Evaluate the iterated integral.

$$\int_{-3}^{3}\int_{1}^{6}\int_{1}^{e} \frac{xz^2}{y}\,dy\,dx\,dz$$

$$\int_{-3}^{3}\int_{1}^{6}\int_{1}^{e} xz^2y^{-1}\,dy\,dx\,dz$$

$$=\int_{-3}^{3}\int_{1}^{6} xz^2\left(\ln(y)\right)\Big|_1^e\,dx\,dz$$

$$=\int_{-3}^{3}\int_{1}^{6} xz^2(1-0)\,dx\,dz$$

$$=\int_{-3}^{3} z^2\int_{1}^{6} x\,dx\,dz$$

$$=\int_{-3}^{3} z^2\left(\frac{1}{2}x^2\right)\Big|_1^6\,dz$$

$$=\int_{-3}^{3} z^2\frac{1}{2}(36-1)\,dz$$

$$=\frac{1}{2}\int_{-3}^{3} 36z^2-z^2\,dz$$

$$=\frac{1}{2}\left(\frac{36}{3}z^3-\frac{1}{3}z^3\right)\Big|_{-3}^{3}$$

$$=\frac{1}{2}\left(12z^3-\frac{1}{3}z^3\right)\Big|_{-3}^{3}$$

$$=\frac{1}{2}\left(12-\frac{1}{3}\right)\left(z^3\right)\Big|_{-3}^{3}$$

$$=\frac{1}{2}\left(12-\frac{1}{3}\right)\left(3^3+3^3\right)$$

Evaluate the iterated integral.

$$\int_0^3 \int_0^2 \int_0^{\sqrt{4-x^2}} \frac{x}{z+1}\, dy\, dx\, dz$$

$$\int_0^3 \int_0^2 \int_0^{\sqrt{4-x^2}} x(z+1)^{-1}\, dy\, dx\, dz$$

$$= \int_0^3 \int_0^2 x(z+1)^{-1}\sqrt{4-x^2}\, dx\, dz$$

$$= \int_0^3 (z+1)^{-1} \int_0^2 x(4-x^2)^{\frac{1}{2}}\, dx\, dz$$

$U = 4 - x^2$ Bands

$\frac{du}{dx} = -2x$ $2 \to 4 - 2^2$ $0 \to 4$

$-\frac{1}{2}du = x\,dx$ $= 0$

$$= \int_0^3 (z+1)^{-1} \int_4^0 u^{\frac{1}{2}} \cdot -\frac{1}{2}\, du\, dz$$

$$= -\frac{1}{2} \int_0^3 (z+1)^{-1} \left(\frac{2}{3}u^{\frac{3}{2}}\right)\Big|_4^0\, dz$$

$$= -\frac{1}{2} \int_0^3 (z+1)^{-1} \cdot -\frac{2}{3} \cdot 4^{\frac{3}{2}}\, dz$$

$$= \frac{8}{3} \int_0^3 (z+1)^{-1}\, dz$$

$U = z + 1$ Bands

$\frac{du}{dz} = 1$ $3 \to 4$ $0 \to 1$

$$dv = dz$$

$$= \frac{8}{3}\int_1^4 v^{-1}\,dv$$

$$= \frac{8}{3}(\ln(4) - \ln(1))$$

$$= \frac{8}{3}\ln(4)$$

Evaluate the triple integral.

$$\iiint_F 1\,dV \text{ where } F = \{(x, y, z) \mid 0 \le x \le 2,\ 0 \le y \le \sqrt{4 - x^2},\ 0 \le z \le \sqrt{4 - x^2}\}$$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2}} 1\,dz\,dy\,dx$$

$$= \int_0^2 4 - x^2\,dx$$

$$= \left(4x - \frac{1}{3}x^3\right)\Big|_0^2$$

$$= 8 - \frac{8}{3}$$

Evaluate the triple integral.

$$\iiint_F 2y^2\cos(z)\,dV \text{ where } F = \left\{(x, y, z) \mid 0 \le y \le \sqrt{\frac{\pi}{2}},\ 0 \le x \le y,\ 0 \le z \le xy\right\}$$

$$\int_0^{\sqrt{\frac{\pi}{2}}} \int_0^y \int_0^{xy} 2y^2\cos(z)\,dz\,dx\,dy$$

$$= 2\int_0^{\sqrt{\frac{\pi}{2}}} y^2 \int_0^y \int_0^{xy} \cos(z)\,dz\,dx\,dy$$

$$= 2\int_0^{\sqrt{\frac{\pi}{2}}} y^2 \int_0^y \sin(z)\Big|_0^{xy}\,dx\,dy$$

$$= 2\int_0^{\sqrt{\frac{\pi}{2}}} y^2 \int_0^y \sin(xy) - \cancel{\sin(0)}\, dx\, dy$$

$$= 2\int_0^{\sqrt{\frac{\pi}{2}}} y^2 \int_0^y \sin(xy)\, dx\, dy$$

$$= 2\int_0^{\sqrt{\frac{\pi}{2}}} y^2 \left(-\cos(xy)\frac{1}{y}\right)\Big|_0^y\, dy$$

$$= -2\int_0^{\sqrt{\frac{\pi}{2}}} y\left(\cos(y^2) - \cos(0)\right) dy$$

$$= -2\int_0^{\sqrt{\frac{\pi}{2}}} y\cos(y^2) - y\ dy$$

$$= -2\int_0^{\sqrt{\frac{\pi}{2}}} y\cos(y^2)\, dy + 2\int_0^{\sqrt{\frac{\pi}{2}}} y\ dy$$

$$\rightarrow \int y\cos(y^2)\, dy$$

$$u = y^2$$

$$\frac{du}{dy} = 2y$$

$$\frac{1}{2}du = y\, dy$$

$$= \frac{1}{2}\int \cos(u)\, dy$$

$$= \frac{1}{2}\sin(u)$$

$$= \frac{1}{2}\sin(y^2)$$

$$= -\left(\sin(y^2)\right)\Big|_0^{\sqrt{\frac{\pi}{2}}} + 2\left(\frac{1}{2}y^2\right)\Big|_0^{\sqrt{\frac{\pi}{2}}}$$

$$= -\left(\sin\left(\frac{\pi}{2}\right) - \cancel{\sin(0)}\right) + \frac{\pi}{2}$$

$$= \frac{\pi}{2} - 1$$

Rewrite the following iterated integral using five different orders of integration.

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{x^2+y^2}^{9} g(x, y, z)\, dz\, dy\, dx$$

$$\int_{-3}^{3}\int_{\square}^{\square}\int_{x^2+y^2}^{9} g(x, y, z)\, dz\, dx\, dy$$

$$\int_{-3}^{3}\int_{x^2}^{9}\int_{\square}^{\square} g(x, y, z)\, dy\, dz\, dx$$

$$\int_{-3}^{3}\int_{y^2}^{9}\int_{\square}^{\square} g(x, y, z)\, dx\, dz\, dy$$

$$\int_{\square}^{\square}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} g(x, y, z)\, dy\, dx\, dz$$

$$\int_{\square}^{\square}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} g(x, y, z)\, dx\, dy\, dz$$

$-3 \le x \le 3$

$-\sqrt{9-x^2} \le y \le \sqrt{9-x^2}$

$x^2+y^2 \le z \le 9$

$\downarrow$

Bonded by

$z = 9$

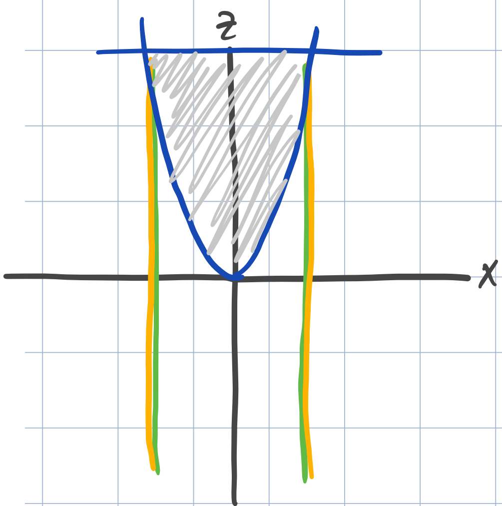
$z = x^2+y^2$

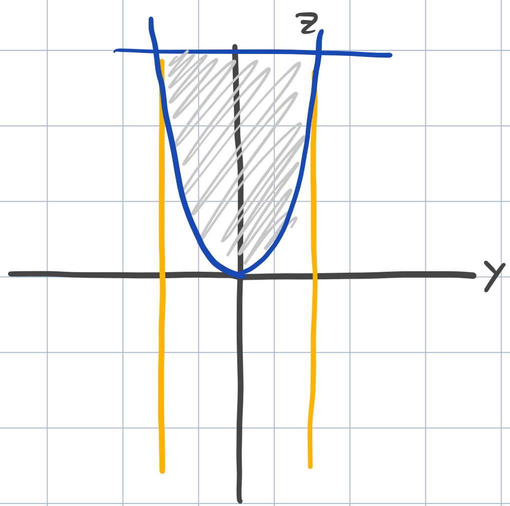
$x^2+y^2 = 9$

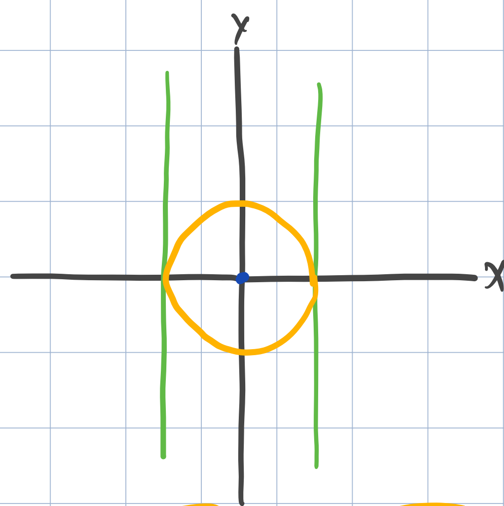
$y = \pm\sqrt{9-x^2}$

$y^2 = 9-x^2$

$x^2+y^2 = 9$







$-3 \le x \le 3$

$x^2 \le z \le 9$

$-3 \le x \le 3$

$-3 \le y \le 3$

$y^2 \le z \le 9$

$-\sqrt{9-x^2} \le y \le \sqrt{9-x^2}$

$-3 \le x \le 3$

$x^2+y^2 \le 6$

Convert to equation bounds

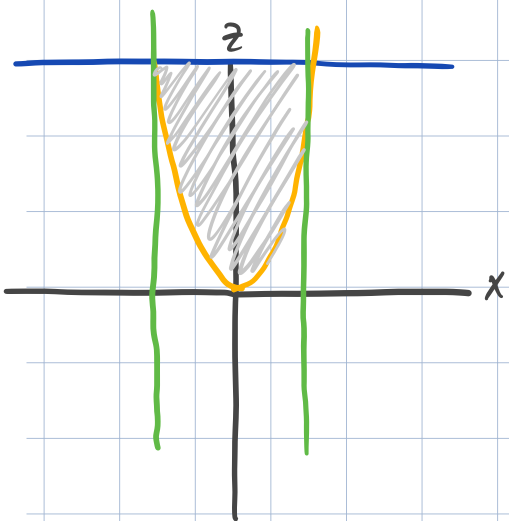
$z = 9$

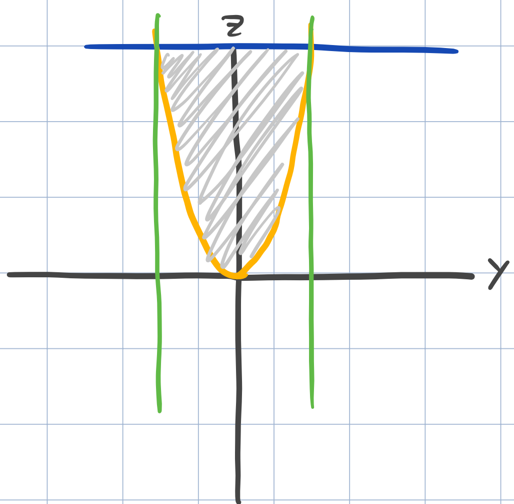
$z = x^2 + y^2$

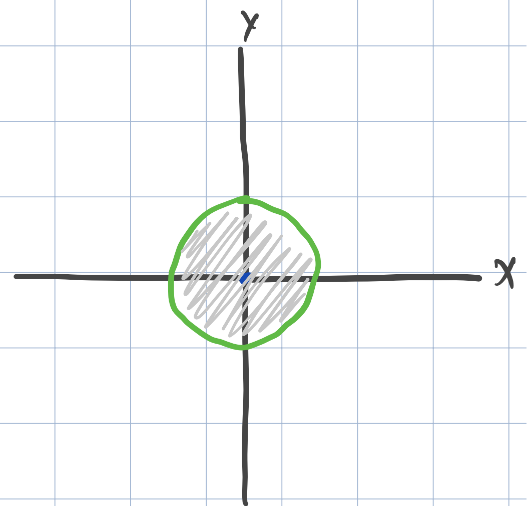
$y = \pm\sqrt{9 - x^2}$

$y^2 = 9 - x^2$

$y^2 + x^2 = 9$







$z = 9$

$z = x^2 + c$

$x^2 = 9$

$x = \pm 3$

$z = 9$

$z = y^2 + c$

$y^2 = 9$

$y = \pm 3$

$y^2 + x^2 = 9$

$c = x^2 + y^2$

Part One

$3 \leq y \leq -3$

$-\sqrt{9 - y^2} \leq x \leq \sqrt{9 - y^2}$

$x^2 + y^2 \leq z \leq 9$

Part Two

$3 \le x \le -3$
$x^2 \le z \le 9$

$z = x^2 + y^2$
$y^2 = z - x^2$
$y = \pm\sqrt{z - x^2}$

$-\sqrt{z-x^2} \le y \le \sqrt{z-x^2}$

Part Three

$-3 \le y \le 3$
$y^2 \le z \le 9$

$z = x^2 + y^2$
$x^2 = z - y^2$
$x = \pm\sqrt{z - y^2}$

$-\sqrt{z-y^2} \le x \le \sqrt{z-y^2}$

Part Four

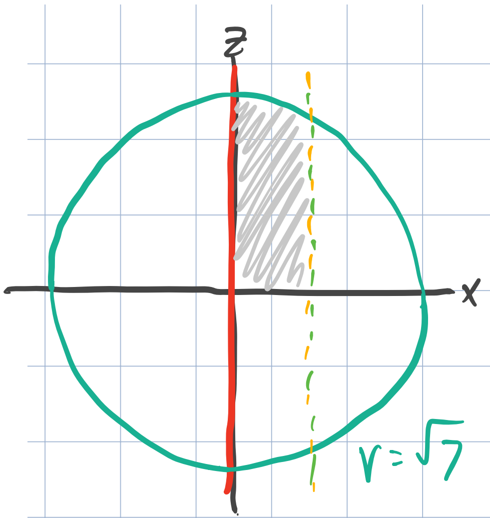
$? \le z \le ?$ ← Range of z to create the rest
$-\sqrt{z} \le x \le \sqrt{z}$ ← Parapola on each x slice
$-\sqrt{z-x^2} \le y \le \sqrt{z-x^2}$ ← Width of each "circle" on each y slice

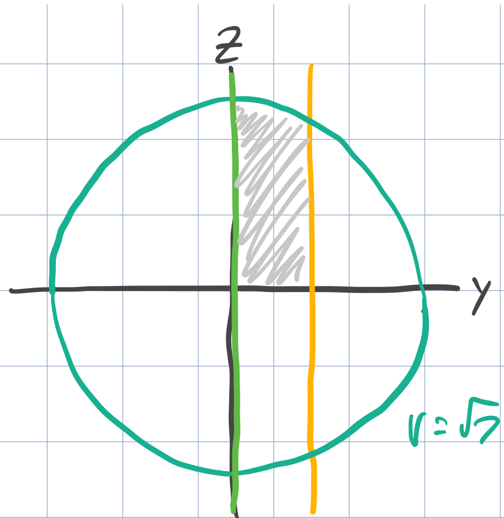
$0 \le z \le 9$

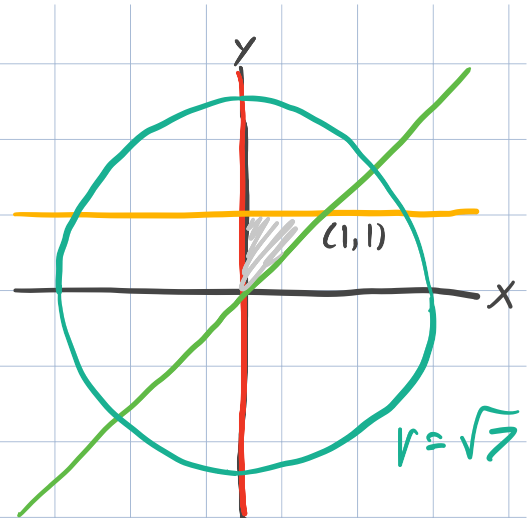
Part Five

Sam as Part Four

Compute the triple integral of $f(x, y, z) = z$ over the region F below the sphere $x^2 + y^2 + z^2 = 7$ and above the triangular region in the xy-plane bounded by $x = 0$, $y = 1$, and $y = x$.







$x = 0$

$z^2 + x^2 = 7$

$y = 1$

$z^2 + y^2 = 7$

$y = 0$

$x = 0$

$y = 1$

$y = x$

$x^2 + y^2 = 7$

Bounds

$0 \le y \le 1$

$0 \le x \le y$

$x^2 + y^2 + z^2 = 7$

$z^2 = 7 - x^2 - y^2$

$z = \pm\sqrt{7 - x^2 - y^2}$ ← only care about positive bc of xy plane bound

$$0 \le z \le \sqrt{7-x^2-y^2}$$

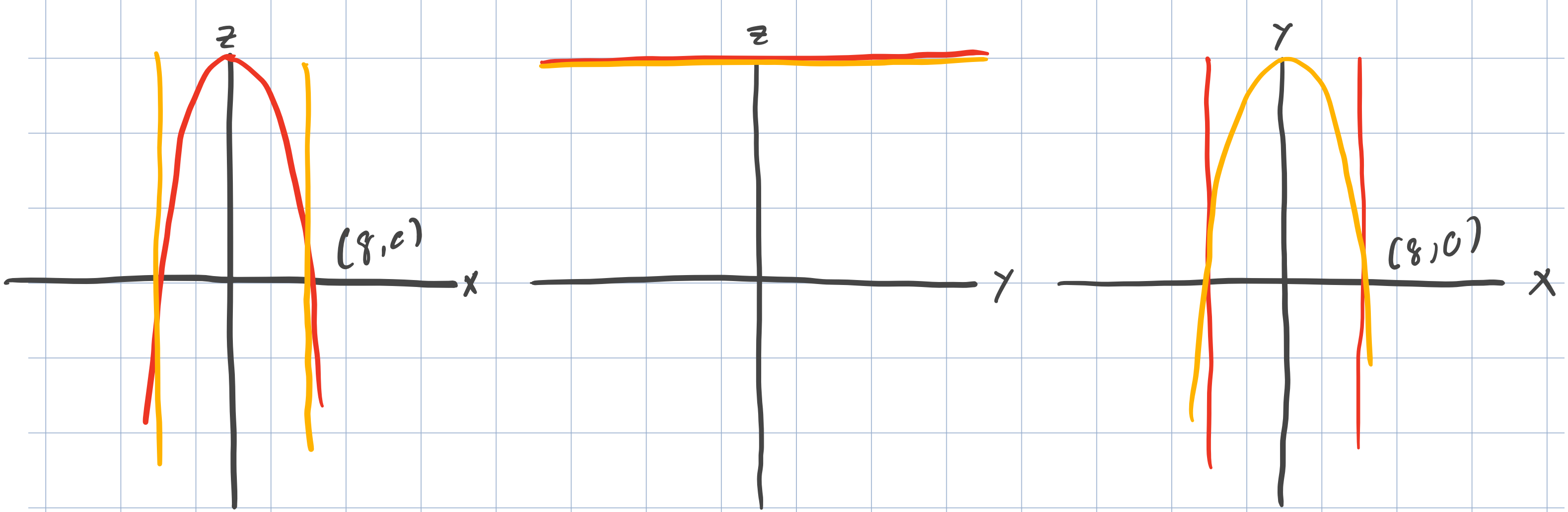
Compute Integral

$$\int_0^1 \int_0^y \int_0^{\sqrt{7-x^2-y^2}} z \, dz\, dx\, dy$$

Evaluated with calculator

$$= \frac{19}{12}$$

Consider the function f given by $f(x, y, z) = xz$. Find the average value of the function f over the region F in the first octant bounded by the coordinate planes and the parabolic cylinders $z = 64 - x^2$ and $y = 64 - x^2$.



$z = 64 - x^2$ $\quad$ $z = 64$ $\quad$ $x = \pm 8$

$y = \pm 8$ $\quad$ $y = 64$ $\quad$ $y = 64 - x^2$

Bounds

$$0 \le x \le 8$$
$$0 \le z \le 64 - x^2$$
$$0 \le y \le 64 - x^2$$

Integral

$$M = \int_0^8 \int_0^{64-x^2} \int_0^{64-x^2} xy \, dy \, dz \, dx$$

$$V = \int_0^8 \int_0^{64-x^2} \int_0^{64-x^2} 1 \, dy \, dz \, dx$$

$$Avg = \frac{M}{V}$$

Evaluated with calculator

$$= \frac{1048576}{\frac{262144}{15}}$$

Find the total mass M and the three moments of inertia I_x, I_y, and I_z of the solid with mass density $\sigma(x, y, z) = x^2 + 8$ kg/m^3 that occupies the unit cube in the first octant given by

$F = \{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$.

M = ____ kg

I_x = ____ kg-m^2

I_y = ____ kg-m^2

I_z = ____ kg-m^2

Part One

$$M = \int_0^1 \int_0^1 \int_0^1 x^2+8 \, dx \, dy \, dz$$

$$= \int_0^1 \int_0^1 \left(\tfrac{1}{3}x^3 + 8x\right)\Big|_0^1 \, dy \, dz$$

$$= \int_0^1 \int_0^1 \tfrac{1}{3} + 8 \, dy \, dz$$

$$= \tfrac{1}{3} + 8 \text{ kg}$$

Part Two

$$M = \int_0^1 \int_0^1 \int_0^1 (x^2+8)(y^2+z^2)\,dx\,dy\,dz$$

Evaluated with calculator

$$= \frac{50}{9}$$

Part Three

$$M = \int_0^1 \int_0^1 \int_0^1 (x^2+8)(x^2+z^2)\,dx\,dy\,dz$$

Evaluated with calculator

$$= \frac{254}{45}$$

Part Far

$$M = \int_0^1 \int_0^1 \int_0^1 (x^2+8)(x^2+y^2)\,dx\,dy\,dz$$

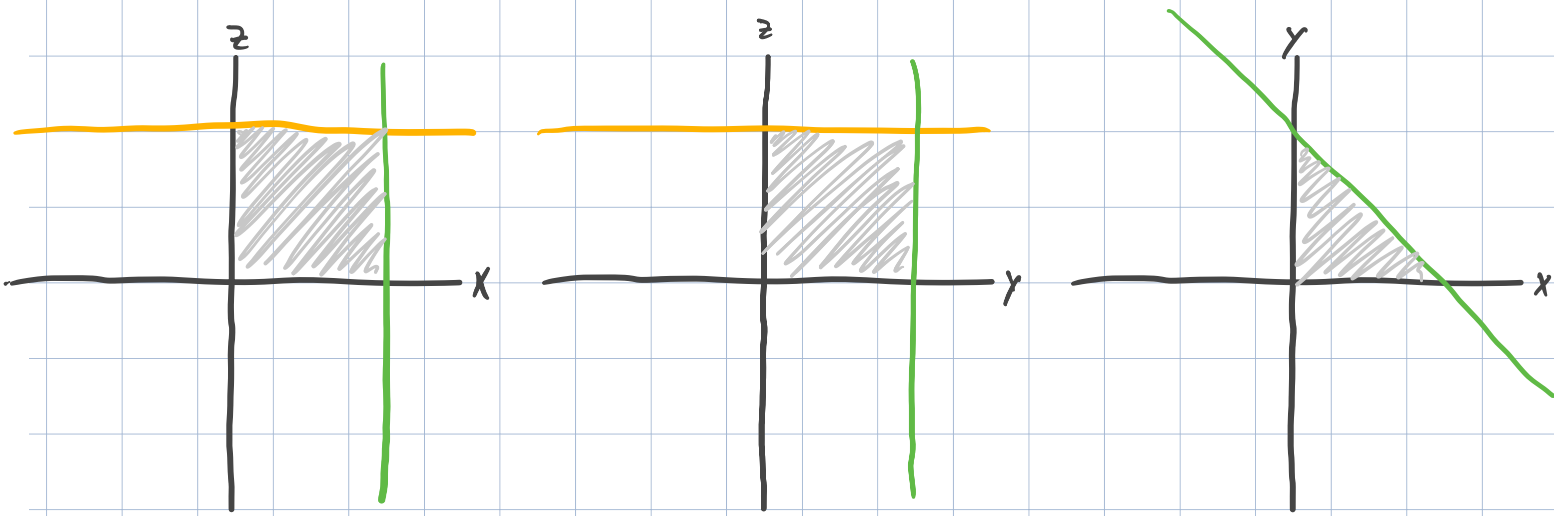
Evaluated with calculator

$$= \frac{254}{45}$$

Find the total mass M and the center of mass of the solid with mass density $\sigma(x, y, z) = kxy^3(6 - z)$ g/cm^3, where $k = 7 \times 10^6$, that occupies the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, $z = 1$, and $x + y = 1$.

$M =$ ____ g

$(\bar{x}, \bar{y}, \bar{z}) =$ (____)



$z = 1$
$x = 1$

$z = 1$
$y = 1$

$y = -x + 1$

Bounds

$0 \leq x \leq 1$
$0 \leq z \leq 1$
$0 \leq y \leq -x + 1$

Part One

$$M = \int_0^1 \int_0^1 \int_0^{-x+1} kxy^3(6-z)\, dy\, dz\, dx$$

Evaluated with calculator

$$= \frac{11}{240} k$$

Part Two

$$M_x = \int_0^1 \int_0^1 \int_0^{-x+1} x(kxy^3(6-z))\, dy\, dz\, dx$$

Evaluated with calculator

$$= \frac{11}{840} k$$

$$\bar{x} = \frac{2}{7}$$

$$M_y = \int_0^1 \int_0^1 \int_0^{-x+1} y(kxy^3(6-z))\, dy\, dz\, dx$$

Evaluated with calculator

$$= \frac{11}{240} k$$

$$\bar{y} = \frac{4}{7}$$

$$M_z = \int_0^1 \int_0^1 \int_0^{-x+1} z(kxy^3(6-z))\, dy\, dz\, dx$$

Evaluated with calculator

$$= \frac{1}{45} k$$

$$\bar{z} = \frac{16}{33}$$

Let F be the solid sphere $0 \le x^2 + y^2 + z^2 \le 1$ of radius 1 centered on the origin and let F_1 be the portion of F that lies in the first octant. Assume that $f(x, y, z)$ is a continuous function that is symmetric with respect to reflections through the coordinate planes. That is:

$$f(-x, y, z) = f(x, y, z),\ f(x, -y, z) = f(x, y, z),\ f(x, y, -z) = f(x, y, z).$$

If $\iiint_{F_1} f(x, y, z)\, dV = 31\pi$, find the value of $\iiint_{F} f(x, y, z)\, dV$.

Most complicated way to say "multibly by 8"

$31\pi \cdot 8$