Determine the domain of $\vec{F}(t) = \langle \tan(t), 5t, \ln(4 - t^2) \rangle$. (Enter your answer using interval notation.)

$\left(-2, -\frac{\pi}{2}\right) \cup \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 2\right)$

Domain of $\tan(t)$: $\ldots \cup \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \ldots$

Domain of $5t$: $(-\infty, \infty)$

Domain of $\ln(4-t^2)$: $(-2, 2)$

Domain of $\vec{F}(t) = \left(-2, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, 2\right)$

Determine the domain of $\vec{F}(t) = \langle \cot(t), 5t^2, \sqrt{4 - t^2} \rangle$. (Enter your answer using interval notation.)

Domain of $\cos(t)$: $\ldots \cup (-\pi, 0) \cup (0, \pi) \cup \ldots$

Domain of $5t^2$: $(-\infty, \infty)$

Domain of $\sqrt{4-t^2}$:

defined when $4 - t^2 > 0$

$t^2 < 4$

$|t| < 2$

$[-2, 2]$

Domain of $\vec{F}(t)$: $[-2, 0) \cup (0, 2]$

Determine if the vector-valued function has a limit at the specified point. Determine if the function is continuous at the specified point. (Your instructors prefer angle bracket notation < > for vectors. If the limit exists enter the limit. If the limit does not exist enter DNE.)

(a) $\vec{F}(t) = \left\langle \frac{\cos(t)}{t}, \tan(t), 2t - 2 \right\rangle$ at $t_0 = \pi$

$\lim_{t \to \pi} \vec{F}(t) =$ [ ]

- ○ The function is continuous at t_0.
- ○ The function is not continuous at t_0.

(b) $\vec{F}(t) = \left\langle \frac{\cos(t)}{t}, \tan(t), 2t - 2 \right\rangle$ at $t_0 = 0$

$\lim_{t \to 0} \vec{F}(t) =$ [ ]

- ○ The function is continuous at t_0.
- ○ The function is not continuous at t_0.

(c) $\vec{F}(t) = \left\langle \frac{e^t - 1}{t}, \tan(t), \frac{1}{t+1} \right\rangle$ at $t_0 = 0$

$\lim_{t \to 0} \vec{F}(t) =$ [ ]

- ○ The function is continuous at t_0.
- ○ The function is not continuous at t_0.

Part A

$$\lim_{t \to \pi} \vec{F}_x(\pi) = \frac{\cos(\pi)}{\pi}$$

$$= -\frac{1}{\pi}$$

$$\lim_{t \to \pi} \vec{F}_y(\pi) = \tan(\pi)$$

$$= 0$$

$$\lim_{t \to \pi} \vec{F}_z(\pi) = 2\pi - 2$$

$$\lim_{t \to \pi} \vec{F}(\pi) = \langle -\tfrac{1}{\pi}, 0, 2\pi - 2 \rangle$$

Continuous at $t = \pi$

Part B

$$\lim_{t \to 0} \vec{F}_x(0) = \frac{\cos(0)}{0}$$

$$= \text{Undefined}$$

$$\lim_{t \to 0} \vec{F}(0) = \text{DNE}$$

Not continuous at $t=0$

Part C

$$\lim_{t \to 0} \vec{F}_x(0) = \frac{e^0 - 1}{0}$$

$$= \frac{0}{0} \rightarrow \text{Use LH}$$

$$= \frac{e^0}{1}$$

$$= 1$$

$$\lim_{t \to 0} \vec{F}_y(0) = \tan(0)$$

$$= \frac{0}{1}$$

$$= 0$$

$$\lim_{t \to 0} \vec{F}_z(0) = \frac{1}{1}$$

$$\lim_{t \to 0} \vec{F}(0) = \langle 1, 0, 1 \rangle$$

Not continuous at $t=0$ because $\vec{F}_x(0) = $ Undefined

Determine if the derivative of the vector-valued function exists at the specified point. (Your instructors prefer angle bracket notation < > for vectors. If the derivative exists at the specified point, enter its value. If the derivative does not exist, enter DNE.)

$\vec{F}(t) = \left\langle \frac{\cos(t)}{t}, \tan(t), 7t - 7 \right\rangle$ at $t_0 = \frac{\pi}{2}$

$$\frac{d}{dt}\vec{F}_x(t) = \frac{d}{dt}\left(\cos(t)\,t^{-1}\right)$$

$$= \cos(t)\left(-t^{-2}\right) + t^{-1}\left(-\sin(t)\right)$$

$$= -\frac{\cos(t)}{t^2} - \frac{\sin(t)}{t}$$

$$\frac{d}{dt}\vec{F}_x\left(\frac{\pi}{2}\right) = \frac{1}{\pi^2} - 0$$

$$= \frac{1}{\pi^2}$$

$$\frac{d}{dt}\vec{F}_y(t) = \frac{d}{dt}\tan(t)$$

$$= \sec^2(t)$$

$$\frac{d}{dt}\vec{F}_y\left(\frac{\pi}{2}\right) = \frac{1}{\cos\left(\frac{\pi}{2}\right)^2}$$

$$= \text{Undefined}$$

$$\frac{d}{dt}\vec{F}\left(\frac{\pi}{2}\right) = \text{DNE}$$

Evaluate the definite integral of the vector-valued function on the specified interval. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{F}(t) = \left\langle \frac{t^3}{5 + t^4}, 2t^2 - 2t + 5, \frac{2}{t} \right\rangle$ on the interval $I = [1, 2]$

$$\int_1^2 \vec{F}_x(t) = \int_1^2 \frac{t^3}{5+t^4}\,dt$$

$$u = 5 + t^4$$

$$\frac{du}{dt} = 4t^3$$

$$\frac{1}{4}\,du = t^3\,dt$$

$$= \int_1^2 u^{-1} \cdot \frac{1}{4}\,du$$

$$= \frac{1}{4} \cdot \ln(u) \Big|_1^2$$

$$= \frac{\ln(5+t^4)}{4} \Big|_1^2$$

$$= \frac{1}{4}\left(\ln(5+2^4) - \ln(5+1^4)\right)$$

$$= \frac{1}{4}(\ln(21) - \ln(6))$$

$$\int_1^2 \vec{F}_y(t) = \int_1^2 2t^2 - 2t + 5\,dt$$

$$= \frac{2}{3}t^3 - \frac{2}{2}t^2 + 5t \Big|_1^2$$

$$= \frac{2}{3}t^3 - t^2 + 5t \Big|_1^2$$

$$= \left(\frac{2}{3}(2)^3 - (2)^2 + 10\right) - \left(\frac{2}{3} - 1 + 5\right)$$

$$= \frac{16}{3} - 4 + \frac{1}{3} + 5$$

$$= \frac{17}{3} + 1$$

$$\int_1^2 \vec{F}_2(t) = \int_1^2 2t^{-1}\, dt$$

$$= 2\ln(t)\Big|_1^2$$

$$= 2(\ln(2) - \ln(1))$$

$$= 2\ln(2)$$

Use the chain rule for differentiation of vector-valued functions to compute $\frac{d}{dt}\vec{F}(g(t))$ for the indicated function. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{F}(u) = \left\langle u, \frac{u^2}{2}, \frac{u^3}{3} \right\rangle$ and $u = \ln(t^4)$

$$\frac{d}{dt}\vec{F}_x(t) = \frac{d}{dt}(\ln(t^4))$$

$$= \frac{1}{t^4} \cdot 4t^3$$

$$= \frac{4t^3}{t^4}$$

$$= \frac{4}{t}$$

$$\frac{d}{dt}\vec{F}_y(t) = \frac{d}{dU}\left(\frac{1}{2}U^2\right)$$

$$= U \cdot \frac{dU}{dt}$$

$$= \ln(t^4)\,\frac{4}{t}$$

$$\frac{d}{dt}\vec{F}_z(t) = \frac{d}{du}\left(\frac{1}{3}u^3\right)$$

$$= u^2 \frac{du}{dt}$$

$$= \ln(t^4)^2 \frac{4}{t}$$

$$\frac{d}{dt}\vec{F}(t) = \left\langle \frac{4}{t}, \ln(t^4)\frac{4}{t}, \ln(t^4)^2\frac{4}{t} \right\rangle$$

Find all antiderivatives of the given vector-valued function. (Your instructors prefer angle bracket notation < > for vectors.)

$\vec{F}(t) = \langle t\cos(t), \cos(t)\sin(t), 7t\rangle$ and $t \in \mathbb{R}$

[ ] $+ \langle C_1, C_2, C_3\rangle$

$$\int \vec{F}_x(t)\,dt = \int t\cos(t)\,dt$$

$u = t$ $\quad dv = \cos(t)\,dt$

$\frac{du}{dt} = 1$ $\quad \frac{dv}{dt} = \cos(t)$

$du = dt$ $\quad v = \sin(t)$

$$= t\sin(t) - \int \sin(t)\,dt$$

$$= t\sin(t) + \cos(t) + C_1$$

$$\int \vec{F}_y(t)\,dt = \int \cos(t)\sin(t)\,dt$$

$u = \sin(t)$

$\frac{du}{dt} = \cos(t)$

$du = \cos(t)\,dt$

$= \int u\,du$

$= \frac{1}{2}u^2 + C_2$

$= \frac{1}{2}\sin(t)^2 + C_2$

$\int \vec{F}_z(t)\,dt = \int 7t\,dt$

$= \frac{7}{2}t^2 + C_3$

Find the antiderivative $\vec{G}(t)$ of $\vec{f}(t) = \left\langle te^{t^2}, \frac{t}{1+t^2}, 2t^2 \right\rangle$ that satisfies $\vec{G}(0) = \langle 1, 2, -4 \rangle$. (Your instructors prefer angle bracket notation < > for vectors.)

$\int \vec{f}_x(t)\,dt = \int te^{t^2}\,dt$

$u = t^2$

$\frac{du}{dt} = 2t$

$\frac{1}{2}du = t\,dt$

$= \frac{1}{2}\int e^u\,du$

$= \frac{1}{2}e^u + C_1$

$$= \frac{1}{2}e^{t^2} + C_1$$

$$\int \vec{F}_y(t)\,dt = \int \frac{t}{1+t^2}\,dt$$

$$u = 1+t^2$$

$$\frac{du}{dt} = 2t$$

$$\frac{1}{2}\,du = t\,dt$$

$$= \frac{1}{2}\int u^{-1}\,du + C_2$$

$$= \frac{1}{2}\ln(1+t^2) + C_2$$

$$\int \vec{F}_z(t)\,dt = \int 2t^2\,dt$$

$$= \frac{2}{3}t^3 + C_3$$

$$G(t) = \langle \frac{1}{2}e^{t^2}, \frac{1}{2}\ln(1+t^2), \frac{2}{3}t^3 \rangle + \langle C_1, C_2, C_3 \rangle$$

Solve for $C_\#$ given $G(0) = \langle 1, 2, -4 \rangle$

$1 = \frac{1}{2}e^0 + C_1$ $\quad$ $2 = \frac{1}{2}\ln(1+0) + C_2$ $\quad$ $-4 = \frac{2}{3}(0)^3 + C_3$

$1 = \frac{1}{2} + C_1$ $\quad$ $C_2 = 2$ $\quad$ $C_3 = -4$

$C_1 = \frac{1}{2}$

$$G(t) = \langle \frac{1}{2}e^{t^2} + \frac{1}{2}, \frac{1}{2}\ln(1+t^2) + 2, \frac{2}{3}t^3 - 4 \rangle$$