Reduce the following to a statement that does not use negation (¬) or implication (→).

NOTE: in this problem, <= is “less than or equal to,” != is “is not equal to”

$\neg\forall x\exists y\forall z([(-z \to y) \wedge (\neg x \to z)] \to [x = z])$

[ ] x [ ] y [ ] z ([(z [ ] y) [ ] (x [ ] z)] [ ] [x [ ] z])

$$:\neg\forall x\,\exists y\,\forall z\left[\,((-z \to y) \wedge (\neg x \to z)) \to (x = z)\right]$$

$$:\neg\forall x\exists y\forall z\left[((\neg\neg z \vee y) \wedge (\neg\neg x \vee z)) \to (x = z)\right]$$

$$:\neg\forall x\exists y\forall z\left[\neg((\neg\neg z \vee y) \wedge (x \vee z)) \vee (x = z)\right]$$

$$:\exists x\neg\exists y\forall z\left[\neg((\neg\neg z \vee y) \wedge (x \vee z)) \vee (x = z)\right]$$

$$:\exists x\forall y\neg\forall z\left[\neg((\neg\neg z \vee y) \wedge (x \vee z)) \vee (x = z)\right]$$

$$:\exists x\forall y\exists z\neg\left[\neg((\neg\neg z \vee y) \wedge (x \vee z)) \vee (x = z)\right]$$

$$:\exists x\forall y\exists z\left[\neg\neg((\neg\neg z \vee y) \wedge (x \vee z)) \wedge \neg(x = z)\right]$$

$$:\exists x\forall y\exists z\left[((\neg\neg z \vee y) \wedge (x \vee z)) \wedge \neg(x = z)\right]$$

$$:\exists x\forall y\exists z\left[((z \vee y) \wedge (x \vee z)) \wedge (x \neq z)\right]$$

Reduce the following to a statement that does not use negation (¬) or implication (→).

$\neg(\forall x\forall y\exists z[[(z \le y) \wedge (y \le x)] \to (y = z)])$

[ ] x [ ] y [ ] z[[(z [ ] y) [ ] (y [ ] x)] [ ] (y [ ] z)])

$$:\neg\forall x\,\forall y\,\exists z\left[\,((z \le y) \wedge (y \le x)) \to (y = z)\right]$$

$$= \exists x \exists y \forall z \neg [((z \leq y) \wedge (y \leq x)) \rightarrow (y = z)]$$

$$= \exists x \exists y \forall z \neg [\neg((z \leq y) \wedge (y \leq x)) \vee (y = z)]$$

$$= \exists x \exists y \forall z [((z \leq y) \wedge (y \leq x)) \wedge \neg(y = z)]$$

$$= \exists x \exists y \forall z [((z \leq y) \wedge (y \leq x) \wedge (y \neq z)]$$

Question:

Let,

$S(x)$ = "x is a student"

$H(x)$ = "x studies hard"

$P(x)$ = "x passes the exam"

$G(x)$ = "x is a gamer"

NOTE: Use Ax instead of $\forall x$ and Ex instead of $\exists x$ (no spaces before variable)

(e.g., Ex S(x) -> H(x))

Express the following in predicate form:

	Statement	Expression
A.	Everyone studies hard	
B.	Students who study hard pass the exams	
C.	No student is a gamer	

$$A = \forall x \, H(x)$$

$$B = \forall x [(S(x) \wedge H(x)) \rightarrow P(x)]$$

$$C = \forall x [S(x) \rightarrow \neg G(x)]$$

Fill in the blanks for the following proof:

Given:

$\forall x P(x)$

$\forall x Q(x)$

$\exists x S(x)$

$\forall x(P(x) \wedge Q(x) \wedge S(x)) \rightarrow B(x)$

Prove:

$\exists x B(x)$

NOTE: Use Ax instead of $\forall x$ and Ex instead of $\exists x$ (no spaces before variable)

Example: "Ap Ed D(p,d)"

(The notation {a/H} means substituting the variable "a" with the constant "H")

Step	Statement	Reason
1.	$\forall x P(x)$	[Choose One]
2.		Given
3.		Given
4.	$\forall x(P(x) \wedge Q(x) \wedge S(x)) \rightarrow B(x)$	[Choose One]
5.		3, Existential Elimination {x/C}
6.	$P(C)$	1, Universal Elimination {x/C}
7.		2, Universal Elimination {x/C}
8.		6, 7, Conjunction
9.		8, 5, Conjunction
10.	$(P(C) \wedge Q(C) \wedge S(C)) \rightarrow B(C)$	4, [Choose One]
11.		9,10, Modus Ponens
12.		11, Existential Introduction

#	Statement	Reason
1	$\forall x P(x)$	Given
2	$\forall x Q(x)$	Given
3	$\exists x S(x)$	Given
4	$\forall x [P(x) \wedge Q(x) \wedge S(x)] \rightarrow B(x)$	Given
5	$S(c)$	3, Existential Elimination

6	$P(c)$	1, Universal Elimination
7	$Q(c)$	2, Universal Elimination
8	$P(c) \wedge Q(c)$	6,7 Conjunction
9	$P(c) \wedge Q(c) \wedge S(c)$	8,5 Conjunction
10	$(P(c) \wedge Q(c) \wedge S(c)) \rightarrow B(c)$	4 Universal Elimination
11	$B(c)$	9,10 Modus Ponens
12	$\exists x B(x)$	11 Existential Introduction

(6 points)

Here is a list of functions and symbols that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Let $D(p, d)$ "Person p owns bird d"

NOTE: Use Ax instead of $\forall x$ and Ex instead of $\exists x$ (no spaces before variable)

Example: "Ap Ed D(p,d)"

Express the following in predicate form:

A There is a bird that belongs to everyone.

Ed Ap D(p,d)

B Everyone has a bird.

Ap Ed D(p,d)

C There is a person who owns all birds.

Ep Ad D(p,d)

D Every bird has an owner.

Ad Ep D(p,d)

E The statement "There is a bird with no owner" is false.

HINT: Your answer must start with a "-" sign. First, write the statement in the quotes, then negate it

-(Ed -Ep D(p,d))

F No one has a bird.

-Ep Ed D(p,d)

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let

$L(x)$ = ‘x is a lion’

$D(x)$ = ‘x is a deer’

$A(x,y)$ = ‘x attacks y’

$R(x,y)$ = ‘x runs from y’

Select the best answer in English form:

$\exists x \exists y L(x) * D(y) * A(x,y)$

- ○ Every Lion attacks one deer
- ○ Some deer attack all lions
- ○ Every lion attacks all deer
- ○ Some deer runs from all lions
- ○ Exactly one lion attacks some deer
- ○ Every lion attacks every deer
- ○ Exactly one lion attacks exactly one deer
- ◉ Some lion attacks some deer

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let

$L(x)$ = ‘x is a lion’

$D(x)$ = ‘x is a deer’

$A(x,y)$ = ‘x attacks y’

$R(x,y)$ = ‘x runs from y’

Select the best answer in English form:

$\forall x \exists y L(x) \rightarrow (D(y) * A(x,y))$

- ○ Every lion attacks the same deer
- ○ Some deer runs from every lion
- ○ All lions attack two deer
- ○ For every lion, there is only one deer it attacks
- ○ Every lion attacks all deer
- ○ All deer runs from the same lion
- ◉ For every lion, there is some deer it attacks

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let

$L(x)$ = ‘x is a lion’

$D(x)$ = ‘x is a deer’

$A(x,y)$ = ‘x attacks y’

$R(x,y)$ = ‘x runs from y’

Select the best answer in English form:

$\exists x \forall y D(x) * L(y) * R(x,y)$

- ○ Some lion attacks some deer
- ○ All deer are attacked by the same lion
- ○ All deer run from all lions
- ◉ There is at least one deer who runs from all lions
- ○ There is at least one deer who runs from some lion
- ○ Only one deer runs from all lions
- ○ Some deer run from some lions
- ○ All lions attack some deer

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let

$L(x)$ = ‘x is a lion’

$D(x)$ = ‘x is a deer’

$A(x,y)$ = ‘x attacks y’

$R(x,y)$ = ‘x runs from y’

Select the best answer in English form:

$\exists x \forall y (L(x) * D(y) * \neg A(x,y))$

- ○ All deer are attacked by some lion
- ○ All deer run from some lion
- ○ The same lion attacks every deer
- ○ There is some lion who attacks every deer
- ○ All deer attack some lion
- ○ The same lion attacks some deer
- ◉ There is some lion who fails to attack any deer

(3 points)

Here is a list of functions and symbols that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Question:

Let,

$S(x)$ = "x is a student"

$H(x)$ = "x studies hard"

$P(x)$ = "x passes the exam"

$G(x)$ = "x is a gamer"

NOTE: Use Ax instead of $\forall x$ and Ex instead of $\exists x$ (no spaces before variable)

(e.g., Ex S(x) -> H(x))

Express the following in predicate form:

	Statement	Expression
A.	Everyone studies hard	Ax H(x)
B.	Students who study hard pass the exams	Ax [(S(x)*H(x))->P(x)]
C.	No student is a gamer	Ax (S(x)->-G(x))

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Let $P(x)$ = "x runs 1 hour every weekday." Select the best answer in English form for the following:

$\exists x P(x)$

- ○ Only one person runs 1 hour every weekday
- ◉ There is a person who runs 1 hour every weekday
- ○ There is a person who runs 1 time
- ○ A person will only run one day for the week
- ○ All people run 1 hour every weekday
- ○ There is a person who walks 1 hour every weekday

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using "Preview My Answer" before clicking "Submit".

Let $P(x)$ = "x runs 1 hour every weekday." Select the best answer in English form for the following:

$\forall x \neg P(x)$

- ○ Everyone does not run every weekday
- ○ There is a person who will not run every weekday
- ○ Everyone walks 1 hour every weekday
- ○ There are no weekdays a person will run
- ◉ No one runs 1 hour every weekday
- ○ No one runs at all
- ○ Not everyone runs 1hr every weekday

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let $P(x)$ = “x runs 1 hour every weekday.” Select the best answer in English form for the following:

$\neg\exists x P(x)$

- ○ There is no one who doesn't run every weekday
- ○ There is no one who walks 1 hour every weekday
- ○ There is no one who runs at all
- ○ There is a person who does not like to run 1 hour every weekday
- ◉ There is no one who runs 1 hour every weekday
- ○ There is a person who runs for only 1 hour
- ○ There is a person who doesn'T run 1hr every weekday

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let $P(x)$ = “x runs 1 hour every weekday.” Select the best answer in English form for the following:

$\neg\forall x \neg P(x)$

- ○ Not all people run 1 hour every weekday
- ◉ Someone runs 1 hour every weekday
- ○ Not everyone runs every hour every weekday
- ○ No one runs 1 hour every weekday
- ○ All people do not run 1 hour every weekday
- ○ Not all people run during the week

(1 point)

Here is a list of functions and symbols that WeBWorK understands. We recommend using “Preview My Answer” before clicking “Submit”.

Let $P(x) = x \geq 2$ where x is an integer. What are the truth values of the following?

Select all the boxes which are true.

- ☑ **A.** $P(4)$
- ☐ **B.** $\forall x P(x)$
- ☐ **C.** $P(0)$
- ☐ **D.** $P(-1)$
- ☑ **E.** $\exists x P(x)$