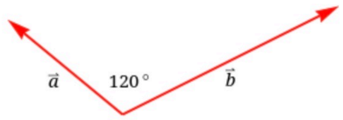
Find the magnitude of the cross product $\vec{a} \times \vec{b}$ of the vectors shown in the figure, where $\|\vec{a}\| = 5$ and $\|\vec{b}\| = 9$.



$$\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin(\theta)$$

$$= 45 \sin(120)$$

Let $\vec{a} = \langle 2, 0, 4 \rangle$, $\vec{b} = \langle -1, 3, 2 \rangle$, and $\vec{c} = \langle 1, 1, 2 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

(a) Compute $\vec{a} \times \vec{b}$.

(b) Compute $\vec{a} \cdot (\vec{b} \times \vec{c})$.

(c) Compute $\vec{a} \times (\vec{b} \times \vec{c})$.

(d) Are vectors $\vec{a}$ and $\vec{b}$ parallel?

- Yes
- No

Part A

$$\vec{a} \times \vec{b} = \langle 0 - 12, -(4+4), 6 - 0 \rangle$$

$$= \langle -12, -8, 6 \rangle$$

$$\begin{array}{c|c|c} 2 & 0 & 4 \\ -1 & 3 & 2 \end{array}$$

Part B

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot \langle 6-2, -(-2-2), -1-3 \rangle$$

$$= \vec{a} \cdot \langle 4, 4, -4 \rangle$$

$$\begin{array}{c|c|c} -1 & 3 & 2 \\ 1 & 1 & 2 \end{array}$$

$= 8 + 0 - 16$

Part C

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times \langle 4, 4, -4 \rangle$

$$\begin{array}{c|c|c} 2 & 0 & 4 \\ 4 & 4 & -4 \end{array}$$

$= \langle 0 - 16, -(-8 - 16), 8 - 0 \rangle$

$= \langle -16, 24, 8 \rangle$

Use the cross product to determine if the three points (1, 3, 4), (−3, 2, 5) and (0, −4, 3) lie on the same line.

- The three points lie on the same line.
- The three points do not lie on the same line.

$\vec{r}_{p2\ to\ p1} = (1, 3, 4) - (-3, 2, 5)$

$= \langle 4, 1, -1 \rangle$

Is $\langle 4, 1, -1 \rangle \times \langle 0, -4, 3 \rangle = 0$

$$\begin{array}{c|c|c} 4 & 1 & -1 \\ 0 & -4 & 3 \end{array}$$

$\langle 3 - 4, -(12 - 0), -16 - 0 \rangle = 0$

$\langle -1, -12, -16 \rangle \neq 0$

No, the points are not on the same line

Find the area of the parallelogram spanned by the vectors $\vec{u}$ and $\vec{v}$.

$\vec{u} = \langle 0, 3, -2 \rangle$ and $\vec{v} = \langle 4, 3, 2 \rangle$

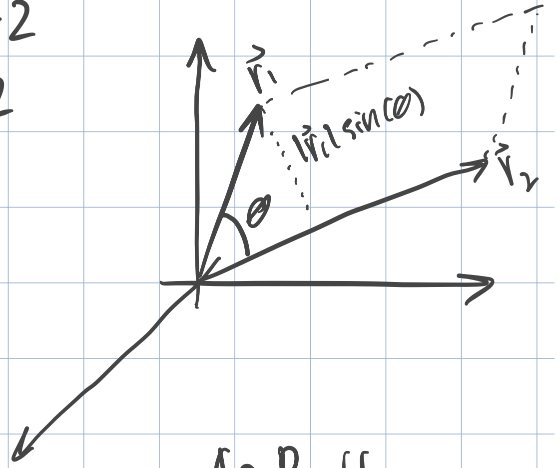
$A = |\vec{u} \times \vec{v}|$

$$\begin{array}{c|c|c} 0 & 3 & -2 \\ 4 & 3 & 2 \end{array}$$

$= |\langle 6+6, -(0+8), 0-12 \rangle|$

$= |\langle 12, -8, -12 \rangle|$

$= \sqrt{12^2 + 8^2 + 12^2}$



$A = B \cdot H$

$= |\vec{r}_1|\sin(\theta) \cdot |\vec{r}_2|$

$= |\vec{r}_1 \times \vec{r}_2|$

Find the area of the triangle spanned by the vectors $\vec{u}$ and $\vec{v}$.

$\vec{u} = \langle -2, -7, 2 \rangle$ and $\vec{v} = \langle -1, -2, -1 \rangle$

Same logic as above but half it

$A = \frac{1}{2} |\vec{u} \times \vec{v}|$

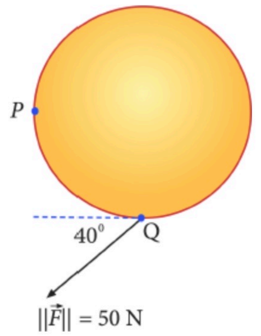
$$\begin{array}{c|c|c} -2 & -7 & 2 \\ -1 & -2 & -1 \end{array}$$

$= \frac{1}{2} |\langle 7+4, -(2+2), 4-7 \rangle|$

$= \frac{1}{2} |\langle 11, -4, -3 \rangle|$

$= \frac{1}{2} \sqrt{11^2 + 4^2 + 3^2}$

A force of magnitude 50 N is applied at the bottom point Q of a disk of radius 5 m that is pinned at P (leftmost point) as shown in the figure.



(a) Find the angle between the force and the vector from P to Q.

[ ] °

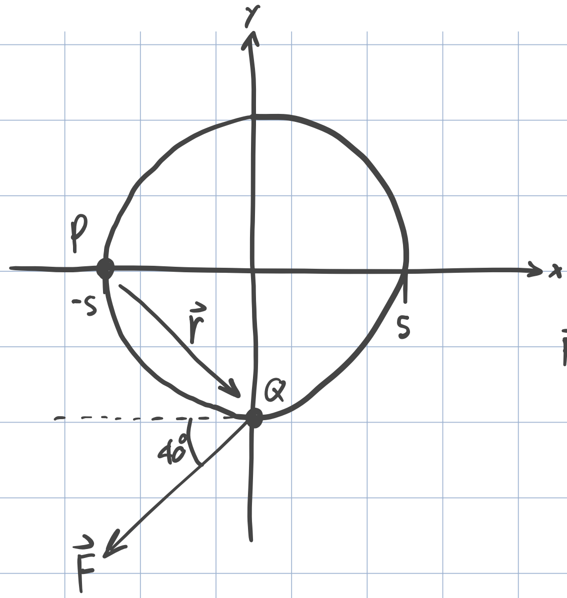
(b) Find the magnitude of the applied torque.

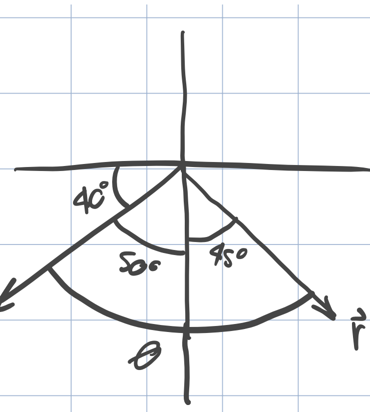
[ ] N-m

Part A

$\theta = 50 + 45$

$= 95°$





Part B

$T = |\vec{F} \times \vec{r}|$

$= 50\sqrt{50}\,\sin(95)$

Calculate the volume of the parallelepiped determined by the position vectors $\vec{a} = \langle 5, 0, 1 \rangle$, $\vec{b} = \langle -3, 4, 5 \rangle$, and $\vec{c} = \langle 2, 3, 5 \rangle$.

[ ] cubic units

$V = (\vec{a} \times \vec{b}) \cdot \vec{c}$

$\begin{array}{c|c|c} 5 & 0 & 1 \\ -3 & 4 & 5 \end{array}$

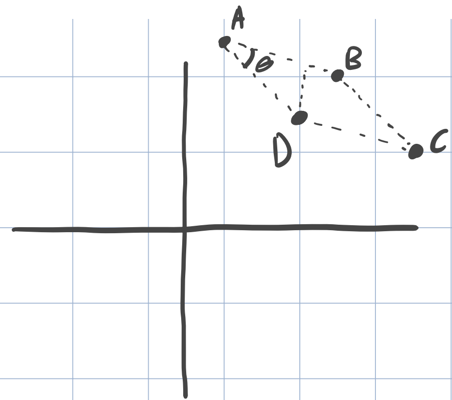
$= \langle 0 - 4, -(25 + 3), 20 \rangle \cdot \vec{c}$

$= \langle -4, -28, 20 \rangle \cdot \vec{c}$

$= -8 - 84 + 100$

$= 8$ cubic units

Compute the area of the parallelogram with vertices *A*(1, 5), *B*(4, 4), *C*(6, 2) and *D*(3, 3).



$\text{Area} = bh$

$b = |\vec{AB}|$

$h = |\vec{AD}| \sin(\theta)$

$\text{Area} = |\vec{AB}||\vec{AD}| \sin(\theta)$

Let $\vec{r}_1 = \vec{AB}$ and $\vec{r}_2 = \vec{AD}$

$\vec{r}_1 = \langle 3, -1, 0 \rangle$

$\vec{r}_2 = \langle 2, -2, 0 \rangle$

$$\begin{array}{c|c|c} 3 & -1 & 0 \\ 2 & -2 & 0 \end{array}$$

$\text{Area} = |\vec{r}_1 \times \vec{r}_2|$

$= |\langle 0-0, 0-0, -6+2 \rangle|$

$= |\langle 0, 0, -4 \rangle|$

$= 4$

Find all vectors $\vec{v} = \langle a, b, c \rangle$ that solve the given vector equation.

$$\vec{v} \times \hat{\imath} = 6\hat{\jmath}$$

- ○ $\vec{v} = \langle a, 6, 0 \rangle$ with $a \in \mathbb{R}$
- ○ No solution exists.
- ○ $\vec{v} = \langle 0, 6, 0 \rangle$
- ○ $\vec{v} = \langle 6, 0, 0 \rangle$
- ○ $\vec{v} = \langle a, 0, 6 \rangle$ with $a \in \mathbb{R}$

$$\langle a, b, c \rangle \times \langle 1, 0, 0 \rangle = 6\langle 0, 1, 0 \rangle$$

$$\begin{array}{c|c|c} a & b & c \\ 1 & 0 & 0 \end{array}$$

$$\langle 0, -(0-c), 0-b \rangle = 6\langle 0, 1, 0 \rangle$$

$$\langle 0, c, -b \rangle = \langle 0, 6, 0 \rangle$$

$$\boxed{\begin{array}{l} 0 = 0 \\ c = 6 \\ 0 = -b \text{ or } b = 0 \end{array}}$$

Not any specification for a, so we knew that it dosnt matter what a is.

$\vec{v} = \langle a, 0, 6 \rangle$

Basically for any valu of a this will be true, so we can let a be a.