Find the interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{(3x)^n}{n+9}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(3x)^{\cancel{n+1}}}{n+10} \cdot \frac{n+9}{\cancel{(3x)^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(3x)(n+9)}{n+10} \right|$$

$$= \left| (3x) \lim_{n\to\infty} \frac{n+9}{n+10} \right|$$

$$= |3x|$$

Convergent when $|3x| < 1$

$|3x| < 1$

$|x| < \frac{1}{3}$

$-\frac{1}{3} \overset{<}{\underset{?}{\leq}} x \overset{<}{\underset{?}{\leq}} \frac{1}{3}$

Check $x = \frac{1}{3}$

$$\sum_{n=0}^{\infty} \frac{(\frac{3}{3})^n}{n+9}$$

$$=\sum_{n=0}^{\infty} \frac{1}{n+9}$$

P Series

Diverges

Check $x = -\frac{1}{3}$

$$\sum_{n=0}^{\infty} \frac{(-\frac{3}{3})^n}{n+9}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+9}$$

Alternating Series Test

$$\lim_{n\to\infty} a_n$$

$$=\lim_{n\to\infty} \frac{(-1)^n}{n+9}$$

$$=0$$

Converges

Interval of Convergence

$$-\frac{1}{3} \leq x < \frac{1}{3}$$

Determine the interval of convergence for the series

$$\sum_{n=0}^{\infty} \frac{6x^n}{n!}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{\cancel{6}x^{\cancel{n}+1}}{(n+1)!}\cdot\frac{n!}{\cancel{6x^n}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x}{(n+1)\cancel{n!}}\cdot\frac{\cancel{n!}}{1}\right|$$

$$=\lim_{n\to\infty}\left|\frac{x}{n+1}\right|$$

$$=\left|x\lim_{n\to\infty}\frac{1}{n+1}\right|$$

$$=0$$

Always Absolutely Convergent

Interval of Convergence

$(-\infty, \infty)$

Determine the interval of convergence for the series

$$\sum_{n=0}^{\infty} (-1)^{n+1}\frac{(5x)^{4n+1}}{(2n+1)!}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(5x)^{4(n+1)+1}}{(2(n+1)+1)!}\cdot\frac{(2n+1)!}{(5x)^{4n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(5x)^{4n+1+4}}{(2n+3)!}\cdot\frac{(2n+1)!}{(5x)^{4n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(5x)^4\cancel{(5x)^{4n+1}}}{(2n+3)!}\cdot\frac{(2n+1)!}{\cancel{(5x)^{4n+1}}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(5x)^4}{\cancel{(2n+1)!}(2n+2)(2n+3)}\cdot\frac{\cancel{(2n+1)!}}{1}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(5x)^4}{(2n+2)(2n+3)}\right|$$

$$=\left|(5x^4)\lim_{n\to\infty}\frac{1}{(2n+2)(2n+3)}\right|$$

$$=0$$

Always Absolutly Convergent

Interval of Convergence

$(-\infty,\infty)$

Determine the interval of convergence

$$\sum_{n=1}^{\infty}\frac{3^n x^n}{n}$$

$$=\sum_{n=1}^{\infty}\frac{(3x)^n}{n}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(3x)^{\cancel{n+1}}}{n+1}\cdot\frac{n}{\cancel{(3x)^n}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(3x)n}{n+1}\right|$$

$$=\left|(3x)\lim_{n\to\infty}\frac{n}{n+1}\right|$$

$$=|3x|$$

Convergent when $|3x|<1$

$|3x|<1$

$|x|<\frac{1}{3}$

$-\frac{1}{3} \overset{<}{?} x \overset{<}{?} \frac{1}{3}$

Check $x=\frac{1}{3}$

$$=\sum_{n=0}^{\infty}\frac{(3\cdot\frac{1}{3})^n}{n}$$

$$=\sum_{n=0}^{\infty}\frac{1}{n}$$

P Series

Diverges

Check $x = -\frac{1}{3}$

$$\sum_{n=0}^{\infty} \frac{(-3 \cdot \frac{1}{3})^n}{n}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n}$$

Alternating Series

$$\lim_{n \to \infty} a_n$$

$$= \lim_{n \to \infty} (-1)^n \frac{1}{n}$$

$$= \pm 0$$

$$= 0$$

Converges

Interval of Convergence

$$-\frac{1}{3} \leq x < \frac{1}{3}$$

$$\left[-\frac{1}{3}, \frac{1}{3}\right)$$

Determine the interval of convergence

$$\sum_{n=5}^{\infty} \frac{(9x)^{n-4}}{\sqrt{n+4}}$$

$$= \sum_{n=0}^{\infty} \frac{(9x)^{n+1}}{\sqrt{n+9}}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(9x)^{n+1+1}}{\sqrt{n+9+1}} \cdot \frac{\sqrt{n+9}}{(9x)^{n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(9x)\cancel{(9x)^{n+1}}}{\sqrt{n+10}} \cdot \frac{\sqrt{n+9}}{\cancel{(9x)^{n+1}}} \right|$$

$$= \left| (9x) \lim_{n\to\infty} \frac{\sqrt{n+9}}{\sqrt{n+10}} \right|$$

$$= |9x|$$

Converges when $|9x| < 1$

$$|9x| < 1$$

$$|x| < \frac{1}{9}$$

$$-\frac{1}{9} \overset{<}{\underset{?}{\leq}} x \overset{<}{\underset{?}{\leq}} \frac{1}{9}$$

Check $x = \frac{1}{9}$

$$\sum_{n=0}^{\infty} \frac{(9 \cdot \frac{1}{9})^{n+1}}{\sqrt{n+9}}$$

$$= \sum_{n=0}^{\infty} \frac{1}{\sqrt{n+9}}$$

Limit Comparison Test

$$a_n = \frac{1}{\sqrt{n+9}}$$

$$b_n = \frac{1}{\sqrt{n}}$$

$$\lim_{n \to \infty} \frac{1}{\sqrt{n+9}} \cdot \frac{\sqrt{n}}{1}$$

$$= \lim_{n \to \infty} \frac{\sqrt{n}}{\sqrt{n+9}}$$

$$= 1$$

Both series converge or diverge

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n}}$$

P Series

Diverges

Check $x = -\frac{1}{9}$

$$\sum_{n=0}^{\infty} \frac{(-9 \cdot \frac{1}{9})^{n+1}}{\sqrt{n+9}}$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} \frac{1^{n+1}}{\sqrt{n+9}}$$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n+9}}$$

Alternating Series Test

$$\lim_{n\to\infty} (-1)^{n+1} \frac{1}{\sqrt{n+9}}$$

$$= \pm 1 \cdot 0$$

$$= 0$$

Converges

Interval of Convergence

$$-\frac{1}{9} \leq x < \frac{1}{9}$$

$$\left[-\frac{1}{9}, \frac{1}{9}\right)$$

Determine the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^2 3^n}$$

$$= \sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)^2 3^{n+1}}$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(x-2)^{\cancel{n+1}+1}}{(n+1+1)^2\,\cancel{3^{n+1+1}}}\cdot\frac{(n+1)^2\,\cancel{3^{n+1}}}{\cancel{(x-2)^{n+1}}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(x-2)}{(n+2)^2 3}\cdot\frac{(n+1)^2}{1}\right|$$

$$=\left|\frac{(x-2)}{3}\lim_{n\to\infty}\frac{n^2+2n+1}{n^2+4n+4}\right|$$

$$=\left|\frac{x-2}{3}\right|$$

Converges when $|x-2|<1$

$$\left|\frac{x-2}{3}\right|<1$$

$$-1\underset{?}{\le}\frac{x-2}{3}\underset{?}{\le}1$$

$$-3\underset{?}{\le}x-2\underset{?}{\le}3$$

$$-1\underset{?}{\le}x\underset{?}{\le}5$$

Check $x=-1$

Converges

Check $x=5$

Converges

Interval of Convergence

$-1 \leq x \leq 5$

$[-1, 5]$

Dertermine the interval of convergence for the series

$$\sum_{n=1}^{\infty} n!(3x-1)^n$$

Ratio Test

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty} \left|\frac{(n+1)!(3x-1)^{\cancel{n+1}}}{1} \cdot \frac{1}{n!\cancel{(3x-1)^n}}\right|$$

$$= \lim_{n\to\infty} \left|\frac{\cancel{n!}(n+1)(3x-1)}{1} \cdot \frac{1}{\cancel{n!}}\right|$$

$$= \lim_{n\to\infty} |(n+1)(3x-1)|$$

$$= |(3x-1) \cdot \infty|$$

$$= \infty$$

Always Diverges

Check $x = \frac{1}{3}$

$$\sum_{n=1}^{\infty} n!\left(3\cdot\tfrac{1}{3}-1\right)^n$$

$$=\sum_{n=1}^{\infty} n!\,(0)^n$$

$$=0$$

Converges

Interval of Convergence

$x = \frac{1}{3}$

$\left[\frac{1}{3}, \frac{1}{3}\right]$

Determine the interval of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n n^2 (x-2)^n$$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(n+1)^2(x-2)^{\cancel{n+1}}}{1}\cdot\frac{1}{n^2\cancel{(x-2)^n}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(n+1)^2(x-2)}{1}\cdot\frac{1}{n^2}\right|$$

$$=\left|(x-2)\lim_{n\to\infty}\frac{(n+1)^2}{n^2}\right|$$

$= |x-2|$

Converges when $|x-2| < 1$

$-1 \underset{?}{<} x-2 \underset{?}{<} 1$

$1 \underset{?}{<} x \underset{?}{<} 3$

Check $x=1$

$$\sum_{n=1}^{\infty} (-1)^n (n)^2 (-1)^n$$

$$= \sum_{n=1}^{\infty} (1)^n n^2$$

$$= \sum_{n=1}^{\infty} n^2$$

Diverges

Check $x=3$

$$\sum_{n=1}^{\infty} (-1)^n (n)^2 (1)^n$$

$$= \sum_{n=1}^{\infty} (-1)^n (n)^2$$

Diverges

Interval of Convergence

$1 < x < 3$

Determine the interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{\cos(n\frac{\pi}{2})(x-5)^n}{n}$$

$$= \frac{\cos(\frac{\pi}{2})(x-5)^1}{1} + \frac{\cos(\pi)(x-5)^2}{2} + \frac{\cos(\frac{3\pi}{2})(x-5)^3}{3} + \frac{\cos(2\pi)(x-5)^4}{4}$$

$$= \frac{0}{1} + \frac{-1(x-5)^2}{2} + \frac{0}{3} + \frac{1(x-5)^4}{4}$$

Eliminate cos by creating a new series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x-5)^{2n}}{2n}$$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-5)^{2(n+1)}}{2(n+1)} \cdot \frac{2n}{(x-5)^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-5)^{2n+2}}{2n+2} \cdot \frac{2n}{(x-5)^{2n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(x-5)^2 \cancel{(x-5)^{2n}}}{2n+2} \cdot \frac{2n}{\cancel{(x-5)^{2n}}} \right|$$

$$= \left| (x-5)^2 \lim_{n\to\infty} \frac{2n}{2n+2} \right|$$

$$= \left| (x-5)^2 \right|$$

Converges when $|(x-5)^2| < 1$

$-1 \underset{?}{\leq} (x-5)^2 \underset{?}{\leq} 1$

Split into its seperate intervals

$-1 \underset{?}{\leq} (x-5)^2$: Always True

$(x-5)^2 \underset{?}{\leq} 1$

$x-5 \underset{?}{\leq} \pm 1$

$-1 \underset{?}{\leq} x-5 \underset{?}{\leq} 1$

$4 \underset{?}{\leq} x \underset{?}{\leq} 6$

Merge intervals

$4 \underset{?}{\leq} x \underset{?}{\leq} 6$

Check $x=4$

$$\sum_{n=1}^{\infty} (-1)^n \frac{(-1)^{2n}}{2n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{3n}}{2n}$$

Alternating Series Test

$$\lim_{n \to \infty} a_n$$

$$= \lim_{n\to\infty} \frac{(-1)^{3n}}{2n}$$

$$= 0$$

Converges

Check $x = 6$

$$\sum_{n=1}^{\infty} (-1)^n \frac{(1)^{2n}}{2n} \qquad \sum_{n=1}^{\infty} (-1)^n \frac{(x-5)^{2n}}{2n}$$

$$= \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n}$$

Alternating Series Test

$$\lim_{n\to\infty} (-1)^n \frac{1}{2n}$$

$$= 0$$

Converges

Interval of Convergence

$4 \le x \le 6$

$[4, 6]$