_0^{0.12} \right)$$

$$2 = k \left(\frac{1}{2} \cdot 0.12^2 \right)$$

$$k = \frac{2500}{9}$$

$$W = \int_{0.05}^{0.1} \frac{2500}{9} x \, dx$$

$$= \frac{2500}{9} \int_{0.05}^{0.1} x \, dx$$

$$= \frac{2500}{9} \left(\frac{1}{2} x^2 \Big|_{0.05}^{0.1} \right)$$

$$= \frac{2500}{9} \cdot \frac{1}{2} (0.1^2 - 0.05^2)$$

$$= \frac{25}{24}$$

Find the work to empty an inverted conical tank if the water starts 1 m down and the cone is 12 m tall and the radius is 2 m by pumping all of the water out a spout located 3 m above the tank.

Density of water = $1000 \, \text{kg/m}^3$
 Gravity = $9.8 \, \text{m/s}^2$

$$P = 1000(9.8) = 9800$$

$$W = \int_a^b P A(y) (h - y) \, dy$$

$A(y) = \pi r^2$
 $= \pi \left(\frac{1}{6} y \right)^2 = \frac{\pi}{36} y^2$
 $h = 15 \quad a = 0 \quad b = 11$
 $\frac{x}{y} = \frac{2}{12}$
 $x = \frac{1}{6} y$
 $W = \int_0^{11} 9800 \pi \left(\frac{1}{36} \right) y^2 (15 - y) \, dy$

