Homework 2

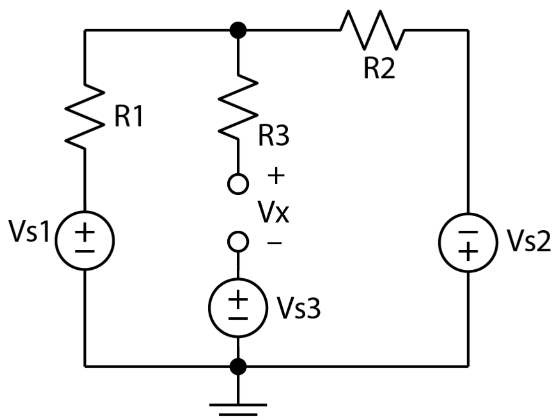
To receive full credits, you must describe the reasoning behind each step, e.g. KVL on L1, or using Ohm's law on R1, etc. Problems without reasoning receive 0 points regardless of providing correct or incorrect result. Problems with clear reasoning and correct result receives full points. Solutions with clear reasoning and incorrect results receive 3/4 of the total points.

<u>Use the branch current method to solve the following problems.</u>

Problem 1: Solve for Vx. (*10 points*)

Vs1 = 2 V   Vs2 = 1 V   Vs3 = 6 V

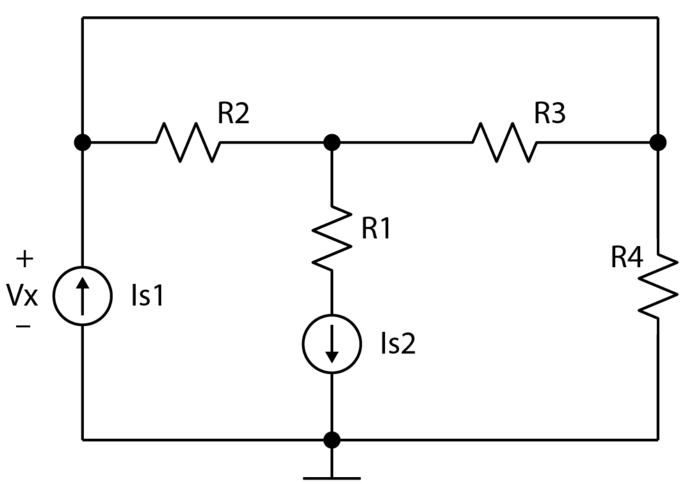
R1 = 1 Ω   R2 = 3 Ω   R3 = 10 Ω



Problem 2: Solve for Vx. (*10 points*)

Is1 = 1 A   Is1 = 2 A

R1 = 10 Ω   R2 = 2 Ω   R3 = 6 Ω   R4 = 5 Ω

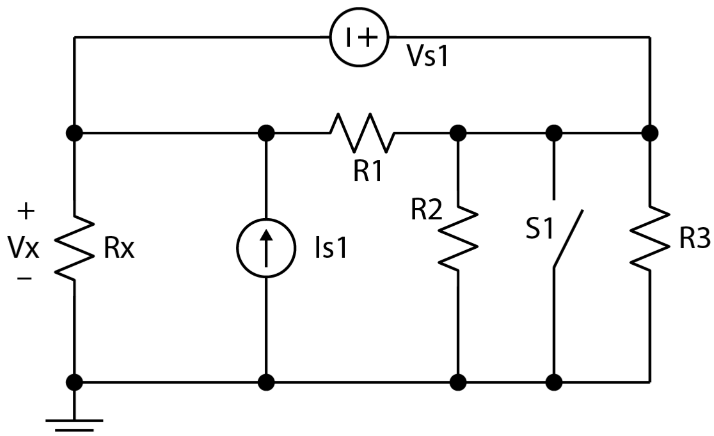


Problem 3: The following circuit has a switch S1.

a) Solve for Vx when the switch S1 is open. (*10 points*)
b) Solve for Vx when the switch S1 is closed. (*10 points*)

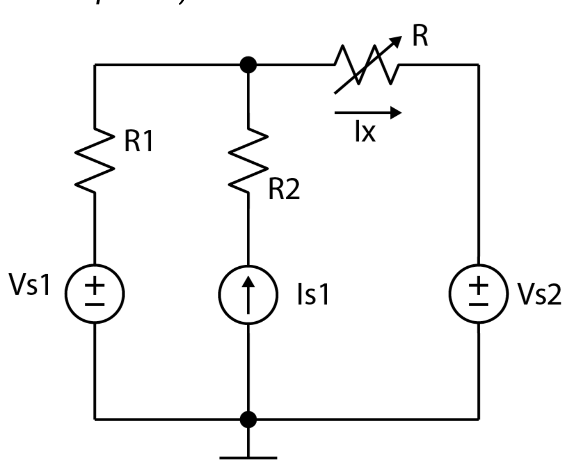
Is1 = 5A Vs1 = 5 V

R1 = 20 Ω R2 = 50 Ω R3 = 100 Ω Rx = 30 Ω



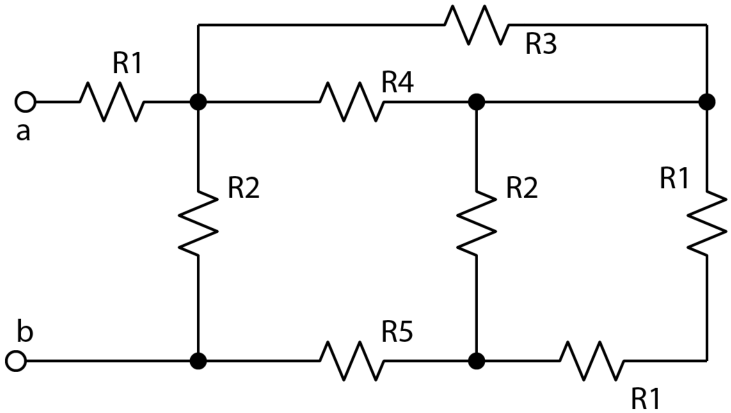
Problem 4: The circuit below contains a potentiometer whose programable resistance is R.

a) Solve for Ix and express the result as a function of R and Vs2. (*20 points*)
   Vs1 = 10 V Is1 = 1 A
   R1 = 5 Ω R2 = 50 Ω
b) Find the value of the potentiometer R such that Ix = 0.5 Is1. Use Vs2 = 10 V. (5 *points*)
c) For the potentiometer value found in "b)", find the value for the Vs2 such that Ix = -0.5 Is1. (*5 points*)



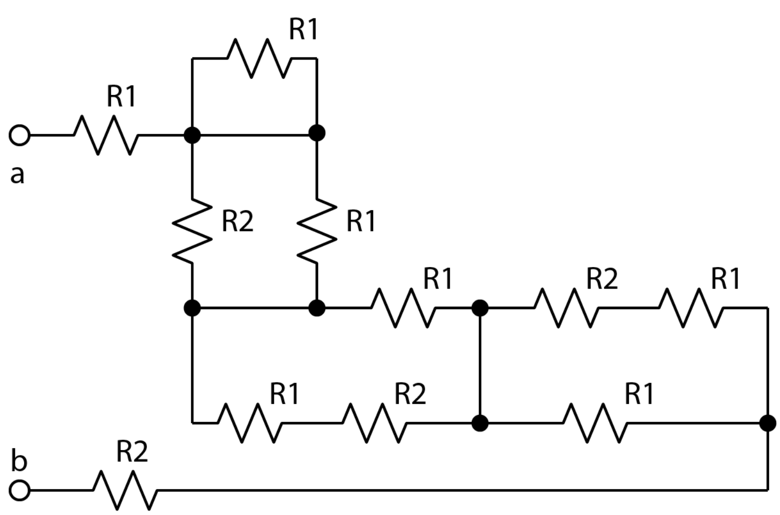
Problem 5: Find the equivalent resistance between points *a* and *b.* (15 points)

R1 = 5 Ω R2 = 10 Ω R3 = 40 Ω R4 = 20 Ω R5 = 25 Ω

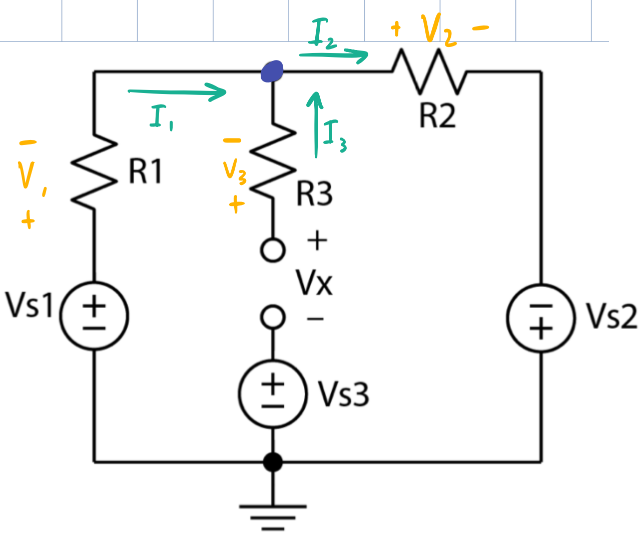


Problem 6: Find the equivalent resistance between points *a* and *b*. (15 points)

R1 = 20 Ω R2 = 10 Ω



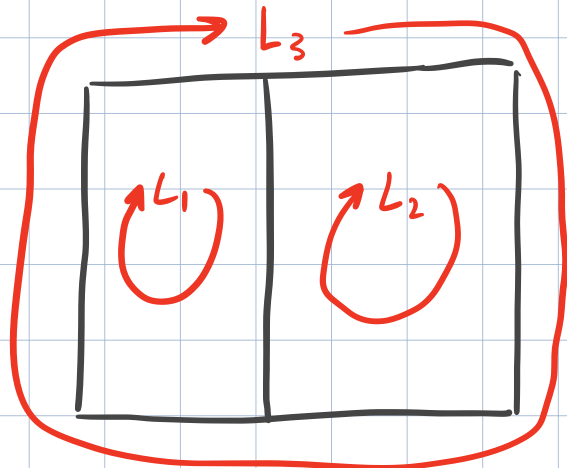
Problem 1



KVL on L_1: $V_{s1} - V_1 + V_3 - V_x - V_{s3} = 0$

KVL on L_2: $V_{s2} + V_{s3} + V_x - V_3 - V_2 = 0$

KVL on L_3: $V_{s1} - V_1 - V_2 + V_{s2} = 0$



Sub with Ohm's Law

$V_{s1} - I_1R_1 + I_3R_3 - V_x - V_{s3} = 0$

$V_{s2} + V_{s3} + V_x - I_3R_3 - I_2V_2 = 0$

$V_{s1} - I_1R_1 - I_2R_2 + V_{s2} = 0$

Because of open circuit $I_3 = 0$ and $I_1 = I_2$

$V_{s1} - I_1R_1 - V_x - V_{s3} = 0$ (1)

$V_{s2} + V_{s3} + V_x - I_1R_2 = 0$ (2)

$V_{s1} - I_1R_1 - I_1R_2 + V_{s2} = 0$ (3)

Solve (1) for I_1

$I_1R_1 = V_{s1} - V_x - V_{s3}$

$$I_1 = \frac{V_{s1} - V_x - V_{s3}}{R_1} \qquad (4)$$

Sub (4) into (2)

$$V_{s2} + V_{s3} + V_x - \left(\frac{V_{s1} - V_x - V_{s3}}{R_1}\right) R_2 = 0$$

$$V_{s2} + V_{s3} + V_x - \frac{V_{s1} R_2}{R_1} + \frac{V_x R_2}{R_1} + \frac{V_{s3} R_2}{R_1} = 0$$

$$V_x + \frac{V_x R_2}{R_1} = -V_{s2} - V_{s3} + \frac{V_{s1} R_2}{R_1} - \frac{V_{s3} R_2}{R_1}$$

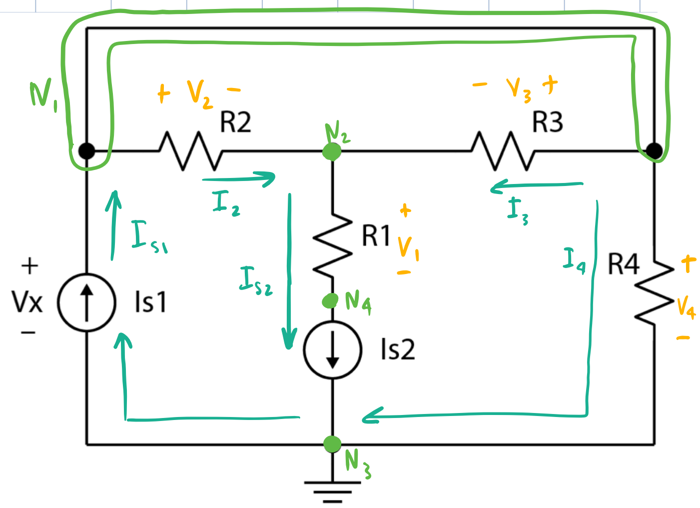
$$V_x \left(1 + \frac{R_2}{R_1}\right) = -V_{s2} - V_{s3} + \frac{V_{s1} R_2}{R_1} - \frac{V_{s3} R_2}{R_1}$$

$$V_x = \frac{-V_{s2} - V_{s3} + \frac{V_{s1} R_2}{R_1} - \frac{V_{s3} R_2}{R_1}}{1 + \frac{R_2}{R_1}}$$

$$= \frac{-V_{s2} - V_{s3} + \frac{R_2}{R_1}(V_{s1} - V_{s3})}{1 + \frac{R_2}{R_1}}$$

$$= -4.75 \text{ V}$$

Problem 2



KCL at N_1: $I_{s1} = I_2 + I_3 + I_4$ (1)

KCL at N_2: $I_2 + I_3 = I_{s2}$ (2)

KCL at N_3: $I_{s2} + I_4 = I_{s1}$ (3)

KVL on $I_{s1} \to R_4$: $V_x - V_4 = 0$ (4)

Sub (4) using Ohm's Law and solve for V_x

$$V_x - R_4 I_4 = 0$$

$$V_x = R_4 I_4 \quad (5)$$

Solve for I_4 using (1) and (2)

$$I_{s1} = I_2 + I_3 + I_4$$

$$\underline{I_2 + I_3 = I_{s2}}$$

$$I_{s1} = I_2 + I_3 + I_4$$

$$\underline{-I_{s2} = -I_2 - I_3}$$

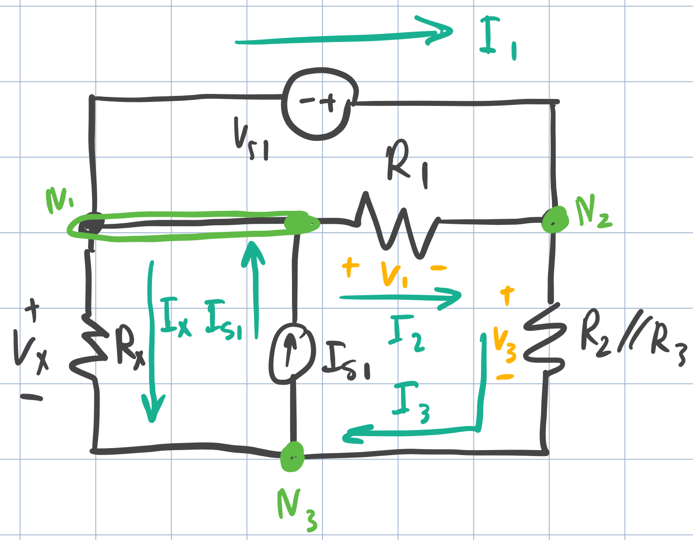
$$I_{s1} - I_{s2} = I_4 \quad (6)$$

Solve for V_x Using (5) and (6)

$$V_x = R_4 (I_{S1} - I_{S2})$$

$$= -5V$$

Problem 3a



Let $R_4 = R_2 // R_3$

3 backyards − 1 current source = 2 unknowns

KCL at N_3: $I_{s1} = I_x + I_3$

KCL at N_2: $I_3 = I_2 + I_1$

Solve for currents in terms of 2 unknowns

$I_3 = I_{s1} - I_x$

$I_2 = I_x + I_{s1} - I_1$

Find KVLs and write in terms of unknown currents

KVL on $V_x \to V_{s1} \to R_2 // R_3$: $V_x + V_{s1} - R_2 // R_3 = 0$

$$V_x + V_{s1} - I_3 R_4 = 0$$

$$V_x + V_{s1} - (I_s - I_x) R_4 = 0$$

$$I_x R_x + V_{s1} - (I_s - I_x) R_4 = 0$$

$$I_x R_x + V_{s1} - I_s R_4 + I_x R_4 = 0$$

$$I_x (R_x + R_4) = I_s R_4 - V_{s1}$$

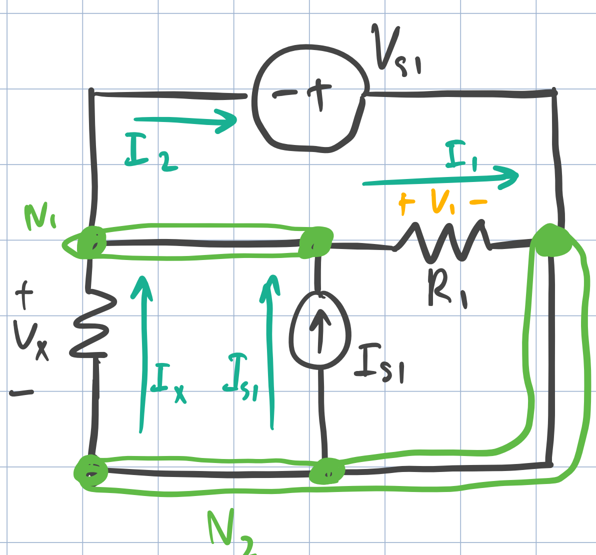
$$I_x = \frac{I_s R_4 - V_{s1}}{R_x + R_4}$$

$$\frac{V_x}{R_x} = \frac{I_s R_4 - V_{s1}}{R_x + R_4}$$

$$V_x = R_x \frac{I_s R_4 - V_{s1}}{R_x + R_4}$$

$$= 76.575 \text{v}$$

Problem 3b

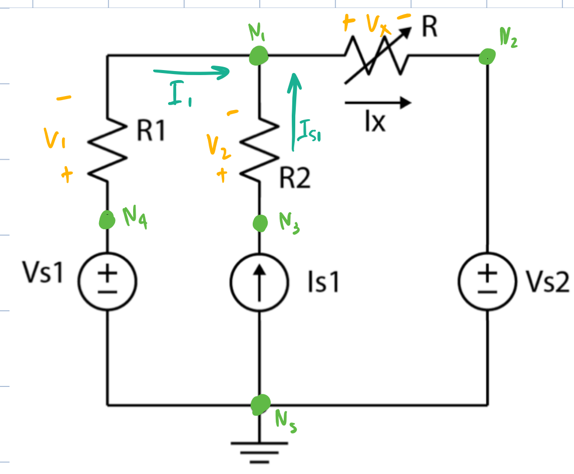


KVL on $V_x \rightarrow V_{s1}$: $V_x + V_{s1} = 0$

$V_x = -V_{s1}$

$= -5\,V$

Problem 4a



KCL at N_1: $I_{s1} = I_1 + I_x$ (1)

KVL on $V_{s1} \rightarrow R_x \rightarrow V_{s2}$: $V_{s1} - V_1 - V_x - V_{s2} = 0$ (2)

$I_x = I_{s1} - I_1$

Solve (1) for I_x

$$I_x = I_{s1} + I_1 \quad (3)$$

Solve (2) for I_1

$$V_{s1} - I_1 R_1 - I_x R_x - V_{s2} = 0$$

$$I_1 R_1 = V_{s1} - I_x R_x - V_{s2}$$

$$I_1 = \frac{V_{s1} - I_x R_x - V_{s2}}{R_1} \quad (4)$$

Sub (4) into (3) and solve for I_x

$$I_x = I_{s1} + \frac{V_{s1} - I_x R_x - V_{s2}}{R_1}$$

$$I_x = I_{s1} + \left(\frac{V_{s1}}{R_1} - \frac{I_x R_x}{R_1} - \frac{V_{s2}}{R_1} \right)$$

$$I_x = I_{s1} + \frac{V_{s1}}{R_1} - \frac{I_x R_x}{R_1} - \frac{V_{s2}}{R_1}$$

$$I_x + \frac{I_x R_x}{R_1} = I_{s1} + \frac{V_{s1}}{R_1} - \frac{V_{s2}}{R_1}$$

$$I_x\left(1 + \frac{R_x}{R_1}\right) = I_{s1} + \frac{V_{s1}}{R_1} - \frac{V_{s2}}{R_1}$$

$$I_x = \frac{I_{s1} + \frac{V_{s1} - V_{s2}}{R_1}}{1 + \frac{R_x}{R_1}}$$

$$= \frac{\frac{I_{s1}R_1 + V_{s1} - V_{s2}}{R_1}}{\frac{R_1 + R_x}{R_1}}$$

$$= \frac{I_{s1}R_1 + V_{s1} - V_{s2}}{\cancel{R_1}} \cdot \frac{\cancel{R_1}}{R_1 + R_x}$$

$$= \frac{I_{s1}R_1 + V_{s1} - V_{s2}}{R_1 + R_x}$$

$$= \frac{5 + 10 - V_{s2}}{5 + R_x}$$

$$= \frac{15 - V_{s2}}{5 + R_x}$$

Problem 4b

$$\frac{1}{2} I_{s1} = \frac{15 - V_{s2}}{5 + R_x}$$

$$5 + R_x = \frac{15 - V_{s2}}{\frac{1}{2} I_{s1}}$$

$$R_x = \frac{15 - V_{S2}}{\frac{1}{2} I_{S1}} - 5$$

$$= 5\,\Omega$$

Problem 4c

$$-\frac{1}{2} I_{S1} = \frac{15 - V_{S2}}{5 + R_x}$$

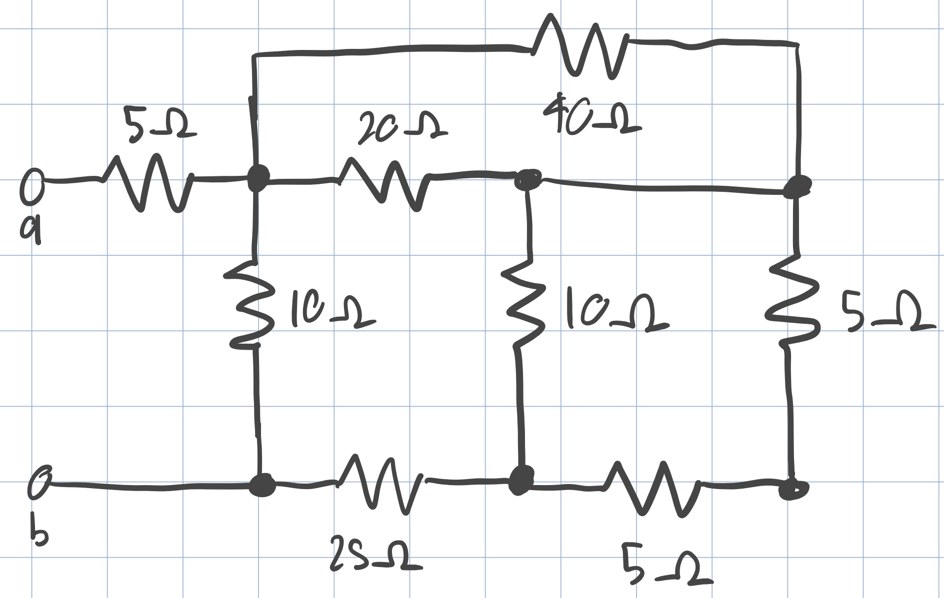
$$\left(-\frac{1}{2} I_{S1}\right)(5 + R_x) = 15 - V_{S2}$$

$$V_{S2} = 15 - \left(-\frac{1}{2} I_{S1}\right)(5 + R_x)$$

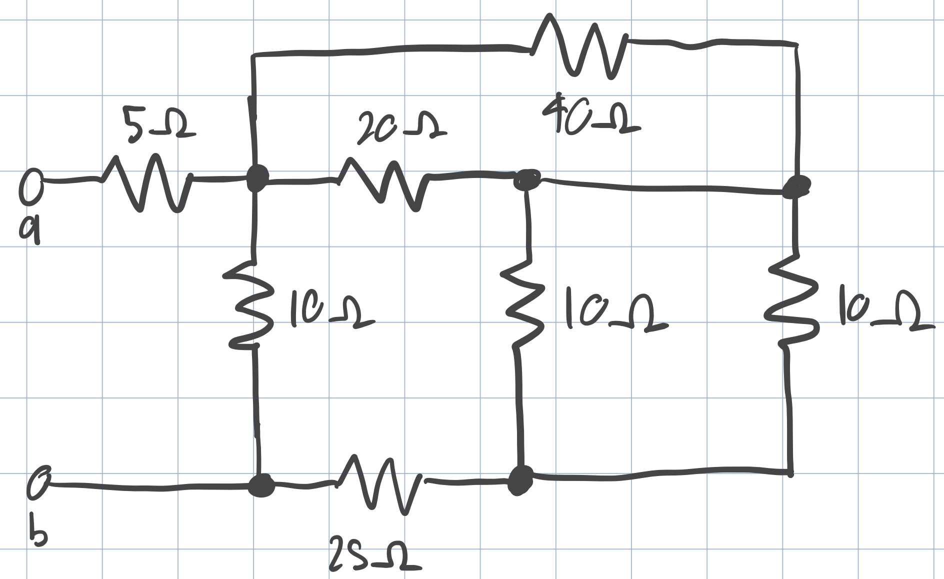
$$= 15 + \frac{1}{2} I_{S1} (5 + R_x)$$

$$= 20\ \text{V}$$

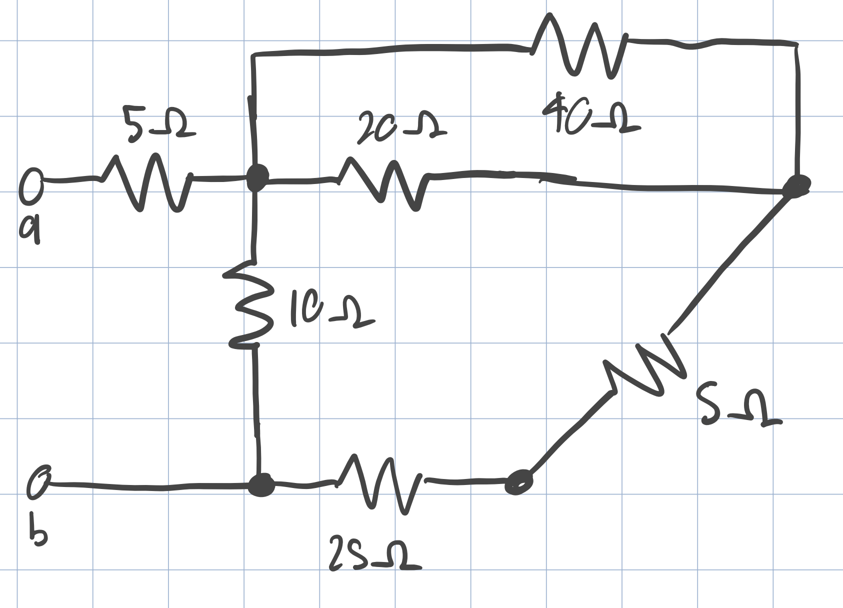
Problem 5



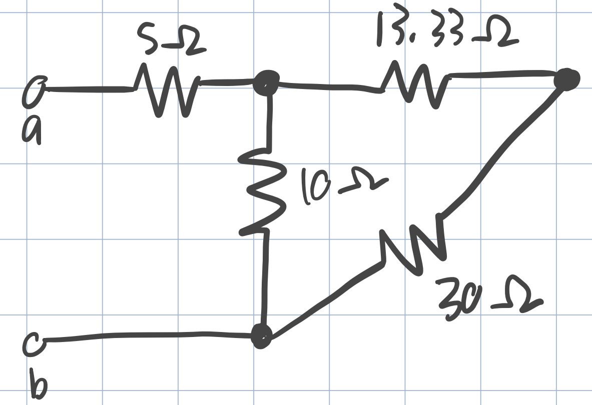
5Ω
20Ω
40Ω
a
10Ω
10Ω
5Ω
b
25Ω
5Ω



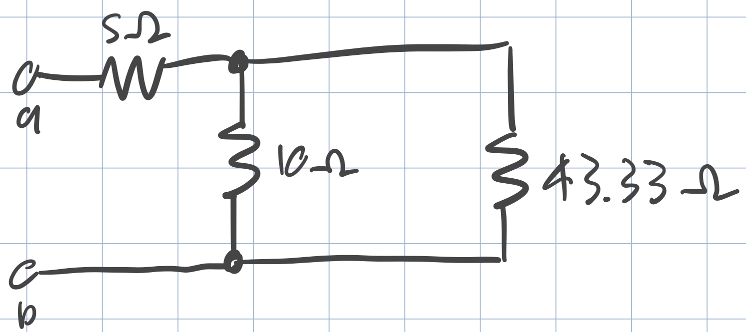
5Ω
20Ω
40Ω
a
10Ω
10Ω
10Ω
b
25Ω



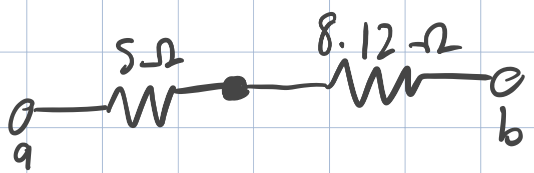
5Ω
20Ω
40Ω
a
10Ω
5Ω
b
25Ω



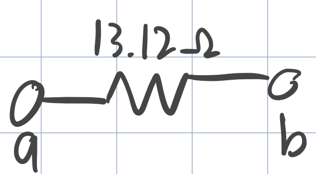
5Ω
13.33Ω
10Ω
30Ω
a
b



5Ω
10Ω
43.33Ω
a
b

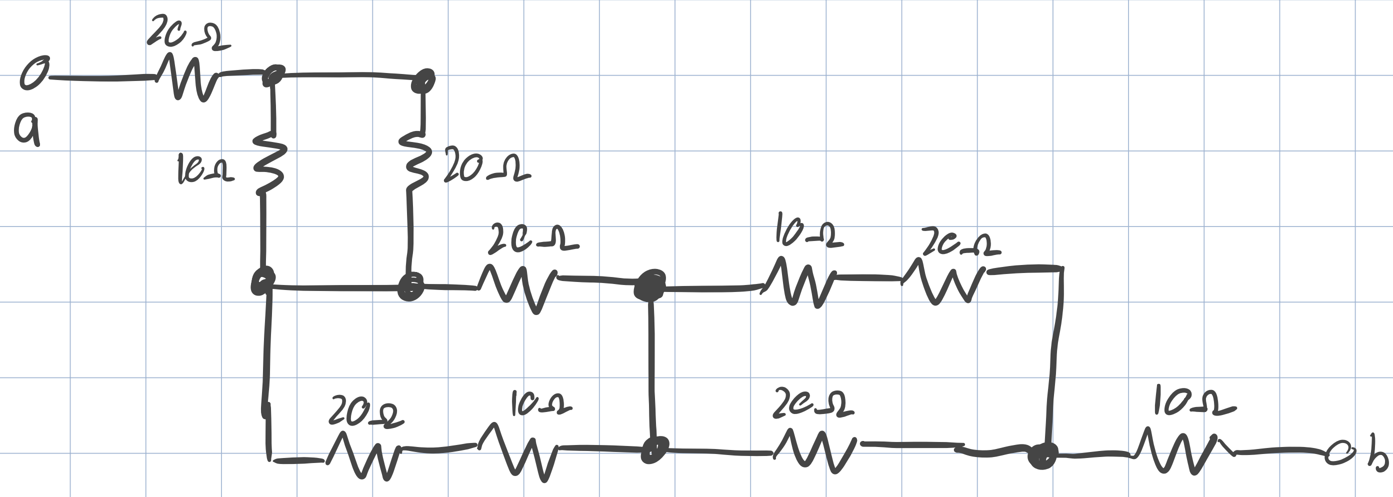


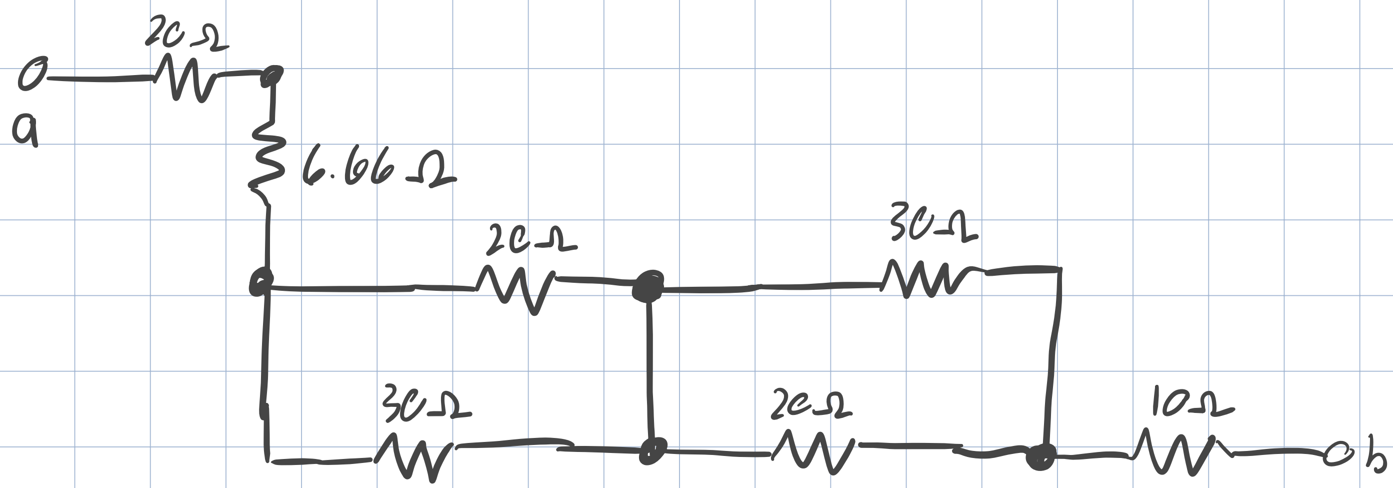
5Ω
8.12Ω
a
b

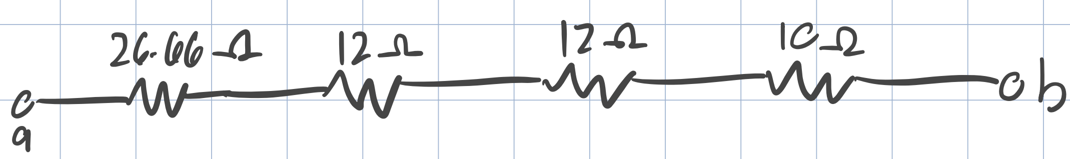


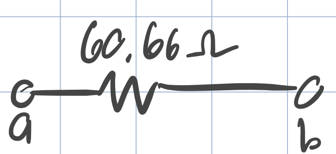
13.12Ω
a
b

Problem 6


a
20Ω
10Ω
20Ω
20Ω
10Ω
20Ω
20Ω
10Ω
20Ω
10Ω
b


a
20Ω
6.66Ω
20Ω
30Ω
30Ω
20Ω
10Ω
b


a
26.66Ω
12Ω
12Ω
10Ω
b


60.66Ω
a
b