Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{(n+3)\sqrt{n}}$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+3}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\sqrt{n+1}}{n+3+1} \cdot \frac{n+3}{\sqrt{n}} \right|$$

$$= \lim_{n\to\infty} \frac{n\sqrt{n} + \cdots}{n\sqrt{n} + \cdots}$$

$$= 1$$

No Conclusion

Alternating Series Test

$$\lim_{n\to\infty} a_n$$

$$= \lim_{n\to\infty} (-1)^n \frac{\sqrt{n}}{n+3}$$

$$= \pm 1 \cdot 0$$

= Conditionally Converges

Determine if the Series Converges or Diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{3n+2}$$

Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty} \frac{1}{3(n+1)+2} \cdot \frac{3n+2}{1}$$

$$= \lim_{n\to\infty} \frac{1}{3n+5} \cdot \frac{3n+2}{1}$$

$$= \lim_{n\to\infty} \frac{3n+2}{3n+5}$$

$$= 1$$

No Conclusion

Alternating Series Test

$$\lim_{n\to\infty} a_n$$

$$= \lim_{n\to\infty} (-1)^n \frac{1}{3n+2}$$

$$= \pm 0$$

= Conditionally Convergent

Determine if the series is convergent or divergent

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+3}{4n+6}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \frac{n+3+1}{4n+6+1} \cdot \frac{4n+6}{n+3}$$

$$= \lim_{n\to\infty} \frac{4n^2+\cdots}{4n^2+\cdots}$$

$$= 1$$

No Conclusion

Divevegence Test

$$\lim_{n\to\infty} a_n$$

$$= \lim_{n\to\infty} (-1)^n \frac{n+3}{4n+6}$$

$$= \pm \frac{1}{4}$$

Diverges

Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{5^n n!}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \frac{1}{5^{n+1}(n+1)!} \cdot \frac{5^n n!}{1}$$

$$= \lim_{n\to\infty} \frac{1}{5 \cdot \cancel{5^n} \cdot (n+1) \cdot \cancel{n!}} \cdot \frac{\cancel{5^n} \cancel{n!}}{1}$$

$$= \lim_{n\to\infty} \frac{1}{5(n+1)}$$

$$= 0$$

Absolutly Convergent

Determine if the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{2+n^2}$$

$$= \sum_{n=1}^{\infty} \frac{1}{2+n^{3/2}}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$=\lim_{n\to\infty} \frac{1}{2+(n+1)^{3/2}} \cdot \frac{2+n^{2/3}}{1}$$

$$=\lim_{n\to\infty} \frac{n^{3/2}+\cdots}{n^{3/2}+\cdots}$$

$$=1$$

No Conclusion

Comparison Convergence Test

$$a_n = \frac{\sqrt{n}}{2+n^2}$$

$$b_n = \frac{\sqrt{n}}{n^2}$$

$a_n \leq b_n$ for all $\mathbb{R}$ so a_n converges if b_n does

$$b_n = \frac{1}{n^{3/2}}$$

$\sum_{n=1}^{\infty} b_n$ is a series and Diverges

Determine if the series converges or diverges

$$\sum_{n=0}^{\infty} \frac{(-16)^n}{n!}$$

$$=\sum_{n=0}^{\infty} (-1)^n \frac{16^n}{n!}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$=\lim_{n\to\infty}\frac{16^{n+1}}{(n+1)!}\cdot\frac{n!}{16^n}$$

$$=\lim_{n\to\infty}\frac{16\cdot\cancel{16^n}}{(n+1)\cancel{n!}}\cdot\frac{\cancel{n!}}{\cancel{16^n}}$$

$$=\lim_{n\to\infty}\frac{16}{n+1}$$

$$=0$$

Absolutly Convergent

Determine if the series converges or diverges

$$\sum_{n=1}^{\infty}\frac{n^2}{(n+4)^2 n}$$

$$=\sum_{n=1}^{\infty}\frac{n}{n^2+8n+16}$$

Limit Comparison Test

$$a_n=\frac{n}{n^2+8n+16}$$

$$b_n=\frac{n}{n^2}$$

$$\lim_{n\to\infty}\frac{n}{n^2+8n+16}\cdot\frac{n^2}{n}$$

$$=\lim_{n\to\infty}\frac{n^3+\cdots}{n^3+\cdots}$$

$= 1$

Both series converge or diverge

$b_n = \frac{n}{n^2}$

$= \frac{1}{n^1}$

P Series and Diverges

Both Diverge

Determine if the series converges or diverges

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n!)^2}{(8n)!}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \frac{((n+1)!)^2}{(8(n+1))!} \cdot \frac{(8n)!}{(n!)^2}$$

$$= \lim_{n\to\infty} \frac{(n \cdot n!)^2}{(8n+8)!} \cdot \frac{(8n)!}{(n!)^2}$$

$$= \lim_{n\to\infty} \frac{n^2 \cdot \cancel{(n!)^2}}{(8n+8)!} \cdot \frac{(8n)!}{\cancel{(n!)^2}}$$

$$= \lim_{n\to\infty} \frac{n^2 (8n)!}{(8n+8)!}$$

$$= \lim_{n \to \infty} \frac{n^2 \cancel{(8n)!}}{\cancel{(8n)!}(8n+8)(8n+7)(8n+6)(8n+5)(8n+4)\cdots}$$

$$= 0$$

Absolutly Convergent