Evaluate $\int \frac{5x-12}{x(x-6)}\,dx$

$\rightarrow \frac{5x-12}{x(x-6)}$

$= \frac{A}{x} + \frac{B}{(x-6)}$

$= \frac{A(x-6)}{x(x-6)} + \frac{Bx}{x(x-6)}$

$\rightarrow A(x-6) + Bx = 5x-12$

$A=2 \qquad B=3$

$= \frac{2}{x} + \frac{3}{x-6}$

$= \int \frac{2}{x} + \frac{3}{x-6}\,dx$

$= \int 2x^{-1} + 3(x-6)^{-1}\,dx$

$= 2\ln(|x|) + 3\ln(|x-6|) + C$

Evaluate $\int \frac{3x^2-21x-28}{(x+1)^2(x-3)}\,dx$

$\rightarrow \frac{3x^2-21x-28}{(x-3)(x+1)^2}$

$= \frac{A}{(x-3)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$

$= \frac{A(x+1)^2}{(x-3)(x+1)^2} + \frac{B(x-3)(x+1)}{(x-3)(x+1)^2} + \frac{C(x-3)}{(x-3)(x+1)^2}$

$\rightarrow A(x+1)^2 + B(x-3)(x+1) + C(x-3) = 3x^2 - 21x - 28$

$A(x^2+2x+1) + B(x^2-2x-3) + C(x-3) = 3x^2 - 21x - 28$

$Ax^2 + 2Ax + A + Bx^2 - 2Bx - 3B + Cx - 3C = 3x^2 - 21x - 28$

$\rightarrow Ax^2 + Bx^2 = 3x^2$

$\rightarrow 2Ax - 2Bx + Cx = -21x$

$\rightarrow A - 3B - 3C = -28$

$1 \rightarrow A + B = 3$

$2 \rightarrow 2A - 2B + C = -21$

$3 \rightarrow A - 3B - 3C = -28$

$1 \rightarrow A + B = 3$

$2 \rightarrow 2A - 2B + C = -21$

$2A + 2B = 6$

$2A - 2B + C = -21$

$4 \rightarrow 4A + C = -15$

$1 \rightarrow A + B = 3$

$3 \rightarrow A - 3B - 3C = -28$

$3A + 3B = 9$

$A - 3B - 3C = -28$

$5 \to 4A - 3C = -19$

$4 \to 4A + C = -15$
$5 \to 4A - 3C = -19$

$4A = -15 - C$
$4A = -19 + 3C$

$-15 - C = -19 + 3C$

$-4C = -4$

$C = 1$

$4 \to 4A + (1) = -15$

$4A = -16$

$A = -4$

$1 \to (-4) + B = 3$

$B = 7$

$$= \frac{-4}{(x-3)} + \frac{7}{(x+1)} + \frac{1}{(x+1)^2}$$

$$= \int \frac{-4}{(x-3)} + \frac{7}{(x+1)} + \frac{1}{(x+1)^2}\, dx$$

$$= -4\int \frac{1}{x-3}\,dx + 7\int \frac{1}{x+1}\,dx + \int \frac{1}{(x+1)^2}\,dx$$

$$= -4\ln(|x-3|) + 7\ln(|x+1|) + \int (x+1)^{-2}\,dx$$

$$= -4\ln(|x-3|) + 7\ln(|x+1|) - \frac{1}{1}(x+1)^{-1} + C$$

$$= -4\ln(|x-3|) + 7\ln(|x+1|) - \frac{1}{x+1} + C$$

Evaluate $\int \frac{25}{(x-1)(x^2+4)}\,dx$

$$= 25\int \underline{\frac{1}{(x-1)(x^2+4)}}\,dx$$

$$\rightarrow \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+4)} = \frac{1}{(x-1)(x^2+4)}$$

$$\frac{A(x^2+4)}{(x-1)(x^2+4)} + \frac{(x-1)(Bx+C)}{(x-1)(x^2+4)} = \frac{1}{(x-1)(x^2+4)}$$

$$A(x^2+4) + (x-1)(Bx+C) = 1$$

$$Ax^2 + 4A + Bx^2 + Cx - Bx - C = 1$$

$\rightarrow Ax^2 + Bx^2 = 0$

$\rightarrow Cx - Bx = 0$

$\rightarrow 4A - C = 1$

$1 \rightarrow A + B = 0$

$2 \rightarrow C - B = 0$

$3 \rightarrow 4A - C = 1$

$2 \to C - B = 0$
$3 \to 4A - C = 1$

$4 \to 4A - B = 1$

$4 \to 4A - B = 1$
$1 \to A + B = 0$

$5A = 1$

$A = \frac{1}{5}$

$1 \to \frac{1}{5} + B = 0$

$B = -\frac{1}{5}$

$2 \to C - \left(-\frac{1}{5}\right) = 0$

$C = -\frac{1}{5}$

$$\frac{\frac{1}{5}}{(x-1)} + \frac{-\frac{1}{5}x - \frac{1}{5}}{(x^2+4)} = \frac{1}{(x-1)(x^2+4)}$$

$$\frac{\frac{1}{5}}{(x-1)} - \frac{\frac{1}{5}(x+1)}{(x^2+4)} = \frac{1}{(x-1)(x^2+4)}$$

$$= 25\int \frac{\frac{1}{5}}{(x-1)} - \frac{\frac{1}{5}(x+1)}{(x^2+4)}\, dx$$

$$= 25\left(\frac{1}{5}\int \frac{1}{(x-1)}\, dx - \frac{1}{5}\int \frac{x+1}{(x^2+4)}\, dx\right)$$

$$=25\left(\frac{1}{5}\ln(|x-1|)-\frac{1}{5}\underline{\int\frac{x+1}{x^2+4}\,dx}\right)$$

$$\rightarrow \int\frac{x+1}{x^2+4}\,dx$$

$$=\underline{\int\frac{x}{x^2+4}\,dx}+\int\frac{1}{x^2+4}\,dx$$

$$\rightarrow \int\frac{x}{x^2+4}\,dx$$

$$u=x^2+4$$

$$\frac{1}{2}du=x\,dx$$

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{1}{2}\ln(|x^2+4|)+C$$

$$=\frac{1}{2}\ln(|x^2+4|)+\underline{\int\frac{1}{x^2+4}\,dx}$$

Use integral calculator because we haven't done anything with inverse trig identities

$$=\frac{1}{2}\ln(|x^2+4|)+\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$$

$$=25\left(\frac{1}{5}\ln(|x-1|)-\frac{1}{5}\left(\frac{1}{2}\ln(|x^2+4|)+\frac{1}{2}\arctan\left(\frac{x}{2}\right)\right)\right)+C$$

Evaluate $\int\frac{x^6-x^3+64}{x^4+8x^2}\,dx$

$$\begin{array}{r}
x^2 \qquad -8 \qquad\qquad\qquad\qquad \\
x^4+0x^3+8x^2 \,\overline{\big)\, x^6+0x^5+0x^4-x^3+0x^2+0x+64} \\
\underline{-x^6-0x^5-8x^4} \qquad\qquad\qquad\quad \\
-8x^4-x^3+0x^2 \qquad\qquad \\
\underline{8x^4+0x^3+64x^2} \qquad\qquad \\
-x^3+64x^2+64
\end{array}$$

$$= \int (x^2-8) + \frac{-x^3+64x^2+64}{x^4+8x^2}\,dx$$

$$= \int x^2-8\,dx + \int \frac{-x^3+64x^2+64}{x^2(x^2+8)}\,dx$$

$$= \left(\tfrac{1}{3}x^3-8x\right) + \underline{\int \frac{-x^3+64x^2+64}{x^2(x^2+8)}\,dx}$$

$$\rightarrow \underline{\int \frac{-x^3+64x^2+64}{x^2(x^2+8)}}\,dx$$

$$\rightarrow \frac{-x^3+64x^2+64}{x^2(x^2+8)}$$

$$\frac{A}{x^2} + \frac{Bx+C}{x^2+8} = \frac{-x^3+64x^2+64}{x^2(x^2+8)}$$

$$\frac{A(x^2+8)}{x^2(x^2+8)} + \frac{x^2(Bx+C)}{x^2(x^2+8)} = \frac{-x^3+64x^2+64}{x^2(x^2+8)}$$

$$A(x^2+8) + x^2(Bx+C) = -x^3+64x^2+64$$

$$Ax^2+8A+Bx^3+Cx^2 = -x^3+64x^2+64$$

$\rightarrow Bx^3 = -x^3$

$\rightarrow Ax^2+Cx^2 = 64x^2$

$\rightarrow 8A = 64$

$B=-1$

$A+C=64$

$A=8$

$C=56$

$$\frac{8}{x^2}+\frac{-x+56}{x^2+8}=\frac{-x^3+64x^2+64}{x^2(x^2+8)}$$

$$=\int \frac{8}{x^2}+\frac{-x+56}{x^2+8}\,dx$$

$$=8\int x^{-2}\,dx-\int\frac{x}{x^2+8}\,dx+56\int\frac{1}{x^2+8}\,dx$$

$$=8\left(-\frac{1}{1}x^{-1}\right)-\int\frac{x}{x^2+8}\,dx+56\int\frac{1}{x^2+8}\,dx$$

$$=-\frac{8}{x}\underline{-\int\frac{x}{x^2+8}\,dx}+56\int\frac{1}{x^2+8}\,dx$$

$$\rightarrow\int\frac{x}{x^2+8}\,dx$$

$u=x^2+8$

$\frac{du}{dx}=2x$

$\frac{1}{2}du=x\,dx$

$$=\frac{1}{2}\int u^{-1}\,dx$$

$$=\frac{1}{2}\ln(|u|)+C$$

$= \frac{1}{2}\ln(|x^2+8|) + C$

$= -\frac{8}{x} - \frac{1}{2}\ln(|x^2+8|) + 56\underline{\int \frac{1}{x^2+8}\,dx}$

$\int \frac{1}{x^2+8}\,dx$ Trig Rule $\rightarrow \int \frac{1}{x^2+1}\,dx = \arctan(x)$

$= \int \frac{1}{8(\frac{x^2}{8}+1)}\,dx$

$= \frac{1}{8}\int \frac{1}{(\frac{x}{\sqrt{8}})^2+1}\,dx$

$u = \frac{x}{\sqrt{8}}$

$\frac{du}{dx} = \frac{1}{\sqrt{8}}$

$\sqrt{8}\,du = dx$

$= \frac{\sqrt{8}}{8}\int \frac{1}{u^2+1}\,du$

$= \frac{1}{\sqrt{8}}\arctan(u) + C$

$= \frac{1}{\sqrt{8}}\arctan(\frac{x}{\sqrt{8}}) + C$

$= -\frac{8}{x} - \frac{1}{2}\ln(|x^2+8|) + 56(\frac{1}{\sqrt{8}}\arctan(\frac{x}{\sqrt{8}})) + C$

$= \frac{1}{3}x^3 - 8x - \frac{8}{x} - \frac{1}{2}\ln(|x^2+8|) + 56(\frac{1}{\sqrt{8}}\arctan(\frac{x}{\sqrt{8}})) + C$

Evaluate $\int_0^1 \frac{x-7}{x^2-5x+6}\,dx$

$$= \int \frac{x-7}{(x-3)(x-2)}\,dx$$

$$x \quad -3, \quad x \quad -2$$

$$x = -3x$$

$$-2 = -2x$$

$$\rightarrow \frac{x-7}{(x-3)(x-2)}$$

$$\frac{A}{x-3} + \frac{B}{x-2} = \frac{x-7}{(x-3)(x-2)}$$

$$\frac{A(x-2)}{(x-3)(x-2)} + \frac{B(x-3)}{(x-3)(x-2)} = \frac{x-7}{(x-3)(x-2)}$$

$$A(x-2) + B(x-3) = x-7$$

$$Ax - 2A + Bx - 3B = x - 7$$

$\rightarrow Ax + Bx = x$

$\rightarrow -2A - 3B = -7$

$1 \rightarrow A + B = 1$

$2 \rightarrow -2A - 3B = -7$

$2A + 2B = 2$

$-2A - 3B = -7$

$-B = -5$

$B = 5$

$A = -4$

$$\frac{-4}{x-3} + \frac{5}{x-2} = \frac{x-7}{(x-3)(x-2)}$$

$$= \int \frac{-4}{x-3} + \frac{5}{x-2}\, dx$$

$$= -4\int \frac{1}{x-3}\, dx + 5\int \frac{1}{x-2}\, dx$$

$$= -4\ln(|x-3|) + 5\ln(|x-2|) + C$$

Evaluate $\int \frac{2x^2+8x+10}{(x^2+1)(x^2+9)}\, dx$

$$= 2\int \frac{x^2+4x+5}{(x^2+1)(x^2+9)}\, dx$$

$$\rightarrow \frac{x^2+4x+5}{(x^2+1)(x^2+9)}$$

$$\frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+9)} = \frac{x^2+4x+5}{(x^2+1)(x^2+9)}$$

$$\frac{(x^2+9)(Ax+B)}{(x^2+1)(x^2+9)} + \frac{(x^2+1)(Cx+D)}{(x^2+1)(x^2+9)} = \frac{x^2+4x+5}{(x^2+1)(x^2+9)}$$

$$(x^2+9)(Ax+B) + (x^2+1)(Cx+D) = x^2+4x+5$$

$$Ax^3 + Bx^2 + 9Ax + 9B + Cx^3 + Dx^2 + Cx + D = x^2+4x+5$$

$\rightarrow Ax^3 + Cx^3 = 0$

$\rightarrow Bx^2 + Dx^2 = x^2$

$\rightarrow 9Ax + Cx = 4x$

$\rightarrow 9B + D = 5$

$1 \rightarrow A + C = 0$

$2 \rightarrow B + D = 1$

$3 \rightarrow 9A + C = 4$

$4 \rightarrow 9B + D = 5$

$2 \rightarrow B + D = 1$

$4 \rightarrow 9B + D = 5$

$-B - D = -1$

$9B + D = 5$

$8B = 4$

$B = \frac{1}{2}$

$2 \rightarrow \frac{1}{2} + D = 1$

$D = \frac{1}{2}$

$1 \rightarrow A + C = 0$

$3 \rightarrow 9A + C = 4$

$-A - C = 0$

$9A + C = 4$

$8A = 4$

$A = \frac{1}{2}$

$$1 \to 2 + C = 0$$

$$C = -\frac{1}{2}$$

$$\frac{\frac{1}{2}x + \frac{1}{2}}{x^2+1} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+9} = \frac{x^2+4x+5}{(x^2+1)(x^2+9)}$$

$$= 2\int \frac{\frac{1}{2}x + \frac{1}{2}}{x^2+1} + \frac{-\frac{1}{2}x + \frac{1}{2}}{x^2+9}\, dx$$

$$= \int \frac{x+1}{x^2+1}\, dx + \int \frac{-x+1}{x^2+9}\, dx$$

$$= \underline{\int \frac{x}{x^2+1}\, dx} + \int \frac{1}{x^2+1}\, dx - \int \frac{x}{x^2+9}\, dx + \int \frac{1}{x^2+9}\, dx\,)$$

$$\to \int \frac{x}{x^2+1}\, dx$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$\frac{1}{2}\, du = x\, dx$$

$$= \frac{1}{2}\int \frac{1}{u}\, du$$

$$= \frac{1}{2}\ln(|u|) + C$$

$$= \frac{1}{2}\ln(|x^2+1|) + C$$

$$= \frac{1}{2}\ln(|x^2+1|) + \int \frac{1}{x^2+1}\, dx - \int \frac{x}{x^2+9}\, dx + \int \frac{1}{x^2+9}\, dx$$

use same process as above

$$= \frac{1}{2}\ln(|x^2+1|) + \int \frac{1}{x^2+1}\,dx - \frac{1}{2}\ln(|x^2+9|) + \int \frac{1}{x^2+9}\,dx$$

$$= \frac{1}{2}\ln(|x^2+1|) + \int \frac{1}{x^2+1}\,dx - \frac{1}{2}\ln(|x^2+9|) + \int \frac{1}{x^2+9}\,dx$$

$$= \frac{1}{2}\ln(|x^2+1|) + \arctan(x) - \frac{1}{2}\ln(|x^2+9|) + \underline{\int \frac{1}{x^2+9}\,dx}$$

$$\rightarrow \int \frac{1}{x^2+9}\,dx$$

$$= \int \frac{1}{9(\frac{x^2}{9}+1)}\,dx$$

$$= \frac{1}{9}\int \frac{1}{(\frac{x}{3})^2+1}\,dx$$

$$v = \frac{x}{3}$$

$$\frac{dv}{dx} = \frac{1}{3}$$

$$3\,dv = dx$$

$$= \frac{3}{9}\int \frac{1}{v^2+1}\,dx$$

$$= \frac{1}{3}\arctan(v) + C$$

$$= \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

$$= \frac{1}{2}\ln(|x^2+1|) + \arctan(x) - \frac{1}{2}\ln(|x^2+9|) + \frac{1}{3}\arctan\left(\frac{x}{3}\right) + C$$

Evaluate $\int \frac{x^3+36}{x^2+36}\,dx$

$$x^2+0x+36 \overline{\big) x^3+0x^2+0x+36}$$ quotient x

$$-x^3-0x^2-36x$$

$$-36x+36$$

$$=\int x+\frac{-36x+36}{x^2+36}\,dx$$

$$=\int x\,dx+36\int\frac{-x+1}{x^2+36}\,dx$$

$$=\frac{1}{2}x^2+36\int\frac{-x+1}{x^2+36}\,dx$$

$$=\frac{1}{2}x^2+36\left(-\underline{\int\frac{x}{x^2+36}\,dx}+\int\frac{1}{x^2+36}\,dx\right)$$

$$\rightarrow \int\frac{x}{x^2+36}\,dx$$

$$u=x^2+36$$

$$\frac{1}{2}du=dx$$

$$=\frac{1}{2}\int\frac{1}{u}\,du$$

$$=\frac{1}{2}\ln(|x^2+36|)+C$$

$$=\frac{1}{2}x^2+36\left(-\frac{1}{2}\ln(|x^2+36|)+\underline{\int\frac{1}{x^2+36}\,dx}\right)$$

$$\rightarrow \int\frac{1}{x^2+36}\,dx$$

$$=\frac{1}{36}\int\frac{1}{\frac{x^2}{36}+1}\,dx$$

$=\frac{1}{36}\int \frac{1}{(\frac{x}{6})^2+1}\,dx$

$u=\frac{x}{6}$

$\frac{du}{dx}=\frac{1}{6}$

$6du=dx$

$=\frac{6}{36}\int \frac{1}{u^2+1}\,dx$

$=\frac{1}{6}\arctan(u)+C$

$=\frac{1}{6}\arctan(\frac{x}{6})+C$

$=\frac{1}{2}x^2+36(-\frac{1}{2}\ln(|x^2+36|)+\frac{1}{6}\arctan(\frac{x}{6}))+C$

Evaluate $\int \frac{6}{x(x-6)^2}\,dx$

$\rightarrow \frac{A}{x}+\frac{B}{(x-6)}+\frac{C}{(x-6)^2}$

$\frac{A(x-6)^2}{x(x-6)^2}+\frac{Bx(x-6)}{x(x-6)}+\frac{Cx}{(x-6)^2}=\frac{6}{x(x-6)^2}$

$A(x-6)^2+Bx(x-6)+Cx=6$

$A(x^2-12x+36)+B(x^2-6x)+Cx=6$

$Ax^2-12Ax+36A+Bx^2-6Bx+Cx=6$

$\rightarrow Ax^2 + Bx^2 = 0$

$\rightarrow -12Ax - 6Bx + Cx = 0$

$\rightarrow 36A = 6$

$1 \rightarrow A + B = 0$

$2 \rightarrow -12A - 6B + C = 0$

$3 \rightarrow 6A = 1$

$A = \frac{1}{6}$

$1 \rightarrow \frac{1}{6} + B = 0$

$B = -\frac{1}{6}$

$2 \rightarrow -12\left(\frac{1}{6}\right) - 6\left(-\frac{1}{6}\right) + C = 0$

$-2 + 1 + C = 0$

$C = 1$

$$\frac{\frac{1}{6}}{x} + \frac{-\frac{1}{6}}{x-6} + \frac{1}{(x-6)^2}$$

$$= \int \frac{\frac{1}{6}}{x} - \frac{\frac{1}{6}}{x-6} + \frac{1}{(x-6)^2}\, dx$$

$$= \frac{1}{6}\int \frac{1}{x}\, dx - \frac{1}{6}\int \frac{1}{x-6}\, dx + \int \frac{1}{(x-6)^2}\, dx$$

$$= \frac{1}{6}\ln(|x|) - \frac{1}{6}\ln(|x-6|) + \int \frac{1}{(x-6)^2}\, dx$$

$\rightarrow \int \frac{1}{(x-6)^2}\,dx$

$u = x-6$

$\frac{du}{dx} = 1$

$du = dx$

$= \int u^{-2}\,du$

$= -\frac{1}{1}u^{-1} + C$

$= -\frac{1}{u}$

$= -\frac{1}{x-6} + C$

$= \frac{1}{6}\ln(|x|) - \frac{1}{6}\ln(|x-6|) - \frac{1}{x-6} + C$