Moments of inertia for some objects of uniform density:

disk $I = (1/2)MR^2$, cylinder $I = (1/2)MR^2$, sphere $I = (2/5)MR^2$

(a) A uniform disk has a moment of inertia that is $(1/2)MR^2$. A uniform disk of mass 13 kg, thickness 0.3 m, and radius 0.7 m is located at the origin, oriented with its axis along the y axis. It rotates clockwise around its axis when viewed from above (that is, you stand at a point on the +y axis and look toward the origin at the disk). The disk makes one complete rotation every 0.2 s.
What is the rotational angular momentum of the disk?

$\vec{L}_{rot}$ = ______ $kg \cdot m^2/s$

What is the rotational kinetic energy of the disk?

K_{rot} = ______ J

(b) A uniform sphere has a moment of inertia that is $(2/5)MR^2$. A sphere of uniform density, with mass 26 kg and radius 0.7 m is located at the origin, and rotates around an axis parallel with the x axis. If you stand somewhere on the +x axis and look toward the origin at the sphere, the sphere spins counterclockwise. One complete revolution takes 0.2 seconds.
What is the rotational angular momentum of the sphere?

$\vec{L}_{rot}$ = ______ $kg \cdot m^2/s$

What is the rotational kinetic energy of the sphere?

K_{rot} = ______ J

(c) A uniform rod has a moment of inertia for rotation around its long axis that is $(1/2)MR^2$. A cylindrical rod of uniform density is located with its center at the origin, and its axis along the z axis. Its radius is 0.06 m, its length is 0.9 m, and its mass is 5 kg. It makes one revolution every 0.04 seconds. If you stand on the +z axis and look toward the origin at the rod, the rod spins clockwise.
What is the rotational angular momentum of the rod?

$\vec{L}_{rot}$ = ______ $kg \cdot m^2/s$

What is the rotational kinetic energy of the rod?

K_{rot} = ______ J

Part A

$$|\vec{L}_{rot}| = I\omega$$

$$= \frac{1}{2}MR^2 \frac{2\pi}{t}$$

$$= 100.060 \ kg \frac{m^2}{s}$$

Use the right hand rule

$$\vec{L}_{rot} = \langle 0, -100.060, 0 \rangle \ kg \frac{m^2}{s}$$

$$K_{rot} = \frac{1}{2}I\omega^2$$

$$= 1571.735 \ J$$

Part B

$$|\vec{L}_{rot}| = \frac{2}{5}MR^2 \cdot \frac{2\pi}{T}$$

$$= 160.096 \text{ kg}\frac{m^2}{s}$$

Use the right hand rule

$$\vec{L}_{rot} = \langle 160.096, 0, 0 \rangle \text{ kg}\frac{m^2}{s}$$

$$K = \frac{1}{2}I\omega^2$$

$$= 2514.775 \text{ J}$$

Part C

$$|\vec{L}_{rot}| = \frac{1}{2}MR^2 \frac{2\pi}{T}$$

$$= 1.414 \text{ kg}\frac{m^2}{s}$$

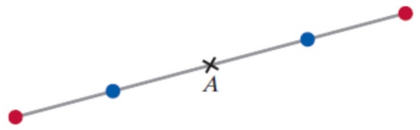
Use the right hand rule

$$\vec{L}_{rot} = \langle 0, 0, -1.414 \rangle \text{ kg}\frac{m^2}{s}$$

$$K = \frac{1}{2}I\omega^2$$

$$= 111.033 \text{ J}$$

Mounted on a low-mass rod of length 0.36 m are four balls (see figure below). Two balls (shown in red on the diagram), each of mass 0.88 kg, are mounted at opposite ends of the rod. Two other balls, each of mass 0.34 kg (shown in blue on the diagram), are each mounted a distance 0.09 m from the center of the rod. The rod rotates on an axle through the center of the rod (indicated by the "×" in the diagram), perpendicular to the rod, and it takes 0.9 s to make one full rotation.



(a) What is the moment of inertia of the device about its center?

[ ] $kg \cdot m^2$

(b) What is the angular speed of the rotating device?

[ ] rad/s

(c) What is the magnitude of the angular momentum of the rotating device?

[ ] $kg \cdot m^2/s$

Part A

$$I = \sum_{i=1}^{n} m_i r_i^2$$

$$= 0.0625 \ kgm^2$$

Part B

$$w = \frac{2\pi}{T}$$

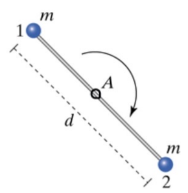
$$= 6.981 \ \frac{rad}{sec}$$

Part C

$$L = I w$$

$$= 0.437 \ kg\frac{m^2}{s}$$

A barbell spins around a pivot at its center at A. The barbell consists of two small balls, each with mass 550 grams (0.55 kg), at the ends of a very low mass rod of length d = 50 cm (0.5 m; the radius of rotation is 0.25 m). The barbell spins clockwise with angular speed 90 radians/s.



We can calculate the angular momentum and kinetic energy of this object in two different ways, by treating the object as two separate balls, or as one barbell.

I: Treat the object as two separate balls

(a) What is the speed of ball 1?
$|\vec{v}|$ = [ ] m/s

(b) Calculate the *translational* angular momentum $\vec{L}_{\text{trans, 1, }A}$ of just one of the balls (ball 1).
$|\vec{L}_{\text{trans, 1, }A}|$ = [ ] kg Â· m²/s
- zero magnitude; no direction
- ⊗ into page
- ⊙ out of page

(c) Calculate the *translational* angular momentum $\vec{L}_{\text{trans, 2, }A}$ of the other ball (ball 2).
$|\vec{L}_{\text{trans, 2, }A}|$ = [ ] kg Â· m²/s
- ⊙ out of page
- zero magnitude; no direction
- ⊗ into page

Part A

$V = \omega r$

$= 22.5$

Part B

$|\vec{L}_{trans}| = |\vec{r}||\vec{p}|\sin(\theta)$

$= rp$

$= rmv$

$= 3.094 \text{ kg} \frac{m^2}{s}$

Part C

Same as part A and B

(d) By adding the translational angular momentum of ball 1 and the translational angular momentum of ball 2, calculate the total angular momentum of the barbell, $\vec{L}_{tot,\,A}$.

$|\vec{L}_{tot,\,A}|$ = [ ] kg Â· m²/s

- ○ zero magnitude; no direction
- ○ ⊗ into page
- ○ ⊙ out of page

(e) Calculate the *translational* kinetic energy of ball 1.

$K_{trans,1} = \frac{1}{2}m|\vec{v}|^2$ = [ ] J

(f) Calculate the *translational* kinetic energy of ball 2.

$K_{trans,2} = \frac{1}{2}m|\vec{v}|^2$ = [ ] J

(g) By adding the translational kinetic energy of ball 1 and the translational kinetic energy of ball 2, calculate the total kinetic energy of the barbell.

K_{total} = [ ] J

II: Treat the object as one barbell

(h) Calculate the moment of inertia I of the barbell.

I = [ ] kg Â· m²

(i) What is the direction of the angular velocity vector $\vec{\omega}$?

- ○ ⊗ into page
- ○ ⊙ out of page
- ○ zero magnitude; no direction

(j) Use the moment of inertia I and the angular speed $|\vec{\omega}|$ = 90 rad/s to calculate the rotational angular momentum of the barbell:

$|\vec{L}_{rot}| = I\,|\vec{\omega}|$ = [ ] kg Â· m²/s

- ○ ⊗ into page
- ○ ⊙ out of page
- ○ zero magnitude; no direction

Part D

$$|\vec{L}_{total}| = |\vec{L}_{trans\ A}| \cdot 2$$
$$= 6.188\ kg\,\frac{m^2}{s}$$

Part E and F and G

$$K_{trans\ A} = \frac{1}{2}mv^2$$

$= 139.219\ J$

$K_{total} = K_{trans\ A} \cdot 2$

$= 278.438\ J$

Part H

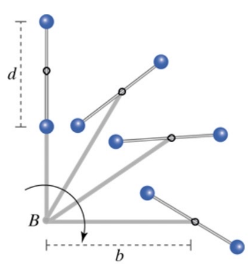
$$I = \sum_{i=1}^{n} m_i r_i^2$$

$= 0.0685\ k\hat{A} \cdot m^2$

Part I

$$|\vec{L}_{rot}| = 6.188\ kg\ \frac{m^2}{s}$$

A barbell consists of two small balls, each with mass 550 grams (0.55 kg), at the ends of a very low mass rod of length d = 35 cm (0.35 m). The center of the barbell is mounted on the end of a low mass rigid rod of length b = 0.525 m (see Figure), and this rod rotates clockwise with angular speed 120 rad/s. In addition, the barbell rotates clockwise about its own center, with an angular speed 100 rad/s.



(a) Calculate $\vec{L}_{rot}$ (both magnitude and direction).

[ ] $kg{\cdot}m^2/s$

- ○ zero magnitude; no direction
- ○ ⊙ out of page
- ○ ⊗ into page

(b) Calculate $\vec{L}_{trans,\ B}$ (both magnitude and direction).

[ ] $kg{\cdot}m^2/s$

- ○ zero magnitude; no direction
- ○ ⊗ into page
- ○ ⊙ out of page

(c) Calculate $\vec{L}_{tot,\ B}$ (both magnitude and direction).

[ ] $kg{\cdot}m^2/s$

- ○ zero magnitude; no direction
- ○ ⊗ into page
- ○ ⊙ out of page

Part A

$$|\vec{L}_{rot}| = I\omega$$

$$I = \sum_{i=1}^{2} m_i r_i^2$$

$$= 0.337$$

$$|\vec{L}_{rot}| = 3.369 \ kg\frac{m^2}{s}$$

Part B

$$|\vec{L}_{trans}| = |\vec{r}||\vec{p}|\sin(\theta)$$

$$= rp$$

$$= rmv$$

$$= rm\omega r$$

$$= m\omega r^2$$

$$= 36.383 \ kg\frac{m^2}{r}$$

Part C

$$|L_{total}| = |\vec{L}_{rot}| + |\vec{L}_{trans}|$$

$= 39.7551 \, kg \frac{m^2}{s}$