Evaluate $\int_{-\infty}^{0} e^x \, dx$

$= \lim_{t \to \infty} \int_{t}^{0} e^x \, dx$

$= \lim_{t \to -\infty} \left(e^x \Big|_t^0 \right)$

$= \lim_{t \to \infty} \left(e^t - e^0 \right)$

$= e^{-\infty} - e^0$

$= 0 - 1$

$= 1$

Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^2+9} \, dx$

$= \int_{-\infty}^{0} \frac{1}{x^2+9} \, dx + \int_{0}^{\infty} \frac{1}{x^2+9} \, dx$

$= \lim_{t \to \infty} \left(\int_{-t}^{0} \underline{\frac{1}{x^2+9}} \, dx + \int_{0}^{t} \underline{\frac{1}{x^2+9}} \, dx \right)$

$\rightarrow \int \frac{1}{x^2+9} \, dx$

$= \frac{1}{9} \int \frac{1}{\frac{x^2}{9} + 1} \, dx$

$= \frac{1}{9} \int \frac{1}{\left(\frac{x}{3}\right)^2 + 1} \, dx$

$u = \frac{x}{3}$

$$\frac{du}{dx} = \frac{1}{3}$$

$$3du = dx$$

$$= \frac{3}{9}\int \frac{1}{u^2+1}\,dx$$

$$= \frac{1}{3}\arctan(u) + C$$

$$= \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

$$= \lim_{t\to\infty}\left(\frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_{-t}^{0} + \frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_{0}^{t}\right)$$

$$= \frac{1}{3}\left(\arctan\left(\frac{x}{3}\right)\Big|_{-\infty}^{0} + \arctan\left(\frac{x}{3}\right)\Big|_{0}^{\infty}\right)$$

$$= \frac{1}{3}\left(\arctan\left(\frac{0}{3}\right) - \arctan\left(\frac{-\infty}{3}\right)\right) + \left(\arctan\left(\frac{\infty}{3}\right) - \arctan\left(\frac{0}{3}\right)\right)$$

$$= \frac{1}{3}\left(\left(0 + \frac{\pi}{2}\right) + \left(\frac{\pi}{2} - 0\right)\right)$$

$$= \frac{1}{3}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= \frac{\pi}{3}$$

Evaluate $\int_1^\infty \frac{1}{x^4+9x^2}\,dx$

$$\rightarrow \int \frac{1}{x^4+9x^2}\,dx$$

$$\rightarrow \frac{1}{x^2(x^2+9)}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+9}$$

$$= \frac{Ax(x^2+9)}{x^2(x^2+9)} + \frac{B(x^2+9)}{x^2(x^2+9)} + \frac{(Cx+D)x^2}{x^2(x^2+9)}$$

$$= Ax(x^2+9) + B(x^2+9) + (Cx+D)x^2$$

$$= Ax^3 + 9Ax + Bx^2 + 9B + Cx^3 + Dx^2$$

$$\rightarrow Ax^3 + Cx^3 = 0$$
$$Bx^2 + Dx^2 = 0$$
$$9Ax = 0$$
$$9B = 1$$

$$A + C = 0$$
$$B + D = 0$$
$$A = 0$$
$$B = \frac{1}{9}$$

$$D = -\frac{1}{9}$$

$$C = 0$$

$$= \frac{0}{x} + \frac{\frac{1}{9}}{x^2} + \frac{0x - \frac{1}{9}}{x^2+9}$$

$$= \frac{1}{9x^2} - \frac{1}{9(x^2+9)}$$

$$= \frac{1}{9}\left(\frac{1}{x^2} - \frac{1}{x^2+9}\right)$$

$$=\frac{1}{9}\int \frac{1}{x^2} - \frac{1}{x^2+9}\,dx$$

$$=\frac{1}{9}\left(\int x^{-2}\,dx - \frac{1}{9}\int \frac{1}{(\frac{x}{3})^2+1}\,dx\right)$$

$$=\frac{1}{9}\left(-\frac{1}{1}x^{-1} - \frac{1}{3}\arctan\left(\frac{x}{3}\right)\right)+C$$

$$=\frac{1}{9}\left(-\frac{1}{x} - \frac{1}{3}\arctan\left(\frac{x}{3}\right)\right)+C$$

$$=\lim_{t\to\infty}\int_1^t \frac{1}{x^4+9x^2}\,dx$$

$$=\lim_{t\to\infty}\left[\frac{1}{9}\left(-\frac{1}{x}\ \frac{1}{3}\arctan\left(\frac{x}{3}\right)\right)\Big|_1^t\right]$$

$$=\frac{1}{9}\lim_{t\to\infty}\left[-\frac{1}{x}-\frac{1}{3}\arctan\left(\frac{x}{3}\right)\Big|_1^t\right]$$

$$=\frac{1}{9}\lim_{t\to\infty}\left[\left(-\frac{1}{t}-\frac{1}{3}\arctan\left(\frac{t}{3}\right)\right)-\left(-\frac{1}{1}-\frac{1}{3}\arctan\left(\frac{1}{3}\right)\right)\right]$$

$$=\frac{1}{9}\left[\left(-\frac{1}{\infty}-\frac{1}{3}\arctan\left(\frac{\infty}{3}\right)\right)-\left(-1-\frac{1}{3}\arctan\left(\frac{1}{3}\right)\right)\right]$$

$$=\frac{1}{9}\left[-\frac{1}{3}\left(\frac{\pi}{2}\right)+1+\frac{1}{3}\arctan\left(\frac{1}{3}\right)\right]$$

$$=\frac{1}{9}\left[-\frac{\pi}{6}+1+\frac{1}{3}\arctan\left(\frac{1}{3}\right)\right]$$

Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec(\theta)\,d\theta$

Vertical Asymtote at $x=\frac{\pi}{2}$

$\rightarrow \int \sec(\theta)\,d\theta$

$= \ln(|\tan(\theta) + \sec(\theta)|) + C$

$= \ln(|\tan(\theta) + \sec(\theta)|)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$

$= \ln(|\tan(\frac{\pi}{2}) + \sec(\frac{\pi}{2})|) - \ln(|\tan(\frac{\pi}{3}) + \sec(\frac{\pi}{3})|)$

$= \ln(|$undefined ...

Limit Diverges

Evaluate $\int_0^9 \frac{1}{(x-1)^{2/3}}\,dx$

Vertical Asymtote at $x=1$

$= \underline{\int_0^1 \frac{1}{(x-1)^{2/3}}\,dx} + \underline{\int_1^9 \frac{1}{(x-1)^{2/3}}\,dx}$

$\rightarrow \int \frac{1}{(x-1)^{2/3}}\,dx$

$= \int (x-1)^{-2/3}\,dx$

$= \frac{3}{1}(x-1)^{1/3} + C$

$= 3\sqrt[3]{x-1} + C$

$= (3\sqrt[3]{x-1}\,\Big|_0^1) + (3\sqrt[3]{x-1}\,\Big|_1^9)$

$= 3(\sqrt[3]{x-1}\,\Big|_0^1 + \sqrt[3]{x-1}\,\Big|_1^9)$

$$=3\left[\left(\sqrt[3]{1-1}-\sqrt[3]{0-1}\right)+\left(\sqrt[3]{9-1}-\sqrt[3]{1-1}\right)\right]$$

$$=3\left[(0+1)+(\sqrt[3]{8}-0)\right]$$

$$=3(1+2)$$

$$=9$$

Evaluate $\int_0^6 \frac{1}{x^2-4x-5}\,dx$

$$=\int_0^6 \frac{1}{(x-5)(x+1)}\,dx$$

$$\begin{array}{ccl} x & & x=-5x \\ -5 & \times & 1=\underline{\;x\;} \\ & & -4x \end{array}$$

Vertical Asymtote at $x=5$, $x=-1$

$$=\int_0^5 \underline{\frac{1}{(x-5)(x+1)}}\,dx+\int_5^6 \underline{\frac{1}{(x-5)(x+1)}}\,dx$$

$$\rightarrow \frac{1}{(x-5)(x+1)}$$

$$=\frac{A}{(x-5)}+\frac{B}{(x+1)}$$

$$=\frac{A(x+1)}{(x-5)(x+1)}+\frac{B(x-5)}{(x-5)(x+1)}$$

$$=A(x+1)+B(x-5)$$

$$=Ax+A+Bx-5B$$

$\rightarrow Ax + Bx = 0$
$A - 5B = 1$

$A + B = 0$
$A - 5B = 1$

$5A + 5B = 0$
$A - 5B = 1$

$6A = 1$

$A = \frac{1}{6}$

$B = -\frac{1}{6}$

$= \frac{\frac{1}{6}}{(x-5)} + \frac{-\frac{1}{6}}{(x+1)}$

$= \frac{1}{6}\left(\frac{1}{(x-5)} - \frac{1}{(x+1)}\right)$

$= \frac{1}{6}\left(\underline{\int_0^5 \frac{1}{x-5} - \frac{1}{x+1}\,dx} + \int_5^6 \frac{1}{x-5} - \frac{1}{x+1}\,dx\right)$

Asymtete at $x=5$

$\rightarrow \int_0^5 \frac{1}{x-5} - \frac{1}{x+1}\,dx$

$= \lim_{t \to 5^+} \int_0^t \frac{1}{x-5} - \frac{1}{x+1}\,dx$

$= \lim_{t \to 5^+} \ln(|x-5|) - \ln(|x+1|)\Big|_0^t$

$$=\lim_{t\to 5^+} \left(\ln(|t-5|)-\ln(|t+1|)\right)-\left(\ln(|0-5|)-\ln(|0+1|)\right)$$

$$=(-\infty-\ln(6))-(\ln(5)-\ln(1))$$

$$=(-\infty)-(\ln(5)-0)$$

$$=-\infty-\ln(5)$$

$$=-\infty$$

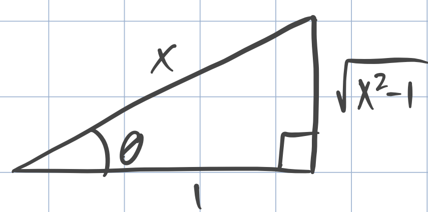
Limit Diverges

Evaluate $\int_1^2 \frac{1}{x\sqrt{x^2-1}}\,dx$

Vertical Asymptote at $x=1$, $x=0$

$$\to \int \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$=\int \frac{1}{\cancel{\sec(\theta)}\cancel{\tan(\theta)}}\cancel{\sec(\theta)}\cancel{\tan(\theta)}\,d\theta$$



$=\int 1\,d\theta$ $\qquad \cos(\theta)=\frac{1}{x}$ $\qquad \underline{\tan(\theta)=\sqrt{x^2-1}}$

$=\theta+C$ $\qquad x=\frac{1}{\cos(\theta)}$ $\qquad \cos(\theta)=\frac{1}{x}$

$=\arccos\left(\frac{1}{x}\right)+C$ $\qquad \underline{x=\sec(\theta)}$ $\qquad \theta=\arccos\left(\frac{1}{x}\right)$

$$=\lim_{t\to 1^+} \int_t^2 \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$=\lim_{t\to 1^+} \arccos\left(\frac{1}{x}\right)\Big|_t^2$$

$$=\lim_{t\to 1^+} \left(\arccos\left(\frac{1}{2}\right)-\arccos\left(\frac{1}{t}\right)\right)$$

$$=\arccos\left(\frac{1}{2}\right)-\arccos(1)$$

Evaluate $\underline{\int_0^1 \frac{1}{x\sqrt{16-x^2}}\,dx}$

Vertical Asymptote at $x=0$, $x=4$

$$\to \int \frac{1}{x\sqrt{16-x^2}}\,dx$$

$$=\int \frac{1}{4\sin(\theta)\cancel{4\cos(\theta)}}\cancel{4\cos(\theta)}\,d\theta$$

$$=\frac{1}{4}\int \frac{1}{\sin(\theta)}\,d\theta$$

$$=\frac{1}{4}\int \frac{\sin(\theta)}{\sin^2(\theta)}\,d\theta$$

$$=\frac{1}{4}\int \frac{\sin(\theta)}{1-\cos^2(\theta)}\,d\theta$$

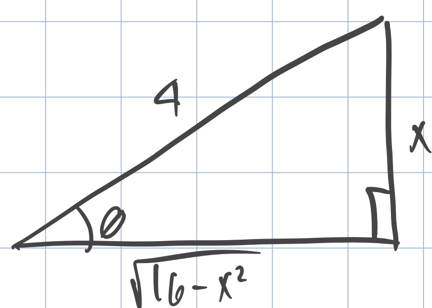
$u=\cos(\theta)$

$\frac{du}{d\theta}=-\sin(\theta)$

$-du=\sin(\theta)d\theta$

$\frac{dx}{d\theta}=\sec(\theta)\tan(\theta)$

$\underline{dx=\sec(\theta)\tan(\theta)d\theta}$



$\sin(\theta)=\frac{x}{4}$ $\qquad$ $\cos(\theta)=\frac{\sqrt{16-x^2}}{4}$

$\underline{x=4\sin(\theta)}$ $\qquad$ $\underline{\sqrt{16-x^2}=4\cos(\theta)}$

$\frac{dx}{d\theta}=4\cos(\theta)$

$\underline{dx=4\cos(\theta)d\theta}$

$$= -\frac{1}{4}\int \frac{1}{1-u^2}\,du$$

$$= -\frac{1}{4}\int \frac{1}{u^2-1}\,du$$

$$= -\frac{1}{4}\int \frac{1}{(u+1)(u-1)}\,du$$

$$\rightarrow \frac{1}{(u+1)(u-1)}$$

$$= \frac{A}{u+1} + \frac{B}{u-1}$$

$$= A(u+1) + B(u-1)$$

$$= Au + A + Bu - B$$

$$\rightarrow Au + Bu = 0$$
$$A - B = 1$$

$$A + B = 0$$
$$A - B = 1$$

$$2A = 1$$

$$A = \frac{1}{2}$$

$$B = -\frac{1}{2}$$

$$= \frac{\frac{1}{2}}{u+1} - \frac{\frac{1}{2}}{u-1}$$

$$=\frac{1}{2}\left(\frac{1}{v+1}-\frac{1}{v-1}\right)$$

$$=-\frac{1}{8}\int \frac{1}{v+1}-\frac{1}{v-1}\,dv$$

$$=-\frac{1}{8}\left(\ln(|v+1|)-\ln(|v-1|)\right)+C$$

$$=-\frac{1}{8}\left(\ln(|\cos(\theta)+1|)-\ln(|\cos(\theta)-1|)\right)+C$$

$$=-\frac{1}{8}\left(\ln\left(\left|\frac{\sqrt{16-x^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-x^2}}{4}-1\right|\right)\right)+C$$

$$=\lim_{t\to 0^+}\int_t^1 \frac{1}{x\sqrt{16-x^2}}\,dx$$

$$=\lim_{t\to 0^+}\left(-\frac{1}{8}\left(\ln\left(\left|\frac{\sqrt{16-x^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-x^2}}{4}-1\right|\right)\right)\Big|_t^1\right)$$

$$=-\frac{1}{8}\lim_{t\to 0^+}\ln\left(\left|\frac{\sqrt{16-x^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-x^2}}{4}-1\right|\right)\Big|_t^1$$

$$=-\frac{1}{8}\lim_{t\to 0^+}\left[\ln\left(\left|\frac{\sqrt{16-1^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-1^2}}{4}-1\right|\right)\right]-\left[\ln\left(\left|\frac{\sqrt{16-t^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-t^2}}{4}-1\right|\right)\right]$$

$$=-\frac{1}{8}\left(\left[\ln\left(\left|\frac{\sqrt{16-1^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-1^2}}{4}-1\right|\right)\right]-\left[\ln\left(\left|\frac{\sqrt{16-0^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-0^2}}{4}-1\right|\right)\right]\right)$$

$$=-\frac{1}{8}\left(\left[\ln\left(\left|\frac{\sqrt{16-1^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-1^2}}{4}-1\right|\right)\right]-\left[\ln(1+1)-\ln(0)\right]\right)$$

$$=-\frac{1}{8}\left(\left[\ln\left(\left|\frac{\sqrt{16-1^2}}{4}+1\right|\right)-\ln\left(\left|\frac{\sqrt{16-1^2}}{4}-1\right|\right)\right]-\left[\ln(2)-(-\infty)\right]\right)$$

$$=-\infty$$

Limit Diverges

Evaluate $\int_5^{\infty} \frac{1}{\sqrt{x-5}}\, dx$

$\rightarrow \int \frac{1}{\sqrt{x-5}}\, dx$

$u = x-5$

$\frac{du}{dx} = 1$

$du = dx$

$= \int u^{-\frac{1}{2}}\, du$

$= 2u^{\frac{1}{2}} + C$

$= 2\sqrt{x-5} + C$

Vertical Asymptote at $x=5$

$= \int_5^{10} \frac{1}{\sqrt{x-5}}\, dx + \int_{10}^{\infty} \frac{1}{\sqrt{x-5}}\, dx$

$= \lim_{t\to 5^-} \int_t^{10} \frac{1}{\sqrt{x-5}}\, dx + \lim_{g\to\infty} \int_{10}^{g} \frac{1}{\sqrt{x-5}}\, dx$

$= \lim_{t\to 5^-} 2\sqrt{x-5}\Big|_t^{10} + \lim_{g\to\infty} 2\sqrt{x-5}\Big|_{10}^{g}$

$= 2\left(\lim_{t\to 5^+} \sqrt{x-5}\Big|_t^{10} + \lim_{g\to\infty} \sqrt{x-5}\Big|_{10}^{g}\right)$

$=2[(\sqrt{10-5}-\sqrt{0})-(\sqrt{\infty}-\sqrt{0})]$

$=-\infty$

Limit Diverges

Evaluate $\int_{10}^{\infty} \frac{1}{(3x-5)^{3/2}}\,dx$

Vertical Asymptote $x=\frac{5}{3}$

$\rightarrow \int (3x-5)^{-3/2}\,dx$

$u=3x-5$

$\frac{du}{dx}=3$

$\frac{1}{3}du=dx$

$=\frac{1}{3}\int u^{-3/2}\,du$

$=\frac{1}{3}(-2u^{-\frac{1}{2}})+C$

$=-\frac{2}{3}(3x-5)^{-\frac{1}{2}}+C$

$=-\frac{2}{3}\frac{1}{\sqrt{3x-5}}+C$

$=\lim_{t\to\infty}\int_{10}^{t}\frac{1}{\sqrt{3x-5}}\,dx$

$$= \lim_{t\to\infty} -\frac{2}{3}\frac{1}{\sqrt{3x-5}}\Big|_{10}^{t}$$

$$= -\frac{2}{3}\left(\lim_{t\to\infty}\frac{1}{\sqrt{3x-5}}\Big|_{10}^{t}\right)$$

$$= -\frac{2}{3}\left(\lim_{t\to\infty}\frac{1}{\sqrt{3t-5}} - \frac{1}{\sqrt{30-5}}\right)$$

$$= -\frac{2}{3}\left(\frac{1}{\sqrt{\infty}} - \frac{1}{\sqrt{25}}\right)$$

$$= -\frac{2}{3}\left(0 - \frac{1}{5}\right)$$

$$= \frac{2}{15}$$