Appendix A

Appendix: Table of Integrals

A.0.1 Basic Forms

$$\int u\,dv = uv - \int v\,du \tag{1}$$

$$\int u^n\,du = \frac{1}{n+1}u^{n+1} + C,\ n \neq -1 \tag{2}$$

$$\int \frac{du}{u} = \ln|u| + C \tag{3}$$

$$\int e^u\,du = e^u + C \tag{4}$$

$$\int a^u\,du = \frac{1}{\ln a}a^u + C \tag{5}$$

$$\int \sin u\,du = -\cos u + C \tag{6}$$

$$\int \cos u\,du = \sin u + C \tag{7}$$

$$\int \sec^2 u\,du = \tan u + C \tag{8}$$

$$\int \csc^2 u\,du = -\cot u + C \tag{9}$$

$$\int \sec u \tan u\,du = \sec u + C \tag{10}$$

$$\int \csc u \cot u\,du = -\csc u + C \tag{11}$$

$$\int \tan u\,du = \ln|\sec u| + C \tag{12}$$

$$\int \cot u\,du = \ln|\sin u| + C \tag{13}$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C \tag{14}$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C \tag{15}$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C \tag{16}$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C \tag{17}$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C \tag{18}$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C \tag{19}$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C \tag{20}$$

A.0.2 Forms Involving $\sqrt{a^2+u^2}$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C \tag{21}$$

$$\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^4}{8}\ln\left|u+\sqrt{a^2+u^2}\right| + C \tag{22}$$

$$\int \frac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C \tag{23}$$

$$\int \frac{\sqrt{a^2+u^2}}{u^2}\,du = -\frac{\sqrt{a^2+u^2}}{u^2} + \ln\left|u+\sqrt{a^2+u^2}\right| + C \tag{24}$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|u+\sqrt{a^2+u^2}\right| + C \tag{25}$$

$$\int \frac{u^2\,du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C \tag{26}$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C \tag{27}$$

$$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C \tag{28}$$

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C \tag{29}$$

A.0.3 Forms Involving $\sqrt{a^2-u^2}$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C \tag{30}$$

$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C \tag{31}$$

$$\int \frac{\sqrt{a^2-u^2}}{u}\,du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C \tag{32}$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2} - \sin^{-1}\frac{u}{a} + C \tag{33}$$

$$\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C \tag{34}$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C \tag{35}$$

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2u}\sqrt{a^2-u^2} + C \tag{36}$$

$$\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C \tag{37}$$

$$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C \tag{38}$$

A.0.4 Forms Involving $\sqrt{u^2-a^2}$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C \tag{39}$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C \tag{40}$$

$$\int \frac{\sqrt{u^2-a^2}}{u}\,du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C \tag{41}$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right| + C \tag{42}$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C \tag{43}$$

$$\int \frac{u^2\,du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C \tag{44}$$

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C \tag{45}$$

$$\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C \tag{46}$$

A.0.5 Forms Involving $a + bu$

$$\int \frac{u\,du}{a+bu} = \frac{1}{b^2}\left(a + bu - a\ln|a+bu|\right) + C \tag{47}$$

$$\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C \tag{48}$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C \tag{49}$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C \tag{50}$$

$$\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C \tag{51}$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C \tag{52}$$

$$\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C \tag{53}$$

$$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C \tag{54}$$

$$\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C \tag{55}$$

$$\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu} + C \tag{56}$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C \text{ if } a > 0 \tag{57}$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C \text{ if } a < 0$$

$$\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}} \tag{58}$$

$$\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}} \tag{59}$$

$$\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2na}{b(2n+3)}\int \frac{u^{n-1}}{\sqrt{a+bu}}\,du \tag{60}$$

$$\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}}{\sqrt{a+bu}}\,du \tag{61}$$

$$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}} \tag{62}$$

A.0.6 Trigonometric Forms

$$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C \tag{63}$$

$$\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C \tag{64}$$

$$\int \tan^2 u\,du = \tan u - u + C \tag{65}$$

$$\int \cot^2 u\,du = -\cot u - u + C \tag{66}$$

$$\int \sin^3 u\,du = -\frac{1}{3}(2+\sin^2 u)\cos u + C \tag{67}$$

$$\int \cos^3 u\,du = \frac{1}{3}(2+\cos^2 u)\sin u + C \tag{68}$$

$$\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C \tag{69}$$

$$\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C \tag{70}$$

$$\int \sec^3 u\,du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u| + C \tag{71}$$

$$\int \csc^3 u\,du = -\frac{1}{2}\csc u\cot u + \frac{1}{2}\ln|\csc u - \cot u| + C \tag{72}$$

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1}u\cos u + \frac{n-1}{n}\int \sin^{n-2}u\,du \tag{73}$$

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1}u\sin u + \frac{n-1}{n}\int \cos^{n-2}u\,du \tag{74}$$

$$\int \tan^n u\, du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\, du \tag{75}$$

$$\int \cot^n u\, du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\, du \tag{76}$$

$$\int \sec^n u\, du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\, du \tag{77}$$

$$\int \csc^n u\, du = -\frac{1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\, du \tag{78}$$

$$\int \sin au \sin bu\, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C \tag{79}$$

$$\int \cos au \cos bu\, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C \tag{80}$$

$$\int \sin au \cos bu\, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C \tag{81}$$

$$\int u \sin u\, du = \sin u - u\cos u + C \tag{82}$$

$$\int u \cos u\, du = \cos u + u\sin u + C \tag{83}$$

$$\int u^n \sin u\, du = -u^n \cos u + n\int u^{n-1}\cos u\, du \tag{84}$$

$$\int u^n \cos u\, du = u^n \sin u - n\int u^{n-1}\sin u\, du \tag{85}$$

$$\int \sin^n u \cos^m u\, du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2} u \cos^m u\, du \tag{86}$$

$$\int \sin^n u \cos^m u\, du = \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \sin^n u \cos^{m-2} u\, du$$

A.0.7 Inverse Trigonometric Forms

$$\int \sin^{-1} u\, du = u\sin^{-1} u + \sqrt{1-u^2} + C \tag{87}$$

$$\int \cos^{-1} u\, du = u\cos^{-1} u - \sqrt{1-u^2} + C \tag{88}$$

$$\int \tan^{-1} u\, du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C \tag{89}$$

$$\int u \sin^{-1} u \, du = \frac{2u^2 - 1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C \tag{90}$$

$$\int u \cos^{-1} u \, du = \frac{2u^2 - 1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C \tag{91}$$

$$\int u \tan^{-1} u \, du = \frac{u^2 + 1}{2} \tan^{-1} u - \frac{u}{2} + C \tag{92}$$

$$\int u^n \sin^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1} \sin^{-1} u - \int \frac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \; n \neq -1 \tag{93}$$

$$\int u^n \cos^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1} \cos^{-1} u + \int \frac{u^{n+1}\, du}{\sqrt{1-u^2}}\right], \; n \neq -1 \tag{94}$$

$$\int u^n \tan^{-1} u \, du = \frac{1}{n+1}\left[u^{n+1} \tan^{-1} u - \int \frac{u^{n+1}\, du}{1+u^2}\right], \; n \neq -1 \tag{95}$$

A.0.8 Exponential and Logarithmic Forms

$$\int u e^{au} \, du = \frac{1}{a^2}(au - 1)e^{au} + C \tag{96}$$

$$\int u^n e^{au} \, du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du \tag{97}$$

$$\int e^{au} \sin bu \, du = \frac{e^{au}}{a^2 + b^2}(a \sin bu - b \cos bu) + C \tag{98}$$

$$\int e^{au} \cos bu \, du = \frac{e^{au}}{a^2 + b^2}(a \cos bu + b \sin bu) + C \tag{99}$$

$$\int \ln u \, du = u \ln u - u + C \tag{100}$$

$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C \tag{101}$$

$$\int \frac{1}{u \ln u} \, du = \ln|\ln u| + C \tag{102}$$

A.0.9 Hyperbolic Forms

$$\int \sinh u \, du = \cosh u + C \tag{103}$$

$$\int \cosh u \, du = \sinh u + C \tag{104}$$

$$\int \tanh u \, du = \ln \cosh u + C \tag{105}$$

$$\int \coth u \, du = \ln |\sinh u| + C \tag{106}$$

$$\int \operatorname{sech} u \, du = \tan^{-1} |\sinh u| + C \tag{107}$$

$$\int \operatorname{csch} u \, du = \ln |\tanh \frac{1}{2} u| + C \tag{108}$$

$$\int \operatorname{sech}^2 u \, du = \tanh u + C \tag{109}$$

$$\int \operatorname{csch}^2 u \, du = -\coth u + C \tag{110}$$

$$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C \tag{111}$$

$$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C \tag{112}$$

A.0.10 Forms Involving $\sqrt{2au - u^2}$

$$\int \sqrt{2au - u^2} \, du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{113}$$

$$\int u\sqrt{2au - u^2} \, du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{114}$$

$$\int \frac{\sqrt{2au - u^2}}{u} \, du = \sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{115}$$

$$\int \frac{\sqrt{2au - u^2}}{u^2} \, du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{116}$$

$$\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{117}$$

$$\int \frac{u \, du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C \tag{118}$$

$$\int \frac{u^n\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C \tag{119}$$

$$\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C \tag{120}$$

